Some Challenges in Computational Algebraic Geometry

David A. Cox^a

^aDepartment of Mathematics and Computer Science, Amherst College, Amherst, MA 01002, USA

There are many challenging problems and open questions in computational algebraic geometry. But the field also faces some challenges of a very different nature. My lecture will discuss three of these intriguing challenges. The first challenge is posed by users in other disciplines: when they use symbolic methods, how do we get them to take the computation seriously? I will illustrate the difficulty using a January 2000 article from the journal Nature. The second challenge concerns the question of what we mean by a computation. Is it an algorithm? A straight-line program? A determinantal formula? I will present several examples concerning resultants and Rees algebras to illustrate of range of what is encompassed by the term "computation." Finally, I will discuss the challenge of loving bad algorithms. I don't mean that we should love all bad algorithms, but rather that there are some essentially useless algorithms that nevertheless have a constructive role to play in computational algebraic geometry. In particular, I will discuss two of my favorite bad algorithms: one for factoring univariate polynomials with integer coefficients, and the other for solving Sudoku puzzles.