Using Constraint Programming for Solving Distance CSP with uncertainty

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Many problems in chemistry, robotics or molecular biology can be expressed as a Distance CSP (Constraint Satisfaction Problem). Sometimes, the parameters of this kind of problems are determined in an experimental way, and therefore they have an uncertainty degree. A classical approach for solving this class of problems is to solve the CSP without considering the uncertainties, and to obtain a set of solutions without knowing the real solution sub-spaces. A better approach is to apply a branch and prune algorithm to generate a set of disjoint boxes that include all the solution sub-spaces, but without information about independent solution sub-spaces or the different types of boxes.

We propose a new methodology built from the combination of both previous approaches and a *feasibility checker* for tackling uncertainties in a CSP formed by distance constraints. A distance constraint c between two points P_i and P_j in a n-dimensional space can be expressed as $c(P_i, P_j) : \sum_{k=1}^{n}(x_{ik} - x_{jk})^2 = d_{ij}^2$, where x_{ik} is the k-th coordinate of the point P_i, and d_{ij} is the distance value between them. In this class of problems, all fixed values are called *parameters*. When a CSP has parameters with interval values, it is called *CSP with uncertainties*.

In our methodololy, the main idea consists in solving the CSP without taking into account the uncertainties, by replacing each parameter with interval value by the middle point of the interval, and applying a SSA (solution separation algorithm) to calculate a set of sub-domains. The SSA calculates the equation of the median plane between each pair of solutions. Then, for each solution found, we solve a new CSP built from the original CSP (with uncertainties) and a set of plane inequations. The sub-domain defined by the conjunction of all inequations is equivalent to the sub-space of a Voronoi diagram for the solution. A branch and prune algorithm is then applied for each CSP built. We combine this algorithm with a *feasibility checker* in order to determine when a box is totally included in the solution sub-space. The best results of this methodology are obtained when the following two hypotheses are verified:

– The problem has a finite number of solutions ρ, without taking into account the uncertainties.
– The problem has only connected sets of solutions around each initial solution, when we consider the uncertainties.

The first condition allows the separation of the initial domain into a set of sub-domains containing only one solution found, while the second one, assures the existence of only one solution sub-space in each sub-domain.