université de BORDEAUX
LABORATOIRE BORDELAIS DE RECHERCHE EN INFORMATIQUE LaBRI
Bordeaux INP ENSEIRB MATMECA

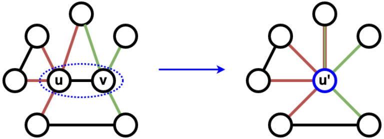
Graph Minor & Graph Drawing

Claire Hilaire

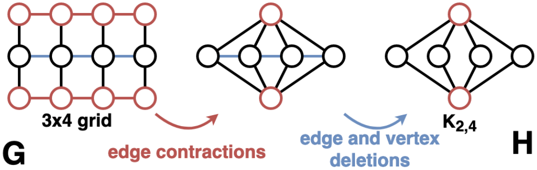
Cyril Gavoille

Context

Graph Minor

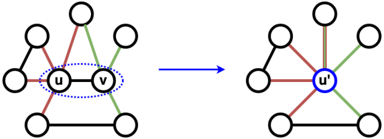


Edge contraction.

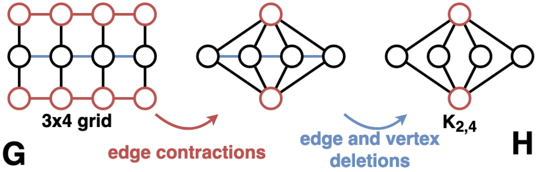


H is a minor of G.

Graph Minor



Edge contraction.

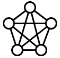
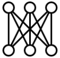


H is a minor of G.

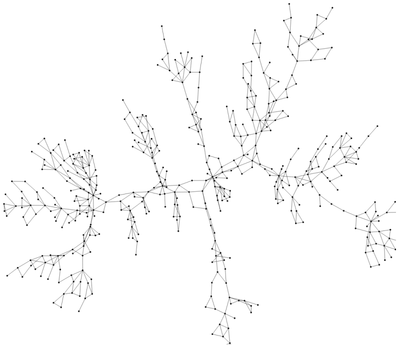
Graph Minor Theorem, Robertson & Seymour (1983-2004)

Every graph family closed under taking minors can be defined by a finite set of forbidden minors.

Wagner's theorem

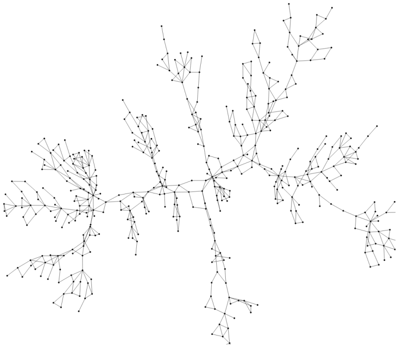
If G excludes   as minors, then G is planar.

Treewidth



Graph G with treewidth $tw(G) = 2$.

Treewidth



Graph G with treewidth $tw(G) = 2$.

Th. Robertson & Seymour (1986)

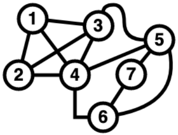
If a graph G excludes a planar graph H as minor, then $tw(G) \leq C_H$.

Th. Robertson & Seymour & Thomas (1994)

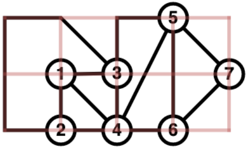
If a **planar** graph G excludes a $r \times r$-grid as minor, then $tw(G) = O(r)$.

Corollary: If a **planar** graph G excludes a planar graph H, then $tw(G) = O(|V(H)|)$.

Graph Drawing

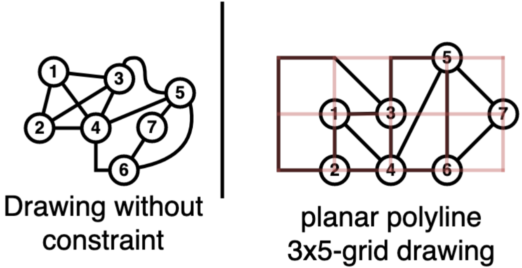


Drawing without constraint



planar polyline 3x5-grid drawing

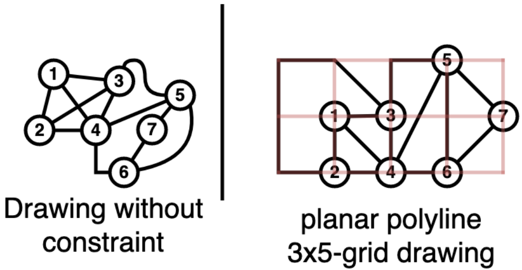
Graph Drawing

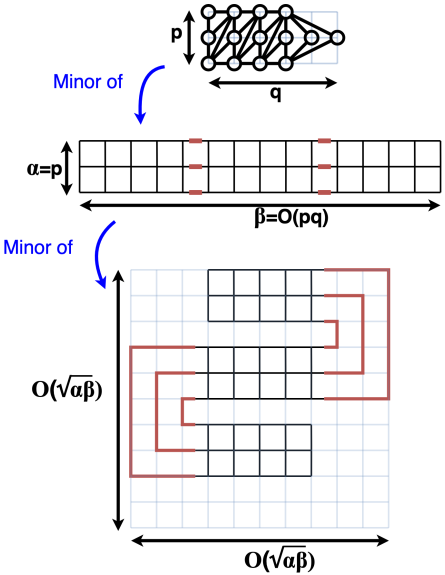


Th. Dieng & Gavoille (2020)

Let H be a graph that has a polyline $p \times q$-grid drawing. If a planar graph G excludes H as minor, then $tw(G) = O(p\sqrt{q})$.

Graph Drawing





Th. Dieng & Gavoille (2020)

Let H be a graph that has a polyline $p \times q$-grid drawing. If a planar graph G excludes H as minor, then $tw(G) = O(p\sqrt{q})$.

Problem

Given a graph H with a planar polyline $p \times q$-grid drawing, what is the minimum area of a grid whose H is minor?

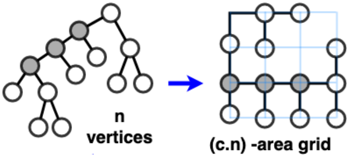
Can we find a grid with a $O(pq)$ area?

Results

If H is a tree (1/2)

Th. Valiant (1981)

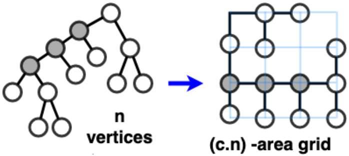
Let T be a tree with n vertices and max degree 3: then T has a planar polyline **orthogonal** drawing on a $O(n)$-area grid.



If H is a tree (1/2)

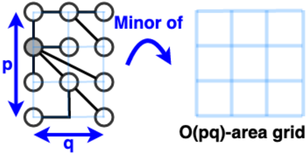
Th. Valiant (1981)

Let T be a tree with n vertices and max degree 3: then T has a planar polyline **orthogonal** drawing on a $O(n)$-area grid.



Theorem 1

Let T be a tree with a drawing on a $p \times q$-grid, then T is a minor of a $O(pq)$-area grid.



Lemma

Let T be a tree with n vertices : then T is a minor of T' with max degree 3 and $\leq 2n - 1$ vertices.

If H is a tree (1/2)

Th. Valiant (1981)

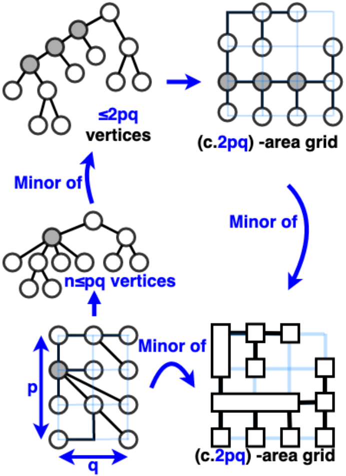
Let T be a tree with n vertices and max degree 3: then T has a planar polyline **orthogonal** drawing on a $O(n)$-area grid.

Theorem 1

Let T be a tree with a drawing on a $p \times q$-grid, then T is a minor of a $O(pq)$-area grid.

Lemma

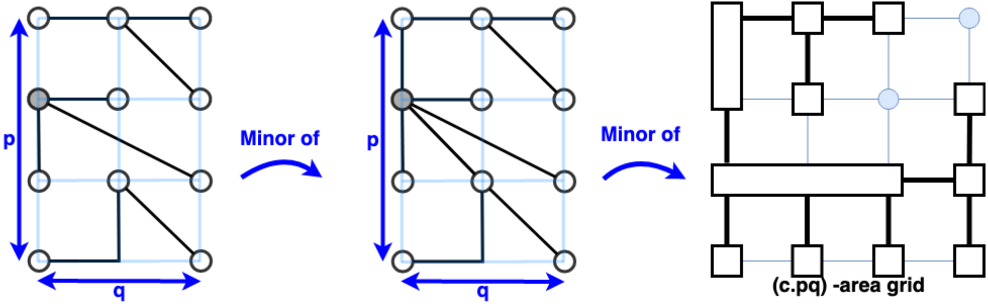
Let T be a tree with n vertices : then T is a minor of T' with max degree 3 and $\leq 2n - 1$ vertices.



If H is a tree (2/2)

Corollary 1

Let F be a forest with a drawing on a $p \times q$-grid, then F is a minor of a $O(pq)$-area grid.



If H is a tree (2/2)

Corollary 1

Let F be a forest with a drawing on a $p \times q$-grid, then F is a minor of a $O(pq)$-area grid.

Corollary 2

Let H be an apex-tree with a drawing on a $p \times q$-grid, then H is a minor of a $O(pq)$-area grid.

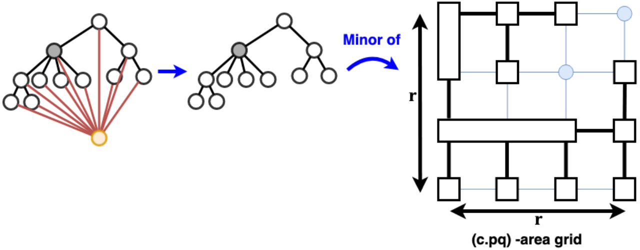
If H is a tree (2/2)

Corollary 1

Let F be a forest with a drawing on a $p \times q$-grid, then F is a minor of a $O(pq)$-area grid.

Corollary 2

Let H be an apex-tree with a drawing on a $p \times q$-grid, then H is a minor of a $O(pq)$-area grid.



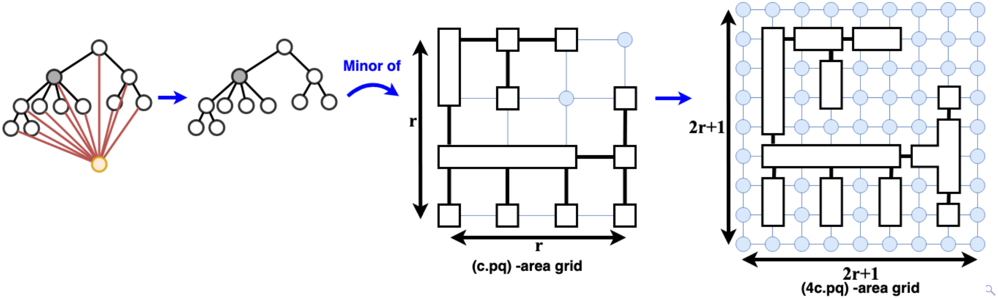
If H is a tree (2/2)

Corollary 1

Let F be a forest with a drawing on a $p \times q$-grid, then F is a minor of a $O(pq)$-area grid.

Corollary 2

Let H be an apex-tree with a drawing on a $p \times q$-grid, then H is a minor of a $O(pq)$-area grid.



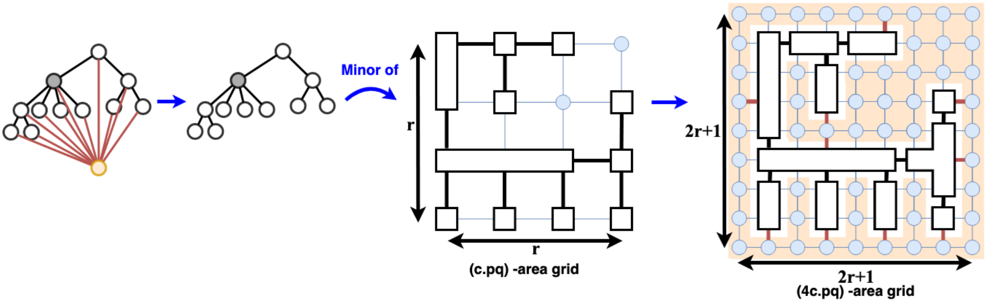
If H is a tree (2/2)

Corollary 1

Let F be a forest with a drawing on a $p \times q$-grid, then F is a minor of a $O(pq)$-area grid.

Corollary 2

Let H be an apex-tree with a drawing on a $p \times q$-grid, then H is a minor of a $O(pq)$-area grid.



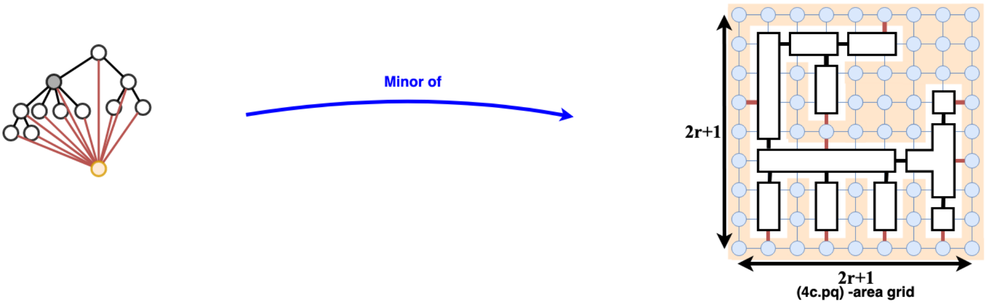
If H is a tree (2/2)

Corollary 1

Let F be a forest with a drawing on a $p \times q$-grid, then F is a minor of a $O(pq)$-area grid.

Corollary 2

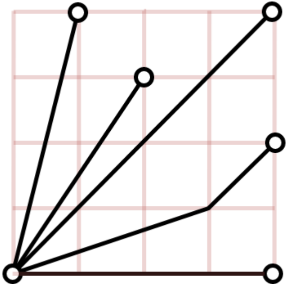
Let H be an apex-tree with a drawing on a $p \times q$-grid, then H is a minor of a $O(pq)$-area grid.



If H is k-monotone

H has a $p \times q$-grid drawing k-**monotone** if:

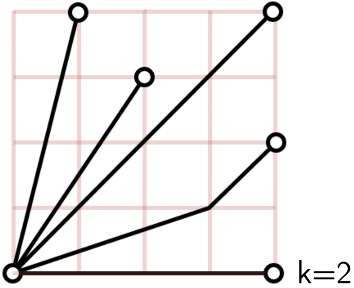
- every edge-segment crosses at most k vertical edges of the grid (not the vertices of the grid),

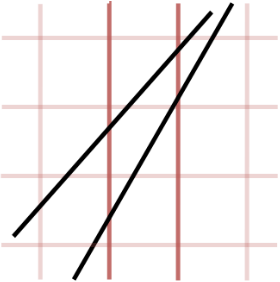


If H is k-monotone

H has a $p \times q$-grid drawing k-**monotone** if:

- every edge-segment crosses at most k vertical edges of the grid (not the vertices of the grid),
- for every 2 consecutive columns, the *monotone* property holds.

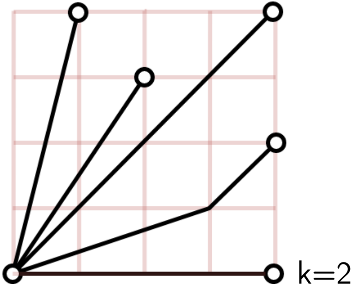


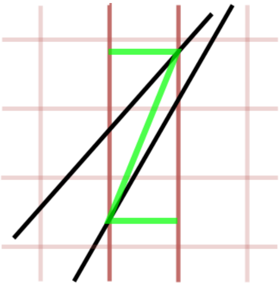


If H is k-monotone

H has a $p \times q$-grid drawing k-**monotone** if:

- every edge-segment crosses at most k vertical edges of the grid (not the vertices of the grid),
- for every 2 consecutive columns, the *monotone* property holds.

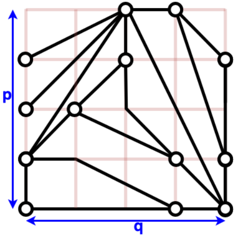




Example

H has a $p \times q$-grid drawing k-**monotone** if:

- every edge-segment crosses at most k vertical edges of the grid (not vertices),
- for every 2 consecutive columns, the *monotone* property holds.

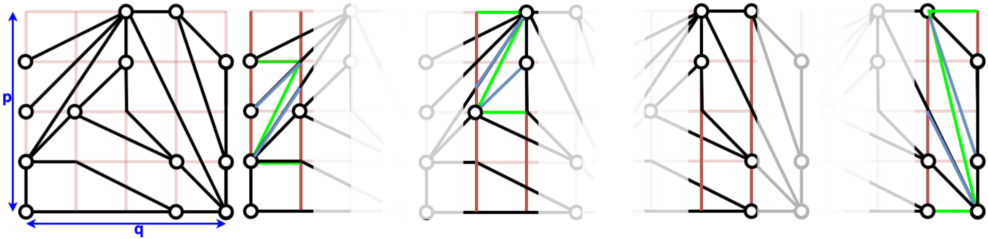


Example of a 1-monotone drawing.

Example

H has a $p \times q$-grid drawing k-**monotone** if:

- every edge-segment crosses at most k vertical edges of the grid (not vertices),
- for every 2 consecutive columns, the *monotone* property holds.

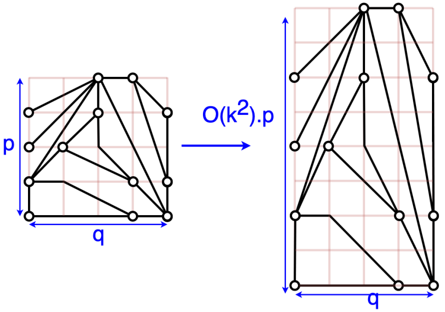


Example of a 1-monotone drawing.

If H is k-monotone

Theorem 2

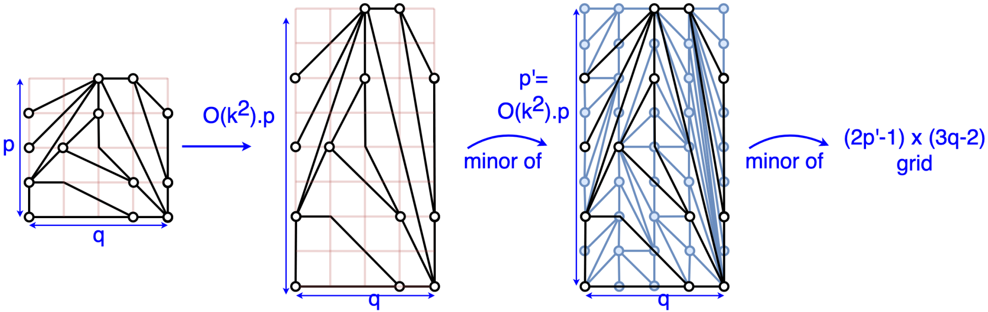
If H has a k-**monotone** $p \times q$-grid drawing, then H is a minor of a $O(k^2.pq)$ area grid.



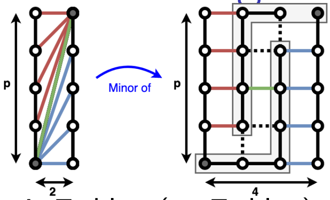
If H is k-monotone

Theorem 2

If H has a k-**monotone** $p \times q$-grid drawing, then H is a minor of a $O(k^2.pq)$ area grid.

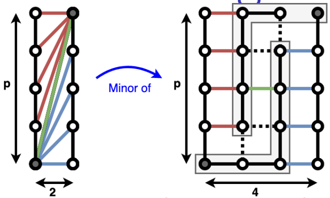


Z-bloc and Tiling

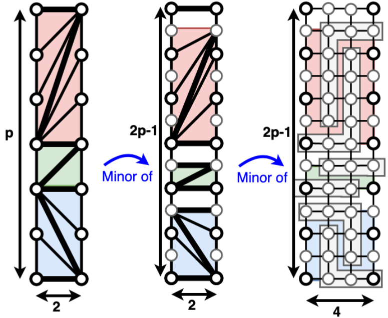


A $\bar{Z}_p$-bloc (or Z_p-bloc).

Z-bloc and Tiling

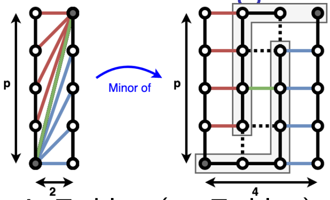


A Z_p-bloc (or Z_p-bloc).

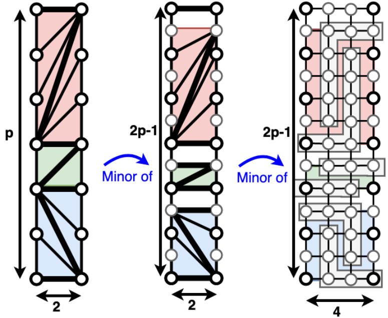


A column of Z-blocs.

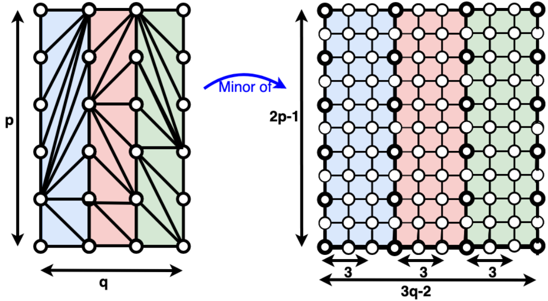
Z-bloc and Tiling



A Z_p-bloc (or Z_p-bloc).



A column of Z-blocs.



A tiling of Z-blocs.

Lemma

If H has a $p \times q$-grid drawing as a minor of a tiling of Z-blocs, then H is minor of the $(2p-1) \times (3q-2)$-grid.

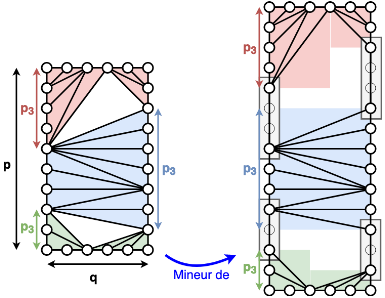
Conclusion

Thank you for your attention.

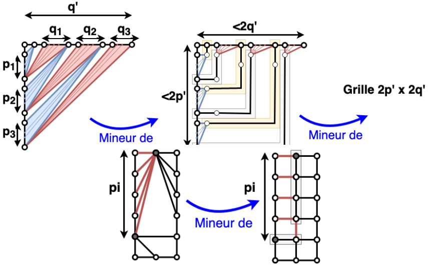
Annexes

Rectangles

if H has a $p \times q$-grid drawing with every vertices on the external face, H is a minor of the $6p \times 6q$ grid.



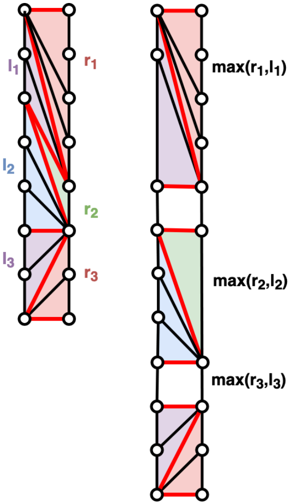
Decomposition of H.



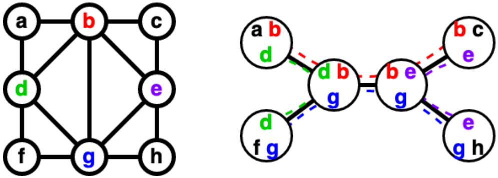
handling the triangles.

Stick graph

Iif H has a $p \times 2$-grid drawing, H if minor of the $2p-1 \times 4$-grid.



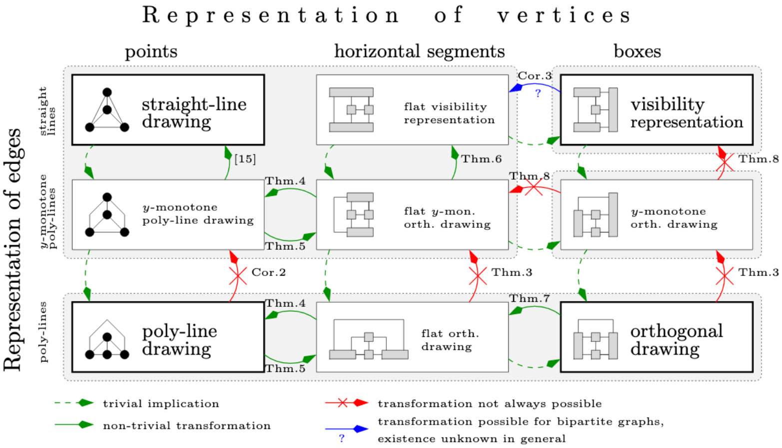
Treewidth



Example of a tree decomposition.

Width: size of the biggest bag -1.
Treewidth: smallest width among all the tree decompositions of the graph.

graph drawings



recap scheme from
Therese C. Biedl. Height-preserving transformations of planar graph drawings, *Lecture Notes in Computer Science*, 8871 :380–391, 2014.