Block elimination distance

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Joint work with

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[3] Computer Engineering Department, Kadir Has University, Istanbul, Turkey.

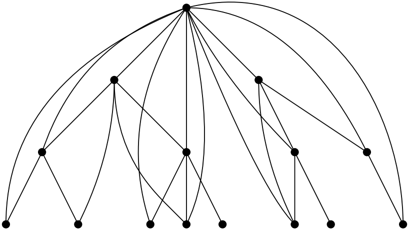
JGA2020, 16-18/11/2020 (en distanciel)

Treedepth

$\mathcal{C}(G) :=$ the set of connected components of G.

Definition

$$\mathrm{td}(G) = \begin{cases} \max\{\mathrm{td}(H) \mid H \in \mathcal{C}(G)\} & \text{if } |\mathcal{C}(G)| > 1 \text{ and } E(G) \neq \emptyset, \\ 1 + \min_{v \in V(G)}\{\mathrm{td}(G \setminus v)\} & \text{if } G \text{ is connected and } E(G) \neq \emptyset, \\ 1 & G \text{ is edgeless} \end{cases}$$



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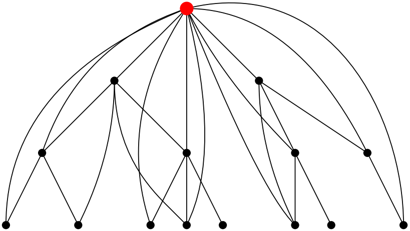

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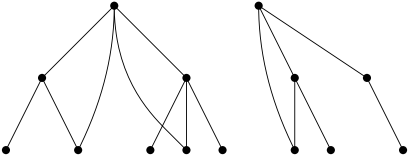

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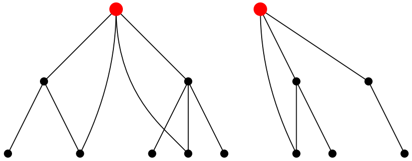

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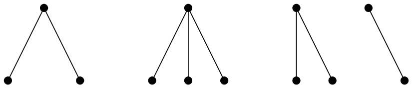

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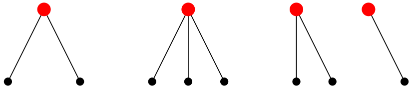

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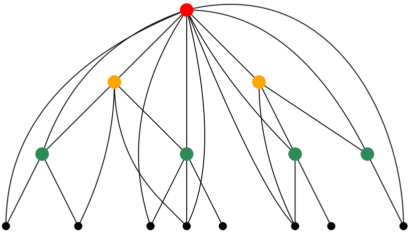

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Elimination distance to a graph class

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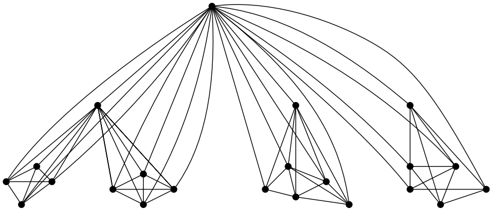

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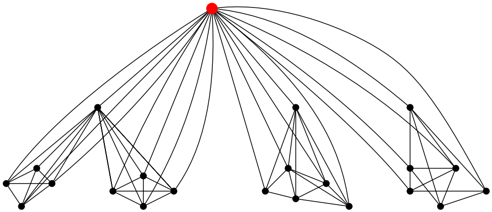

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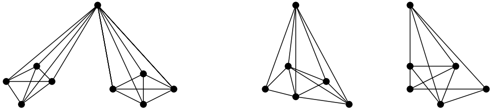

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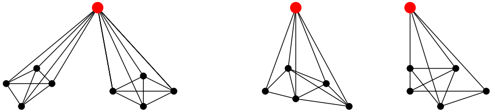

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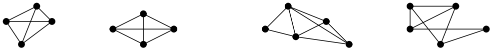

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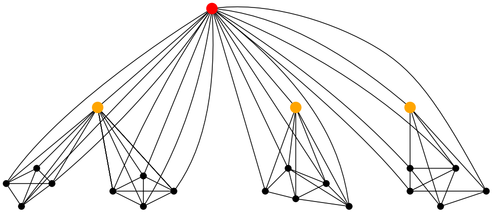

$\mathrm{ed}_{\mathcal{P}}(G) = 3$

Minor-Closed Graph Classes

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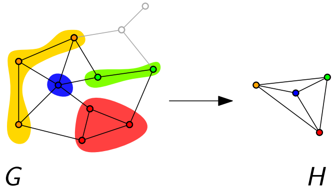

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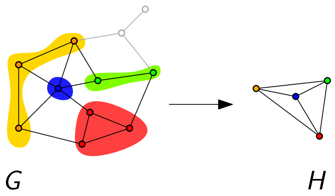

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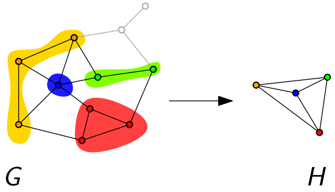

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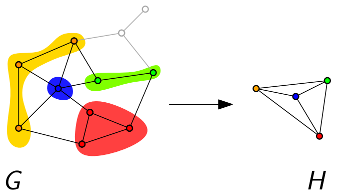

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Robertson & Seymour Theorem

If $\mathcal{G}$ is minor-closed, then obs($\mathcal{G}$) is a finite set.

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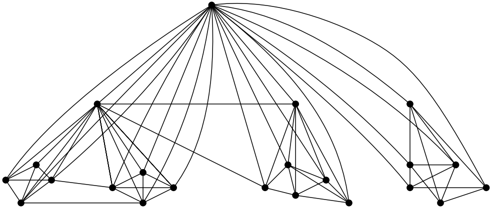

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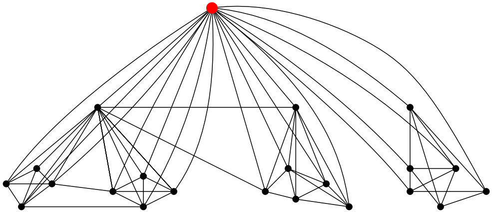

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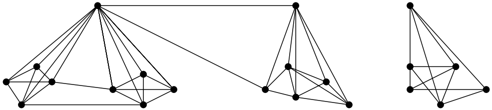

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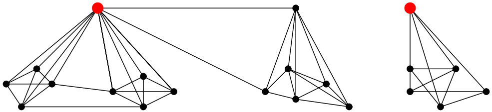

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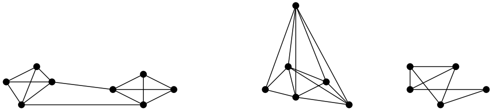

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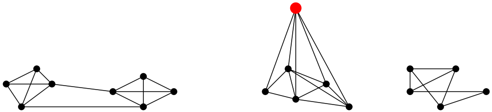

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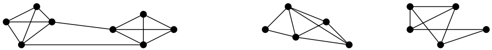

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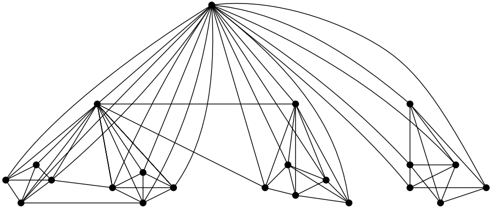

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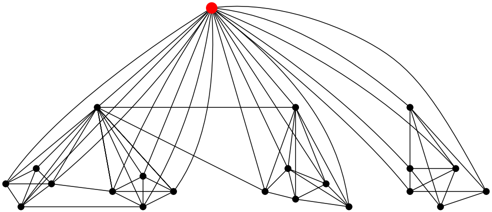

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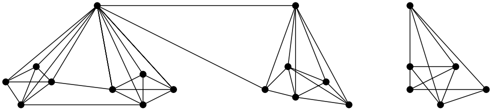

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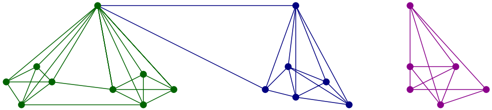

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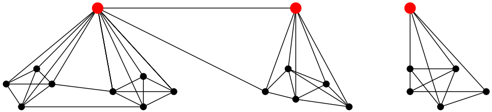

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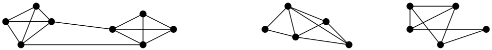

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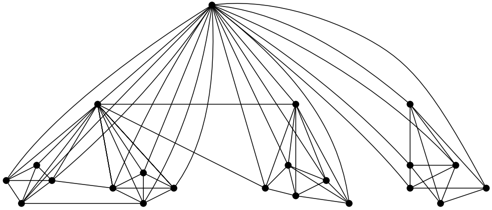

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Our first result

$$\mathcal{G}^{(k)} := \{G \mid \mathrm{bed}_{\mathcal{G}}(G) \leq k\}.$$

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We prove the following intermediate result:

Lemma

There is an algorithm that, given $\mathrm{obs}(\mathcal{G})$, outputs $\mathrm{obs}(\mathrm{bcl}(\mathcal{G}))$.

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Lemma $+$ algorithm of Adler et al. $\rightarrow$ **Theorem 1**

Block elimination distance to edgeless graphs

"Biconnected analogue" of treedepth.

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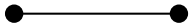
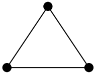
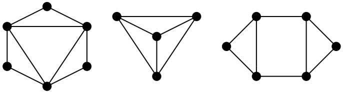
Observation: $\{G \mid \mathrm{btd}(G) \leq 2\} = \mathcal{E}^{(2)} = \{\text{forests}\}$.

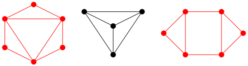
Obstruction sets for bounded btd

We study the set obs$(\mathcal{E}^{(k)})$ for every $k \in \mathbb{N}$.

	obs$(\mathcal{E}^{(k)})$
k $= 1$	
k $= 2$	
k $= 3$	

Our second result

We completely identify the outerplanar graphs in obs($\mathcal{E}^{(k)}$).



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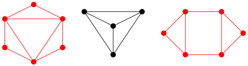
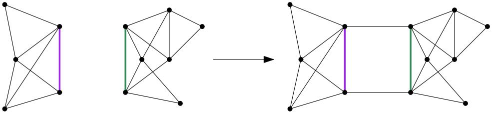


We define the following operations:

- Parallel join:



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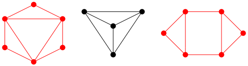

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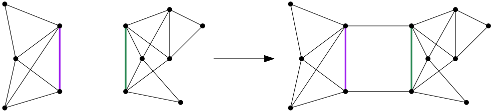
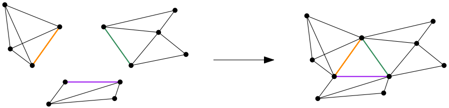


- Triangle gluing:



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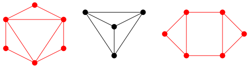

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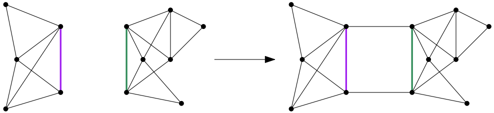

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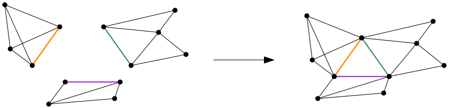

Theorem 2

Every outerplanar graph in obs($\mathcal{E}^{(k)}$) can be obtained by parallel join or triangle gluing of outerplanar graphs in obs($\mathcal{E}^{(k-1)}$).

Conclusion

Open Problems:

- Explicit function f for computing $\text{obs}(\mathcal{Z}_k(\mathcal{G})$ and $\text{obs}(\mathcal{G}^{(k)})$?
- Optimize the parametric dependence (on k) of the algorithms.

Thank You!