ABC(T)-graphs

Journées Graphes et Algorithmes 2020

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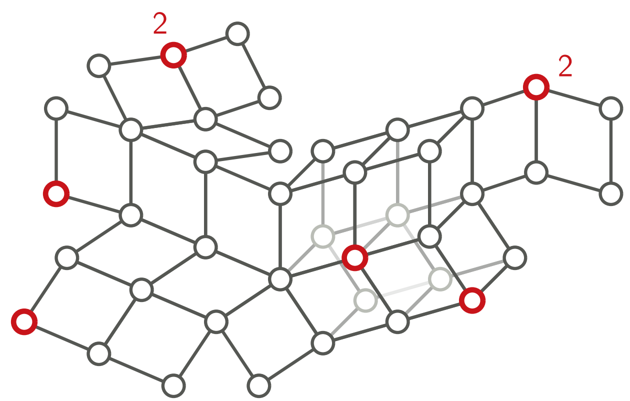

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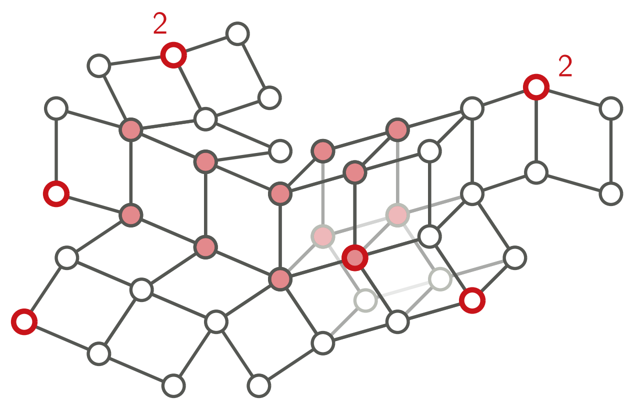

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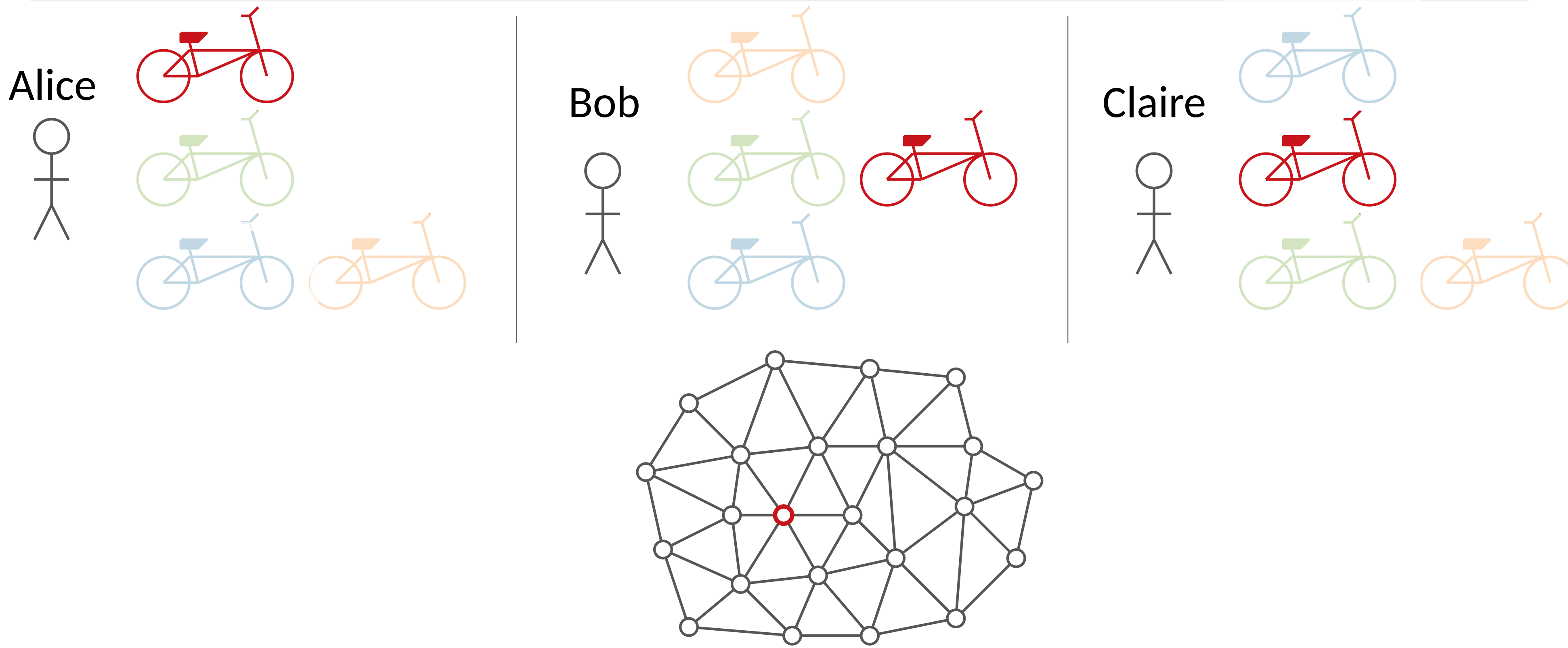

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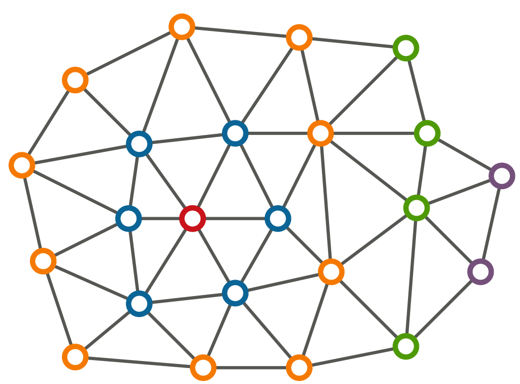

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Consensus function on graphs : $L : V^* \to 2^V \setminus \emptyset$

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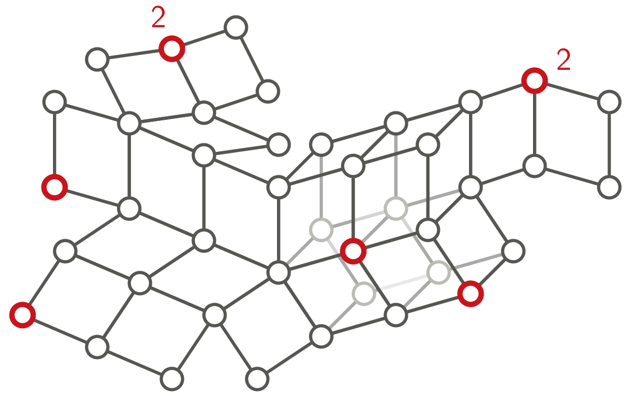

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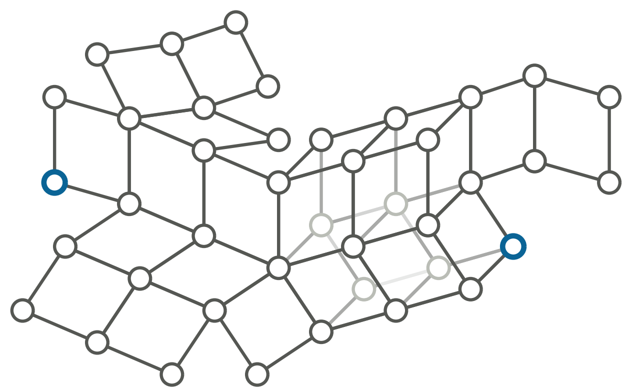

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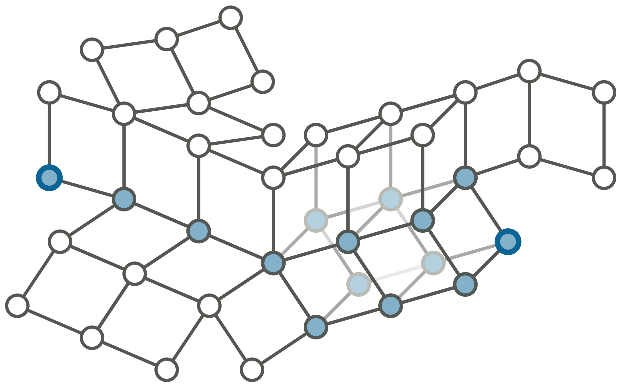

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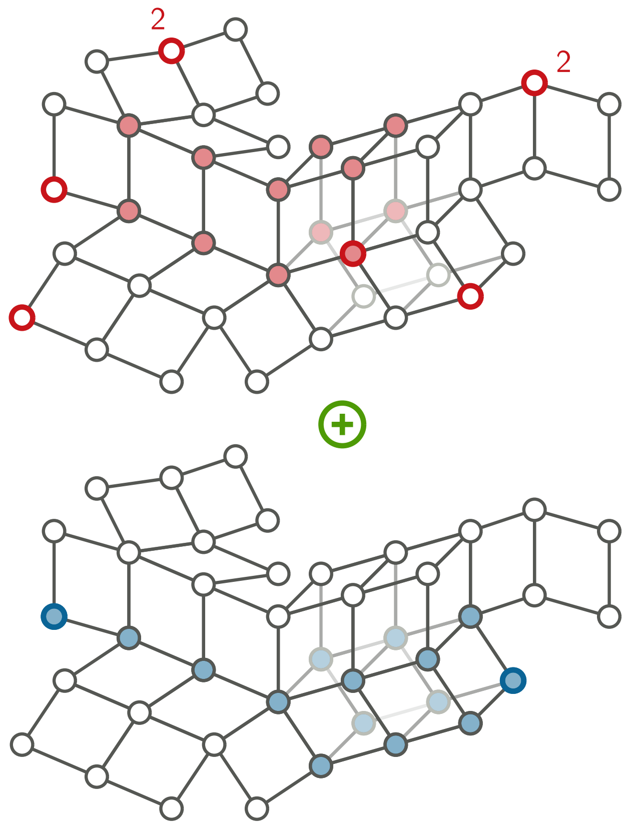

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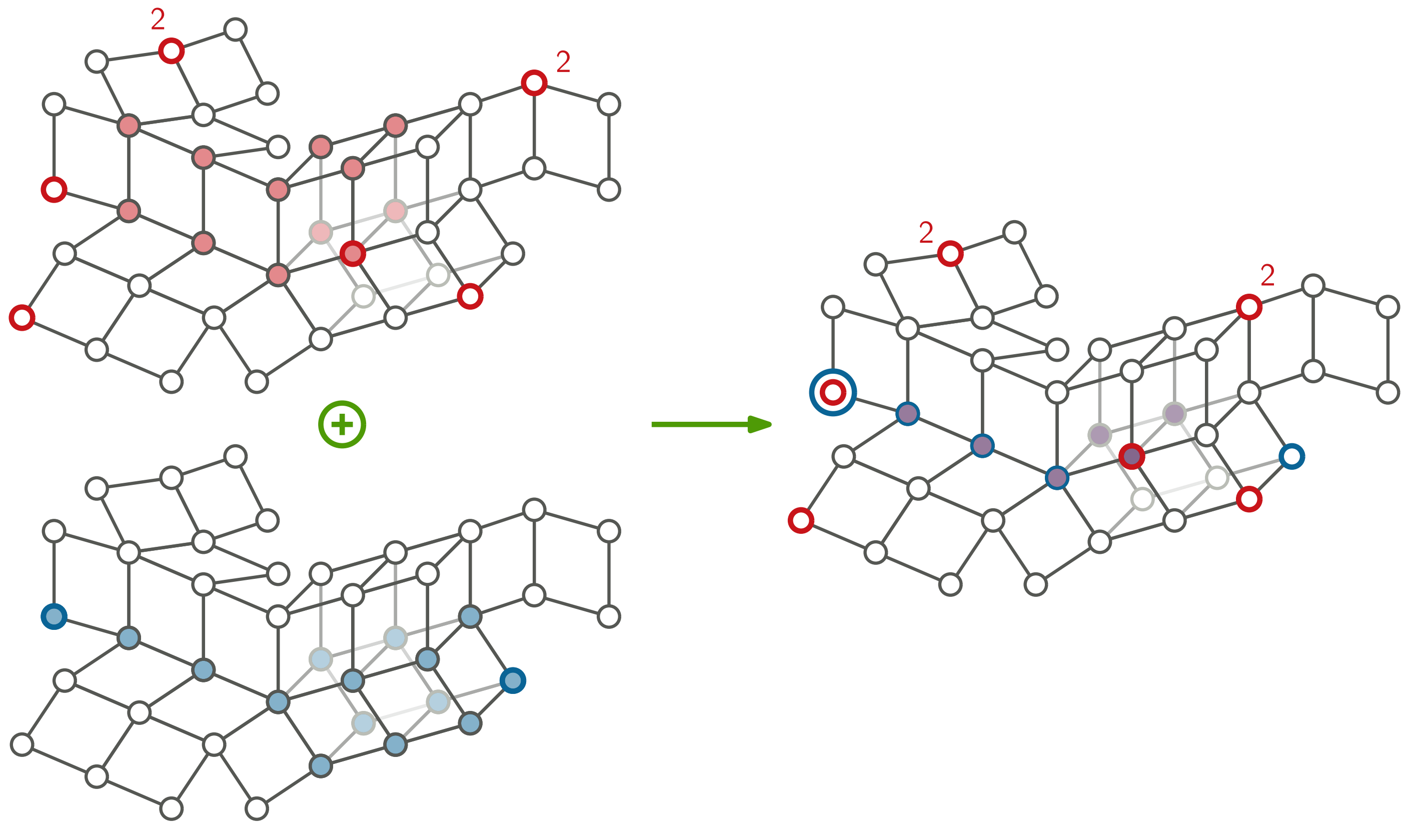

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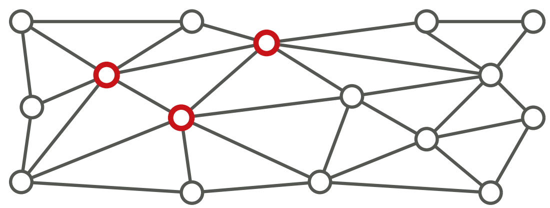

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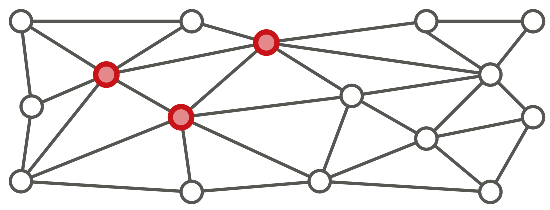

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Theorem A: Graphs with connected medians are ABCT graphs

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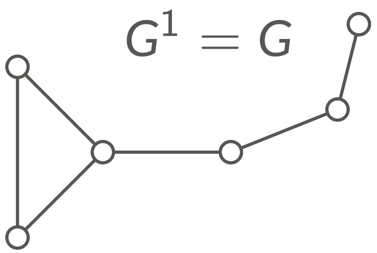

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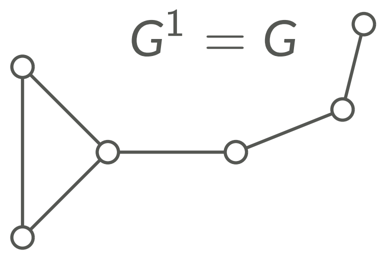

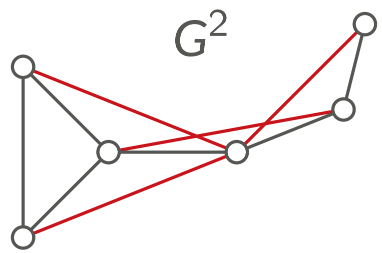

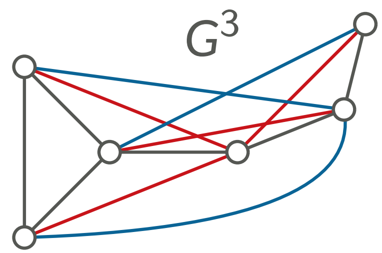

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Modular graphs : For each $u, v, w \in V$, $I(u,v) \cap I(u,w) \cap I(v,w) \neq \emptyset$

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1. The following conditions are equivalent:
   - (i) $\text{Med}^{loc}_{G^p}(\pi) = \text{Med}(\pi)$
   - (ii) F_π is p-step weakly-convex
   - (iii) all median sets $\text{Med}(\pi)$ are connected in G^p
2. The problem to decide whether a graph have G^p connected medians can be solved in polynomial time

Theorem: The following classes of graphs have G^2-connected medians :

- Chordal graphs
- Bridged graphs
- Bipartite absolute retracts
- Benzenoid systems

Corollary : If $u \notin Med(\pi)$, $\exists v$ with $d(u,v) \leq p$ s.t $F_\pi(v) < F_\pi(u)$

Modular graphs : For each $u, v, w \in V$, $I(u,v) \cap I(u,w) \cap I(v,w) \neq \emptyset$

Theorem B: Modular graphs with G^2-connected medians are ABC-graphs

Main Results

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Theorem A: Graphs with connected medians are ABCT-graphs

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Theorem A: Graphs with connected medians are ABCT-graphs

Theorem B: Modular graphs with G^2-connected medians are ABC-graphs

ABCT-property (1)

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Claim 1: If L is a consensus function that respects the ABCT axioms then for any $uv \in E$:

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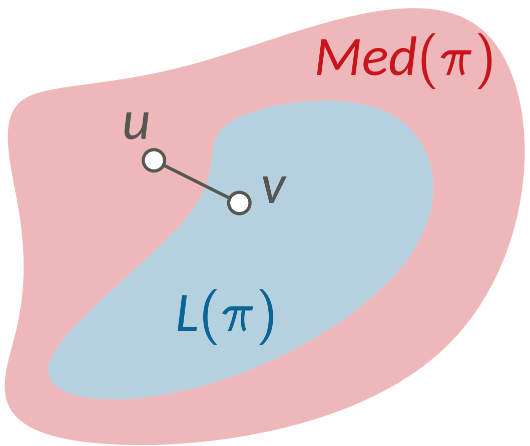
$\exists u \sim v \in Med(\pi), u \notin L(\pi), v \in L(\pi)$



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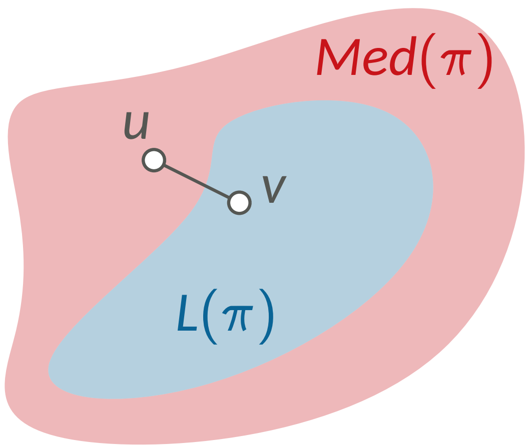

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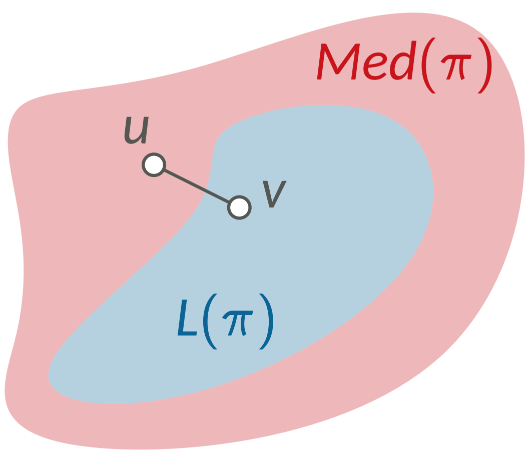
$L(\pi) = \text{Med}(\pi)$



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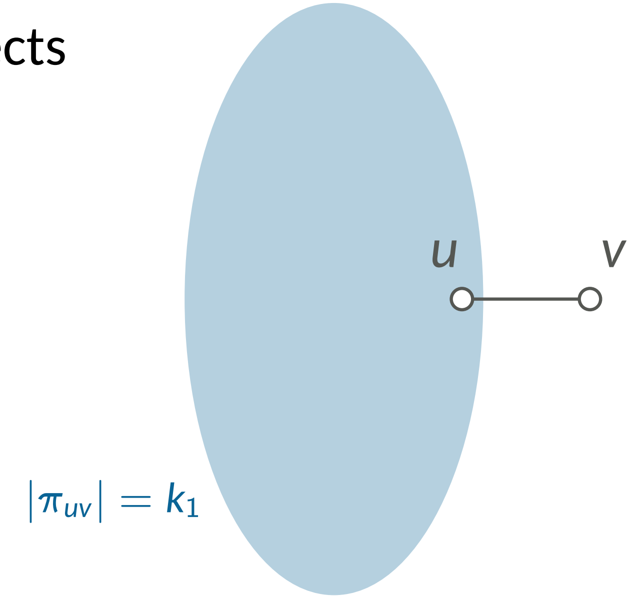

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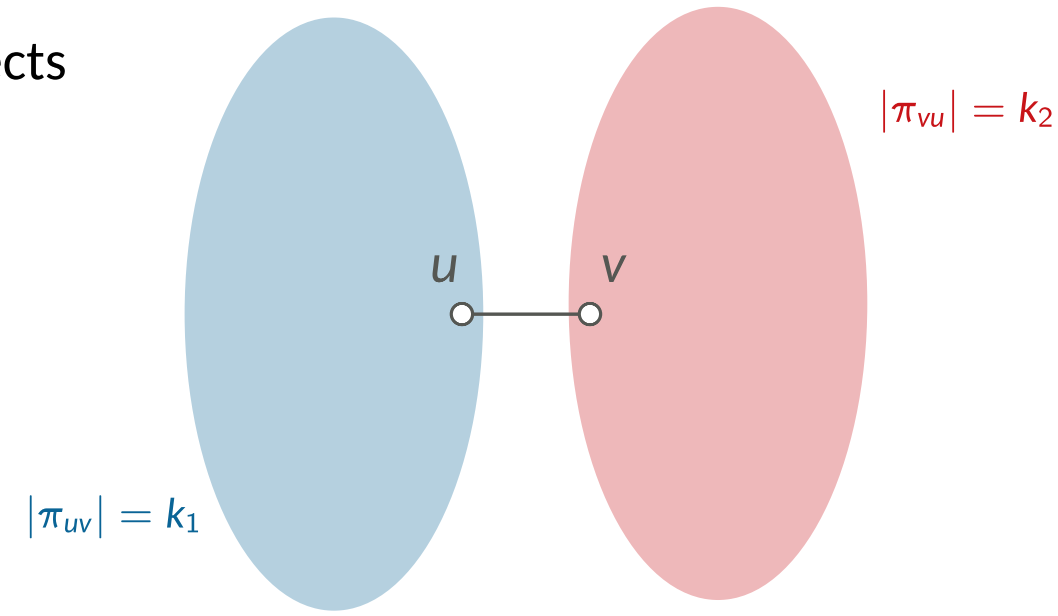

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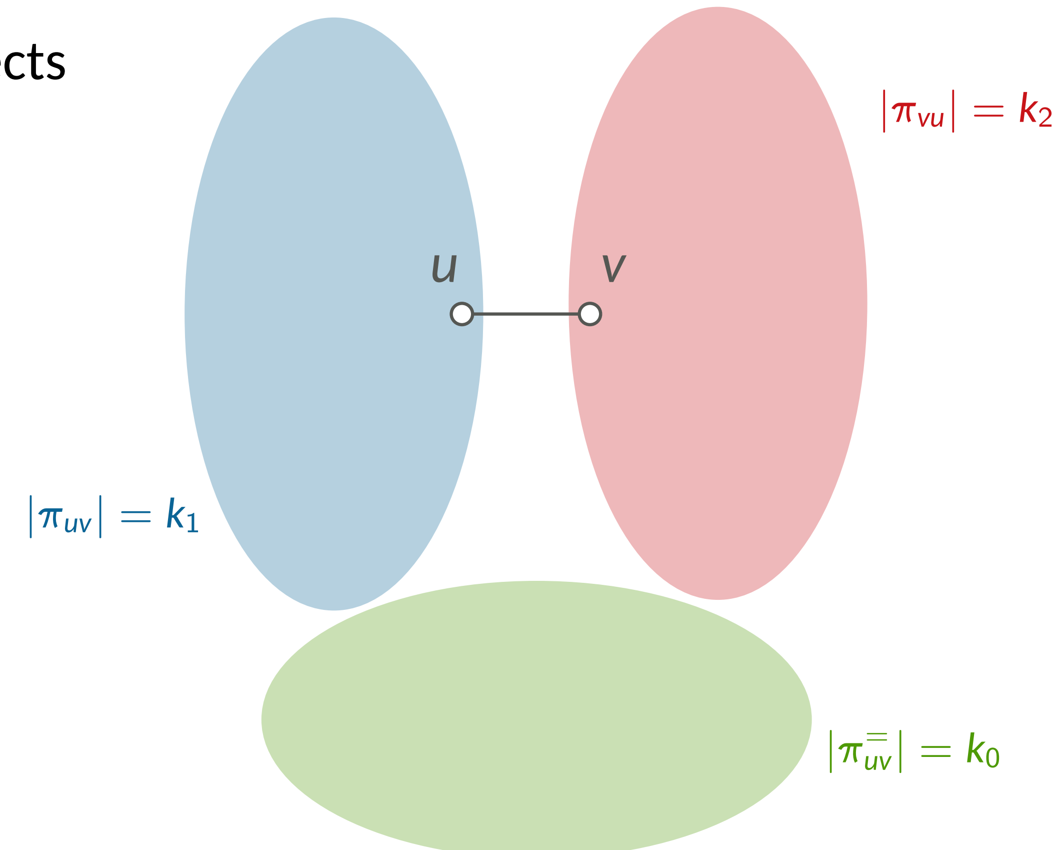

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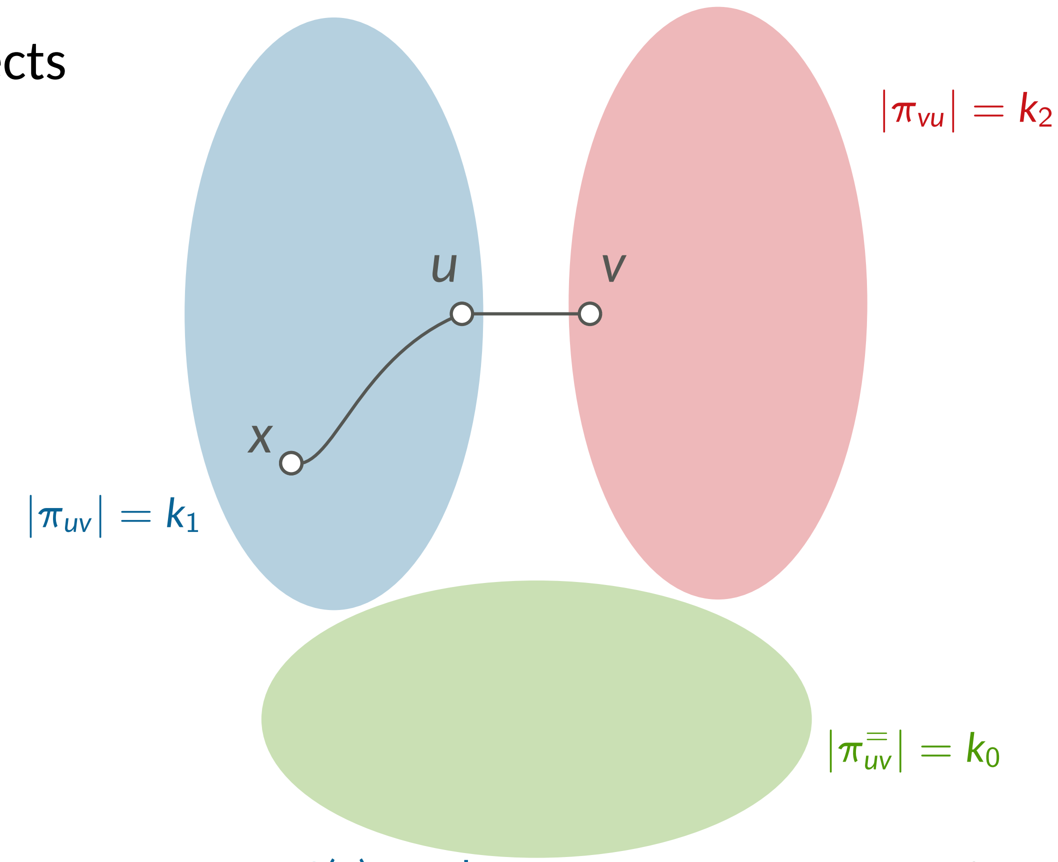

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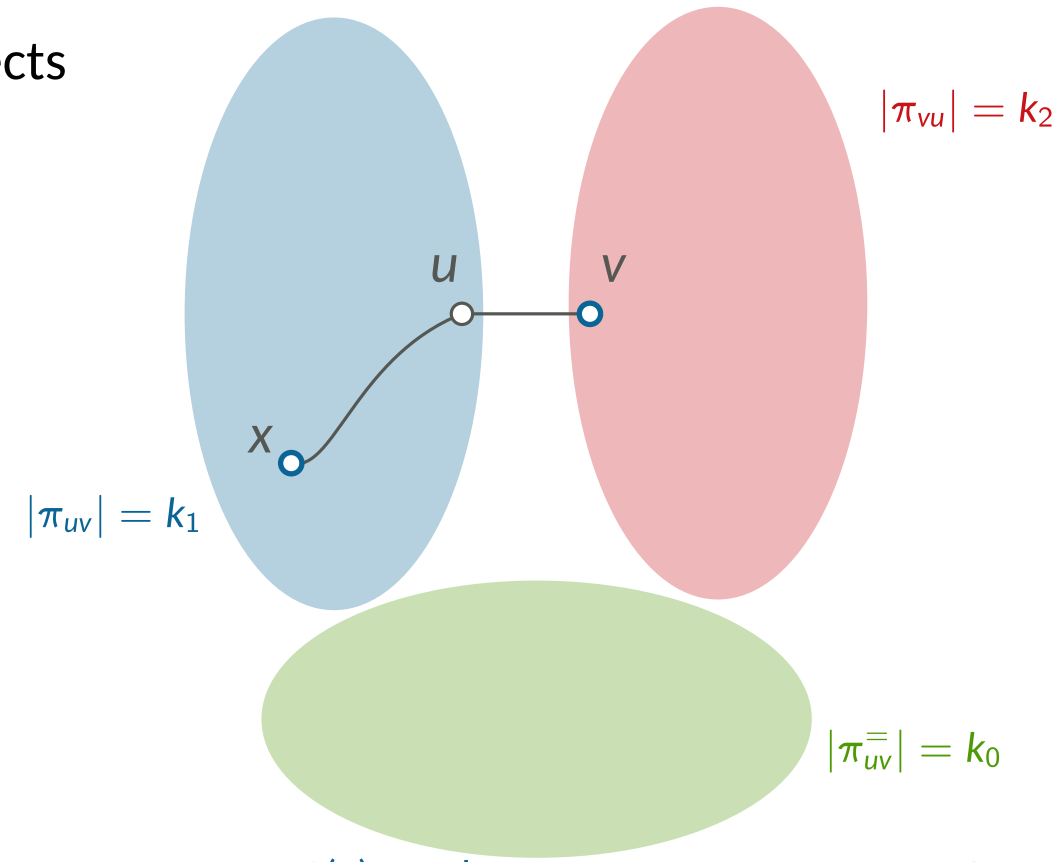

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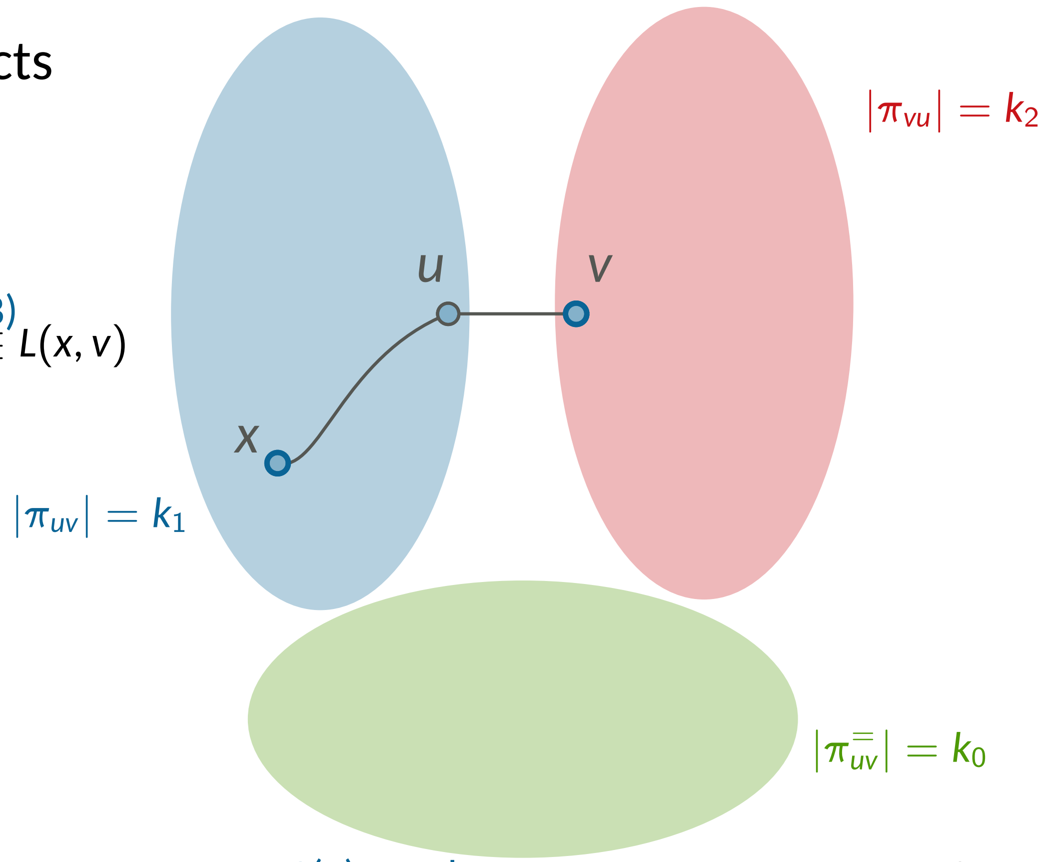

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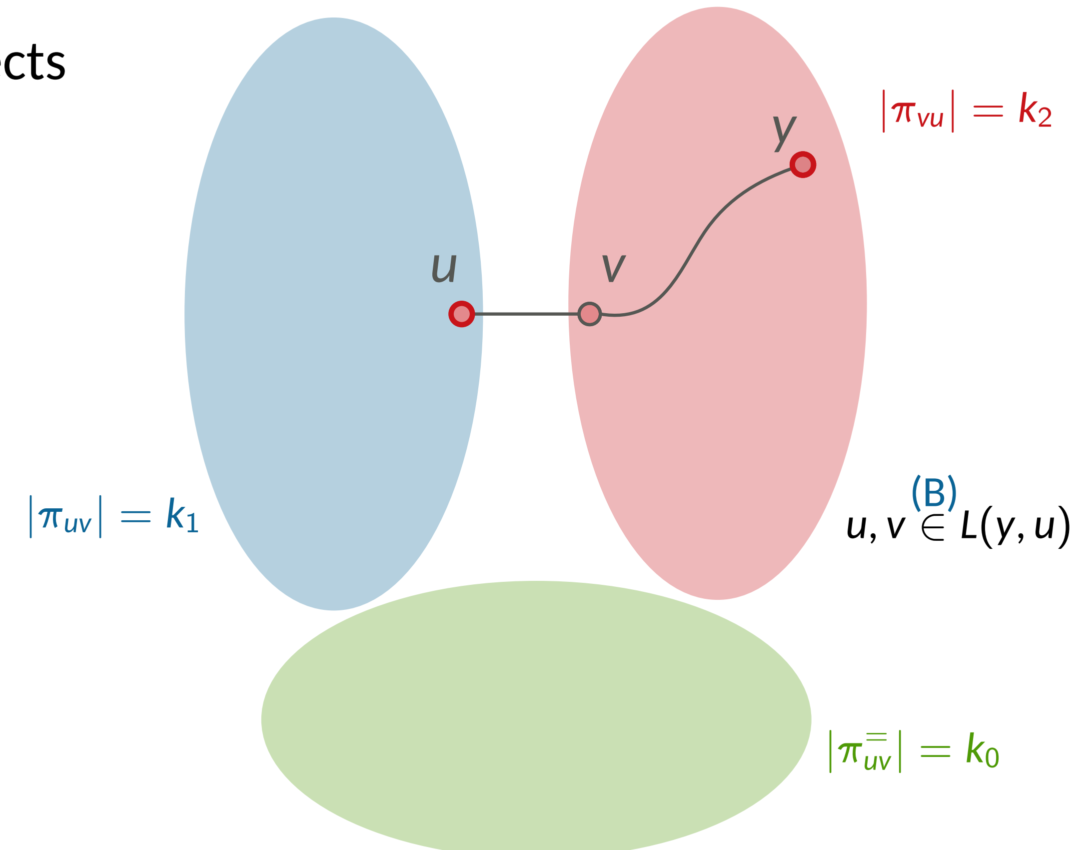

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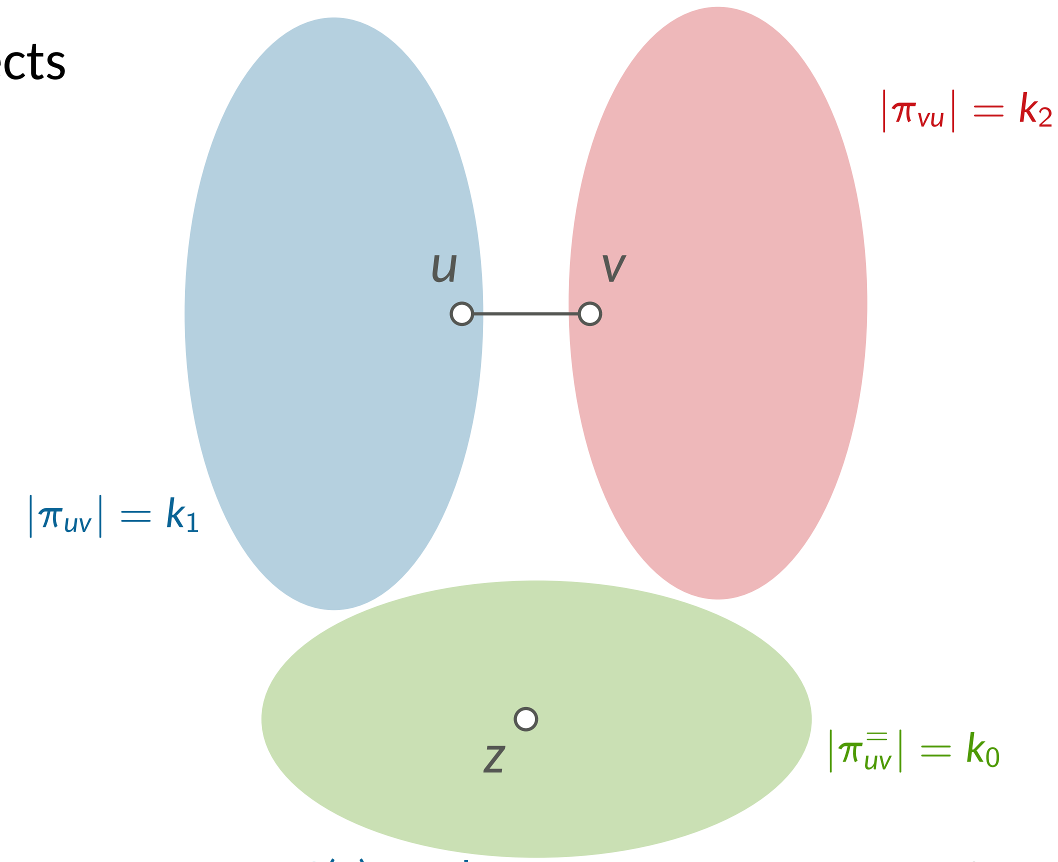

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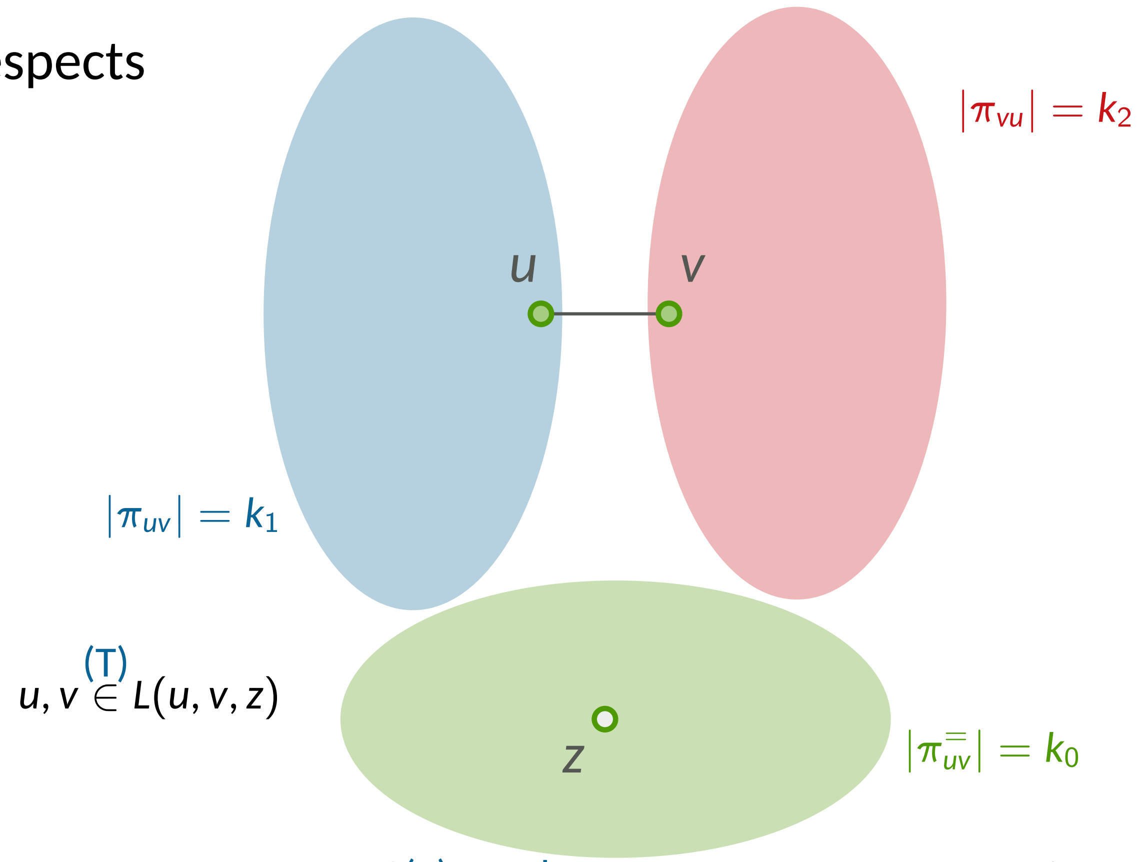

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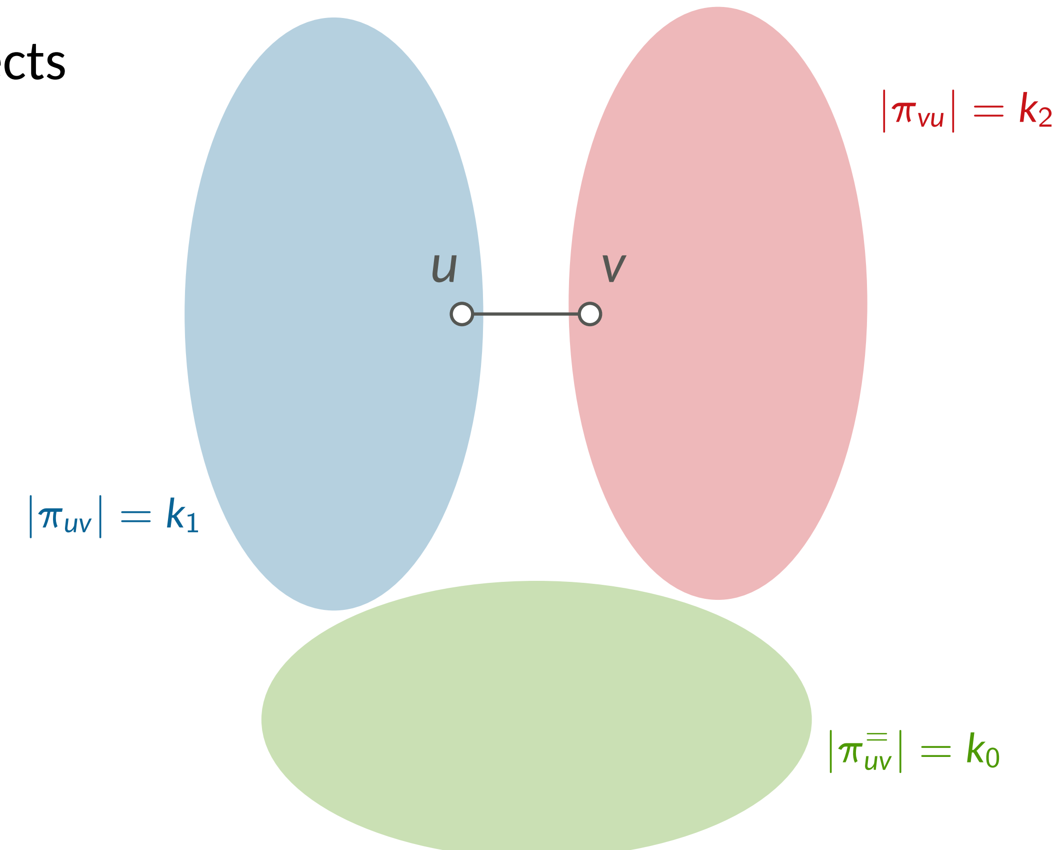

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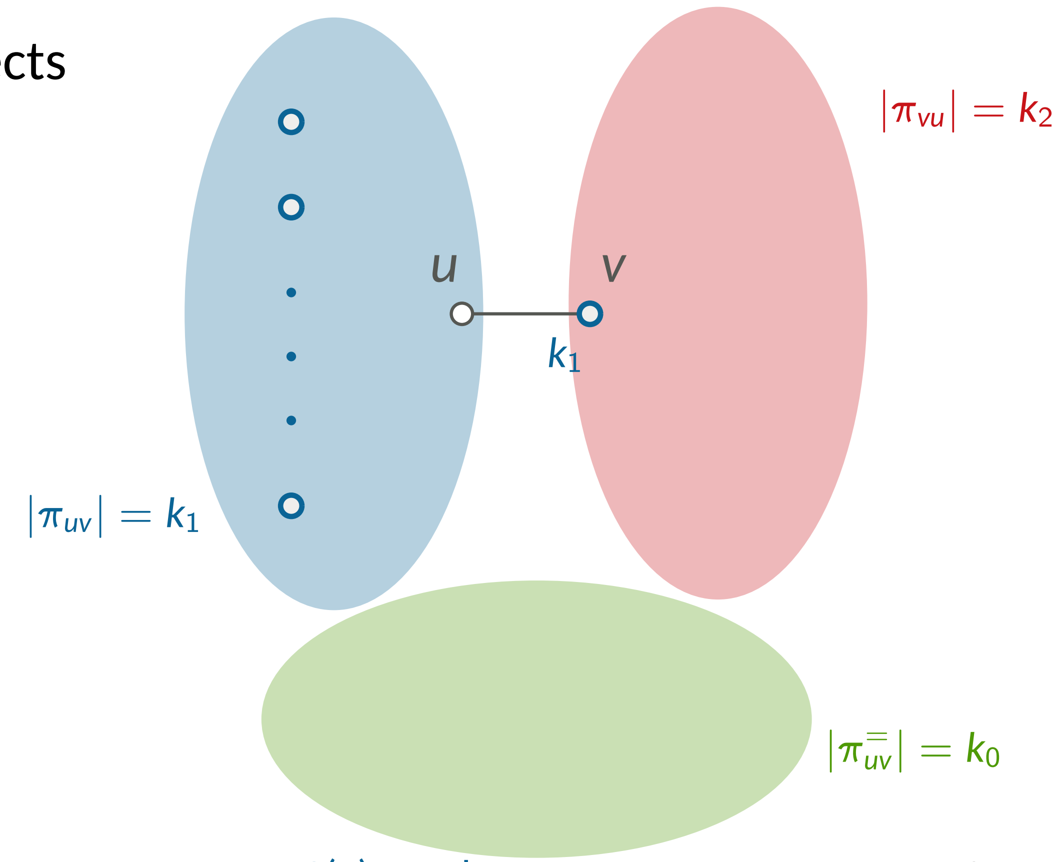

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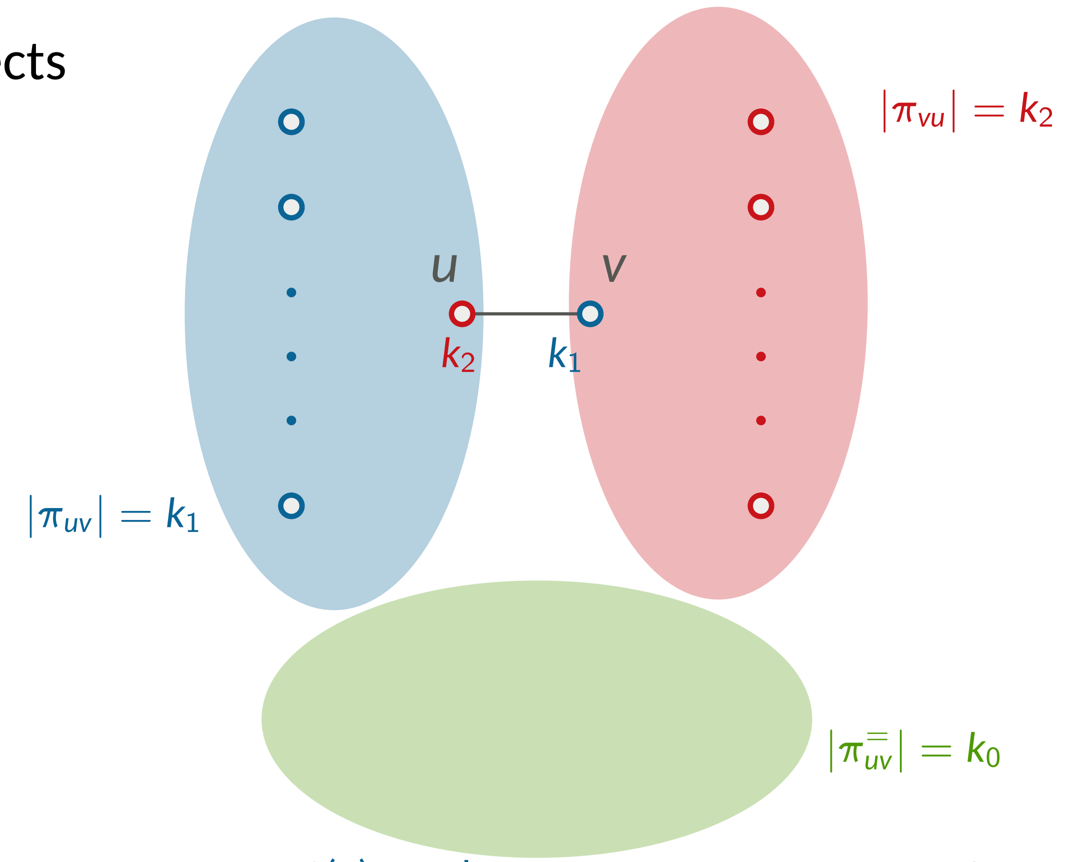

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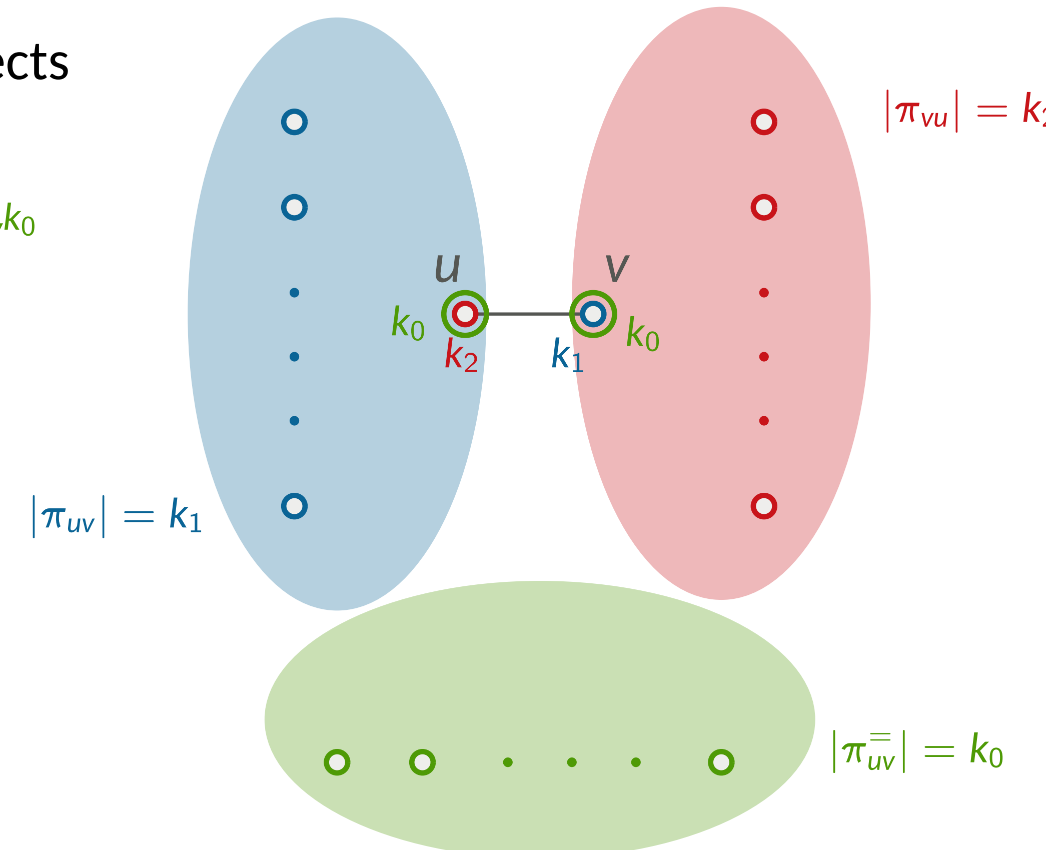

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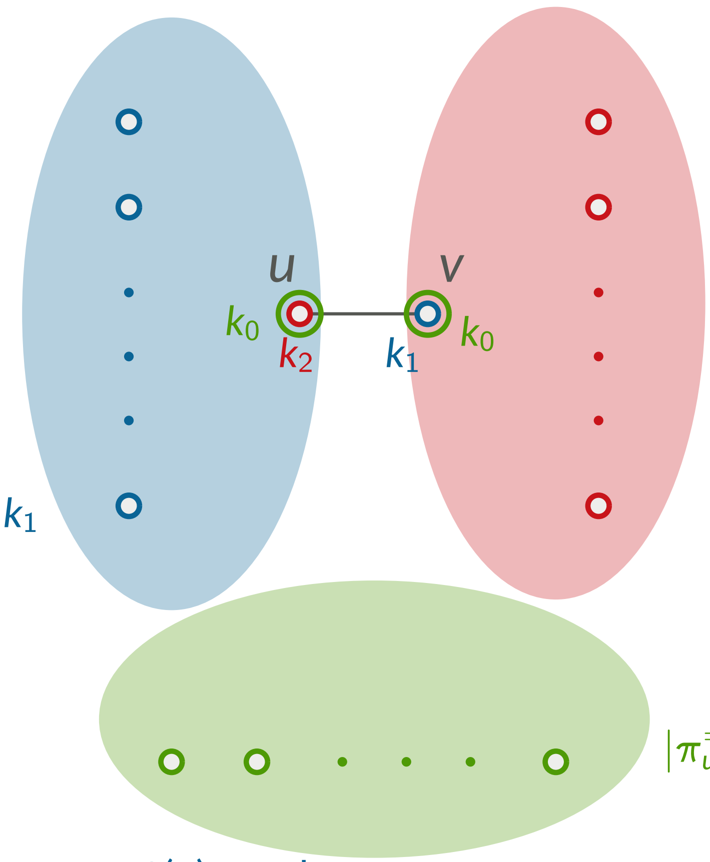
$$u, v \overset{(C)}{\in} L(\pi')$$



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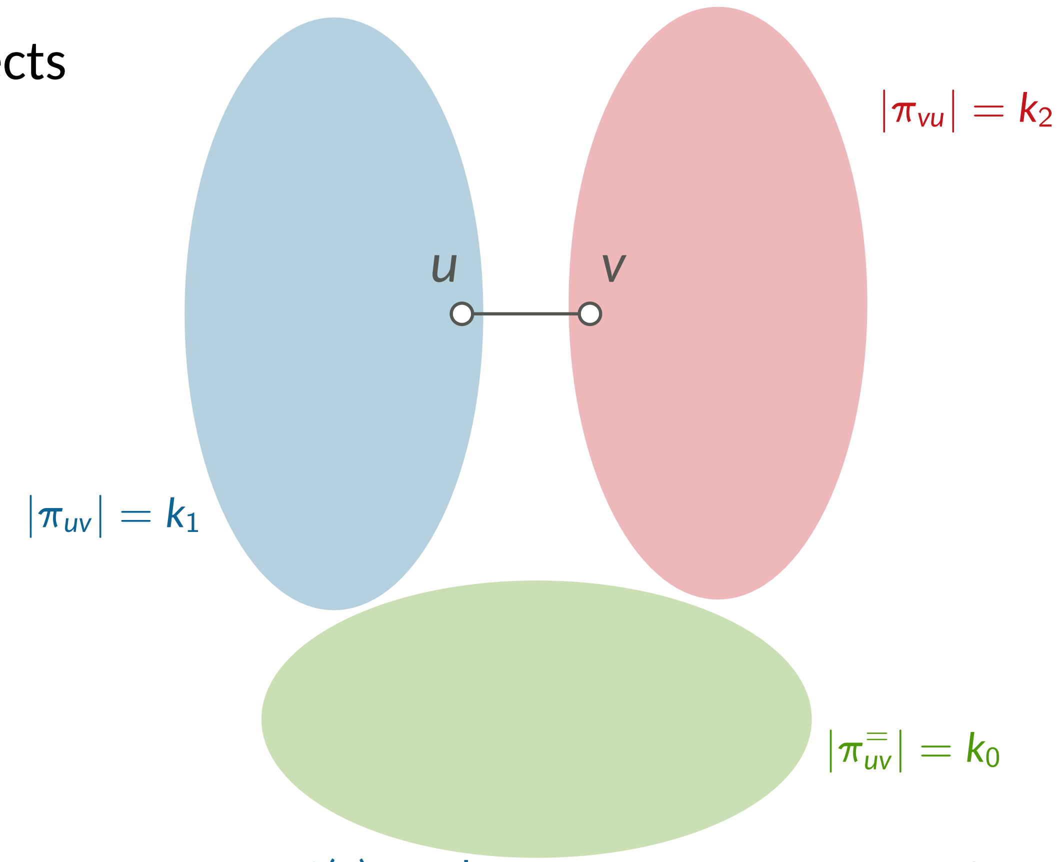
If $F_\pi(u) = F_\pi(v)$



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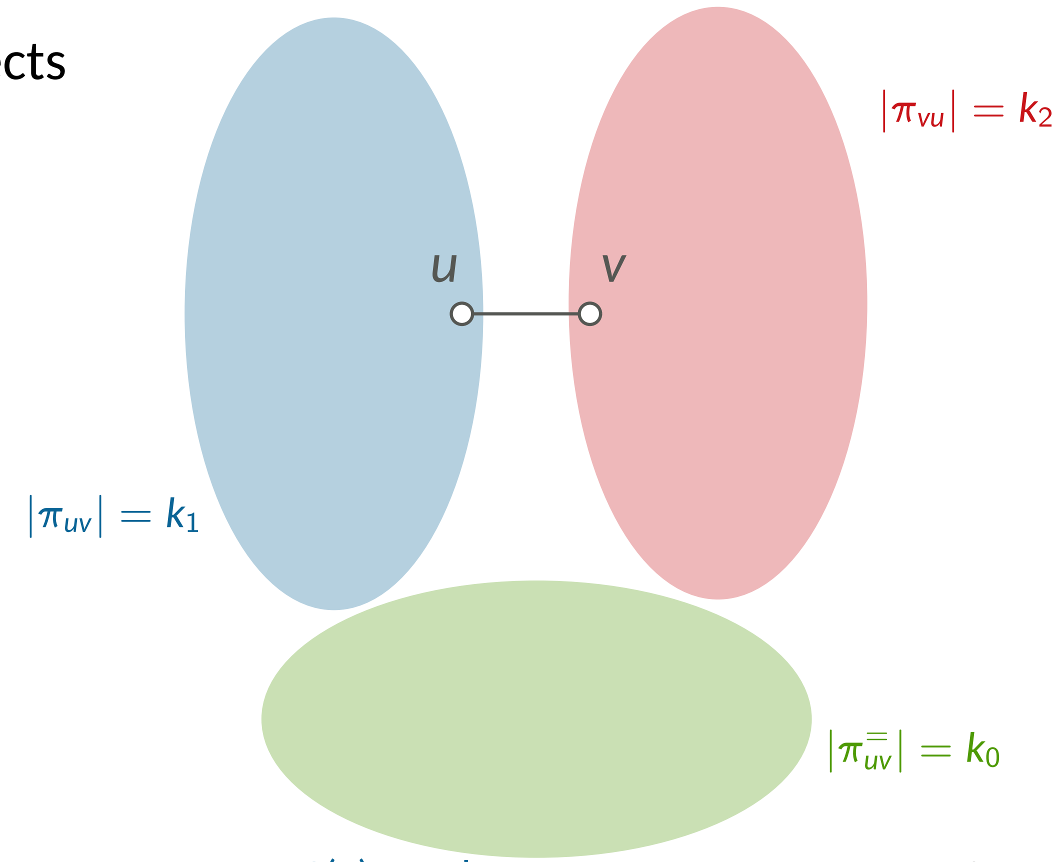

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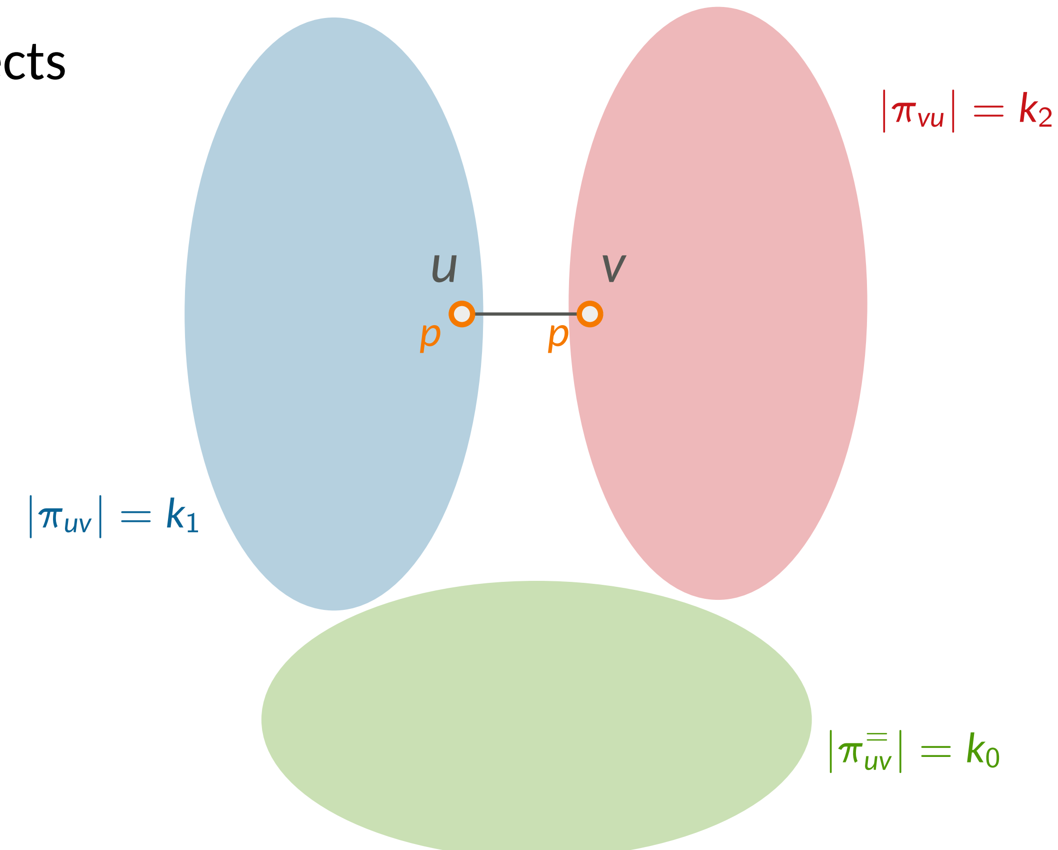

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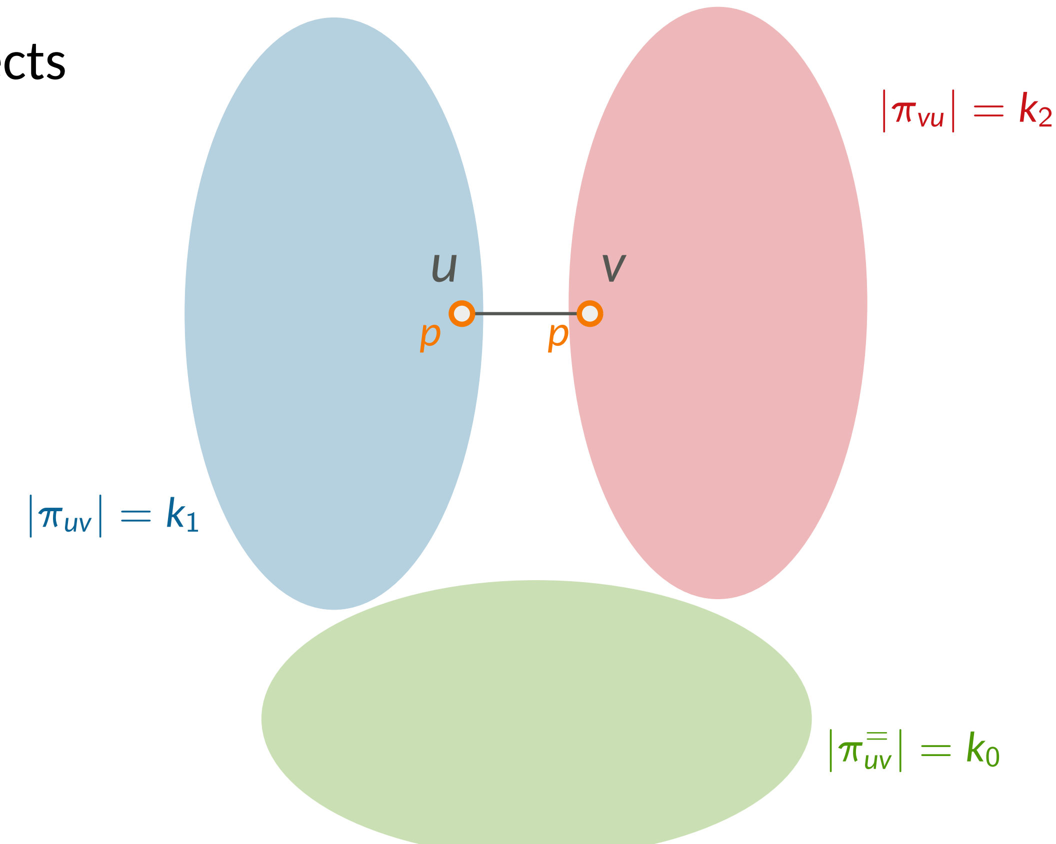
$L(\pi u^{k_2+k_0} v^{k_1+k_0}) = L(\pi\, u^p v^p)$

$u, v \overset{(B)}{\in} L(u, v)$



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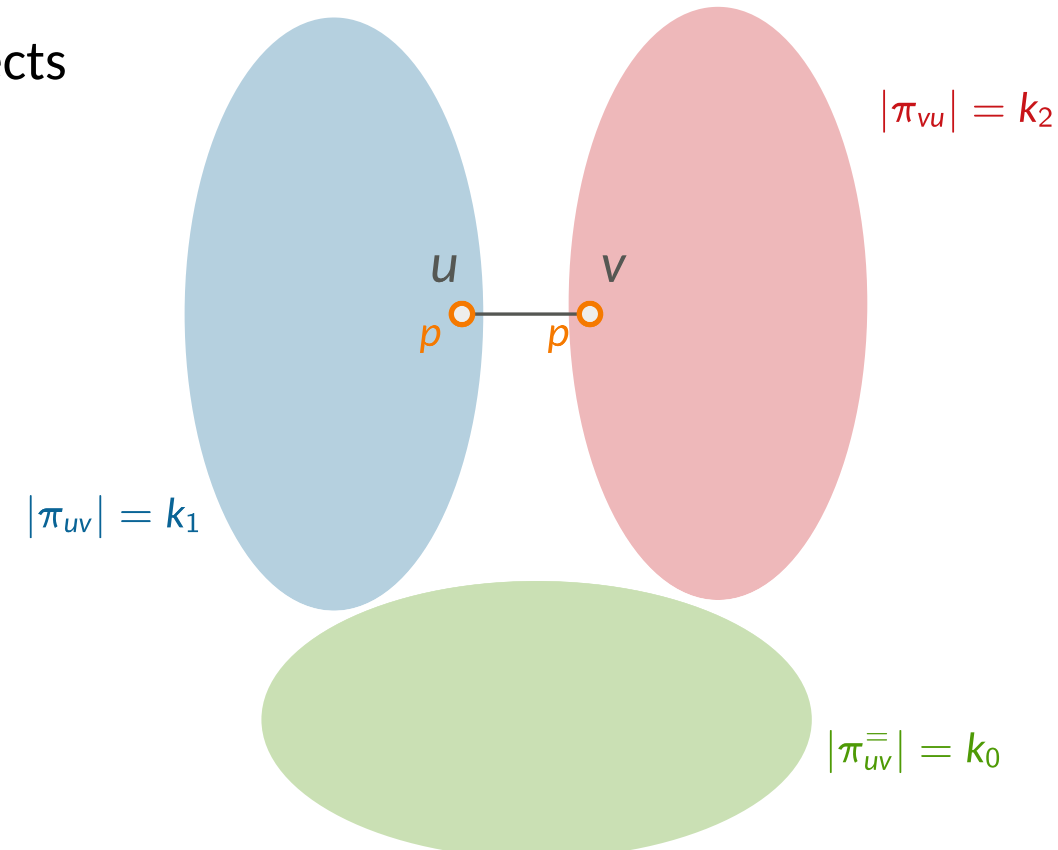

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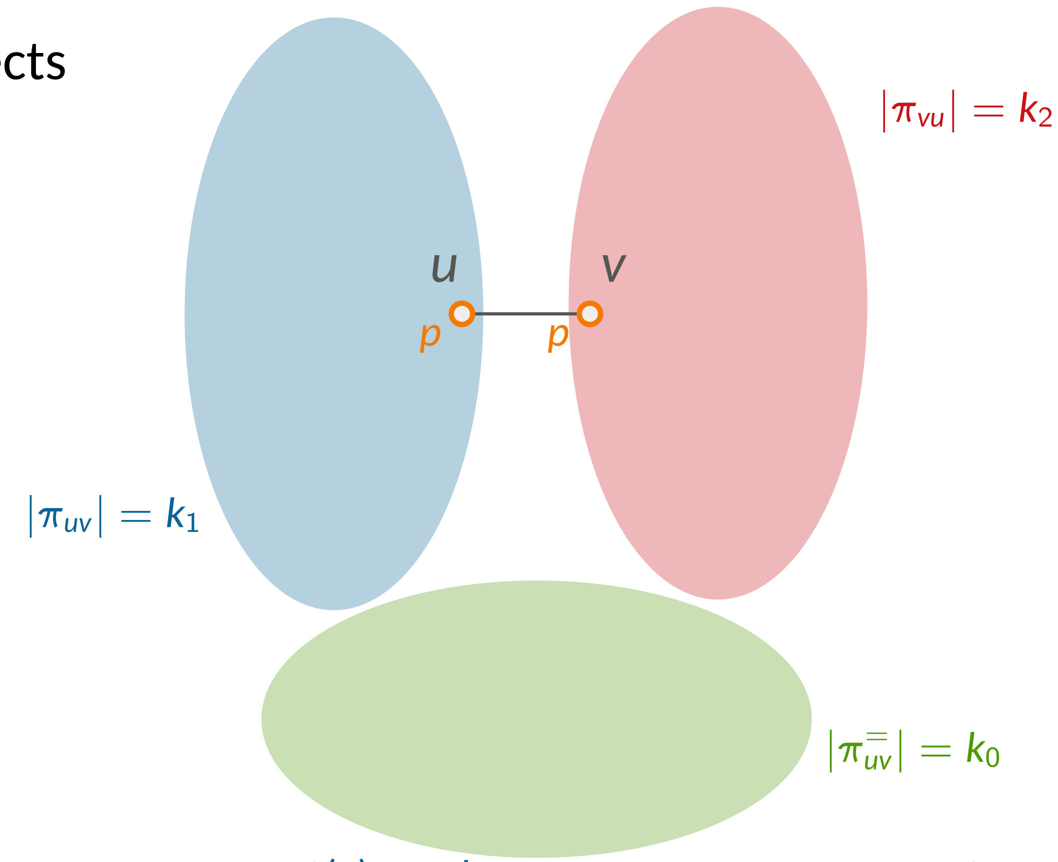
$u, v \overset{(B)}{\in} L(u, v)$ $u, v \overset{(C)}{\in} L(u^p v^p)$

$u \in L(\pi)$ $\overset{(C)}{\text{iff}}$ $v \in L(\pi)$



ABCT-property (3)

Theorem A: Graphs with connected medians are ABCT-graphs

ABC-property (1)

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Modular graphs : For each $u, v, w \in V$, $I(u, v) \cap I(u, w) \cap I(v, w) \neq \emptyset$

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Claim 2: If L is a consensus function that respects the ABC axioms then for any u, v $d(u, v) = 2$ in G, a modular graph:

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Theorem B: Modular graphs with G^2-connected medians are ABC-graphs

ABC-property (2)

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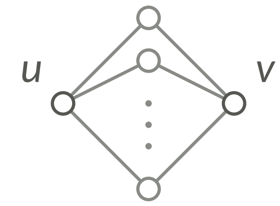

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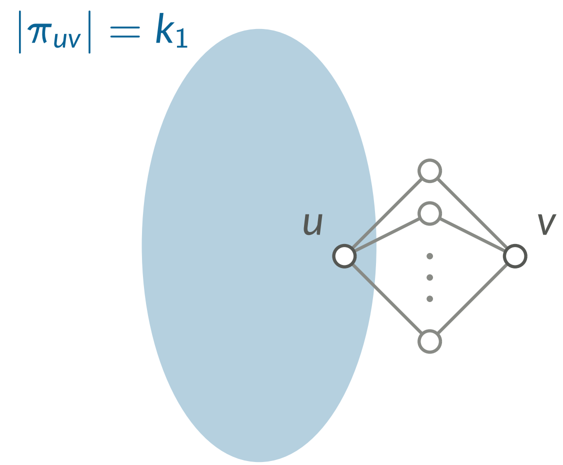

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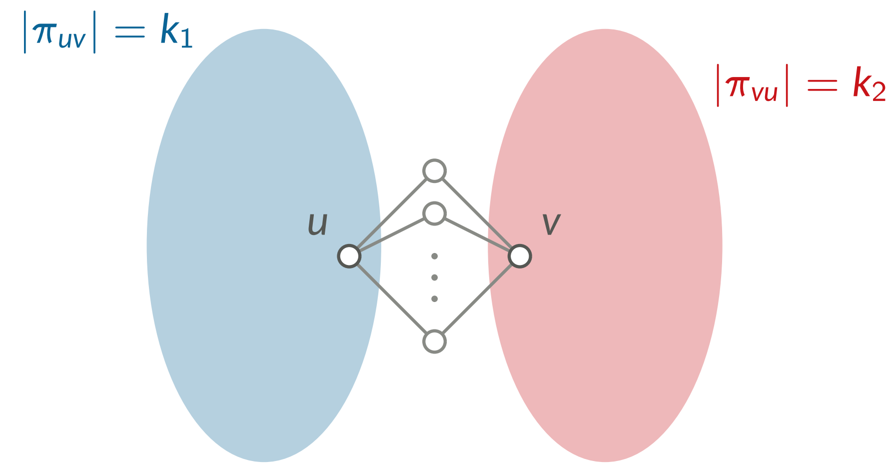

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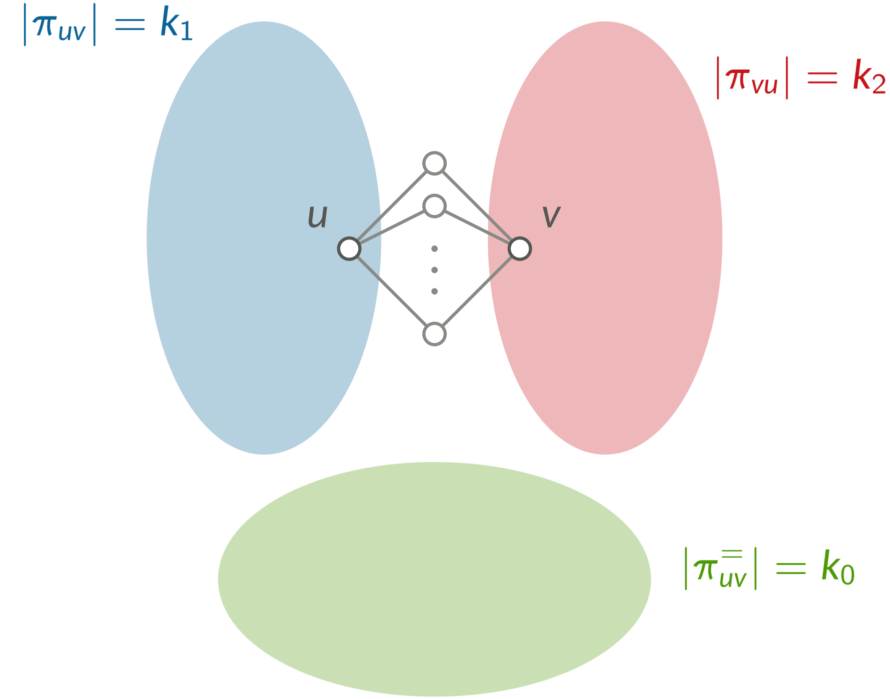

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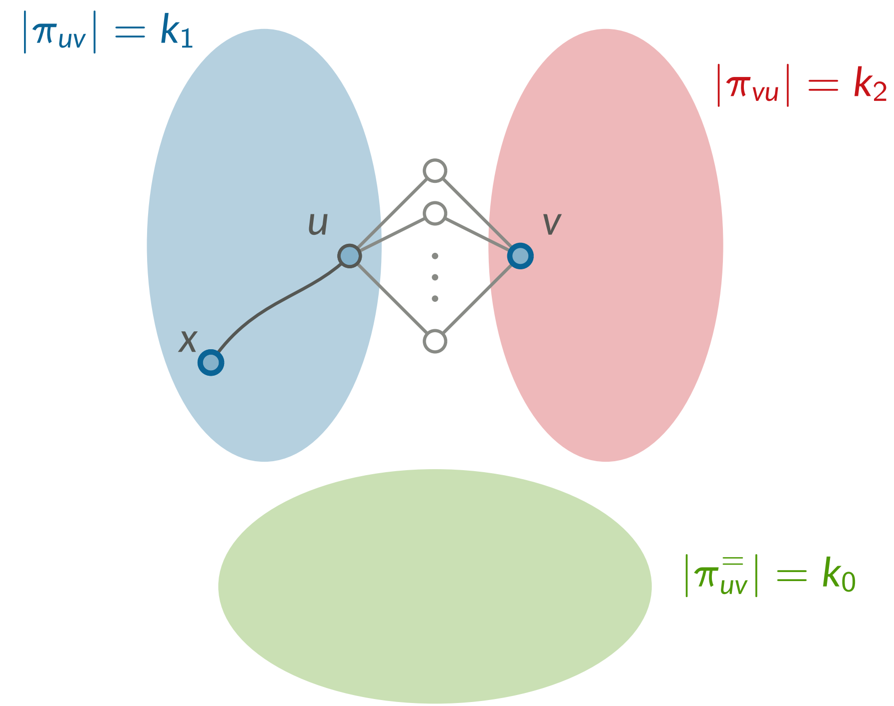

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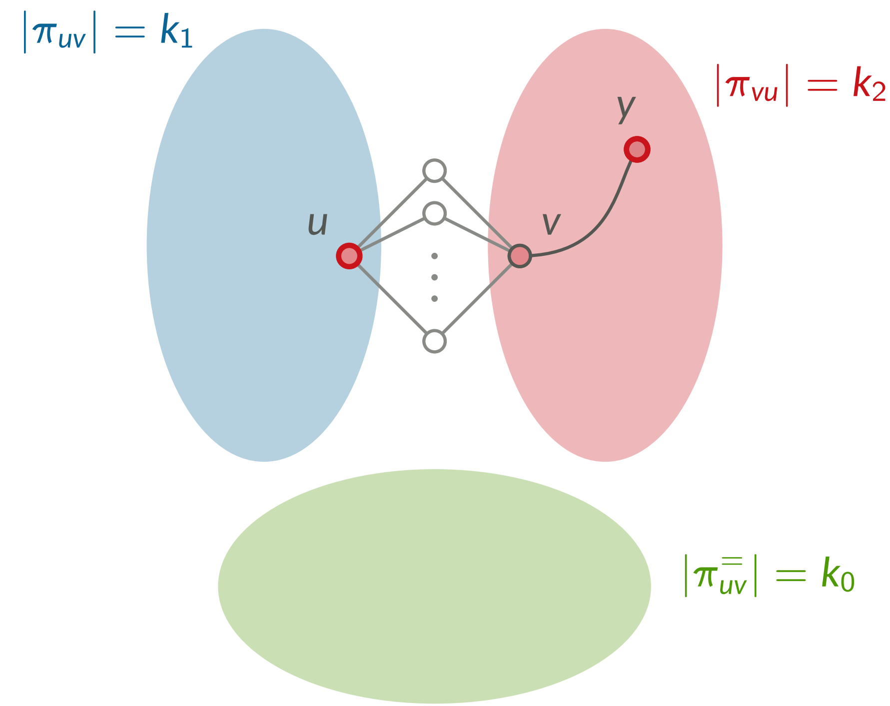

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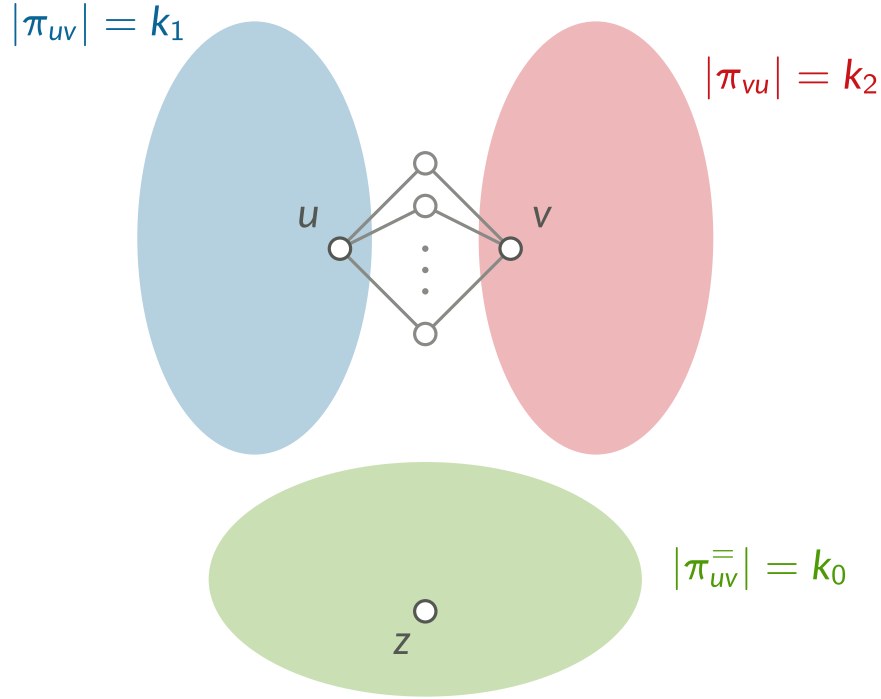

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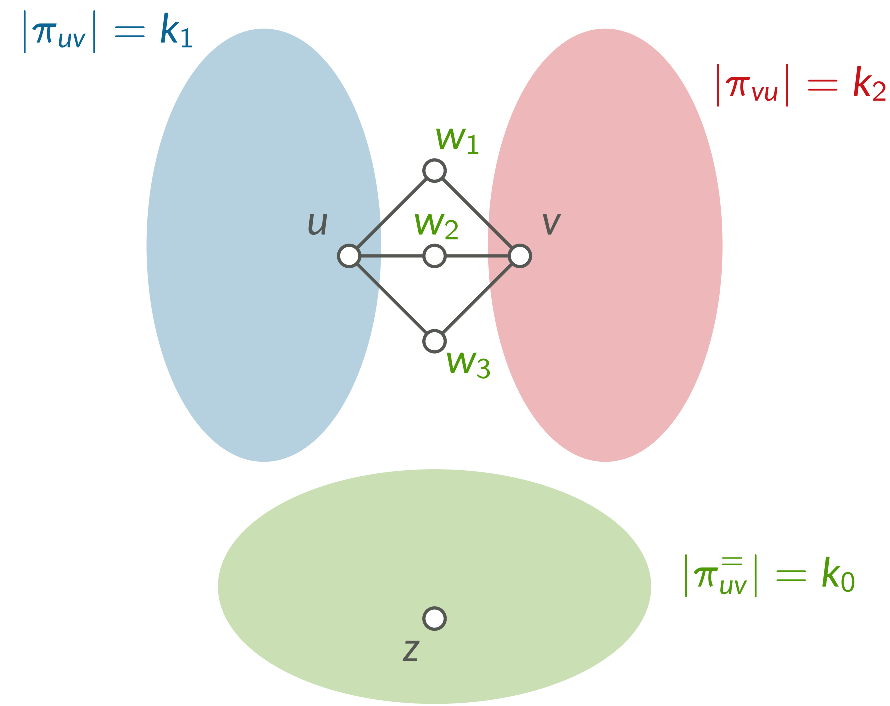

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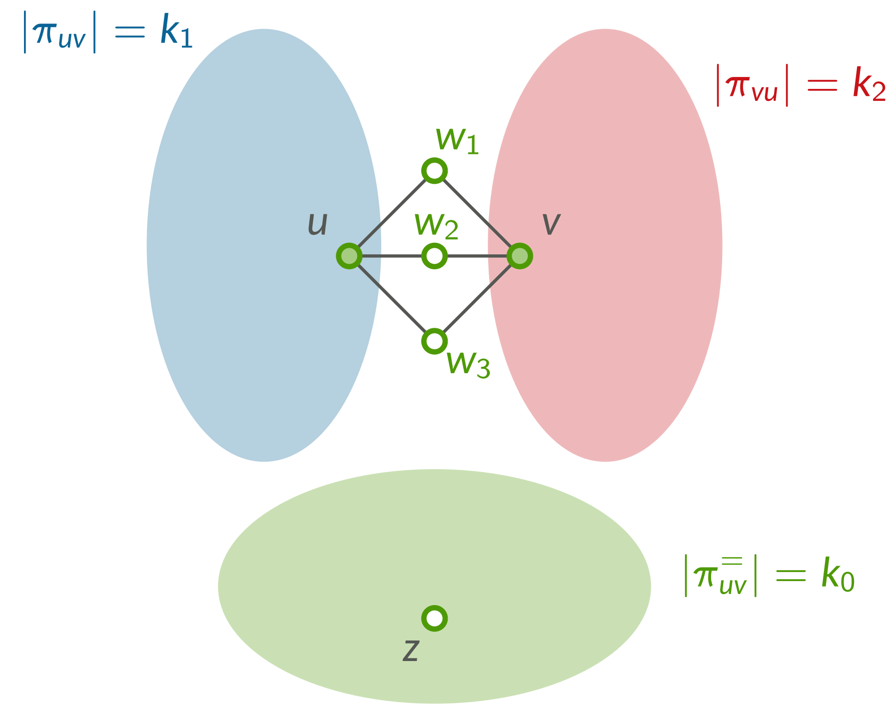

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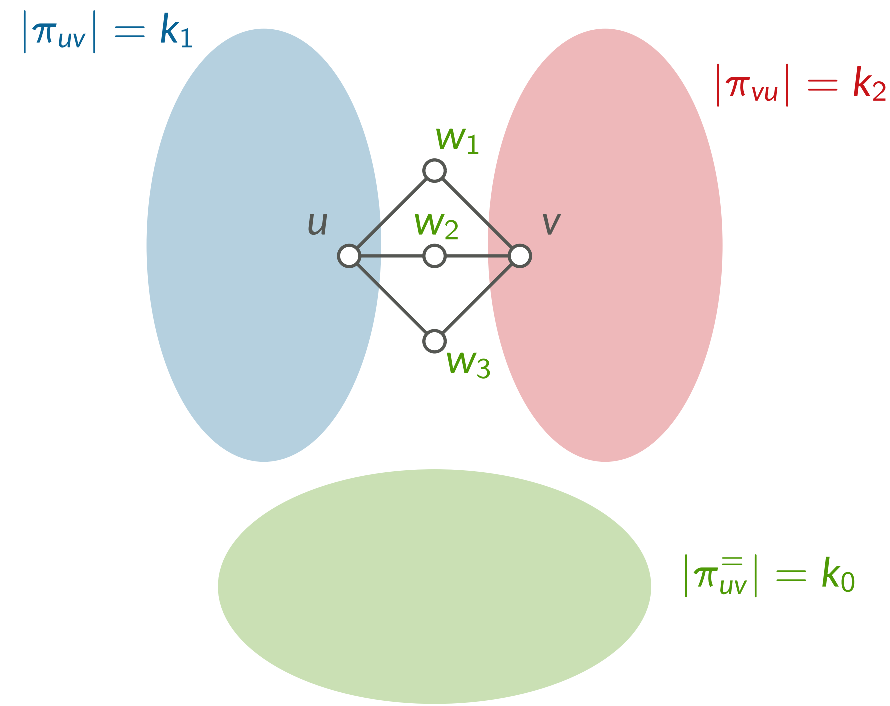

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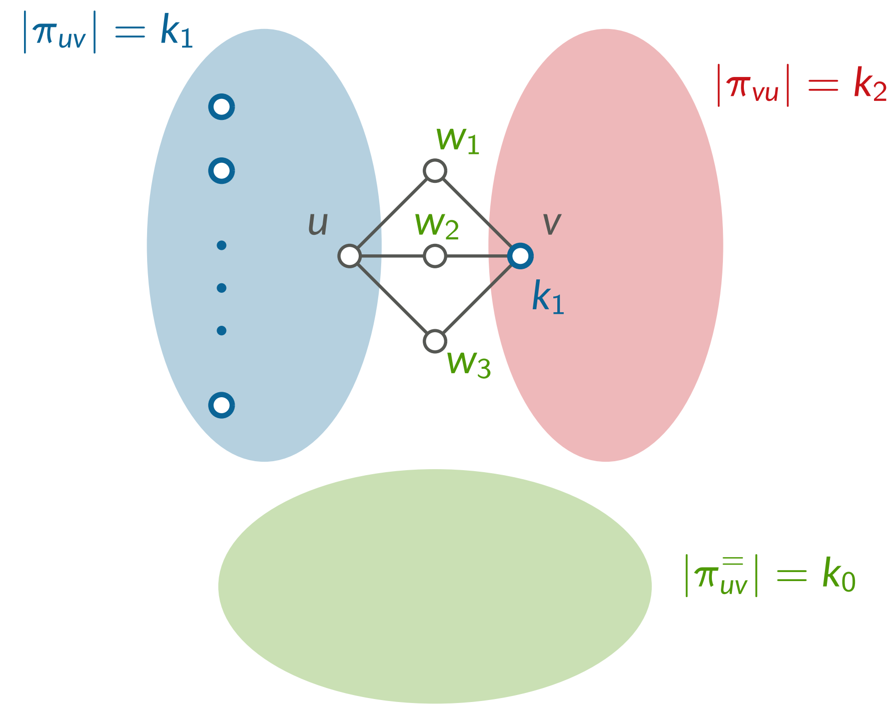
$\pi' = \pi_{uv} v^{k_1}$



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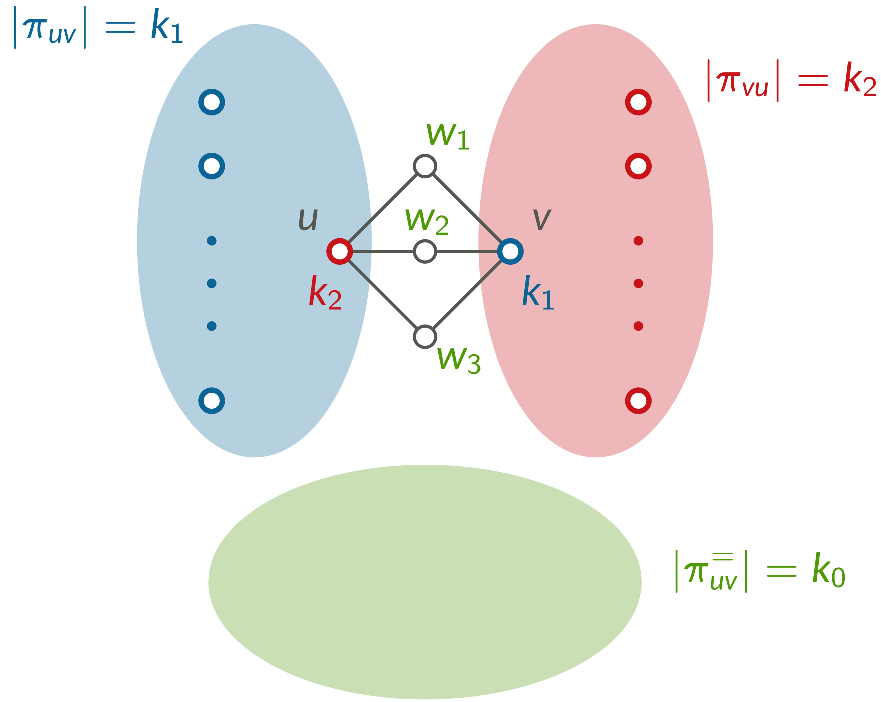

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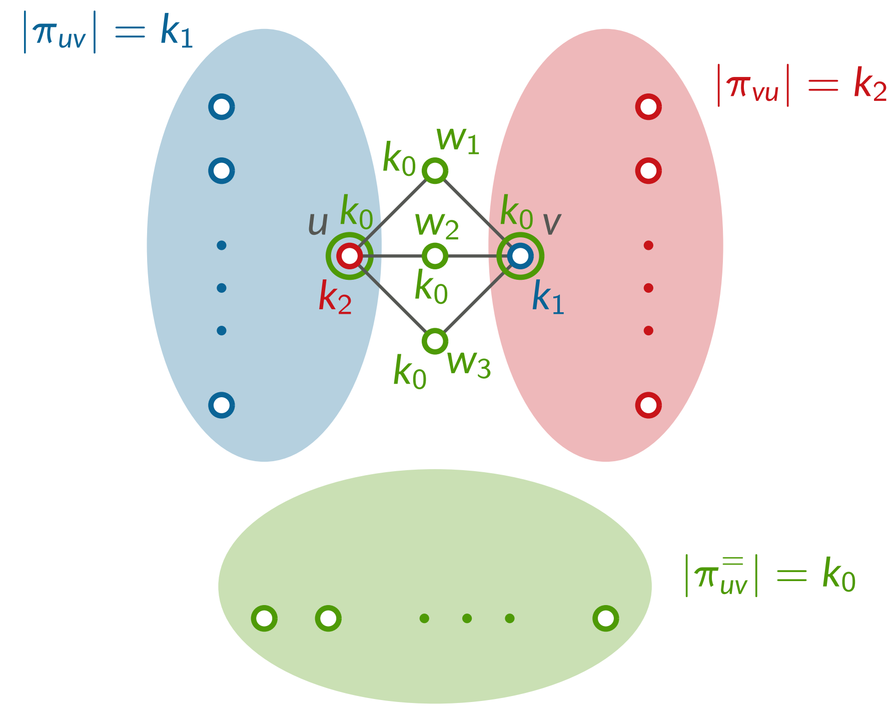

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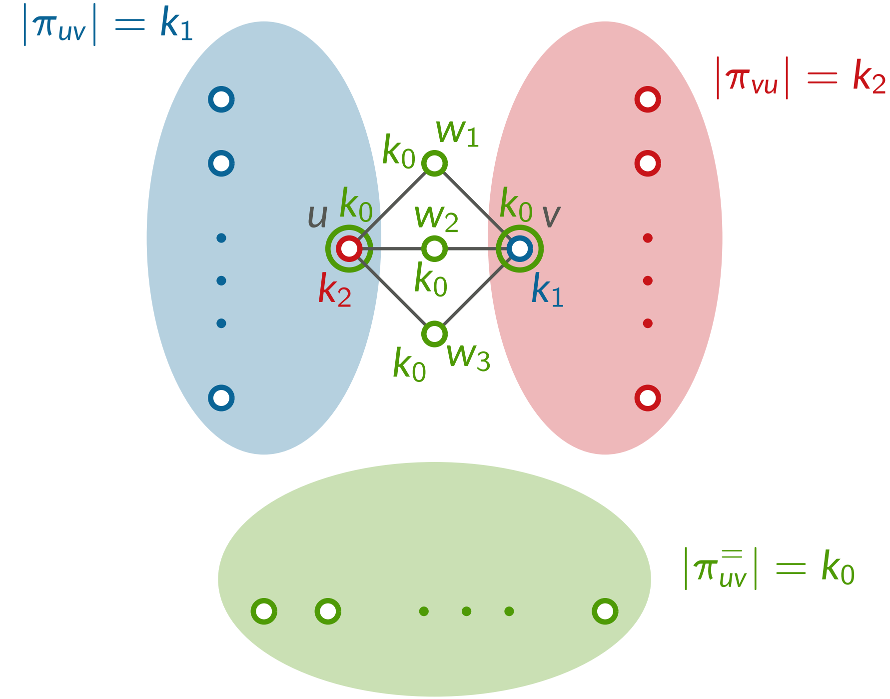
$$= \pi u^{k_2+k_0} v^{k_1+k_0} w_1^{k_0} w_2^{k_0} w_3^{k_0}$$



ABC-property (3)

Theorem B: Modular graphs with G^2-connected medians are ABC-graphs

Conclusion

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Main results:

Conclusion

Main results: Characterization of graphs with G^p-connected medians

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Graphs with connected medians are ABCT-graphs

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Modular graphs with G^2-connected medians are ABC-graphs

Questions : Are chordal graphs ABCT-graphs ?

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Graphs with connected medians are ABCT-graphs

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Questions : Are chordal graphs ABCT-graphs ?

Are ABC-graphs triangle-free graphs ?

Thank You !