An Efficient Recursive Algorithm for Atlas Construction

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Abstract. Atlas construction is a fundamental problem in Medical Image Computing. Every disease assessment task requires a template "reference" to compare to in order to assess the amount of changes in the anatomy or function. The key then is to define this "reference" in a meaningful fashion. The "reference" also commonly referred to as an atlas is normally defined as the most representative of the population of given data. Statistically, this is chosen to be an average or a weighted average of the given data set. Since the control population consists of distinct subjects, the task of estimating an unbiased atlas is posed as a groupwise non-rigid diffeomorphic registration of the given data to this unknown average defined as the minimizer of the sum of squared geodesic distances cost function. This is a hard joint minimization over the space of diffeomorphisms and atlases, and is computationally very expensive. In this paper, we present an efficient alternative which involves arbitrarily choosing one of the given data sets as a reference and estimating the diffeomorphisms from the given pool of data to this reference. Then, efficiently estimating the Fréchet mean (FM) of these diffeomorphisms and applying this FM-diffeomorphism to the chosen reference yields the desired unbiased atlas. We prove that the atlas obtained in this manner is the same as the one obtained using the conventional groupwise registration approach mentioned above. The key advantage of our approach over conventional groupwise registration approach is that we do not require any optimization over the space of atlases, thereby reducing computational cost dramatically. Further, our approach is a recursive approach and thus is amenable to updates when the data pool is augmented with new data without the need to compute the atlas from scratch. We present several real data experiments demonstrating the computational advantages of our proposed approach over state-of-the-art.

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1 Introduction

An atlas is an informative representative of a population of “objects” (images in our context). Constructing an atlas is a key ingredient to many applications including but not limited to image alignment [22], image segmentation [23,27] and statistical analysis [2,10]. Hence, over the past decade, algorithms for atlas construction have attracted substantial attention in Medical Image Analysis research. In particular, an unbiased diffeomorphic atlas construction algorithm was first proposed in [13]. This algorithm spawned a flurry of activity in the area of atlas construction resulting in many variants being proposed in the recent past. Some of the recent methods for constructing multiple atlases on heterogeneous data are [19,26]. A popular approach for atlas construction is to compute the arithmetic mean of group-wise registered images. But, these algorithms [13,16,19] often generate a blurred atlas. Recently Xie et al. [26] proposed a multiple atlas construction framework which yields “sharp” atlases. Other methods for generating “sharp” atlases also exist in literature and we refer the reader to [4,25]

Since most of the popular atlas construction (sequential) algorithms (implemented on standard multi-core desktops) are computationally intensive (typically taking tens of hours to days of CPU time), there is a dire need to develop a time efficient atlas construction algorithm. In this work, we propose a computationally efficient atlas construction algorithm. We further assume that the images in the given data pool can be diffeomorphically registered to each other and to atlas being sought. This is not an uncommon assumption and was made in Younnes [28] for tackling the diffeomorphic registration problem. Now, instead of doing averaging on the image space, we compute the average over the space of diffeomorphisms. By using methods in [15], we can map a subset of “interesting” diffeomorphisms to the Hilbert sphere. We propose an efficient mean estimator on the hypersphere and consistency of this estimator is shown in an accompanying manuscript published in this workshop[20]. Using this fast estimator, we obtain an efficient way to compute the atlas of an image population. In the experiments section, we have shown the significant time gain of our proposed atlas construction algorithm over state-of-the-art. Now, we present a brief survey of existing atlas construction algorithms.

A popular way to construct an atlas is based on an unbiased group-wise registration method. This type of formulation [13] requires the solution to a hard non-convex optimization problem involving two high dimensional unknowns namely, the atlas and the non-rigid transformations between the unknown atlas and input image data. It is solved using an alternating strategy, involving, fixing the average and estimating the transformations required to transform all the input data sets to this average, then fixing the transformations and estimating the average. This is a hard optimization problem and local solutions can be unsatisfactory at times. Another, popular atlas construction algorithm is $iCluster$ [19], which computes the atlas by fitting Gaussian mixture model to the input images. Then, they use an Expectation-Maximization (EM) algorithm to construct the atlas. In this algorithm, they also compute the arithmetic mean of the groupwise registered images. Both of these algorithms [13,19] lead to a blurred atlas which is the result of arithmetic mean of images. Note that, the arithmetic mean of the images need not necessarily lie on the underlying manifold (from which the images were sampled), hence if the images are not tightly clustered on the manifold, arithmetic mean is a *poor* choice. Hence, in Xie et al. [26] they proposed a multiple atlas construction method which produces a "sharp" atlas. In this method, they use a graph representation of the underlying manifold. Then, they used a graph partitioning followed by computing the FM of images belonging to each cluster. Though this algorithm produced a "sharp" atlas, like other atlas construction algorithm it is computationally expensive. For other recent image atlas construction methods we refer the reader to [8,9,11,3,24,5] and for shape atlases to [6] and the references therein.

The rest of the paper is organized as follows. In section 2, we describe our proposed method in detail. Several real data experimental results along with comparisons are presented in section 3. Finally, we conclude in section 4.

2 Methodology

In this section, we present our proposed method of incremental atlas construction from multiple images. *We will use the term incremental, recursive and inductive interchangeably throughout the paper*. We would like to emphasize that our proposed method is applicable to scalar, vector and tensor-valued fields. However, for simplicity, here we present a formulation for the case of scalar-valued fields, i.e., intensity images. Let $\mathcal{I}$ denote the space of images, we define images in $\mathcal{I}$ as $\mathbf{L}^2$ functions on a image domain $\sigma \subset \mathbf{R}^d$. Given a set of images $\mathcal{C} = \{I_1, \cdots, I_n\} \subset \mathcal{I}$, our goal is to construct an atlas from these n images. We will assume that the set of images are rigidly registered by randomly choosing a reference image and rigidly registering the rest to this reference image. Now, given the set of images in the same coordinate system, an atlas can be defined as the minimizer of the sum of squared (geodesic) distances from it to the rest of the given images in the data pool. Let, $d_{\mathcal{I}}(I_i, I_j)$ denote the "distance" between two images I_i and I_j. Note that here we use "distance" loosely without having defined the underlying metric or even the underlying manifold. In the spirit of Xie et al. [26], we will assume that the images are samples from an unknown manifold. Now, let I^* be the atlas of the n input images, then $I^* = \arg\min_\mu \sum_{i=1}^n d^2_{\mathcal{I}}(\mu, I_i)$. Note that this is primarily the Fréchet mean (FM) [7] of the n images. Though this formulation is simple, it is computationally expensive as the space of images $\mathcal{I}$ is huge, and the minimization of the sum of squared distances formulation involves searching over the entire space $\mathcal{I}$. This provides us sufficient motivation to seek an alternative time-efficient algorithm to compute the atlas. Below, we present a simple yet illustrative example, that captures the essence of our proposed approach to atlas construction.

Motivating Example: Given the following real numbers -1, -5, 0, 2, 8 and 8, the Fréchet mean (FM) (arithmetic mean in this simple case) is 2. But instead of averaging on the numbers, we can instead do the following. We randomly choose one of the numbers as the reference number. Then compute the FM (arithmetic mean in this case) of the differences between each of the numbers and the reference. For example, let us choose -5 as the reference number, then the differences between -5 and each of the numbers are 4, 0, 5, 7, 13 and 13 respectively. The average of these six numbers is 7. Then, if we add this mean-difference to our chosen reference, i.e., -5 we get the mean of the numbers, i.e., 2.

At the outset, this approach seems more complicated than simply using the standard arithmetic mean of six numbers. But, if instead of an input of real numbers, our input is a set of images, then developing a time-efficient algorithm for computing the FM of the "differences", will make this second approach more time-efficient. But, two important questions that surface naturally are, first, like in our toy example above, will the second approach still yield the true FM of images? Second, what is the "difference" term in context of images? In the context of image registration and atlas construction, it is natural to interpret this "difference" as the transformation required to register the two images. *In this work, we will assume that the images in the given data pool whose atlas is to be constructed can all be diffeomorphically registered with each other and with the atlas to be constructed.* The answer to the first question is given by the theorem given below. *Using the second approach, the key advantage that we gain in atlas construction over conventional approaches such as [13] and variants thereof is that the hard joint optimization over the space of diffeomorphisms and atlases is now transformed to one over the space of diffeomorphisms. This leads to an enormous savings in computation time as evidenced through the experiments in section 3.* We now state and prove the aforementioned theorem.

Let σ be an open subset of $\mathbf{R}^d$ and G a group of diffeomorphisms on σ. Consider a set of images $\mathcal{J} \in \mathcal{I}$ on which G has an action, i.e., for every $I \in \mathcal{J}$ and every $\phi \in G$, the result of the action of ϕ on $\mathcal{J}$ is denoted by $\phi.\ I \in \mathcal{J}$, where . is the group operator. Let $\mathcal{C} \subset \mathcal{J}$ be a set of n images, i.e., $\mathcal{C} = \{I_1, \cdots, I_n\}$. Let $I_{ref} \in \mathcal{C}$ be an arbitrarily chosen reference image. Let, T_i be the diffeomorphism from I_{ref} to I_i, i.e., $I_{ref}(x) = I_i(T_i(x)), \forall i$. Further, given an arbitrary image $\mu \in \mathcal{J}$, let T_μ be the diffeomorphism from I_{ref} to μ. Let $d_{\mathcal{I}}$ and $d_{\mathcal{T}}$ be the geodesic distance functions on the space of images and transformations respectively. Let us define the relation $\sim$ between two objective functions f_1 and f_2 iff $\exists$ a bijection between $\mathfrak{F}(f_1)$ and $\mathfrak{F}(f_2)$, where $\mathfrak{F}(.)$ is the set of solutions of the corresponding objective function. It is easy to show that $\sim$ is an equivalence relation. Then we have,

Theorem 1.

$$arg\min_{\mu} \sum_{i=1}^{n} d_{\mathcal{I}}^2(\mu, I_i) \sim \arg\min_{T_\mu} \sum_{i=1}^{n} d_{\mathcal{T}}^2(T_i, T_\mu) \tag{1}$$

Proof.

$$\begin{aligned}
\arg\min_{\mu} \sum_{i=1}^{n} d_{\mathcal{I}}^2(\mu, I_i) &= \arg\min_{\mu} \sum_{i=1}^{n} d_{\mathcal{I}}^2(I_i(T_i \circ T_\mu^{-1}(x)), I_i(x)) \\
&\sim \arg\min_{T_\mu} \sum_{i=1}^{n} d_{\mathcal{I}}^2(I_i(T_i \circ T_\mu^{-1}(x)), I_i(x)) \\
&= \arg\min_{T_\mu} \sum_{i=1}^{n} d_{\mathcal{I}}^2(I_i(T_i \circ T_\mu^{-1}(x)), I_i(T_\mu \circ T_\mu^{-1}(x)) \\
&\sim \arg\min_{T_\mu} \sum_{i=1}^{n} d_{\mathcal{T}}^2(T_i, T_\mu)
\end{aligned}$$

The above equalities hold based on the following two claims.

Claim 1: $\arg\min_{\mu} \sum_{i=1}^{n} d_{\mathcal{I}}^2(I_i(T_i \circ T_{\mu}^{-1}(x), I_i(x)) \sim \arg\min_{T_{\mu}} \sum_{i=1}^{n} d_{\mathcal{I}}^2(I_i(T_i \circ T_{\mu}^{-1}(x), I_i(x))$

Proof. Let $S_1 = \arg\min_{\mu} \sum_{i=1}^{n} d_{\mathcal{I}}^2(I_i(T_i \circ T_{\mu}^{-1}(x), I_i(x))$ and $S_2 = \arg\min_{T_{\mu}} \sum_{i=1}^{n} d_{\mathcal{I}}^2(I_i(T_i \circ T_{\mu}^{-1}(x), I_i(x))$. We have to prove that $\exists$ a bijection between S_1 and S_2. Let us first prove the cardinality of S_1 and S_2 are same. Let $I \in S_1$, then, it is easy to see that $T_I \in S_2$, which proves $S_1 \subset S_2$. The other way of double containment is similar to prove. Hence, the cardinalities are same. And given $I \in \mathcal{I}$, $\exists!$ a T_I such that $I(x) = I_{ref}((T_I)^{-1}(x))$. And for a given T_I, the choice of I is also unique. Hence, $\exists$ a bijection between S_1 and S_2. This proves the claim. ■

Claim 2: $\arg\min_{T_{\mu}} \sum_{i=1}^{n} d_{\mathcal{I}}^2(I_i(T_i \circ T_{\mu}^{-1}(x), I_i(T_{\mu} \circ T_{\mu}^{-1}(x)) \sim \arg\min_{T_{\mu}} \sum_{i=1}^{n} d_{\mathcal{T}}^2(T_i, T_{\mu})$

Proof. It's easy to see that $\arg\min_{T_{\mu}} \sum_{i=1}^{n} d_{\mathcal{I}}^2(I_i(T_i(y), I_i(T_{\mu}(y)) \sim \arg\min_{T_{\mu}} \sum_{i=1}^{n} d_{\mathcal{T}}^2(T_i, T_{\mu})$. And by transitivity of the relation, $\sim$, our claim holds. ■

Now, from the proofs of the two claims, the proof of the theorem follows. ■

So by Theorem 1, in order to compute the FM of the given images, we first compute the FM of the diffeomorphisms (between the images and an arbitrarily chosen reference I_{ref}) and apply this mean diffeomorphism on the arbitrarily chosen reference. Hence, if T^* is the FM of the diffeomorphisms and I^* is the atlas, $I^*(x) = I_{ref}((T^*)^{-1}(x))$.

Note that the above hypothesis that, $\exists$ a diffeomorphism between any two images in $\mathcal{J}$ simply means that members of $\mathcal{J}$ are of same topology and diffeomorphically related. In practise, for the atlas construction problem, the given image data pool from which the atlas is being constructed can be assumed to have the same structures of interest, since, it is meaningful to construct atlas from a population of say, "normal" human brains but it is not meaningful to construct an atlas from a population consisting of "normal" human brains and brains with pathology as they maybe of a different topology.

Corollary 1. *Given the hypothesis as above, let a set of n images $\mathcal{C} = \{I_1, \cdots, I_n\}$, then, eqn. 1 in Thm. 1 holds for any transformations $GL(m)$ (with appropriate m), where $GL(m)$ denotes the general linear group consisting of $m \times m$ invertible matrices.*

Proof. The proof follows from Thm. 1. ■

Corollary 2. *Given the hypothesis as above, let a set of n images $\mathcal{C} = \{I_1, \cdots, I_n\}$, then, eqn. 1 in Thm. 1 holds for all local affine transformations.*

Proof. For local affine transformations, the transformation N_i for each image is a product of $GL(m)$ matrices (with appropriate m). Hence, the proof follows from Cor. 1. ■

Now, our next concern is how to *efficiently* compute FM on the space of diffeomorphisms. Let M be the image domain and let $d\mu$ be an associated Riemannian volume form on M. Let $Diff(M)$ and $Diff_{\mu}(M)$ denote the infinite dimensional group of diffeomorphisms on M and its infinite dimensional subgroup of volume preserving diffeomorphisms on M. Khesin et al. in [15] showed that the right invariant $\dot{H}^1$ metric (see [15] for definition of this metric) on $Diff(M)$ descends to a non-degenerate Riemannian metric on the homogeneous space of densities on M, $Dens(M) = Diff(M)/Diff_\mu(M)$. Further, they proved that equipped with the $\dot{H}^1$ metric, the space $Dens(M)$ is isometric to a subset of an infinite dimensional sphere in the Hilbert space. We use this result in our work here and compute the FM of this class of diffeomorphisms (points in $Dens(M)$) identified with those on the Hilbert sphere.

Now, we can use the square root of the density parameterization to map a point in $Dens(M)$ to the infinite dimensional unit Hilbert Sphere. But, note that our goal was to compute FM on the space of diffeomorphisms, i.e., $Diff(M)$. Hence, we need to justify why working on the quotient space $Diff(M)/Diff_\mu(M)$ instead of $Diff(M)$ is an acceptable choice. The atlas construction problem is normally posed as follows: Given a population of images acquired from distinct subjects, the goal is to construct a representative image or an atlas. It is reasonable to assume that in constructing the atlas from distinct subject scans, volume preserving diffeomorphisms are highly unlikely and ought to be treated as nuisance transformations. Hence, they ought to be quotiented out. Moreover, by quotienting out the volume preserving diffeomorphisms (volumorphisms), the computed atlas becomes invariant to any volumorphisms i.e., rigid transformations etc. This is an additional advantage of our proposed atlas construction scheme. Now, given that we have mapped the points from $Dens(M)$ to the unit Hilbert Sphere, we propose an efficient scheme to compute FM on the unit Hilbert sphere.

A common approach to computing the FM of a finite sample set of points on a Riemannian manifold is to find the global optimum (if it exists) of the sum of squared geodesic distances cost function. A popular approach to solve this problem involves the use of the gradient descent method. An alternative way to compute the FM is to develop a recursive/inductive definition that does not involve optimizing the aforementioned cost function. Where applicable, a recursive algorithm can take advantage of the closed form solution to compute the FM of two points as the base of the recursion and recurse through the number of points in the given set. This will yield a much faster way to compute the FM if and when the convergence of the algorithm can be proved.

Note that in Euclidean space, the recursive form of computing the arithmetic mean (which yields the same solution as the minimization of sum of squared distances) involves only two points in each recursion step and can be geometrically interpreted as moving an appropriate distance away from the already computed mean (old mean) towards the new-mean on the straight line joining the old mean and the new data point. This geometric procedure can be readily extended to any Riemannian manifold using geodesics. To this end, we make use of the closed form expression – derived using the sphere metric on the hypersphere – for the geodesic between two points on the hypersphere. More precisely, after computing the estimate of the FM of k points, denoted by M_k, the $k+1^{th}$ estimate lies on the geodesic between M_k and the $k+1^{th}$ point S_{k+1}. This readily yields an algorithm for computing the FM that does not require any function optimization, a considerable advantage often realized as gains in computation time of several orders in magnitude over non-incremental algorithms based on minimization of sum of squared geodesic distances. *Since, the FM on a sphere is unique only when all* *the data lie with in an injectivity radius of* $\pi/2$ *[18,14,1], we will make this assumption through the rest of this paper.*

Let $\{S_1, \cdots, S_n\} \subset S^\infty$, then we define the inductive (recursive) estimator of the FM by the recursion in Eqn. 2. A proof of convergence of this algorithm is presented in [20].

$$M_1 = S_1 \quad (2)$$

$$M_{k+1} = \Gamma\Big(M_k, S_{k+1}, \omega_{k+1}\Big) \quad (3)$$

Here, $\Gamma\Big(X, Y, .\Big)$ denotes the geodesic between X and Y. The geodesic $\Gamma(X, Y, t)$ on S^∞ is defined as follows:

$$\Gamma(X, Y, t) = Exp_X(t\ Exp_X^{-1}(Y)) \quad (4)$$

where, Exp and Exp^{-1} are the Riemannian exponential and inverse exponential mapping as defined below.

- **Exponential Map**: Given a vector $\mathbf{v} \in T_X S^\infty$, the Riemannian Exponential map on S^N is defined as $Exp_X(\mathbf{v}) = \cos(|\mathbf{v}|)X + \sin(|\mathbf{v}|)\mathbf{v}/|\mathbf{v}|$. The Exponential map gives the point which is located on the great circle along the direction defined by the tangent vector $\mathbf{v}$ at a distance $|\mathbf{v}|$ from X.M
- **Inverse Exponential Map**: The tangent vector $\mathbf{v} \in T_X S^\infty$ directed from X to Y is given by, $Log_X(Y) = \frac{\theta}{\sin(\theta)}(Y - X\cos(\theta))$ where, $\theta = \arccos(X^T Y)$.

After computing the FM on hypersphere, we lift it back to the space of diffeomorphisms, $Diff(M)$ using the formulation proposed in [21] to get the mean diffeomorphism. Note that this mean diffeomorphism is unique upto volume preserving transformations. And finally we apply this mean diffeomorphism to the reference image to get the atlas image. As claimed above, this formulation is easily generalized to tensor field data. We call this atlas construction procedure as the *incremental atlas construction algorithm*, $iAcA$. We summarize the steps of our algorithm in the following:

Algorithm 1 Algorithm for *incremental atlas construction*

1: *Input*: a population of n images $\{I_1, \cdots, I_n\}$.
2: *Output*: an atlas image, I^* of the population.
3: Step 1. Arbitrarily choose any one of the given images as the reference, denoted by I_{ref} .
4: Step 2. Compute the diffeomorphisms $\{T_i\}$ from I_{ref} to I_i, where T_i is the diffeomorphism to I_i.
5: Step 3. Map each of these diffeomorphisms, T_i, to the hypersphere (of appropriate dimension) using the scheme proposed in [21,15]. Let the points on the hypersphere be $\{S_i\}$.
6: Step 4. Compute the inductive FM, S^*, of $\{S_i\}$ using Eqn. 2.
7: Step 5. Map S^* onto $Diff(M)$ using the method in [21]. Let the FM diffeomorphism be T^*.
8: Step 6. Apply T^* on I_{ref} to get the atlas image I^*.

We now list a few advantages of $iAcA$.

- The key advantage of $iAcA$ over popular atlas construction methods such as the one in [13] and variants thereof perform a hard multi-variate optimization involving

joint search over very large spaces of diffeomorphisms and atlases respectively. In contrast, $iAcA$ involves a search only over the space of diffeomorphisms. This leads to a much simpler and more time efficient alternative.
- The $iAcA$ is very time efficient as demonstrated via the experimental results in section 3.
- $iAcA$ yields an atlas that is invariant to volume preserving transformations.
- In many medical imaging applications, it is customary to augment the data pool as and when new scans are acquired. In such situations, it is more time efficient to update the already computed atlas rather than to compute the atlas from scratch. Due to it's recursive nature, $iAcA$ achieves this optimally.

3 Experimental Results

In this section, we present experimental results of our atlas construction algorithm, $iAcA$, and compare its performance with two other atlas construction algorithms, one for constructing atlases from fields of ensemble average propagators (EAPs) derived from diffusion MR scans acquired from rat spinal cords [5] and another for 2D shapes from the MPEG-7 database [17]. For the MPEG-7 data, we used the atlas construction algorithm in the well known *ANTS* [3] software. We report the time taken by both of these algorithms. All the computation time required for various algorithms reported in this paper, were measured on an Intel-7 quad-core processor, 16GB RAM desktop. Accuracy is hard to assess on real data sets and will be focus of our future work. *We would like to point out that though ANTS is a highly optimized toolbox written in C++, our $iAcA$ code was written in MATLAB, which is not efficient for non-matrix operations. Code optimization to achieve further time savings with $iAcA$ implementation will be the focus of our future work.*

3.1 Atlas construction from diffusion MR scans of rat spinal cords

In this section, we used HARDI data acquired from several rat spinal cords. The HARDI scans were acquired using a 3T Phillips MR scanner with the following parameters: $b-$values: 0, $1500\,\frac{s}{mm^2}$, 22 gradient directions and voxel size = $2 \times 2 \times 2\,mm^3$. We have constructed the EAP field atlas from EAP fields derived from 7 control rat data sets. The EAP fields from each HARDI data set was first estimated using the approach in [12]. Sample slices from the HARDI scans of the rat spinal cords as well as the atlases constructed by the method in [5] and $iAcA$ respectively are shown in Fig. 1. In this figure, the top row (left to right) depicts the zero gradient image, S_0 from the HARDI scans of three rat spinal cords. Second row, left to right, depicts a slice from the estimated EAP fields superposed on the corresponding slice of the S_0 images. Last row, left to right, depicts a slice from the atlas EAP fields (superimposed on the corresponding S_0 images) estimated by $iAcA$ and the approach in [5].

Class	Time (s)	
	$iAcA$	ANTS
Spinal_Cord	109800.0	302400.0
apple	2092.0	27280.0
heart	2272.9	34800.0
car	2304.6	37134.0

Table 1: Computational time for atlas construction

From a visual inspection view point, the atlas computed from $iAcA$ appears to be much sharper and better than that from the method in [5]. The time required by these two atlas construction methods are presented in Table 1. From this table, it is evident that computationally, $iAcA$ is significantly faster compared to the approach in [5].

3.2 Atlas construction on MPEG-7 data

We randomly chose 3 classes of objects from the MPEG-7 database namely, heart, apple and car shapes. Each of these classes contains 20 two-dimensional images from which we construct an atlas for each shape class. In Fig. 2, we present 10 random images of each of these three classes and the atlases constructed by *iAcA* and *ANTS* respectively. For *ANTS*, we have used *Greedy Symmetric Normalization* for non-rigid registration. Further, the initial atlas is chosen to be the default i.e., the arithmetic mean of the population of the class. In the figure, for all these three subjects, the leftmost image in the bottom row is the atlas constructed by $iAcA$ and this is followed by the atlas constructed using ANTS. The first, second, fourth and fifth rows consist of images of 10 sample data from the respective classes. The computation time required for these two algorithms is reported in Table 1, which clearly depicts the superior time efficiency of $iAcA$ over *ANTS*. Further, from a visual inspection view point, the atlas constructed by $iAcA$ appears to be of higher quality (sharper). Our future work will focus quantitatively validating the accuracy of the constructed atlases.

4 Conclusions

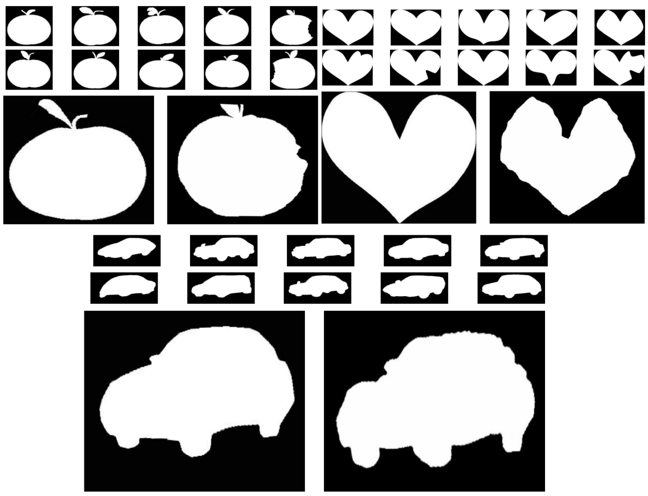

Fig. 2: Apple, heart and car shape atlas construction. Rows 1,2,4 & 5 depict samples from the data pool. Rows 3 & 6 depict the atlas obtained using $iAcA$ and *ANTS* repectively.

In this paper, we presented a novel incremental atlas construction algorithm called *iAcA*. The key advantage of this algorithm over conventional unbiased groupwise registration based atlas construction approach (which requires a joint optimization over the space of diffeomorphisms and atlases) is that, it needs an optimization only over the space of diffeomorphisms to register $(n-1)$ pairs of data sets from the given pool of n data sets. A reference data set is arbitrarily chosen from the given pool and all the other data are diffeomorphically registered to this reference. We then compute the FM of these diffeomorphisms after quotienting out the volumorphisms. The FM computation is achieved

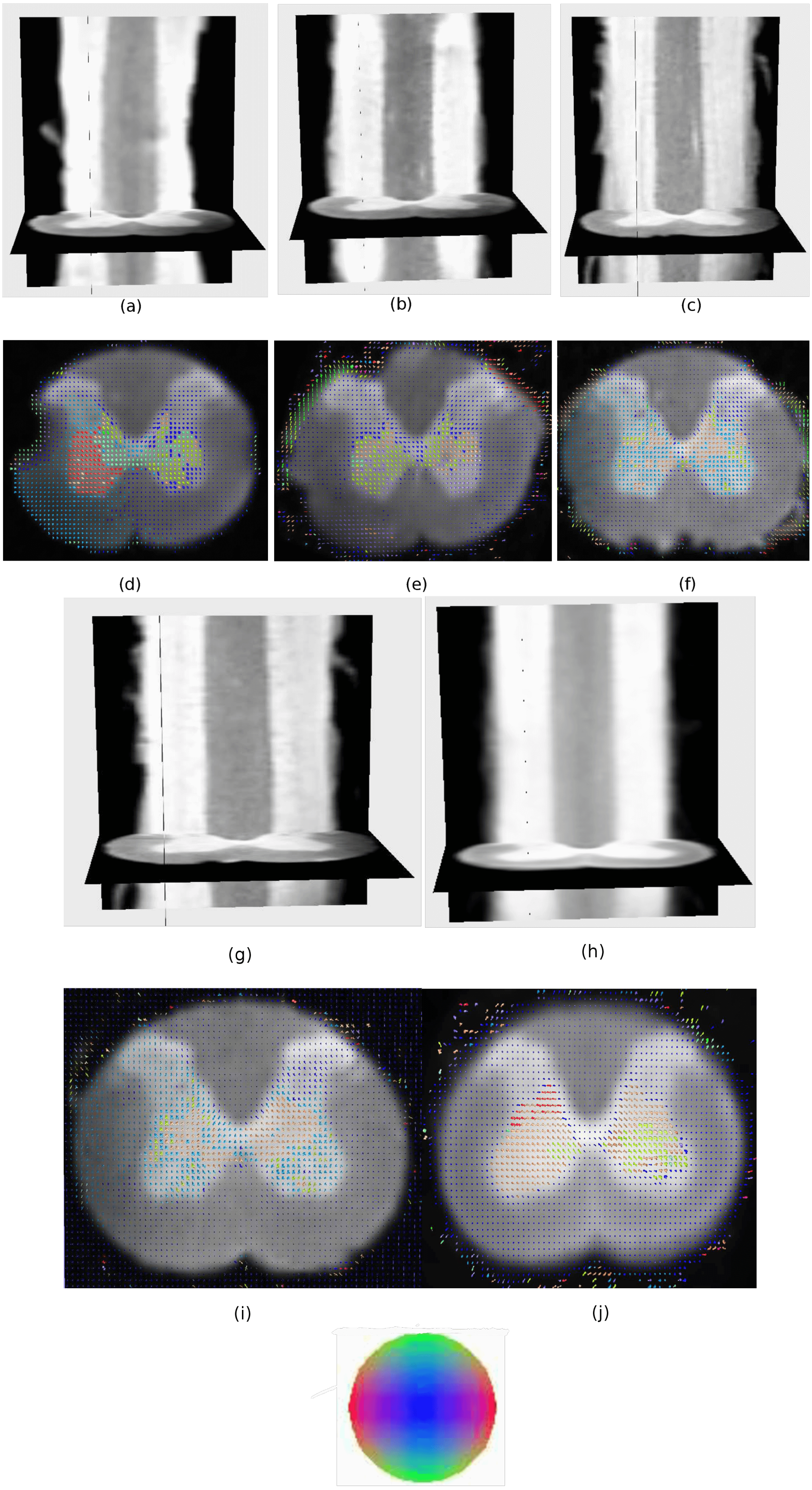

Fig. 1: Constructed Atlases from Spinal Cord 3D data. [a-c]: sample S_0 images from the population of controls, [d-f]: corresponding sample EAP fields superposed on respective S_0 images, [g,i]: S_0 and EAP atlas using $iAcA$, [h,j]: S_0 and EAP atlas using [5]. Last row shows the color ball used to color the EAPs.

recursively and does not require any optimization. This Fm is then applied to the chosen reference to obtain the desired atlas. We demonstrated dramatic savings in computational cost using our approach (over state-of-the-art) for the task of atlas construction from diffusion MR scans of rat spinal cords and MPEG-7 shape data sets. Our future efforts will focus on a thorough quantitative validation of the constructed atlases.

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