Inferring Network Characteristics via Moment-Based Estimators

Sara Alouf   Philippe Nain   and   Don Towsley

Abstract—**In this work we develop simple inference models based on finite capacity single server queues for estimating the buffer size and the intensity of cross traffic at the bottleneck link of a path between two hosts. Several pairs of moment-based estimators are proposed to estimate these two quantities. The best scheme is then identified by using simulation models in ns-2 [10].**

Keywords—**Queueing, measurement, monitoring, simulation.**

I. Introduction

The huge expansion of the Internet coupled to the emergence of new (in particular, multimedia) applications pose challenging problems in terms of performance and control of the network. These include the design of efficient congestion control and recovery mechanisms, and the ability of the network to offer good Quality of Service (QoS) to the users. In the current Internet, there is a single class best effort service which does not promise anything to the users in terms of performance guarantees. The forthcoming deployment in the Internet of differentiated services (known as *diffserv* [1]) will be a first (long awaited) step towards the support of various types of applications and business requirements. It is however doubtful that Diffserv – which will mark each packet to receive a particular forwarding treatment, or per-hop behavior, at each network node – or the RED mechanism for congestion avoidance in gateways [5] alone will solve all QoS issues raised by real-time applications. Diffserv and RED are two instances of a general approach that aims at adding more intelligence in the *network*. A more ambitious component of this approach is captured in the concept of *active networking* [16] that aims at exploiting mobile code and programmable infrastructure to provide rapid and specialized service introduction.

A complementary approach for providing QoS guarantees is to add intelligence to the *applications*. The idea is to provide applications with enough knowledge of the network so that they can use this information to adapt their transmission rates to current network conditions. Since it is impossible to monitor every link on the Internet, static (e.g. bandwidth of a link) and dynamic (e.g. available bandwidth on a path) network internal characteristics have to be estimated from measurements delivered by the network (e.g. packet losses in RTCP feedback). Our work falls into the latter category.

The estimation of network characteristics from measurements has been carried out in a number of cases. For instance, the *packet-pair* technique can be used to estimate both the bottleneck bandwidth [1], [3], [12] and the available bandwidth along a path connecting two hosts on the Internet [3], [6], [12]. The steady-state throughput of a bulk transfer TCP flow can be estimated as a function of loss rate and round-trip time using the so-called TCP-friendly formula [8], [9], [11]. Although the previous estimates have been devised under the assumption that there exists a single bottleneck link on a path connecting two hosts, experiments reported in the previous references indicate that these estimates still perform reasonably well when this assumption is violated.

In this paper we develop a simple inference (queueing) model, based on the single bottleneck link assumption, that will allow us to simultaneously estimate the bandwidth capacity, the traffic intensity (hereafter called the *cross traffic*) and the buffer size at the bottleneck link. As already pointed out, knowledge of the available capacity at the bottleneck link can be used by an application to adapt its transmission rate. We expect this quantity to be better estimated when taking explicitly into account the cross traffic and the buffer size at the bottleneck link (these quantities are not taken care of in the packet-pair algorithm). We also believe that estimates of the intensity of the cross traffic and of the buffer size at the bottleneck node are useful quantities on their own and can be used by an application for instance, to estimate the maximum size of a burst. We expect these estimates to be used in a number of contexts: adapting an application rate to the congestion in the networks, optimizing the amount of FEC in continuous media applications such as IP telephony, [2] and congestion control, [15].

The paper is organized as follows: two inference mod-

Sara Alouf, INRIA, B.P. 93, 06902, Sophia Antipolis Cedex, France. Email: salouf@sophia.inria.fr

Philippe Nain, INRIA, B.P. 93, 06902, Sophia Antipolis Cedex, France. Email: nain@sophia.inria.fr

Don Towsley, Department of Computer Science University of Massachusetts, Amherst, MA 01003, USA. Email: towsley@cs.umass.edu

[1] http://www.ietf.org/html.charters/diffserv-charter.html

els based on the M/M/1/K and on the M/D/1/K queues are introduced in Sections II and III, respectively. In each case, several QoS metrics of interest are identified and expressed in terms of the unknown parameters (buffer size, cross traffic intensity, service capacity). In Section IV, we restrict ourselves to the estimation of the intensity of the cross traffic and of the buffer size (i.e. we assume that the service capacity is available and provided, for instance, by the packet-pair technique) and we propose eleven schemes that can be used to compute these estimates. Section V reports simulation results obtained with the ns-2 simulator from which we were able to select the best scheme out of these eleven schemes. Concluding remarks and directions for future research are given in Section VI.

II. THE M+M/M/1/K QUEUE

A. The model

We model an Internet connection by a single server queue representing its bottleneck node, following [1]. The buffer is finite with room for K customers ($K \geq 1$) including the customer in service. The incoming traffic at the bottleneck is modeled as two independent Poisson sources: the probe traffic generated by a foreground source with rate γ, and the cross traffic generated by a background source with rate λ. This background source can be seen as the superposition of many heterogeneous sources. The packets length being variable in the Internet networks, some packets being small such as the acknowledgments of TCP traffic, while others containing data are larger, and service times being more likely proportional to the packets length, we propose to model the service times as i.i.d. random variables with exponential distribution with mean $1/\mu$, further independent of the arrival processes. This assumption represents the variability of the packet sizes. We are aware of the strength of this assumption but we assumed it for the tractability it gives to the study of the model.

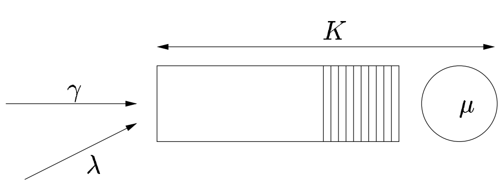

Fig. 1. The inference model

The traffic intensity is defined as

$$\rho = \frac{\lambda + \gamma}{\mu}. \tag{1}$$

We are interested in the behavior of the system from the perspective of the foreground customers. This includes stationary measures such as expected delay, loss probability, server occupation and a number of additional statistics associated with the foreground loss process, namely, the probability of two consecutive losses and the probability of two consecutive successes. It is important to notice that these stationary metrics do not pertain exclusively to the foreground source, but to the background source as well, due to the Poisson assumption and its memoryless property.

Let $\{Q_n\}_{n=1}^{\infty}$ be the process of the number of packets in the buffer at time of the n-th arrival from foreground source, and let $Q = \lim_{n\to\infty} Q_n$[2]. The distribution of Q is $\pi_i = P(Q = i)$. It is known that [7]

$$\pi_i = \frac{(1-\rho)\rho^i}{1-\rho^{K+1}}, \qquad i = 0, 1, \ldots, K. \tag{2}$$

B. The loss probability

We focus here on the loss process. We define $X_n = \mathbf{1}\{Q_n = K\}$ and $X = \lim_{n\to\infty} X_n$. A customer is lost whenever it arrives to a full buffer. In other words, customer n is lost whenever $X_n = 1$ and is not lost otherwise. Let $\{a_n\}_{n=1}^{\infty}$ and $\{d_n\}_{n=1}^{\infty}$ be the arrival times to the system and the departure times from the system, respectively, of the n-th foreground customer, $n = 1, 2, \ldots$. When a packet is lost, it never reaches the destination. We shall assume that $d_n = \infty$ if $X_n = 1$.

The probability that a foreground customer arrives to find the system full, and is lost, is

$$\begin{aligned} P_L := P(X = 1) &= P(Q = K) \\ &= \frac{(1-\rho)\,\rho^K}{1-\rho^{K+1}}. \end{aligned} \tag{3}$$

Observe that the expression for P_L can be used to give the following expression for K in terms of ρ and P_L,

$$K = \frac{1}{\log \rho} \log\left(\frac{P_L}{1-\rho\,(1-P_L)}\right). \tag{4}$$

C. The server occupation

The second metric of interest is the occupation U of the server, defined as the probability of a non-empty queue as seen by a foreground customer. In order to express U, we introduce the following indicator $Y_n = \mathbf{1}\{Q_n > 0\}$ and define $Y = \lim_{n\to\infty} Y_n$. The server occupation is then

$$U := P(Y = 1) = P(Q > 0)$$

[2] A word on the notation. Let $\{Z_n\}_n$ be a sequence of random variables taking values in $[0, \infty)$. Assume that $\lim_{n\to\infty} P[Z_n \leq x]$ exists for all $x \geq 0$. Then $Z = \lim_{n\to\infty} Z_n$ designates any random variable such that $P[Z \leq x] = \lim_{n\to\infty} P[Z_n \leq x]$

$$= \rho \left(\frac{1-\rho^K}{1-\rho^{K+1}} \right) \quad (5)$$

$$= \rho\,(1-P_L). \quad (6)$$

Again, we can derive from the expression for U the following expression for K in terms of ρ and U,

$$K = \frac{1}{\log \rho} \log \left(\frac{1-U/\rho}{1-U} \right). \quad (7)$$

D. The expected response time

When available, the expected response time is also a relevant metric. To express this quantity, we first define T_n as the response time of the n-th foreground packet. It follows that $T_n = d_n - a_n$. Again, let $T = \lim_{n\to\infty} T_n$ then, the expected response time is

$$R := E[T \,|\, X=0] = \frac{\sum_{i=0}^{K-1} (i+1)\,\pi(i)}{\mu(1-\pi_K)} \quad (8)$$

since a customer waits for an average time $(i+1)/\mu$ if there were already i customers in the queue; $1-\pi_K$ is the probability of a success. Using (2) we can write

$$R = \frac{1}{\mu\,(1-\pi_K)} \left(\frac{1-\rho}{1-\rho^{K+1}} \sum_{i=0}^{K-1} (i+1)\,\rho^i \right).$$

Hence,

$$R = \frac{1}{\mu\,(1-\pi_K)} \left[\frac{1-\rho^K}{(1-\rho)\,(1-\rho^{K+1})} - \frac{K\,\rho_K}{1-\rho^{K+1}} \right]$$

$$= \frac{1}{\mu\,(1-\rho)} - \frac{K}{\mu}\,\frac{\pi_K}{(1-\rho)\,(1-\pi_K)}. \quad (9)$$

The last expression for R, which was derived using (2), will prove useful. We can express R only in terms of ρ and K by replacing π_K given by (3) in (9). This gives

$$R = \frac{1}{\mu\,(1-\rho)} - \frac{K}{\mu}\,\frac{\rho^K}{1-\rho^K}. \quad (10)$$

E. The conditional loss probability

The next metric we are going to study is the conditional loss probability or, in other words, the probability that two consecutive losses occur. It is expressed as follows

$$q_L := P(Q_n = K \,|\, Q_{n-1} = K). \quad (11)$$

In order to be able to derive a closed form expression for q_L, we define $N(t)$ to be the queue length of the system at time $t \geq 0$ with the foreground source removed ($\gamma = 0$). Let $P_{i,k}(t) = P(N(t) = k \,|\, N(0) = i)$. Hence (11) rewrites

$$q_L = \gamma \int_0^\infty e^{-\gamma t} P_{K,K}(t)\,dt = \gamma\,P^*_{K,K}(\gamma) \quad (12)$$

where $P^*_{K,K}(\gamma)$ is the Laplace transform of $P_{K,K}(t)$.

Proposition II.1: The following result is proved in the appendix:

$$P^*_{i,K}(\gamma) = \frac{a^{i+1}\,(1-a)^{-1} - b^{i+1}\,(1-b)^{-1}}{\lambda\,(b^{K+1} - a^{K+1})}$$

for $i = 0, 1, \ldots, K$, where

$$a = \frac{\lambda+\mu+\gamma+\sqrt{(\lambda+\mu+\gamma)^2 - 4\lambda\mu}}{2\lambda}$$

$$b = \frac{\lambda+\mu+\gamma-\sqrt{(\lambda+\mu+\gamma)^2 - 4\lambda\mu}}{2\lambda}.$$

□

From proposition II.1 we get that

$$P^*_{K,K}(\gamma) = \frac{(1-a)^{-1} - (b/a)^{K+1}(1-b)^{-1}}{\lambda\,((b/a)^{K+1} - 1)}$$

which in turn implies from (12) that

$$q_L = \left(\frac{\gamma}{\lambda}\right) \left(\frac{(1-a)^{-1} - (b/a)^{K+1}(1-b)^{-1}}{(b/a)^{K+1} - 1} \right). \quad (13)$$

Expression (13) for q_L can be inverted to give

$$K = \frac{\log\left(\frac{\gamma}{1-a} + q_L\lambda\right) - \log\left(\frac{\gamma}{1-b} + q_L\lambda\right)}{\log b - \log a} - 1. \quad (14)$$

F. The conditional non-loss probability

Another metric can also be calculated. It is the conditional probability that a foreground packet arrives to find room in the buffer given that the previous foreground customer was also admitted. We shall refer to this probability as the *conditional non-loss probability* and will denote it by q_N. We have

$$q_N := P(Q_{n+1} \neq K \,|\, Q_n \neq K)$$

$$= \sum_{i=0}^{K-1} \frac{P(Q_{n+1} \neq K, Q_n = i, Q_n \neq K)}{P(Q_n \neq K)}$$

$$= \sum_{i=0}^{K-1} \frac{P(Q_{n+1} \neq K \,|\, Q_n = i)\,\pi(i)}{1-\pi(K)}$$

$$= \sum_{i=0}^{K-1} \frac{(1 - P(Q_{n+1} = K \,|\, Q_n = i))\,\pi(i)}{1-\pi(K)}.$$

Recall the definition of $P_{i,k}(t) = P(N(t) = k \,|\, N(0) = i)$, where $N(t)$ is the queue-length at time t when $\gamma = 0$ (no foreground customers). Since the n-th foreground customer is accepted in the system when $Q_n = i < K$, we have

$$P(Q_{n+1} = K \,|\, Q_n = i) = \int_0^\infty P_{i+1,K}(t)\,\gamma\,e^{-t\gamma}\,dt$$

$$= \gamma\,P^*_{i+1,K}(\gamma)$$

for $i = 0, 1, \ldots, K-1$, with $P^*_{j,k}(s) = \int_0^\infty e^{-st} P_{j,k}(t)dt$. Therefore,

$$q_N = 1 - \gamma \sum_{i=0}^{K-1} \frac{P^*_{i+1,K}(\gamma)\,\pi(i)}{1-\pi(K)}. \qquad (15)$$

Using proposition (II.1) which gives an expression for $P^*_{j,K}(\gamma)$, (15) rewrites

$$q_N = 1 - \left(\frac{\gamma}{\lambda}\right)\left(\frac{1-\rho}{1-\rho^K}\right)\left(\frac{1}{b^{K+1}-a^{K+1}}\right) \times \left[\frac{a^2\,(1-(\rho a)^K)}{(1-a)(1-\rho a)} - \frac{b^2\,(1-(\rho b)^K)}{(1-b)(1-\rho b)}\right]. \qquad (16)$$

Since

$$P(Q_{n+1}=K) = P(Q_{n+1}=K \,|\, Q_n = K)\,P(Q_n=K) + P(Q_{n+1}=K \,|\, Q_n \neq K)\,P(Q_n \neq K)$$

we deduce that P_L, q_L and q_N are linked by the following relationship

$$P_L\,(1-q_L) = (1-P_L)\,(1-q_N). \qquad (17)$$

III. The M+M/D/1/K queue

A. The model

We still consider the model introduced in Section II-A but we now relax the assumption that the service times are exponentially distributed. Instead we will assume that the service times are constant and all equal to $1/\mu$. In Section II-A, we motivated our choice for exponentially distributed service times by the fact that various packet lengths are possible. Taking into consideration that packet lengths may not be so variable to justify the choice of an exponential distribution, we study here the other extreme case: the service times are taken to be constant (i.e. all packets have the same length) with value $\sigma = 1/\mu$. Recall the definition of the traffic intensity given in (1).

Again, let $\{Q_n\}_{n=1}^\infty$ be the process of the number of packets in the queue at time of the n-th arrival from foreground source and let $Q = \lim_{n\to\infty} Q_n$. Some preliminary results must be introduced before computing the stationary distribution of Q.

Let $\mathcal{F}(\theta) = \mathbf{E}\,[\exp(-\theta\sigma)]$ ($\Re(\theta) \geq 0$) be the Laplace-Stieltjes transform (LST) of the service time distribution. Since we consider a constant service time, this transform rewrites as $\mathcal{F}(\theta) = \exp(-\theta\sigma)$. For $\rho \geq 0$ and $|z| \leq 1$, define

$$\mathcal{G}_\rho(z) = \mathcal{F}\left(\frac{\rho\,(1-z)}{\bar{\sigma}}\right) - z = e^{-\rho\,(1-z)} - z. \qquad (18)$$

For $\rho \geq 0$, we denote by $z_0(\rho)$ the zero of $\mathcal{G}_\rho(z)$ having the smallest modulus. Also, we denote by $[z^n]f$ the coefficient of z^n in the Taylor series expansion of f.

To express the stationary distribution of Q, we base our calculus on Cohen's analysis of the M/G/1 queue with finite waiting room [4, Chapter III.6]. Introduce the parameter B defined as

$$B = 1 + \frac{\rho}{2\pi i}\oint_{D_r} \frac{1}{\mathcal{G}_\rho(z)} \cdot \frac{dz}{z^{K-1}} \qquad (19)$$

with D_r any circle in the complex plane with center 0 and radius strictly less than $|z_0(\rho)|$. According to the results obtained by Cohen [4, page 575], we have

$$P(Q=0) = \frac{1}{B}$$

$$P(Q=j) = \frac{1}{2\pi i B}\oint_{D_r}\left(\frac{1-z}{\mathcal{G}_\rho(z)} - 1\right)\frac{dz}{z^j},$$

$$P(Q=K) = \frac{1}{2\pi i B}\oint_{D_r}\left(\frac{\rho-1}{\mathcal{G}_\rho(z)} + \frac{1}{1-z}\right)\frac{dz}{z^{K-1}}.$$

where $j = 1, \ldots, K-1$. The integrals in the r.h.s. of (19) and the preceding two equations can be evaluated using the theorem of residues. The distribution of Q is then given by

$$\pi_0 = \frac{1}{1+\rho\,\alpha_K(\rho)} \qquad (20)$$

$$\pi_1 = \frac{\alpha_2(\rho)-1}{1+\rho\,\alpha_K(\rho)} \qquad (21)$$

$$\pi_j = \frac{\alpha_{j+1}(\rho)-\alpha_j(\rho)}{1+\rho\,\alpha_K(\rho)}, \quad j = 2, \ldots, K-1 \qquad (22)$$

$$\pi_K = \frac{1+(\rho-1)\,\alpha_K(\rho)}{1+\rho\,\alpha_K(\rho)}. \qquad (23)$$

where

$$\alpha_j(\rho) = [z^{j-2}]\frac{1}{\mathcal{G}_\rho(z)} = \frac{1}{(j-2)!}\left(\frac{d^{(j-2)}}{dz^{j-2}}\frac{1}{\mathcal{G}_\rho(z)}\right)_{z=0}.$$

When the service times are constant, an analytical expression for $\alpha_j(\rho)$ can be derived. To this end, start from

$$\frac{1}{\mathcal{G}_\rho(z)} = \frac{1}{e^{-\rho\,(1-z)} - z} = e^{\rho\,(1-z)}\sum_{i\geq 0}(z e^{\rho\,(1-z)})^i,$$

where the last equality is true for z such that $|ze^{\rho\,(1-z)}| < 1$, which yields

$$\frac{1}{\mathcal{G}_\rho(z)} = \sum_{i\geq 0} z^i e^{\rho\,(i+1)}\,e^{-\rho\,(i+1)z}$$

$$= \sum_{i\geq 0}\sum_{k\geq 0} z^i e^{\rho\,(i+1)}\,\frac{(-\rho\,(i+1)z)^k}{k!}$$

$$= \sum_{n\geq 0}\left(\sum_{i+k=n}\frac{e^{\rho\,(i+1)}\,(-1)^k\,\rho^k\,(i+1)^k}{k!}\right)z^n.$$

Hence,

$$\alpha_j(\rho) = \sum_{i+k=j-2} \frac{e^{\rho\,(i+1)}(-1)^k \rho^k (i+1)^k}{k!}, \; j \geq 2. \quad (24)$$

B. The loss probability

Recall the definition of X_n introduced in Section II-B, $X_n = \mathbf{1}\{Q_n = K\}$ and $X = \lim_{n\to\infty} X_n$. A customer n is lost whenever $X_n = 1$ and is not lost otherwise.

The probability that a foreground customer is lost is the probability that it finds the system full upon arrival, namely,

$$\begin{aligned} P_L &:= P(X=1) = P(Q=K) \\ &= \frac{1+(\rho-1)\,\alpha_K(\rho)}{1+\rho\,\alpha_K(\rho)}. \end{aligned} \quad (25)$$

C. The server occupation

The occupation U of the server was defined as the probability of a non-empty queue. The server occupation is

$$\begin{aligned} U &:= P(Q>0) \\ &= \frac{\rho\,\alpha_K(\rho)}{1+\rho\,\alpha_K(\rho)}. \end{aligned} \quad (26)$$

D. The expected response time

Applying Little's formula and the PASTA property to the queue, we find that the expected response time R is given by

$$R = \frac{\sum_{j=1}^{K} j\,\pi(j)}{(\lambda+\gamma)(1-\pi_K)}. \quad (27)$$

Using (21), (22) and (23), (27) rewrites

$$R = \frac{K + (K\,\rho - 1)\,\alpha_K(\rho) - 1 - \sum_{j=2}^{K-1} \alpha_j(\rho)}{\rho\,\mu\,\alpha_K(\rho)} \quad (28)$$

with $\alpha_j(\rho)$ defined in (24).

IV. Using the inference models

A. An inference question

Until now we have introduced two models for a connection. In the first model we were able to identify five metrics describing the quality of service provided to the foreground source, given in (3), (5), (10), (13) and (16). In the second model we were only able to find three QoS metrics, given in (25), (26) and (28). Since $\rho = (\lambda+\gamma)/\mu$, all these equations are expressed in terms of the parameters λ, μ and K (γ is assumed to be known throughout the paper).

The problem is therefore the following: How can we infer estimates $\hat{\lambda}_n$, $\hat{\mu}_n$ and $\hat{K}_n$ of parameters λ, μ and K, respectively, from the observations collected by the first n probe packets?

If the parameters λ, μ and K are unknown, then equations (3), (5), (10), (13) and (16) leave us with nine schemes to compute these three constants in the M/M/1/K case (there are ten possible schemes but relation (17) reduces that number to nine); no scheme is available in the M/D/1/K as only equations (25) and (26) can be used.

If we now assume that only λ and K are unknown (μ being estimated for instance, using a *packet-pair* based technique [1], [3], [12]), then ten schemes in the M/M/1/K case and one scheme in the M/D/1/K case can be used to compute these two constants. These eleven schemes are listed in Table I, where the notation X_Y denotes the scheme obtained by using the metrics X and Y in the M/M/1/K case and where $P_L\_U\_D$ denotes the scheme obtained by using the metrics P_L and U in the M/D/1/K case.

TABLE I
Schemes for estimating λ and K.

Scheme		Equations to use
P_L_U (referred to as scheme **I**)		(3), (5)
P_L_R	**(II)**	(3), (10)
$P_L\_q_L$	**(III)**	(3), (13)
$P_L\_q_N$	**(IV)**	(3), (16)
U_R	**(V)**	(5), (10)
U_q_L	**(VI)**	(5), (13)
U_q_N	**(VII)**	(5), (16)
R_q_L	**(VIII)**	(10), (13)
R_q_N	**(IX)**	(10), (16)
$q_L\_q_N$	**(X)**	(13), (16)
$P_L\_U\_D$	**(XI)**	(25), (26)

B. Solving for the equations

From now on we will restrict ourselves to the estimation of parameters λ and K, thereby assuming that μ and γ are known. In this case, each scheme identified in Section IV-A involves two QoS metrics. For instance, the loss probability P_L and the server occupation U in scheme **I**, the expected response time R and the conditional loss probability q_L in scheme **VIII**, etc. Assume that both QoS metrics involved in a scheme can be evaluated from measurements collected at the sender/receiver (cf. Section IV-C). Then, estimators for λ and K will be obtained by "solving" the scheme w.r.t. the variables λ and K.

If we want to apply a certain scheme to estimate the buffer size and the intensity of the cross traffic, we must establish existence and uniqueness of its solution (λ, K). To be more precise, consider for instance scheme **I**. Then, for any (observed/measured) values of P_L and U with $0 \leq P_L < 1$ and $0 \leq U < 1$, we want to find a single pair (λ, K) satisfying the set of equations defined by (3) and (5). This existence and uniqueness property holds for scheme **I** as shown below, as well as for schemes **II** and **V** (proofs not provided for sake of conciseness). As to the other schemes we have not been able to show that property, but in all experiments that have been carried out and that are reported in Section V, each scheme always gave us a unique solution. We now briefly discuss the solution to scheme **I**, show the existence and uniqueness of the solution for scheme **II**, and indicate how a solution can be found for scheme **XI**.

B.1 Solving for scheme I

Equations involved here are (3) and (5), namely,

$$\begin{aligned} P_L &= \frac{(1-\rho)\,\rho^K}{1-\rho^{K+1}} \\ U &= \rho\left(\frac{1-\rho^K}{1-\rho^{K+1}}\right) = \rho\,(1-P_L). \end{aligned}$$

Hence,

$$\rho = \frac{U}{1-P_L} \quad \text{i.e.} \quad \lambda = \frac{\mu\, U}{1-P_L} - \gamma \tag{29}$$

and

$$K = \frac{\log(P_L/(1-U))}{\log(U/(1-P_L))} \tag{30}$$

by combining (29) and (7). Therefore, the set of equations (3) and (5) in the variables λ and K has a unique solution given in (29) and (30), respectively.

It is interesting to investigate the sensitivity of λ and K with respect to the variables P_L and U. To do so, let us compute the differentials of λ and K considered as functions of P_L and U. From (29) and (30) we find

$$\begin{aligned} d\lambda &= \frac{\mu}{1-P_L}\left(dU + \frac{U}{1-P_L}\,dP_L\right) \\ dK &= \frac{1}{\log^2(U/(1-P_L))}\,(B\,dU + C\,dP_L) \end{aligned}$$

where

$$\begin{aligned} B &= \frac{\log(U/(1-P_L))}{1-U} - \frac{\log(P_L/(1-U))}{U} \\ C &= \frac{\log(U/(1-P_L))}{P_L} - \frac{\log(P_L/(1-U))}{1-P_L}. \end{aligned}$$

We conclude from the above that λ will be more sensitive to the variations of P_L (resp. U) than to the variations of U (resp. P_L) whenever $\rho = U/(1-P_L) > 1$ (resp. $\rho < 1$). As for K, it follows primarily U's variations since it is easily seen that $B > C$, except when $\rho = 1$ ($U = 1 - P_L$) in which case λ and K are equally influenced by P_L and U's variations.

B.2 Solving for scheme II

Assume that one knows R and P_L and that ρ and K are unknown. The expression for P_L can be used to give the following expression for K in terms of ρ and P_L. We find from (3)

$$K = \frac{\log(P_L/(1-\rho\,(1-P_L)))}{\log(\rho)}. \tag{31}$$

Plugging now this value of K in (9) gives

$$R = \frac{1}{\mu\,(1-\rho)} - \frac{P_L\,\log(P_L/(1-\rho\,(1-P_L)))}{\mu\,(1-\rho)\,(1-P_L)\,\log(\rho)}.$$

Observe that necessarily $\rho < 1/(1-P_L)$. For $0 < x < 1/(1-P_L)$, introduce the mapping

$$f(x) = \frac{1}{\mu\,(1-x)} - \frac{P_L\,\log(P_L/(1-x\,(1-P_L)))}{\mu\,(1-x)\,(1-P_L)\,\log(x)} - R\,.$$

If one can show that the equation $f(x) = 0$ has a unique solution in $(0, 1/(1-P_L))$, then this solution will give us ρ, hence λ, and subsequently K by using (31). Proposition IV.1 shows that this is indeed the case.

Proposition IV.1: For any constants $\mu > 0$, $P_L \in [0,1)$ and $R \geq 1/\mu$, the equation $f(x) = 0$ has a unique solution in $[0, 1/(1-P_L)]$. □

Sketch of the proof. Define the mappings

$$\begin{aligned} &h(x) := ((1-a)P_L^2 + (1+2a)P_L - (2+a))x^2 \\ &+ (1-P_L)^2 x^3 + (1 + 2a\,(1-P_L)^2)\,x - a(1-P_L) \end{aligned}$$

and

$$k(x) := (1-P_L)^2\,x^2 + (P_L^2 + P_L - 2)x + 1.$$

where $a := \frac{1-R\mu}{R\mu\,(1-P_L)} \leq 0$.

When $a < 0$ it can be shown that the polynomial $h(x)$ has a unique zero in $[0, 1/(1-P_L)]$, denoted as $\rho(a)$. When $a = 0$, then $h(x)$ has two zeros in $[0, 1/(1-P_L)]$: $x = 0$ and $x = \rho(0)$ where $\rho(0) \neq 0$ is the unique zero of the polynomial $k(x)$ in $[0, 1/(1-P_L)]$. The following properties then hold for all $a \leq 0$:

- if $\rho(a) < 1$ then $f(x)$ has a unique zero in the interval $[0, 1/(1-P_L)]$; this zero is located in the interval $[0, \rho(a)]$;

- if $\rho(a) > 1$ then $f(x)$ has a unique zero in the interval $[0, 1/(1-P_L)]$; this zero is located in the interval $[\rho(a), 1/(1-P_L)]$;
- if $\rho(a) = 1$ then $x = 1$ is the unique zero of $f(x)$ in the interval $[0, 1/(1-P_L)]$

which concludes the proof.

B.3 Solving for scheme **XI**

This scheme still involves P_L and U, but this time these quantities have to be computed for the M+M/D/1/K queue. More precisely, cf. (25) and (26),

$$P_L = \frac{1+(\rho-1)\,\alpha_K(\rho)}{1+\rho\,\alpha_K(\rho)} \tag{32}$$

$$U = \frac{\rho\,\alpha_K(\rho)}{1+\rho\,\alpha_K(\rho)} \tag{33}$$

with $\alpha_K(\rho)$ given in (24).

Recall that we want to solve the system of equations (32)-(33) w.r.t. the variables ρ and K. We readily observe from (32)-(33) that

$$\rho = \frac{U}{1-P_L} \tag{34}$$

and

$$\alpha_K(\rho) = \frac{1-P_L}{1-U}. \tag{35}$$

If all coefficients $\{\alpha_j(\rho), j \geq 2\}$ in the Taylor series expansion of $1/\mathcal{G}_\rho(z)$ are different, then (34)-(35) will return a unique solution (ρ, K).

For given ρ, we computed the coefficients $\alpha_j(\rho)$ for a certain range of j and compared the results with the r.h.s. of (35); then K was chosen as the integer j for which $\alpha_j(\rho)$ was the closest to (the measured value of) $(1-P_L)/(1-U)$.

C. *Calculating the moment-based estimators*

We have at our disposal the first n samples of $\{X_i\}_i$, $\{Y_i\}_i$, $\{a_i\}_i$, $\{d_i\}_i$, and we know γ and μ. Let $\hat{U}(n)$, $\hat{P}_L(n)$, $\hat{R}(n)$, $\hat{q}_L(n)$ and $\hat{q}_N(n)$ denote the estimators of U, P_L, R, q_L and q_N, respectively. They are defined as $(n = 1, 2, \ldots)$

$$\hat{P}_L(n) := \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}(X_i = 1) \tag{36}$$

$$\hat{U}(n) := \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}(Y_i = 1) \tag{37}$$

$$\hat{R}(n) := \frac{\sum_{i=1}^{n} \mathbf{1}(X_i = 0)\,(d_i - a_i)}{\sum_{i=1}^{n} \mathbf{1}(X_i = 0)}, \tag{38}$$

$$\hat{q}_L(n) := \frac{\sum_{i=1}^{n-1} \mathbf{1}(X_i = 1, X_{i+1} = 1)}{\sum_{i=1}^{n-1} \mathbf{1}(X_i = 1)}, \tag{39}$$

$$\hat{q}_N(n) := \frac{\sum_{i=1}^{n-1} \mathbf{1}(X_i = 0, X_{i+1} = 0)}{\sum_{i=1}^{n-1} \mathbf{1}(X_i = 0)}. \tag{40}$$

$\hat{P}_L(n)$ and $\hat{U}(n)$ are estimated over all packets, $\hat{R}(n)$ and $\hat{q}_N(n)$ are estimated over successful packets and $\hat{q}_L(n)$ is estimated over lost packets only. It is expected to have a slowly convergence when estimating q_L, hence, intuitively, we can say that all schemes involving this metric will not perform well.

D. *Desirable properties of an estimator*

If a comparison is to be made among several estimators, it is useful to have in mind the main properties of a good estimator. Namely, an estimator is preferably unbiased and consistent. Unbiasedness has been proved for $\hat{P}_L(n)$, $\hat{U}(n)$ and $\hat{R}(n)$, while $\hat{q}_L(n)$ and $\hat{q}_N(n)$ turn out to be biased (see Section IV-D.1). Consistency for each metric is much more complicated to show. The major difficulty in establishing such a property is due to the fact that the rv's $\{X_i\}_i$ are correlated, since the queue is finite and since the samples are taken from consecutive foreground packets rather than random packets.

D.1 Study of the mean values

Using the identity $E[\mathbf{1}(A)] = P(A)$ that holds for any event A, we find

$$E[\hat{P}_L(n)] = P_L$$
$$E[\hat{U}(n)] = U$$
$$E[\hat{R}(n)] = R$$
$$E[\hat{q}_L(n)] = q_L - \frac{\text{cov}[\hat{q}_L(n), \hat{P}_L(n-1)]}{P_L}$$
$$E[\hat{q}_N(n)] = q_N + \frac{\text{cov}[\hat{q}_N(n), \hat{P}_L(n-1)]}{P_L}.$$

The last two equalities follow from (39) and (40) when expressed as follows

$$\hat{q}_L(n) = \frac{\sum_{i=1}^{n-1} \mathbf{1}(X_i = 1, X_{i+1} = 1)}{(n-1)\,\hat{P}_L(n-1)}$$
$$\hat{q}_N(n) = \frac{\sum_{i=1}^{n-1} \mathbf{1}(X_i = 0, X_{i+1} = 0)}{(n-1)\,(1-\hat{P}_L(n-1))}.$$

Clearly, $\hat{P}_L(n)$, $\hat{U}(n)$ and $\hat{R}(n)$ are unbiased estimators whereas $\hat{q}_L(n)$ and $\hat{q}_N(n)$ are biased but the bias depends on the size of the samples. Moreover, if $\hat{P}_L(n)$ is consistent then the bias approaches 0 as $n \to \infty$ for both estimators.

The overall performance of the estimators is presented in Table II. For each experiment (50 different experiments have been done; see details in Section V), we have computed the relative error between each estimator in (36)-(40) and its corresponding theoretical value. This computation has been performed after the generation of 50000 probe packets. For a given estimator, we therefore have a collection of 50 relative errors. These 50 numbers have been ordered and their average value has been computed. Table II now reads as follows. Consider the metric P_L (1st line): in 66% of the experiments the relative error was within 6.1% of the theoretical value (in the table this quantity is referred to as the percentage of hits), the average value being approximately equal to 3. In all cases, the relative error is larger than 0.08% of the theoretical value. The 4th column in Table II gives, for each QoS metric, the minimum/average/maximum values of the empirical variance obtained after 50000 probe packets. More precisely, only the experiments for which the relative error was smaller than the maximum value indicated in column 3 were used in this computation, the others being discarded. For instance, in the 5th line of Table II, we can read that the empirical variance for q_N lies in the interval $(3 \times 10^{-5}, 6.7)$ with an average value of 1.40. We see from this table that the top three estimators (in terms of unbiasedness and consistency) are, in decreasing order, R, U and q_N. Further comments on Table II can be found in Section V-C.

TABLE II

Overall performance of the estimators for 50000 probe packets.

	Hits	Relative error min/avg/max (%)	Empirical variance min/avg/max
P_L	66%	0.08/3/6.1	8×10^{-4}/0.65/3.5
U	98%	0.0003/0.62/5.4	5×10^{-6}/0.47/4.7
R	94%	0.0005/0.88/3.6	2×10^{-8}/0.007/0.2
q_L	48%	0.05/2.92/8.9	6×10^{-4}/3.50/33.6
q_N	100%	0.004/1.92/6.3	3×10^{-5}/1.40/6.7

V. Simulation results and analysis

A. Trace generation

The data sets $\{a_i\}_i$, $\{d_i\}_i$, $\{X_i\}_i$ and $\{Y_i\}_i$ were extracted from traces generated by simulation models in ns-2. Overall 50 simulations have been performed. We have concentrated our work on the simple case of a single queue (i.e. we didn't try to use our estimates in the case where there are several bottleneck links), to test the behavior of both inference models under various background traffic patterns. Several types of background traffic have been considered:

- *(T1)* a Poisson flow of packets with exponentially distributed packet size
- *(T2)* a superposition of 100 Poisson-like flows. The packet length is constant for each flow and its value is taken from an exponential distribution
- *(T3)* an aggregation of 250 FTP over TCP flows
- *(T4)* an aggregation of 1000 FTP over TCP flows
- *(T5)* an aggregation of 100 On/Off flows, where the On and Off times were taken from a Pareto distribution
- *(T6)* an aggregation of 250 On/Off flows, where the On and Off times were taken from a Pareto distribution.

In all experiments foreground packets arrive according to a Poisson process and have exponentially distributed packet size except in the case when the background traffic is of type (T2); in the latter case, the foreground source is also of type (T2).

Note that the Poisson assumption for the background traffic is everywhere violated except in case (T1). Indeed, as we know that in general traffic is not Poisson in today's networks [13], it is important to test the robustness of our estimators when the background traffic is not Poisson (this assumption has been made only for mathematical tractability) and exhibits correlations across several time scales (like in case (T5)-(T6); see [17]).

The rate of the foreground traffic (the probe packets) was equal to 250 pkts/s in all experiments (except in case (T1) where values of 125, 250 and 500 pkts/s were chosen). On the network side, the server rate was either equal to 1500 pkts/s or to 6500 pkts/s and the buffer size was either equal to 10, 30, 65, 100, 150 or to 1000 packets (recall that in ns-2 the size of the buffer is defined in number of packets regardless of their size).

Below, we give the ranges of values obtained for the five metrics over all 50 experiments:

- P_L ranged from 1.7×10^{-4} to 0.637
- U ranged from 0.892 to 1
- R ranged from 0.0107 to 0.67 seconds
- q_L ranged from 0.105 to 0.64
- q_N ranged from 0.367 to 0.9998.

As for the rate of exogenous traffic intensity λ, measured as the number of background packets arriving to the bottleneck link over the run time, its value ranged from 1593.2 to 17437 packets per millisecond, giving the range 0.965 - 2.758 for the traffic intensity ρ.

B. Estimating foreground traffic intensity and buffer size

Having at our disposal $\{a_i\}_i$, $\{d_i\}_i$, $\{X_i\}_i$ and $\{Y_i\}_i$ for the n first packets, the moment-based estimators are computed according to formulas (36), (37), (38), (39) and (40). At this point, $\hat{P}_L(n)$, $\hat{U}(n)$, $\hat{R}(n)$, $\hat{q}_L(n)$ and $\hat{q}_N(n)$ are plugged into equations (3), (5), (10), (13), (16), (25) and

(26). The eleven pairs of equations referred to as schemes **I** through **XI** are then solved numerically using a C program including the NAG[3] C library.

Results are reported in Tables II-VI.

C. Analysis of the results

Coming back to Table II, we observe that metrics R, U and q_N exhibit the highest percentage of hits thereby suggesting that schemes involving these metrics may perform better than the others. This is not what we have found though, the best scheme being in general P_L_R. We explain this result by the fact that estimators P_L and R are probably the ones containing the least redundant information (as opposed to scheme U_R, for instance). From now on, only results pertaining to scheme P_L_R are presented.

TABLE III

Relative errors of the estimates for 50000 probe packets returned by the scheme P_L_R, when the cross traffic is a single Poisson source.

	λ	6600	6600	6600	17068
	γ	124	248	496	125
K	μ	6968	6968	6967	6962
10	$\hat{\lambda}$	0.6	0.004	0.1	0.1
	$\hat{K}$	0.7	0.9	1.4	0.4
30	$\hat{\lambda}$	0.1	0.1	0.3	0.9
	$\hat{K}$	0.7	1.6	0.6	0.05
65	$\hat{\lambda}$	0.04	0.2	0.7	1.0
	$\hat{K}$	1.1	1.0	0.3	0.1
100	$\hat{\lambda}$	0.04	0.1	0.1	0.5
	$\hat{K}$	2.8	2.7	0.2	0.01
150	$\hat{\lambda}$	0.1	0.1	0.02	0.2
	$\hat{K}$	8.3	4.1	0.4	0.03

Tables III-V report the relative errors between the estimate of parameter λ (resp. K), denoted as $\hat{\lambda}$ (resp. $\hat{K}$), returned by scheme P_L_R and the measured value λ (resp. the true value K), for various cross traffic patterns (Poisson traffic as defined in (T1) in Table III, a superposition of On/Off sources with Pareto On and Off time distribution or a superposition of FTP/TCP flows as defined in (T3)-(T6) in Table IV, Poisson-like flows as defined in (T2) in Table IV).

Results contained in Table III are fairly good for moderate values of K (all relative errors are within 1.6% of the correct values when $K \leq 65$); for larger values of K, it appears that the convergence to the true values is slower.

[3]NAG is a copyright of The Numerical Algorithms Group Ltd

TABLE IV

Relative errors of the estimates returned by the scheme P_L_R for 120000 probe packets

Cross traffic		On/Off		FTP/TCP	
Nb of sources		100	250	250	1000
	λ	6812	17437	1641	1870
	γ	247	246	248	248
K	μ	6663	6532	1502	1491
10	$\hat{\lambda}$	3.8	1.0	7.0	2.8
	$\hat{K}$	7.5	1.1	5.9	0.5
30	$\hat{\lambda}$	5.2	0.6	7.6	3.2
	$\hat{K}$	17.7	0.5	2.2	0.04
65	$\hat{\lambda}$	6.0	0.6	8.3	3.0
	$\hat{K}$	29.5	0.4	1.0	0.4
100	$\hat{\lambda}$	5.4	0.4	7.6	3.2
	$\hat{K}$	31.4	0.4	1.4	0.6
150	$\hat{\lambda}$	4.7	0.05	6.7	3.0
	$\hat{K}$	33.2	0.1	1.3	0.6

TABLE V

Relative errors of the estimates returned by the scheme P_L_R for 120000 probe packets: case of Poisson-like flows, $\lambda = 6677.5$, $\gamma = 227.3$ and $\mu = 6374$.

K	10	30	65	100	150
$\hat{\lambda}$	3.1	1.1	0.04	0.1	0.1
$\hat{K}$	9.2	8.5	6.9	5.4	4.0

Notice that in any case the estimates should converge to the true values as the number of probe packets increases since the model considered in Table III is the M/M/1/K queue.

Of more interest are the results in Table IV since they have been obtained when the assumption that the cross traffic is Poisson is violated. We see that the quality of the estimates increases as the number of source increases (results obtained after 120000 probe packets). For 250 On/Off sources all estimates for λ (resp. K) are within 1% (resp. 1.1%) of the correct value; for 1000 FTP/TCP flows all estimates for λ (resp, K) are within 3.2% (resp. 0.6%), still a good result.

As for the set of simulations where all sources (i.e. foreground *and* background sources) were Poisson-like (type (T2)) relative errors are reported in Table V. In this case, the service time was constant for each source, which differs from the theoretical model, thereby explaining the error in the estimation of K, the estimation of λ being satisfactory. Finally, Table VI shows the performance of scheme P_L_R over all 50 simulations (each simulation lasts exactly 500 seconds). We see from this table that the estimate for λ (resp. K) is always within 9% of the exact value in 98% (resp. 84%) of the experiments. Only for large values of K ($K \geq 1000$) the scheme works poorly and may return no value for $\hat{K}$.

TABLE VI

Percentage of hits for scheme P_L_R over all simulations.

Results of estimation for scheme P_L_R	
$\hat{\lambda}$, error within 1 %	52 %
$\hat{K}$, error within 1 %	40 %
$\hat{\lambda}$, error within 5 %	76 %
$\hat{K}$, error within 5 %	70 %
$\hat{\lambda}$, error within 9 %	98 %
$\hat{K}$, error within 9 %	84 %
wrong estimation for λ	2 %
wrong estimation for K	10 %
no estimation for (K)	6 %

From this experimental study we conclude that the inference model M+M/M/1/K returns reasonably good results even when the Poisson assumption on the background traffic is violated. This result should nevertheless be validated on a real network, which is the object of ongoing research.

D. *From simulation to reality*

Till now, we have always assumed to have access to the first n samples of $\{a_i\}_i$, $\{d_i\}_i$, $\{X_i\}_i$ and $\{Y_i\}_i$. In this work, we have carried out simulations to generate traffic traces, and the samples were extracted from the traces. However, ultimately, we must extract the samples from the real network (e.g. the Internet). How can we do this?

Usually, real-time applications use the Real-Time Transport Protocol (RTP) [14] together with UDP and IP. RTP provides end-to-end network transport functions suitable for this kind of applications, over multicast or unicast network services. RTP consists of two parts, a data part and a control part referred to as RTCP, the Real-Time Control Protocol. The feedback information is carried in RTCP packets referred to as Receiver Reports (RR).The rate at which they are multicast is controlled so that the load created by the control information is a small fraction of that created by data traffic. The RR sent by a destination includes several information: the highest sequence number received, the number of packets lost, the estimated packet interarrival jitter, and timestamps. At the sender, the $\{a_i\}_i$ are available, the $\{d_i\}_i$ are retrieved from the timestamps present in the RR and the $\{X_i\}_i$ are retrieved from the highest sequence number received at the destination, also given in the RR. As for the $\{Y_i\}_i$, they are hard to obtain, hence we can try to estimate U differently, by considering that a packet finds the bottleneck server empty if its response time is minimal.

E. *Example of a possible application*

A real-time application uses a FEC-based error control algorithm for Internet telephony. The optimal amount of redundant information is somehow dependent on the *available bandwidth* $\mu - \lambda$ and the *buffer capacity* at the bottleneck of the connection. The application should then estimate the parameters μ, λ and K. It can use the Packet Bunch Mode (PBM) algorithm proposed by Paxson in [12] to estimate the bottleneck bandwidth along the unicast path. After that it should transmit probes using RTP together with UDP at a Poisson rate γ. Having used the Receiver Reports (RR) to retrieve $\hat{P}_L$ and $\hat{R}$, the application should calculate $\hat{\lambda}$ and $\hat{K}$ as in scheme P_L_R with the pair of equations (3) and (10) (cf Section IV-B.2).

VI. CONCLUSION

In this work, we have proposed two simple models for a connection, based on a single server queue with finite waiting room, to infer the *buffer size* and the *intensity of cross traffic* at the bottleneck link of a path between two hosts. We have quantified several parameters of both models and obtained eleven pairs of *moment-based* estimators. Using traces generated by the network simulator ns-2, estimated values for both parameters have been calculated according to the characteristics of the *a priori* models. Pairs of estimators have been discarded while others have proved to give good results. However, the pair of estimators we have “elected” as the best one need to be tested on an experimental network, or even better, on the Internet, in order to evaluate its performance under realistic network traffic conditions.

Acknowledgements: We thank Tian Bu (Univ. of Massachusetts) for helping us simulating a Poisson source with ns-2 and for useful discussions at an early stage of this work.

APPENDIX

A Proof of Proposition (II.1)

Recall the definition $P_{i,k}(t) = P(N(t) = k \mid N(0) = i)$ where $N(t)$ is the queue length of the system with the foreground source removed ($\gamma = 0$) at time $t \geq 0$. We have the following differential equations:

$$\frac{d}{dt}P_{i,0}(t) = -\lambda P_{i,0}(t) + \mu P_{i,1}(t) \quad (41)$$

$$\frac{d}{dt}P_{i,K}(t) = -\mu P_{i,K}(t) + \lambda P_{i,K-1}(t) \quad (42)$$

$$\frac{d}{dt}P_{i,k}(t) = -(\lambda+\mu)P_{i,k}(t) + \lambda P_{i,k-1}(t)$$
$$+ \mu P_{i,k+1}(t), \quad \text{for } k = 1, 2, \ldots, K-1 \qquad (43)$$

for $i = 0, \ldots, K$. Define $P_i(z,t) = \sum_{k=0}^{K} P_{i,k}(t) z^k$. Then,

$$z\frac{d}{dt}P_i(z,t) = z\frac{d}{dt}\sum_{k=0}^{K} P_{i,k}(t)z^k = z\sum_{k=0}^{K}\frac{d}{dt}P_{i,k}(t)z^k.$$

Using equations (41) – (43) and after some algebra we get

$$\frac{z}{1-z}\frac{d}{dt}P_i(z,t) = (\mu - \lambda z)P_i(z,t) - \mu P_{i,0}(t)$$
$$+ \lambda z^{K+1} P_{i,K}(t). \qquad (44)$$

Now we consider the Laplace transform of $P_i(z,t)$, $P_i^*(z,s) = \int_0^\infty e^{-st}P_i(z,t)\,dt$. Replacement of this in (44) along with the use of the following relation

$$\int_0^\infty e^{-st}\frac{d}{dt}P_i(z,t)\,dt = sP_i^*(z,s) - P_i(z,0)$$

and some algebraic manipulations yields

$$P_i^*(z,s) = \frac{z^{i+1} - \mu(1-z)P_{i,0}^*(s)}{sz - (1-z)(\mu - \lambda z)}$$
$$+ \frac{\lambda(1-z)z^{K+1}P_{i,K}^*(s)}{sz - (1-z)(\mu - \lambda z)} \qquad (45)$$

where $P_{i,k}^*(s) = \int_0^\infty e^{-st}P_{i,k}(t)dt$, $k = 0, 1, \ldots, K$. The denominator of the right-hand side of (45) contains two zeros,

$$z_1(s) = \frac{\lambda + \mu + s - \sqrt{(\lambda+\mu+s)^2 - 4\lambda\mu}}{2\lambda}$$
$$z_2(s) = \frac{\lambda + \mu + s + \sqrt{(\lambda+\mu+s)^2 - 4\lambda\mu}}{2\lambda}$$

for $\Re(s) \geq 0$. As $P_i^*(z,s)$ is analytic, the zeros of the denominator must also be zeros of the numerator. More precisely, the numerator must satisfy

$$z_k^{i+1}(s) - [1 - z_k(s)][\mu P_{i,0}^*(s) - \lambda z_k(s)^{K+1}P_{i,K}^*(s)] = 0.$$

These two equations for $k = 1, 2$ can be solved to yield

$$P_{i,0}^*(s) = \frac{\frac{z_2(s)^{i+1}z_1(s)^{K+1}}{1-z_2(s)} - \frac{z_1(s)^{i+1}z_2(s)^{K+1}}{1-z_1(s)}}{\mu\left(z_1(s)^{K+1} - z_2(s)^{K+1}\right)}$$
$$P_{i,K}^*(s) = \frac{\frac{z_2(s)^{i+1}}{1-z_2(s)} - \frac{z_1(s)^{i+1}}{1-z_1(s)}}{\lambda\left(z_1(s)^{K+1} - z_2(s)^{K+1}\right)}.$$

Let a and b be defined as $a = z_2(\gamma)$ and $b = z_1(\gamma)$, we get

$$P_{i,K}^*(\gamma) = \frac{a^{i+1}(1-a)^{-1} - b^{i+1}(1-b)^{-1}}{\lambda\left(b^{K+1} - a^{K+1}\right)}$$

and the proof is concluded.

References

[1] J.-C. Bolot. End-to-end packet delay and loss behavior in the internet. In *Proc. of ACM SIGCOMM*, pages 289–298, September 1993.

[2] J.-C. Bolot, S. Fosse-Parisis, and D. Towsley. Adaptive fec-based error control for Internet telephony. In *Proc. INFOCOM'99, New York, NY*, volume 3, pages 1453–1460, March 1999.

[3] R. Carter and M. Crovella. Measuring bottleneck link speed in packet-switched networks. *Performance Evaluation*, 27&28:297–318, October 1996.

[4] J. W. Cohen. *The Single Server Queue*. North-Holland publishing company, 1982.

[5] S. Floyd and V. Jacobson. Random early detection gateways for congestion avoidance. *IEEE/ACM Trans. on Networking*, 1(4):397–413, August 1993.

[6] V. Jacobson. Congestion avoidance and control. In *Proc. of ACM SIGCOMM*, pages 314–329, August 1988.

[7] L. Kleinrock. *Queueing Systems*, volume 1. Wiley, New York, 1975.

[8] J. Mahdavi and S. Floyd. Tcp-friendly unicast rate based flow control, January 1996.

[9] M. Mathis, J. Semske, J. Mahdavi, and T. Ott. The macroscopic behavior of the tcp congestion avoidance algorithm. *Computer Comm. Review*, 27(3), July 1997.

[10] S. McCanne and S. Floyd. *The LBNL/UCB network simulatorSoftware on line at http://www-nrg.ee.lbl.gov/ns/*. Lawrence Berkeley Lab., Univ. of California, Berkeley. Software on line at http://www-nrg.ee.lbl.gov/ns/.

[11] J. Padhye, V. Firoiu, D. Towsley, and J. Kurose. Modeling tcp throughput: A simple model and its empirical validation. In *Proc. of ACM SIGCOMM*, pages 303–314, August 1988.

[12] V. Paxson. *Measurements and analysis of end-to-end Internet dynamics*. PhD thesis, University of California at Berkeley, April 1997.

[13] V. Paxson and S Floyd. Wide-area traffic: the failure of poisson modeling. In *Proc. of ACM SIGCOMM*, pages 257–268, 1994.

[14] H. Schulzrinne, S. Casner, R. Frederick, and V. Jacobson. RTP: a transport protocol for real-time applications. RFC 1889, Network Working Group, January 1996.

[15] D. Sisalem and H. Schulzrinne. The loss-delay based adjustment algorithm: a tcp-friendly adaptation scheme. In *Proc. NOSSDAV'98, Cambridge, UK*, July 1998.

[16] D. J. Wetherall. *Service introduction in an active network*. PhD thesis, Dept. Electrical Engineering and Computer Science, MIT, February 1999. available via http://www.cs.washington.edu/homes/djw/papers/ants-thesis.ps.gz.

[17] W. Willinger, M.S. Taqqu, and R. Sherman. Proof of a fundamental result in self-similarity in traffic modeling. *Computer Comm. Review*, 27(2):5–23, April 1997.