GEOMETRICA

Computational Geometry and Topology

Jean-Daniel Boissonnat

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## Two sites, one team

P. Alliez  
J-D. Boissonnat (head)  
D. Cohen-Steiner  
O. Devillers  
M. Teillaud  
M. Yvinec (vice-head)

<table>
<thead>
<tr>
<th>Year</th>
<th>7 Ph.D. students</th>
<th>2 Postdocs</th>
<th>11 Ph.D. Theses defended in 2006-2010</th>
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<tbody>
<tr>
<td>2003</td>
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### Saclay (2006)
F. Chazal (vice-head)  
S. Oudot  
M. Glisse

<table>
<thead>
<tr>
<th>Year</th>
<th>3 Ph.D. Students</th>
<th>1 Postdoc</th>
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<td>2006</td>
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Visit of Athens University

**GEOMETRICA**
Research directions

- **Mesh generation and geometry processing**
  - Meshing curved domains
  - Surface reconstruction
  - Mesh optimization

- **Topological and geometric inference**
  - Geometric sampling theory
  - Computational topology
  - Point cloud analysis

- **Geometric data structures and robust computation**
  - Triangulations
  - Higher dimensions

- **Software development: CGAL**
Two selected results

Mesh generation

- Anisotropic mesh generation with guarantees
  C. Wormser [Ph.D. 2007, Google Zurich]

Geometric inference

- Optimal transport and geometric structure extraction from point sets
  Q. Mérigot [Ph.D. 2009, CNRS]
CGALmesh

Curved objects

(RINSERM)

Multi-domains

(IRCAD)

Sharp features

<table>
<thead>
<tr>
<th>Size (mm)</th>
<th>#Tetrahedra</th>
<th>Time (s)</th>
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<tbody>
<tr>
<td>16</td>
<td>20K</td>
<td>0.88</td>
</tr>
<tr>
<td>8</td>
<td>160K</td>
<td>6.97</td>
</tr>
<tr>
<td>4</td>
<td>1.2M</td>
<td>54</td>
</tr>
<tr>
<td>2</td>
<td>10M</td>
<td>431</td>
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with S. Tayeb [engineer ADT]
Anisotropic mesh generation

Definition-Motivation

Anisotropic meshes
Mesh elements are elongated according to prescribed directions

Requirements on shapes are described through a metric field

Why anisotropic meshes?
Anisotropic meshes enhance the trade off accuracy / mesh complexity

- interpolation/approximation
- solving anisotropic PDE
Anisotropic mesh generation

Main approaches

Many heuristics
ellipsoidal bubble packing, biting ellipses, anisotropic centroidal Voronoi, pliant method, anisotropic mesh adaptation ...

Few methods with guarantees

- Anisotropic Voronoi diagrams (2d)
  [Labelle & Shewchuk 2003]

- Anisotropic Delaunay meshes
  [SoCG 2007]

each simplex is Delaunay for the metrics attached to all its vertices
Anisotropic Delaunay meshes

- **Stars**
  Compute the star of each vertex \( v \) in the Delaunay triangulation associated to the metric of \( v \)

- **Inconsistencies**
  a simplex may appear in the stars of some (but not all) of its vertices

- **Star conciliation**
  if the metric field is smooth, inconsistencies can be removed by carefully refining the mesh
Anisotropic Delaunay Meshes of Surfaces
Anisotropic mesh generation

Discussion

- An algorithm with theoretical guarantees (in any dim)
- Implementation in progress ⇒ CGALmesh
- Star stitching is a useful paradigm
  triangulation of submanifolds of dim $k \ll d$ [SoCG 2010]
  Delaunay triangulation of Riemannian manifolds
Two selected results

Mesh generation

- Anisotropic mesh generation with guarantees

  C. Wormser [Ph.D. 2007, Google Zurich]

Geometric inference

- Optimal transport and geometric structure extraction from point sets
  - distance functions for noisy data
  - Voronoi covariance measures

  Q. Mérigot [Ph.D. 2009, CNRS]
Distance function based inference

If $P$ is a dense sample of a shape $S$, appropriate offsets of $P$ have the same topology as $S$.

This depends only on two properties of distance functions:

- Robustness with respect to the Hausdorff distance, i.e.
  \[ \|d_K - d_{K'}\|_\infty \leq d_H(K, K') \]
  for two compact sets $K, K'$

- Squared distance functions are 1-semiconcave, i.e.
  \[ \|x\|^2 - d_K^2(x) \]
  is a convex function
Distance function to a probability measure

- The previous approach obviously fails if there are outliers.
- Notion of distance function to a mass distribution, robust with respect to the transport distance.
- The square of this function is 1-semiconcave, making it suitable for shape inference from data corrupted by outliers:
  - Sublevel-sets are topologically correct [FoCM11]
  - Zero-set of a signed version leads to robust reconstruction [SGP10]
Voronoi covariance measures

- Convolving covariance matrices of the Voronoi cells yields a tensor-valued measure on the data points, robust under Hausdorff perturbations of the data [FoCM10]

- Features locations and directions can be found using eigenanalysis of the local covariance measures [SPM09]
Feature preserving surface reconstruction

- Infer feature graph from data
- Compute implicit surface fitting the data
- Mesh surface using constraint-preserving Delaunay refinement [SGP10]
Geometric inference in the presence of noise

Discussion

- Measure-theoretic framework
- Rather simple to implement even in high dimensions (nearest neighbour search and Monte Carlo methods)
- Does not take into account the statistical nature of noise
Perspectives

3 main scientific objectives

- 3D Geometry processing
  - ERC Grant IRON
- Geometric inference
  - ICT-FET CG-Learning
- Geometric algorithms and data structures
  - ANR Présage

Software development

- CGAL
  - CGAL-HiD
Objective 1: 3D Geometry Processing

- Technological paradox: Data increasingly heterogeneous and defect-laden
- Enduring challenge: processing step before meshing in computational engineering pipeline

New spin-off project-team (P. Alliez)
Objective 2: Geometric Inference

Understanding geometric structures in high dimensional data

- Geometry + statistics
- Complexity, algorithms and implementation
- Data analysis: clustering, segmentation, classification,...
Objective 3: Geometric Algorithms and Data Structures

- Probabilistic methods (smoothed analysis)
- Non-Euclidean geometries (hyperbolic geometry)
- Higher dimensions (CGAL HiD)
Collaborations with Athens

European projects

- (2001-2004) ECG: Effective Computational Geometry for Curves and Surfaces
- (2010-2013) CGL : Computational Geometric Learning

INRIA Collaborative Research Actions (ARC)

- (2005-2006) Arcadia: Arrangements of Quadrics, algorithms, implementation and applications
Collaborations with Athens

Master students

- (2003) Athanasios Kakargias (Monique Teillaud, in Galaad)
- (2005) Constantinos Tsirogiannis (Monique Teillaud, in Galaad)
- (2009) Vissarion Fisikopoulos (Monique Teillaud)

Post-docs

- (2006) Elias P. Tsigeridas, 3 month visit (Monique Teillaud)

PhD

- Monique Teillaud is a member of the “Three persons committee” for Vissarion Fisikopoulos.
Thank you for your attention!