GEOMETRICA

Computational Geometry and Topology

Jean-Daniel Boissonnat

February 15, 2011

Two sites, one team

Sophia-Antipolis (2003)

P. Alliez [01]
J-D. Boissonnat (head)
D. Cohen-Steiner [04]
O. Devillers
M. Teillaud
M. Yvinec (vice-head)

7 Ph.D. students
2 Postdocs

Saclay (2006)

F. Chazal (vice-head) [07]
S. Oudot [07]
M. Glisse [09]

3 Ph.D. Students
1 Postdoc

11 Ph.D. Theses defended in 2006-2010

Research directions

- Mesh generation and geometry processing
  - Meshing curved domains
  - Surface reconstruction
  - Mesh optimization
- Topological and geometric inference
  - Geometric sampling theory
  - Computational topology
  - Point cloud analysis
- Geometric data structures and robust computation
  - Triangulations
  - Higher dimensions
- Software development: CGAL

Two selected results

Mesh generation

- Anisotropic mesh generation with guarantees

C. Wormser [Ph.D. 2007, Google Zurich]

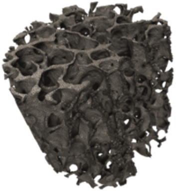
Geometric inference

- Optimal transport and geometric structure extraction from point sets

Q. Mérigot [Ph.D. 2009, CNRS]

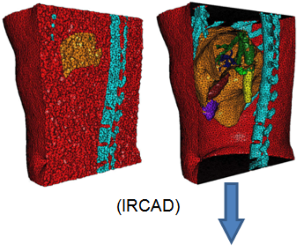
CGALmesh

Curved objects



(INSERM)

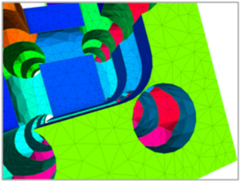
Multi-domains

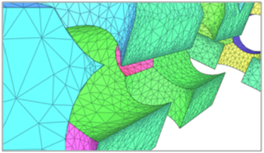


(IRCAD)

Size (mm)	#Tetrahedra	Time (s)
16	20K	0.88
8	160K	6.97
4	1.2M	54
2	10M	431

Sharp features





(Distene)

with S. Tayeb [engineer ADT]

Anisotropic mesh generation

Definition-Motivation

Anisotropic meshes

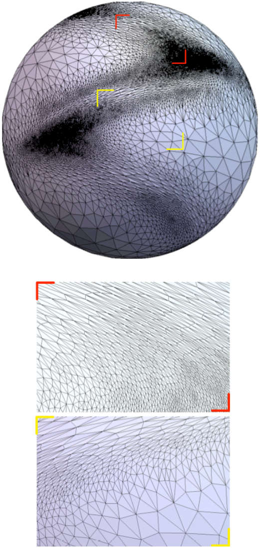
Mesh elements are elongated according to prescribed directions

Requirements on shapes are described through a metric field

Why anisotropic meshes ?

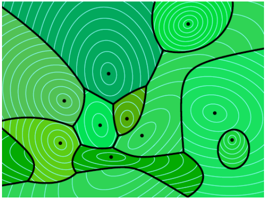
Anisotropic meshes enhance the trade off accuracy / mesh complexity

- interpolation/approximation
- solving anisotropic PDE



Anisotropic mesh generation

Main approaches

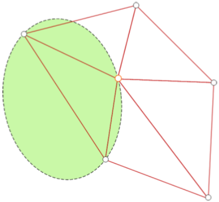


Many heuristics

ellipsoidal bubble packing, biting ellipses, anisotropic centroidal Voronoi, pliant method, anisotropic mesh adaptation ....

Few methods with guarantees

- Anisotropic Voronoi diagrams (2d)
  [Labelle & Shewchuk 2003]
- Anisotropic Delaunay meshes
  [SoCG 2007]

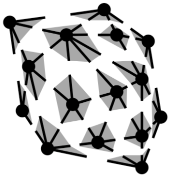


each simplex is Delaunay for the metrics attached to all its vertices

Anisotropic Delaunay meshes

- **Stars**

  Compute the star of each vertex v in the Delaunay triangulation associated to the metric of v

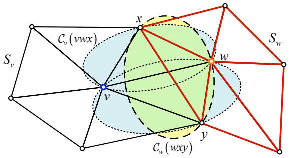


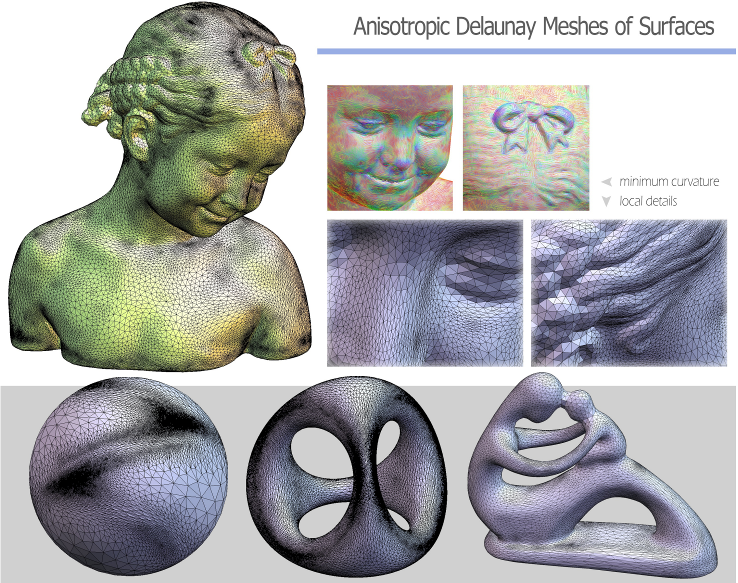
- **Inconsistencies**

  a simplex may appear in the stars of some (but not all) of its vertices

- **Star conciliation**

  if the metric field is smooth, inconsistencies can be removed by carefully refining the mesh




Anisotropic Delaunay Meshes of Surfaces
minimum curvature
local details

Anisotropic mesh generation

Discussion

- An algorithm with theoretical guarantees (in any dim)
- Implementation in progress $\Rightarrow$ CGALmesh
- Star stitching is a useful paradigm

  triangulation of submanifolds of dim $k \ll d$ [SoCG 2010]

  Delaunay triangulation of Riemannian manifolds

Two selected results

Mesh generation

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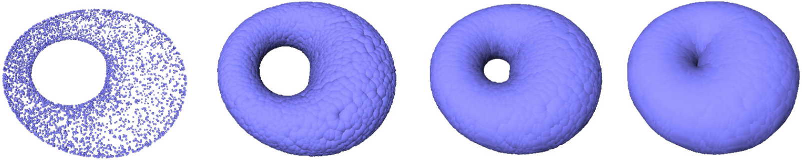
Geometric inference

- Optimal transport and geometric structure extraction from point sets
  - distance functions for noisy data
  - Voronoi covariance measures

Q. Mérigot [Ph.D. 2009, CNRS]

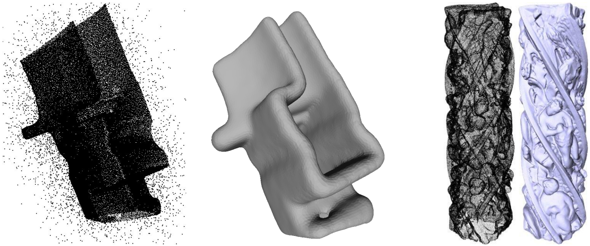
Distance function based inference

[DCG, CGTA, CGF 2009]



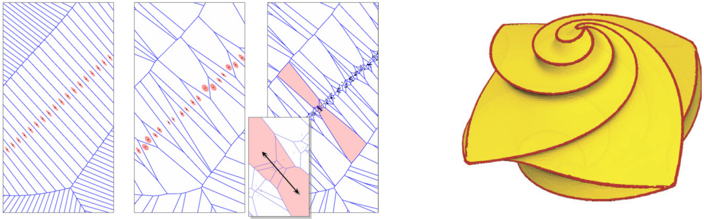
- If P is a dense sample of a shape S
  appropriate offsets of P have the same topology as S
- This depends only on two properties of distance functions:
  - Robustness with respect to the Hausdorff distance, *i.e.* $||d_K - d_{K'}||_\infty \leq d_H(K, K')$ for two compact sets K, K'
  - Squared distance functions are 1-semiconcave, *i.e.* $||x||^2 - d_K^2(x)$ is a convex function

Distance function to a probability measure



- The previous approach obviously fails if there are outliers
- Notion of distance function to a mass distribution, robust with respect to the transport distance
- The square of this function is 1-semiconcave, making it suitable for shape inference from data corrupted by outliers
  - Sublevel-sets are topologically correct [FoCM11]
  - Zero-set of a signed version leads to robust reconstruction [SGP10]

Voronoi covariance measures



- Convolving covariance matrices of the Voronoi cells yields a tensor-valued measure on the data points, robust under Hausdorff perturbations of the data [FoCM10]
- Features locations and directions can be found using eigenanalysis of the local covariance measures [SPM09]

Feature preserving surface reconstruction

- Infer feature graph from data
- Compute implicit surface fitting the data
- Mesh surface using constraint-preserving Delaunay refinement [SGP10]

Geometric inference in the presence of noise

Discussion

- Measure-theoretic framework
- Rather simple to implement even in high dimensions (nearest neighbour search and Monte Carlo methods)
- Does not take into account the statistical nature of noise

Perspectives

3 main scientific objectives

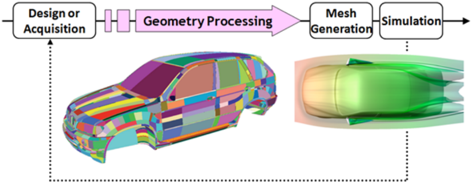
- 3D Geometry processing — ERC Grant IRON
- Geometric inference — ICT-FET CG-Learning
- Geometric algorithms and data structures — ANR Présage

Software development

- CGAL — CGAL-HiD

Objective 1: 3D Geometry Processing

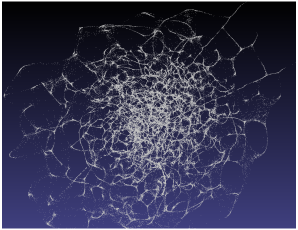
- Technological paradox: Data increasingly heterogeneous and defect-laden
- Enduring challenge: processing step before meshing in computational engineering pipeline



New spin-off project-team (P. Alliez)

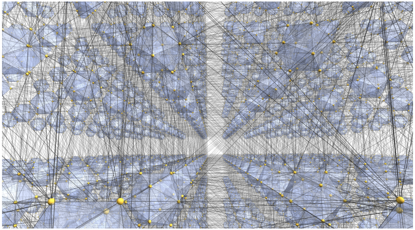
Objective 2 : Geometric Inference

Understanding geometric structures in high dimensional data



- Geometry + statistics
- Complexity, algorithms and implementation
- Data analysis: clustering, segmentation, classification,...

Objective 3 : Geometric Algorithms and Data Structures



- Probabilistic methods (smoothed analysis)
- Non-Euclidean geometries (hyperbolic geometry)
- Higher dimensions (CGAL HiD)

Collaborations with Athens

European projects

- (2001-2004) ECG: Effective Computational Geometry for Curves and Surfaces
- (2005-2008) ACS: Algorithms for Complex Shapes with certified topology and numerics
- (2010-2013) CGL : Computational Geometric Learning

INRIA Collaborative Research Actions (ARC)

- (2005-2006) Arcadia: Arrangements of Quadrics, algorithms, implementation and applications

Collaborations with Athens

Master students

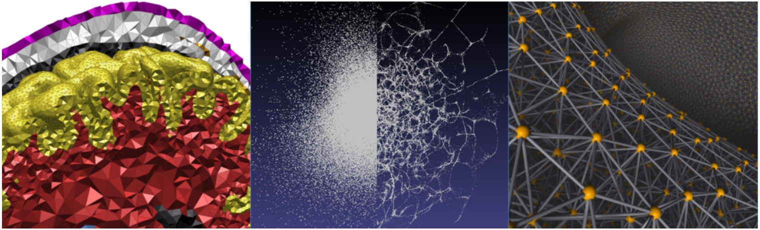
- (2003) Athanasios Kakargias (Monique Teillaud, in Galaad)
- (2005) Constantinos Tsirogiannis (Monique Teillaud, in Galaad)
- (2009) Vissarion Fisikopoulos (Monique Teillaud)

Post-docs

- (2006) Elias P. Tsigaridas, 3 month visit (Monique Teillaud)

PhD

- Monique Teillaud is a member of the "Three persons committee" for Vissarion Fisikopoulos.



Thank you for your attention!