GALAAD
Geometry, Algebra and Algorithms

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GALAAD: joint team between INRIA and UNSA

Permanent staff

- L. Busé (CR1 INRIA)
- A. Galligo (Prof. UNSA)
- E. Hubert (CR1 INRIA)
- B. Mourrain (DR2 INRIA)

Current Ph. D. Student

- Marta Abril Bucero (ANR, 2011-2014)
- Matthieu Collowald (MESR, 2011-2014)
- Abdallah Lachaal (EU Terrific, 2011-2014)

Current post-doc.:

- A. Bernardi (IEF Marie Curie, 2010-2012)
- N. Botbol (ITN Marie Curie SAGA, 2011-2012)

Temp. Engineer:

- M. Perrinel (INRIA, ADT IJD)

Collaborator:

- I.Z. Emiris (Prof. Univ. of Athens).

Assitant:

- S. Honnorat (INRIA, IA)

Among former Ph.D. students:

- A. Mantzaflaris (Ph.D. SAGA 2008-2011)
- T. Lu Baa (Ph.D. 2008-2011) Post. Doct. in NKUA.

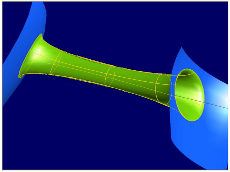
Our approach

- centered on **algebraic** representations of the **geometry**;
- in order to provide **rich, compact, high quality models**;
- and **efficient tools** for exploiting these models;

Topics of investigation:

1. **Algebraic algorithms for geometric computing**
2. **Symbolic-numeric analysis**
3. **Algebraic representations for geometric modeling**

I Algebraic algorithms for Geometric Computing



- points, curves, surfaces, volumes, ...
- represented by parametric, implicit models
- require specific algebraic methods.

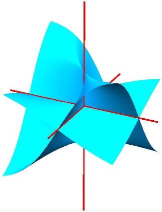
Problems:

- Change from parametric to implicit representations,
- **Intersection** points or curves, **autointersection**, **singular** points,
- **Closest point** or distance computation, Collision.
- **Pipes**, **canal surfaces**, **offset**, **fillet**, **blending**, . . .
- **Medial axis** or skeleton structure.
- . . .

Study

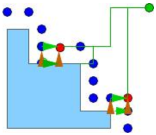
☞ systems polynomial equations, algorithms for their solution in geometric applications.

Resultant and syzygy methods:



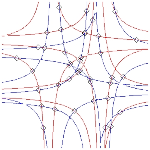
- *Adapted resultant matrices for the self-intersection problem (generic case);*
- *Matrix representations of rational hypersurfaces and of rational curves;*
- *Intersection problems with rational curves; reduction of non-square to square one-dimensional pencils of matrices;*
- *Implicit equation of rational ruled surfaces and of canal surfaces;*
- *Singularity of rational curves as invariant factors of a one-dimensional pencil of matrices;*
- *Link between adjoint curves to a rational plane curve and some equations of a Rees algebra;*

Algebraic solvers using Border basis:



- Effective rep. of $\mathcal{A} = \mathbb{K}[x_1, \ldots, x_n]/(f_1, \ldots, f_s)$;
- Generalisation of Gröbner basis; better for approximate computation;
- Further analyzis of algebraic properties;
- Intrisinc characterisation; Hilbert Scheme equations;
- Extension to real algebraic geometry and optimisation problems.

Subdivision methods:



- Univariate subdivision solvers; Bernstein or monomial basis; Continued Fraction approximation; one of the most efficient implementations;
- Multivariate Bernstein basis; preconditionning and reduction; sleeves; practical efficiency;
- Multivariate continued fraction;
- Complexity analysis; new (quasi-optimal) bounds for separation; condition number of systems.

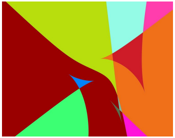
II Symbolic-numeric analysis

- Input data may be known with uncertainty;
- Computation may be performed approximately;
- Non-linearity leads to unstability;

Problems:

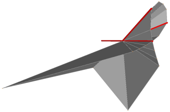
- Analyze familly of input data of the same "shape";
- Certification of the result even if approximate computation is used;
- Develop robust (stable) algebraic algorithms for geometric analysis;
- Analyze the conditionement of the problems, of the methods;

Topology and arrangements of curves:



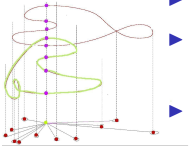
- *New approach for topology;*
- *Isolate first the extremal and singular points;*
- *Subdivision based on regularity test from cell boundary;*
- *Use topological degree to analyse the real branches at a singular point;*

Topology of algebraic surfaces:



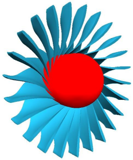
- *Exploit properties of subresultants;*
- *Compute an explicit Whitney stratification;*
- *Improved complexity bounds compared to Cylindrical Algebraic Decomposition;*
- *Analysis of iterated resultants and discriminants;*

Algebraic decomposition and absolute factorisation:



- *Use monodromy or Wood theorem to split fiber points;*
- *Construct a smallest algebraic extension field to represent the coefficients;*
- *Practical efficiency: absolute factorisation of pol. of degree 400.*

III Algebraic representations for geometric modeling

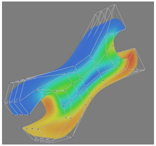


- Implicit or parametric representation;
- Piecewise algebraic descriptions (splines) associated to subdivisions;

Problems:

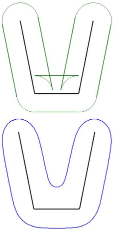
- **Interpolation** of points or curves by surfaces;
- Curves or surfaces **fitting**;
- Surfaces **filling** holes with boundary constraints;
- Approximation of data/observations by **compact algebraic models**;
- Representation/analysis of **functions** on a given geometry;

Tensor decomposition:



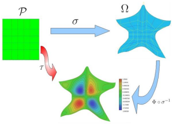
- *Decompose a tensor as a minimal sum of tensors of rank* 1 *(product or power of linear forms);*
- *Geometry of secants of Veronese and Segre varieties;*
- *New algebraic method extending Sylvester approach;*
- *New links with truncated moment problems;*

Tubular and convolution surfaces:



- *Envelop of spheres; implicit equations; curves in the space of spheres;*
- *Skeleton based Geometric Modeling for Computer Graphics;*
- *General formulae for convolution surfaces based on sets of line segments and circular arcs;*
- *Efficient formulae for convolution surfaces based on planar polygons.*

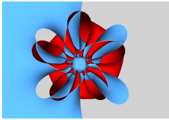
Isogeometry:



- *Use the same function basis for the geometry and the simulation; exact description of the geometry;*
- *High order numerical scheme;*
- *Optimisation of the parameterisation for a given problem;*
- *Local refinement of function spaces;*
- *Interaction with shape optimisation;*

Software developments

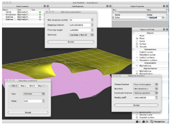
MATHEMAGIX



http://www.mathemagix.org

- Free computer algebra system with an interpreter, a compiler, a front-end mathematical editor TEXMACS, a geometric modeler AXEL.
- Algebraic and numeric computation; most efficient known algorithms.
- Efficient dedicated autonomous C++ packages as plugins in the interpreter; Connections with GMP, MPFR, LAPACK, CDD, ...
- Collaborative project with J. van der Hoeven and G. Lecerf (CNRS, Palaiseau); Ph. Trébuchet (LIP6); . . .
- GPL Licence; 400 000 lines of code; Automatic tools for configuration, documentation; nightly tests;

AXEL



http://axel.inria.fr

- Algebraic Geometric modeler: polynomial parametrisation, B-Spline, implicit curves and surfaces, . . . ; intersection, self-intersection, implicitization, topology and arrangement of implicit (singular) curves or surfaces, . . .
- Isogeometric analysis tools; link with `gotools` (SINTEF), `isogeometric toolbox` (EXCITING).
- Plugins for optimization and PDE solvers (R. Duvigneau, OPALE), algebraic knot invariants (M. Hodorog, RICAM, Linz, Austria), ray GPU tracing (J. Seland, SINTEF, Oslo, Norway).
- GPL Licence; 200 000 lines of code;

National collaborations

- ANR: GEOLMI (Geometry and Algebra of Linear Matrix Inequalities with Systems Control Applications).

International collaborations

- European projects: **SAGA** (ShApe Geometry and Algebra) ITN Marie-Curie; **Exciting** (Isogeometry); **TERRIFIC**, STREP, “Factory of the Future”; DECONSTRUCT, IEF Marie-Curie.
- Recent billateral collab.: Barcelona, Buenos Aires, Hong Kong, Seoul, ...