MESH-ADAPTIVE COMPUTATION OF LINEAR AND NON-LINEAR ACOUSTICS

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Methodology. Pressure deviation can propagate over long distances. Two typical examples are shock waves and linear acoustics. Shock waves involve the sonic boom, which can be seen as a steady shock wave (in a wind tunnel, typically), and the blast wave (typically unsteady). Linear acoustic waves refer also either to a transient wave of bounded duration, or to a periodic vibration. A important context in the study of these different type of waves is the case where we are interested only by the effect of the wave on a sensor ocupying a very small locus in the region affected by the pressure perturbation. Further simplifying, we can be interested by a single scalar measure of this effect, for example the total energy E_{tot} received by the sensor during a given time interval. If the pressure perturbation is emitted at a very long distance, in an open and complex spatial domain, the numerical simulation of this phenomenon, that would be necessary to predict E_{tot}, can be extremely computer intensive, if not impossible. Now, many events in the simulation are useless for our target, some others not very important, some others of crucial importance. The local mesh fineness should reflect this hierarchy.

The issue we address in this work is the application of mesh adaptation for designing the best mesh for obtaining a scalar output like E_{tot} with a given accuracy.

The technique adopted in this work is the anisotropic mesh-adaptation introduced by Alauzet *et al.* in [1] combined with a goal-oriented method. The choice of anisotropic mesh is motivated by its strong influence on the accuracy of many CFD predictions. This technique allows (i) to substantially reduce the number of degrees of freedom, thus impacting favorably the CPU time, (ii) to automatically capture the anisotropy of the physical phenomena, and (iii) to access to high order asymptotic convergence. Recent works have shown a fertile development of metric-based, or Hessian-based methods [6, 5], which rely on an ideal representation of the *interpolation error* and of the *mesh*. Getting rid of error iso-distribution and prefering L^p error minimization allow to take into account discontinuities with higher-order convergence [11].

Metric-based methods aim at controlling the interpolation error but this purpose is not often so close to the objective that consists in obtaining the best solution of a PDE. Further, in many engineering applications, a specific scalar output needs to be accurately evaluated, e.g. lift, drag, heat flux. To address this need, the *goal-oriented* mesh adaptation focuses on deriving the best mesh to observe a given output functional. Goal-oriented methods result from a series of papers dealing with *a posteriori estimates* ([13, 3, 7, 12]. Extracting informations concerning mesh anisotropy from an *a posteriori* estimate is a difficult task. Starting from *a priori estimates*, Loseille *et al.* proposed in [10] a fully anisotropic goal-oriented mesh adaptation (FAGOMA) technique. This latter method combines goal oriented rationale and the application of Hessian-based analysis to truncation error. This method was described for the simulation of a steady sonic boom.

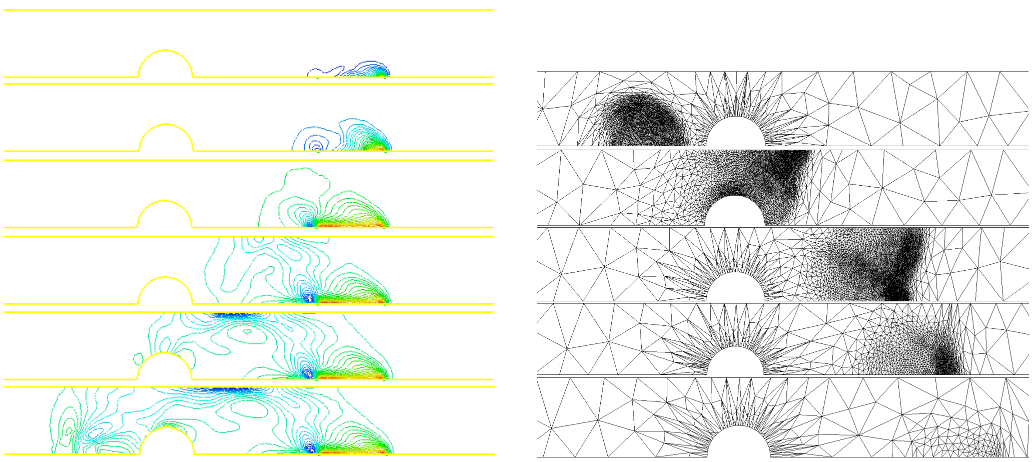

Figure 1: Nonlinear blast wave past a cylinder, adjoint state and resulting adaptive mesh.

Mesh adaptation for *unsteady flows* is also an active research field and brings an attracting increase in simulation efficiency. Complexity of the algorithms is larger than for steady case: time discretisation needs also to be adapted, possibly in a manner that combines well with mesh adaptation. Meshes can be moved as in [2], locally refined [4] or rebuild as in [9]. Many error sensors, *a posteriori* estimates and goal-oriented methods have been considered in the literature, see a typical contribution in [3]. A transient fixed-point (TFP) method was proposed in [1]. The Hessian criteria at the different time steps was synthetized with the metric intersection method

[1, 8]. In [8] it is proposed to minimise the $L^\infty(L^p)$ norm (supremum in time of the L^p spatial norm) of the Hessian-based criterion. The present work combines the FAGOMA of [10] and the TFP advances of [1, 8].

Nonlinear blast wave. We present some preliminary results on the 2D test case of a channel flow past a semi-circular object. We consider a "blast-like" initialization inside a circle on the left part of domain's bottom. The cost function measures the impulse over a target surface lying on the right bottom, two diameters after the circular obstacle, see Fig.1.

Linear acoustics wave. In a first investigation of acoustics waves, we consider a sound source located at the left-bottom of a square domain. We are interested by the mesh-adaptive calculation of the impact of it on a micro **M** located on the center of the same domain bottom. The role expected from mesh adaptation is to reduce as much as possible mesh fineness in the parts of computational domain where accuracy loss does not influence the quality of sound prediction on the micro. This is illustrated in Fig.2.

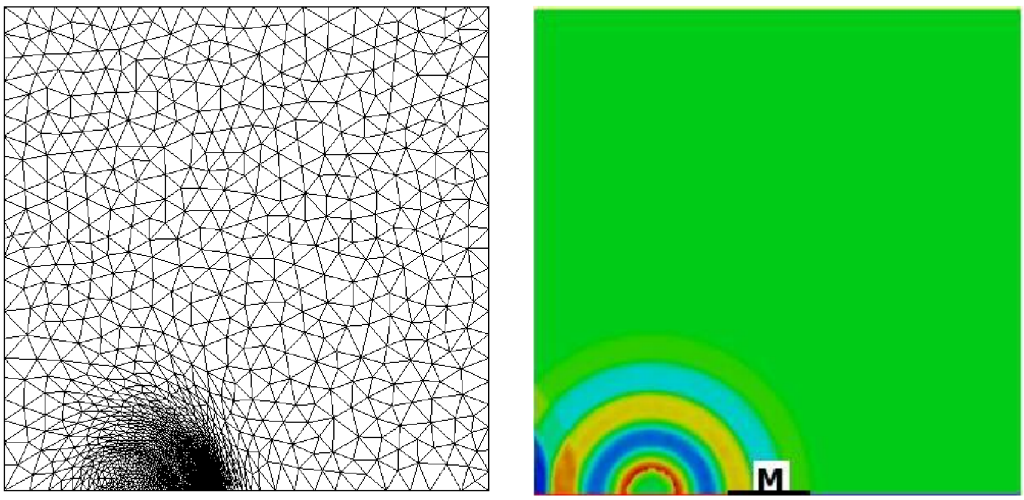

Figure 2: Influence of an acoustic wave on a micro located close to it. Mesh resolution concentrates between source and micro **M**.

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