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# A Half-Stochastic Model for Noise Simulation in Free Turbulent Flows

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**The paper presents a way of noise prediction for turbulent flows basing on semi-deterministic turbulence modeling. This approach exploits the solution of Reynolds averaged Navier-Stokes (RANS) equations. The mathematical model is constructed on the base of complete system of Navier-Stokes equations by introducing the triple flow decomposition into mean part and two types of pulsation components. Both the linear part with respect to noise components and the part modeling the source terms are extracted in accordance with the Lighthill analogy. To predict the turbulent pulsation components considered as source elements, a half-stochastic model is introduced. It uses the results of simulating the mean turbulent flow field with the help of "$k$-$\varepsilon$" turbulence model. A peculiarity of the model developed is the assumption of single-peak spectra in shear layers and the attempt to predict the peak frequencies. To determine the characteristic frequencies, a special procedure of flow exposure to stochastic acoustic radiation from artificial white noise sources is proposed. The technique developed is demonstrated on the example of plane mixing layer and jet predictions.**

## Introduction

The paper concerns with one approach to the prediction of acoustic noise generation and propagation in turbulent gas flows. The way is developed in the framework of complete Navier-Stokes (NS) equations taken as a base for the mathematical description of noise dynamics in turbulent gas flows. Such approach allows for avoiding assumptions or simplifications at the start in contrast with the models basing on a single wave-type equation (such as the Lighthill, Phillips, Lilley, Blochintsev equations) for the description of acoustic phenomenon.

The direct numerical simulation of the Navier-Stokes equations (lettered by DNS) seems not the best way for the noise prediction since there is a difficulty in adequate prediction of gasdynamic phenomenon of different scales by the means of unified model. Moreover, the complete nonlinear formulation appears very expensive to provide the necessary accuracy or to offer the usage of highly refined meshes needed for resolving turbulent phenomenon of different scales.

Similar drawbacks can be also related to such known ways of gasdynamics predictions as the numerical solution of RANS equations and Large Eddy Simulation (LES). From the aeroacoustics standpoint, these ways may be even more inconvenient since the resulting flow parameters are somewhat averaged (filtered) which prevents from predicting spectral characteristics. Although in this sense, the LES seems more advantageous in the case of extremely high performance computing when it is possible to use a very little filter width. Another inconvenient detail is that RANS equations and LES are unable in full measure to consider a stochastic nature of sound.

All the above arguments lead to the third way which is currently in wide use for acoustic noise prediction. Generally, it consists in the derivation of governing equations for the pulsation parameters for turbulent flow fields. The paper marks that such approach opens up new opportunities from the theoretical physics standpoint for studying turbulence.[1] This statement can be extended to the numerical simulation. This is also evidenced by the fact that the recent numerical investigations pay considerably more attention to the direct solution of the gas dynamics problems described by the equations for pulsation parameters,[2],[3].[4] At the same time, this strategy contributes to the solution of noise generation problem.

This paper presents a version of implementing the third way of numerical study of turbulent flows. The modification under consideration uses the solutions of RANS equations as input data for the prediction of sound dynamics. A special attention is paid to the construction of acoustic noise sources.

The technique developed is close to the one expressed in the paper[5] in a compact form as a sequence 'DNS - RANS - NLDE - LEE+S' (where NLDE means Non Linear Disturbance Equations and LEE+S - Linearized Euler Equations with source terms). (The authors try to follow the abbreviations used in.[5]) This

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approach seems advantageous since it is the most general and provides a variety of implementations among which undoubtedly are adequate enough solutions. In fact, in the cases of using the decomposition of fluctuations to turbulent and acoustic parts, a difficulty is the necessity of adequate modeling of fields of turbulent pulsation. A number of statistical models developed for this purpose have been proposed.[6–8] Among them the SNGR model[2,5] is elaborated for simulating the fluctuations of turbulent velocity components has originated a half-stochastic model presented in the paper. The half-stochastic model for the reconstruction of turbulent velocity components is proposed in the paper. It implies a reasonable knowledge on flow spectral properties (namely, peak frequencies and mean spectrum width) To specify these spectral characteristics, a special numerical experiment on flow exposure to acoustic radiation from artificial sources is pursued.[9] To simulate such characteristics of noise sources as their location, amplitudes and phases, the data on the time-averaged flow field (including the information on sources correlation) obtained at solving the RANS equations closed by standard $k-\varepsilon$ turbulence model is used.

## Mathematical Formulation

Let the 2D Navier-Stokes equations be written in the following vector form

$$\frac{\partial \mathbf{Q}}{\partial t}+\frac{\partial \mathbf{E}}{\partial x}+\frac{\partial \mathbf{F}}{\partial y}=\frac{\partial \mathbf{E}_\nu}{\partial x}+\frac{\partial \mathbf{F}_\nu}{\partial y} \tag{1}$$

where $\mathbf{Q}$ is a vector of conservative gas dynamics parameters

$$\mathbf{Q}=\begin{pmatrix}\rho\\ m\\ n\\ E\end{pmatrix}=\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ \rho\dfrac{u^2+v^2}{2}+\rho\varepsilon\end{pmatrix} \tag{2}$$

**General decomposition. Non Linear Disturbance Equations (NLDE)**

Let us decompose a flow into two components $\xi=\bar{\xi}+\xi'$ as

$$\begin{aligned}\rho&=\bar{\rho}+\rho' \qquad & u&=\tilde{u}+u''\\ p&=\bar{p}+p' \qquad & v&=\tilde{v}+u''\end{aligned} \tag{3}$$

where the usual time averaging is applied to density and pressure while the Favre averaging is applied to velocity components. Always here the solutions of steady RANS (SRANS) equations are considered that is

$$\overline{f(\mathbf{x})}=\lim_{T\to+\infty}\frac{1}{T}\int_0^T f(\mathbf{x},\tau)d\tau. \tag{4}$$

After implementing this decomposition to (1) and averaging it on time one can derive the SRANS system with the main four equations in the following form

$$\frac{\partial \overline{\mathbf{E}}}{\partial x}+\frac{\partial \overline{\mathbf{F}}}{\partial y}=\frac{\partial \overline{\mathbf{E}}_\nu}{\partial x}+\frac{\partial \overline{\mathbf{F}}_\nu}{\partial y}-\overline{\mathbf{S}_{\mathrm{RANS}}} \tag{5}$$

where $\overline{\mathbf{S}_{\mathrm{RANS}}}=(0,\ s_2,\ s_3,\ s_4)^T$ and

$$s_2=\frac{\partial\overline{\rho u''^2}}{\partial x}+\frac{\partial\overline{\rho u''v''}}{\partial y},\qquad s_3=\frac{\partial\overline{\rho u''v''}}{\partial x}+\frac{\partial\overline{\rho v''^2}}{\partial y},$$

$$\begin{aligned}s_4=&\frac{\partial}{\partial x}\left(\tilde{u}\frac{3\overline{\rho u''^2}+\overline{\rho v''^2}}{2}+\tilde{v}\overline{\rho u''v''}+\frac{\overline{\rho u''^3}+\overline{\rho u''v''^2}}{2}\right)\\ &+\frac{\partial}{\partial y}\left(\tilde{v}\frac{\overline{\rho u''^2}+3\overline{\rho v''^2}}{2}+\tilde{u}\overline{\rho u''v''}+\frac{\overline{\rho u''^2v''}+\overline{\rho v''^3}}{2}\right).\end{aligned}$$

At solving SRANS equations, one predicts the source term $\overline{\mathbf{S}_{\mathrm{RANS}}}$ on the base of a specific turbulence model chosen. In particular, in the case of $k-\varepsilon$ turbulence model used here the source term is described as a function on the Reynolds stresses predicted following to the Bussinesque hypothesis.

In general, the governing equations for the flow pulsation components are derived as a result of the subtraction of SRANS (5) from the NS equations (1). It can be schematically shown as

$$\langle DNS+FlowDecomposition\rangle-\langle SRANS\rangle=\langle NLDE\rangle \tag{6}$$

In doing so, the NLDE has the following form

$$\frac{\partial \mathbf{Q}'^\star}{\partial t}+\frac{\partial \mathbf{E}'^\star}{\partial x}+\frac{\partial \mathbf{F}'^\star}{\partial y}=\overline{\mathbf{S}_{\mathrm{RANS}}}, \tag{7}$$

where

$$\begin{aligned}\mathbf{Q}'^\star&=\mathbf{Q}\left(\bar{\xi}+\xi'\right)-\overline{\mathbf{Q}}\left(\bar{\xi}\right),\\ \mathbf{E}'^\star&=\mathbf{E}\left(\bar{\xi}+\xi'\right)-\overline{\mathbf{E}}\left(\bar{\xi}\right),\\ \mathbf{F}'^\star&=\mathbf{F}\left(\bar{\xi}+\xi'\right)-\overline{\mathbf{F}}\left(\bar{\xi}\right),\end{aligned}$$

$$\mathbf{Q}'^\star=\begin{pmatrix}0\\ \tilde{u}\rho'+\bar{\rho}u''+\rho'u''\\ \tilde{v}\rho'+\bar{\rho}v''+\rho'v''\\ q_4'^\star\end{pmatrix}, \tag{8}$$

$$\begin{aligned}q_4'^\star=&\frac{\tilde{u}^2+\tilde{v}^2}{2}\rho'+\tilde{u}\bar{\rho}u''+\tilde{v}\bar{\rho}v''+\frac{p'}{\gamma-1}\\ &+\tilde{u}\rho'u''+\tilde{v}\rho'v''+(\bar{\rho}+\rho')\frac{u''^2+v''^2}{2}\end{aligned}$$

When it is also introduced that $\mathbf{A}_x=\overline{\mathbf{A}}_x+\mathbf{A}_x'^\star$ and $\mathbf{A}_y=\overline{\mathbf{A}}_y+\mathbf{A}_y'^\star$ the NLDE (7) can be represented as

$$\begin{aligned}\frac{\partial \mathbf{Q}'^\star}{\partial t}+\mathbf{A}_x\frac{\partial \mathbf{Q}'^\star}{\partial x}+\mathbf{A}_x'^\star\frac{\partial \overline{\mathbf{Q}}}{\partial x}&\\ +\mathbf{A}_y\frac{\partial \mathbf{Q}'^\star}{\partial y}+\mathbf{A}_y'^\star\frac{\partial \overline{\mathbf{Q}}}{\partial y}&=\overline{\mathbf{S}_{\mathrm{RANS}}}\end{aligned} \tag{9}$$

where $\mathbf{A}_x=\mathbf{A}_x(\rho,u,v,p)$ and $\mathbf{A}_y=\mathbf{A}_y(\rho,u,v,p)$ are the standard flux Jacobian matrices.

### Decomposition of fluctuations. Linear Acoustic Equations with Sources (LAE+S)

The next stage of flow decomposition concerns with the pulsation parameters which are also divided into two components $\xi' = \xi'_{(1)} + \xi'_{(2)}$.

It is evident that the further considerations strongly depend on the mathematical basis of such decomposition and correspondingly on the nature of separated fluctuations. To predict acoustic noise from turbulent flows, it is naturally to try to separate the fluctuations relating respectively to turbulent and acoustic phenomena, although it is not always possible to establish a reasonable distinction between such disturbances. We also share the intention to separate turbulent and acoustic disturbances, however from the general considerations formally mark them as (1) and (2). In the paper the decomposition is performed under the assumption that $|\xi'_{(2)}| \ll |\xi'_{(1)}|$ in the turbulence regions (the regions where turbulent kinetic energy is considerable), and $\xi'_{(1)} = 0$ in the rest domain parts. Thus, the following decomposition is introduced

$$\begin{array}{ll} \rho' = \rho'_{(2)} & u'' = u''_{(1)} + u''_{(2)} \\ p' = p'_{(2)} & v'' = v''_{(1)} + v''_{(2)} \end{array} \tag{10}$$

It is remarkable that the pulsation components both on pressure and density are entirely related to the fluctuations of the second type. This is done due to the following two considerations. First, the practice shows that the relative values of density and pressure disturbances appear significantly smaller than the corresponding values of fluctuations of velocity components. Second, there is a wish to exclude acoustic waves from the fluctuations of the first type and move them to the ones of the second type.

Let us now introduce the decomposition (10) into (9) and rearrange the terms using the following representations

$$\begin{aligned} \mathbf{Q} &= \overline{\mathbf{Q}} + \mathbf{Q}'_{(1)} + \mathbf{Q}'_{(2)} + \mathbf{Q}'_{(1)(2)} + \mathbf{Q}^{\star(2)}_{(1)} + \mathbf{Q}^{\star}, \\ \mathbf{A}_x &= \overline{\mathbf{A}}_x + \mathbf{A}'_{x(1)} + \mathbf{A}'_{x(2)} + \mathbf{A}'_{x(1)(2)} + \mathbf{A}^{\star(2)}_{x(1)} + \mathbf{A}^{\star}_x, \\ \mathbf{A}_y &= \overline{\mathbf{A}}_y + \mathbf{A}'_{y(1)} + \mathbf{A}'_{y(2)} + \mathbf{A}'_{y(1)(2)} + \mathbf{A}^{\star(2)}_{y(1)} + \mathbf{A}^{\star}_y \end{aligned}$$

where the notation $\mathbf{Q}'_{(i)}, (i = 1, 2)$ designates all the terms linear on pulsation components $\xi'_{(i)}$, $\mathbf{Q}'_{(1)(2)}$ - all the bilinear terms that is the expressions linear on the terms of type $\xi'_{(1)}\eta'_{(2)}$, $\mathbf{Q}^{\star(k)}_{(i)}$ - all the nonlinear terms with the $k$ nonlinearity order on pulsation components $\xi'_{(i)}$ and $\mathbf{Q}^{\star}$ - the rest nonlinear terms.

If then keeping only the main significant terms, one can transform the NLDE (9) in the following way

$$\begin{aligned} &\frac{\partial \mathbf{Q}'_{(2)}}{\partial t} + \overline{\mathbf{A}}_x \frac{\partial \mathbf{Q}'_{(2)}}{\partial x} + \mathbf{A}'_{x(2)} \frac{\partial \overline{\mathbf{Q}}}{\partial x} \\ &\quad + \overline{\mathbf{A}}_y \frac{\partial \mathbf{Q}'_{(2)}}{\partial y} + \mathbf{A}'_{y(2)} \frac{\partial \overline{\mathbf{Q}}}{\partial y} \\ &\quad = \overline{\mathbf{S}_{\mathrm{RANS}}} - \mathbf{S}'_{(1)} - \mathbf{S}^{\star(2)}_{(1)} - \mathbf{S}'_{(1)(2)} \end{aligned} \tag{11}$$

where

$$\begin{aligned} \mathbf{S}'_{(1)(2)} = {} & \frac{\partial \mathbf{Q}'_{(1)(2)}}{\partial t} + \overline{\mathbf{A}}_x \frac{\partial \mathbf{Q}'_{(1)(2)}}{\partial x} + \mathbf{A}'_{x(1)(2)} \frac{\partial \overline{\mathbf{Q}}}{\partial x} \\ & + \mathbf{A}'_{x(1)} \frac{\partial \mathbf{Q}'_{(2)}}{\partial x} + \mathbf{A}'_{x(2)} \frac{\partial \mathbf{Q}'_{(1)}}{\partial x} \\ & + \overline{\mathbf{A}}_y \frac{\partial \mathbf{Q}'_{(1)(2)}}{\partial y} + \mathbf{A}'_{y(1)(2)} \frac{\partial \overline{\mathbf{Q}}}{\partial y} \\ & + \mathbf{A}'_{y(1)} \frac{\partial \mathbf{Q}'_{(2)}}{\partial y} + \mathbf{A}'_{y(2)} \frac{\partial \mathbf{Q}'_{(1)}}{\partial y}, \end{aligned}$$

$$\begin{aligned} \mathbf{S}'_{(1)} = {} & \frac{\partial \mathbf{Q}'_{(1)}}{\partial t} + \overline{\mathbf{A}}_x \frac{\partial \mathbf{Q}'_{(1)}}{\partial x} + \mathbf{A}'_{x(1)} \frac{\partial \overline{\mathbf{Q}}}{\partial x} \\ & + \overline{\mathbf{A}}_y \frac{\partial \mathbf{Q}'_{(1)}}{\partial y} + \mathbf{A}'_{y(1)} \frac{\partial \overline{\mathbf{Q}}}{\partial y}, \end{aligned}$$

$$\begin{aligned} \mathbf{S}^{\star(2)}_{(1)} = {} & \frac{\partial \mathbf{Q}^{\star(2)}_{(1)}}{\partial t} \\ & + \overline{\mathbf{A}}_x \frac{\partial \mathbf{Q}^{\star(2)}_{(1)}}{\partial x} + \mathbf{A}^{\star(2)}_{x(1)} \frac{\partial \overline{\mathbf{Q}}}{\partial x} + \mathbf{A}'_{x(1)} \frac{\partial \mathbf{Q}'_{(1)}}{\partial x} \\ & + \overline{\mathbf{A}}_y \frac{\partial \mathbf{Q}^{\star(2)}_{(1)}}{\partial y} + \mathbf{A}^{\star(2)}_{y(1)} \frac{\partial \overline{\mathbf{Q}}}{\partial y} + \mathbf{A}'_{y(1)} \frac{\partial \mathbf{Q}'_{(1)}}{\partial y}. \end{aligned}$$

The two unknown vectors $\mathbf{Q}'_{(i)} = \overline{\mathbf{P}}\mathbf{U}'_{(i)}$, $\mathbf{U}'_{(i)} = \left(\rho'_{(i)}, u''_{(i)}, v''_{(i)}, p'_{(i)}\right)^T$, $i = 1, 2$ used in (11) are conservative variables vectors linearized on pulsation components (1) or (2) correspondingly. They consist only of the terms linear on fluctuations of physical variables.

$$\mathbf{Q}'_{(i)} = \begin{bmatrix} \rho'_{(i)} \\ \tilde{u}\rho'_{(i)} + \bar{\rho}u''_{(i)} \\ \tilde{v}\rho'_{(i)} + \bar{\rho}v''_{(i)} \\ \dfrac{\tilde{u}^2 + \tilde{v}^2}{2}\rho'_{(i)} + \bar{\rho}\tilde{u}u''_{(i)} + \bar{\rho}\tilde{v}v''_{(i)} + \dfrac{p'_{(i)}}{\gamma - 1} \end{bmatrix} \tag{12}$$

It is clearly seen that the left hand side of equation (11) describes the transport of fluctuations (2) through nonuniform mean flow field. It implicitly involves the term modeling the refraction of fluctuations by the mean flow. This can be shown when taken into account that

$$\mathbf{A}'_{x(i)} \frac{\partial \overline{\mathbf{Q}}}{\partial x} = \frac{\partial \overline{\mathbf{A}}_x}{\partial x} \mathbf{Q}'_{(i)}, \ \mathbf{A}'_{y(i)} \frac{\partial \overline{\mathbf{Q}}}{\partial y} = \frac{\partial \overline{\mathbf{A}}_y}{\partial y} \mathbf{Q}'_{(i)}, \tag{13}$$

introduced the Jacobian matrices for physical variables $\overline{\mathbf{A}^o_x} = \overline{\mathbf{P}}^{-1}\overline{\mathbf{A}}_x\overline{\mathbf{P}}$, $\overline{\mathbf{A}^o_y} = \overline{\mathbf{P}}^{-1}\overline{\mathbf{A}}_y\overline{\mathbf{P}}$ and then performed the following derivations

$$\begin{aligned}&\frac{\partial \mathbf{Q}'_{(2)}}{\partial t} + \overline{\mathbf{A}}_x\frac{\partial \mathbf{Q}'_{(2)}}{\partial x} + \frac{\partial \overline{\mathbf{A}}_x}{\partial x}\mathbf{Q}'_{(2)} + \overline{\mathbf{A}}_y\frac{\partial \mathbf{Q}'_{(2)}}{\partial y}\\ &+\frac{\partial \overline{\mathbf{A}}_y}{\partial y}\mathbf{Q}'_{(2)} = \overline{\mathbf{P}}\Bigg[\frac{\partial \mathbf{U}'_{(2)}}{\partial t} + \overline{\mathbf{A}^o_x}\frac{\partial \mathbf{U}'_{(2)}}{\partial x}\\ &+\overline{\mathbf{A}^o_y}\frac{\partial \mathbf{U}'_{(2)}}{\partial y} + \overline{\mathbf{P}}^{-1}\left(\frac{\partial \overline{\mathbf{P}\mathbf{A}^o_x}}{\partial x} + \frac{\partial \overline{\mathbf{P}\mathbf{A}^o_y}}{\partial y}\right)\mathbf{U}'_{(2)}\Bigg]\end{aligned} \tag{14}$$

If only the main terms are kept in the term $\overline{\mathbf{S}_{\mathrm{RANS}}}$ it can be rewrite as

$$\overline{\mathbf{S}_{\mathrm{RANS}}} = \begin{pmatrix} 0 \\ \dfrac{\partial R_{xx}}{\partial x} + \dfrac{\partial R_{xy}}{\partial y} \\ \dfrac{\partial R_{xy}}{\partial x} + \dfrac{\partial R_{yy}}{\partial y} \\ s_4 \end{pmatrix} \tag{15}$$

$$\begin{aligned}s_4 = &\frac{\partial}{\partial x}\left(\tilde{u}\frac{3R_{xx}+R_{yy}}{2} + \tilde{v}R_{xy}\right)\\ &+\frac{\partial}{\partial y}\left(\tilde{v}\frac{R_{xx}+3R_{yy}}{2} + \tilde{u}R_{xy}\right)\end{aligned}$$

where the following notations designing the back values of Reynolds stresses are used

$$R_{xx} = \overline{\rho u''^2} \qquad R_{xy} = \overline{\rho u''v''} \qquad R_{yy} = \overline{\rho v''^2}. \tag{16}$$

The term $\mathbf{S}'_{(1)(2)}$ in the left hand side of (11) contains the products of $\xi'_{(1)}\eta'_{(2)}$ type and can be considered as a source term describing the interactions between the fluctuations of two types. If the decomposition (10) is accepted this term looks like

$$\mathbf{S}'_{(1)(2)} = (s_1, s_2, s_3, s_4)^T \tag{17}$$

where

$$\begin{aligned}s_1 &= \frac{\partial}{\partial x}\left(u''_{(1)}\rho'_{(2)}\right) + \frac{\partial}{\partial y}\left(v''_{(1)}\rho'_{(2)}\right)\\ s_2 &= \frac{\partial}{\partial t}\left(u''_{(1)}\rho'_{(2)}\right) + 2\frac{\partial}{\partial x}\left(\tilde{u}u''_{(1)}\rho'_{(2)} + \bar{\rho}u''_{(1)}u''_{(2)}\right)\\ &+\frac{\partial}{\partial y}\left(\left(\tilde{u}v''_{(1)} + \tilde{v}u''_{(1)}\right)\rho'_{(2)} + \bar{\rho}\left(v''_{(1)}u''_{(2)} + u''_{(1)}v''_{(2)}\right)\right)\\ s_3 &= \frac{\partial}{\partial t}\left(v''_{(1)}\rho'_{(2)}\right) + 2\frac{\partial}{\partial y}\left(\tilde{v}v''_{(1)}\rho'_{(2)} + \bar{\rho}v''_{(1)}v''_{(2)}\right)\\ &+\frac{\partial}{\partial x}\left(\left(\tilde{u}v''_{(1)} + \tilde{v}u''_{(1)}\right)\rho'_{(2)} + \bar{\rho}\left(v''_{(1)}u''_{(2)} + u''_{(1)}v''_{(2)}\right)\right)\end{aligned}$$

$$\begin{aligned}s_4 = &\\ &\frac{\partial}{\partial t}\left(\left(\tilde{u}u''_{(1)} + \tilde{v}v''_{(1)}\right)\rho'_{(2)} + \bar{\rho}\left(u''_{(1)}u''_{(2)} + v''_{(1)}v''_{(2)}\right)\right)\\ &+\frac{\partial}{\partial x}\Bigg[\left(\frac{3\tilde{u}^2+\tilde{v}^2}{2}u''_{(1)} + \tilde{u}\tilde{v}v''_{(1)}\right)\rho'_{(2)} + \frac{\gamma}{\gamma-1}u''_{(1)}p'_{(2)}\\ &+\bar{\rho}\tilde{u}\left(3u''_{(1)}u''_{(2)} + v''_{(1)}v''_{(2)}\right) + \bar{\rho}\tilde{v}\left(v''_{(1)}u''_{(2)} + u''_{(1)}v''_{(2)}\right)\Bigg]\\ &+\frac{\partial}{\partial y}\Bigg[\left(\frac{\tilde{u}^2+3\tilde{v}^2}{2}v''_{(1)} + \tilde{u}\tilde{v}u''_{(1)}\right)\rho'_{(2)} + \frac{\gamma}{\gamma-1}v''_{(1)}p'_{(2)}\\ &+\bar{\rho}\tilde{v}\left(u''_{(1)}u''_{(2)} + 3v''_{(1)}v''_{(2)}\right) + \bar{\rho}\tilde{u}\left(v''_{(1)}u''_{(2)} + u''_{(1)}v''_{(2)}\right)\Bigg]\end{aligned}$$

The term $\mathbf{S}'_{(1)}$ in the right hand side of (11) models the transport of fluctuations (1) through nonuniform mean flow field including their refraction and looks like the transport terms in the left hand side with the replacement of (2) by (1).

The term $\mathbf{S}^{\star(2)}_{(1)}$ in the right hand side of (11) describes the impact of second order nonlinear terms on fluctuations (1) and has the following form

$$\mathbf{S}^{\star(2)}_{(1)} = (0, s_2, s_3, s_4) \tag{18}$$

where

$$\begin{aligned}s_2 &= \frac{\partial}{\partial x}\left(\bar{\rho}u''^2_{(1)}\right) + \frac{\partial}{\partial y}\left(\bar{\rho}u''_{(1)}v''_{(1)}\right)\\ s_3 &= \frac{\partial}{\partial x}\left(\bar{\rho}u''_{(1)}v''_{(1)}\right) + \frac{\partial}{\partial y}\left(\bar{\rho}v''^2_{(1)}\right)\\ s_4 &= \frac{\partial}{\partial t}\left(\bar{\rho}\frac{u''^2_{(1)} + v''^2_{(1)}}{2}\right)\\ &+\frac{\partial}{\partial x}\left[\bar{\rho}\left(\tilde{u}\frac{3u''^2_{(1)} + v''^2_{(1)}}{2} + \tilde{v}u''_{(1)}v''_{(1)}\right)\right]\\ &+\frac{\partial}{\partial y}\left[\bar{\rho}\left(\tilde{v}\frac{u''^2_{(1)} + 3v''^2_{(1)}}{2} + \tilde{u}u''_{(1)}v''_{(1)}\right)\right]\end{aligned}$$

The similar term $\mathbf{S}^{\star(2)}_{(2)}$ corresponding to the fluctuations of the second type is neglected here due to the assumption of its smallness.

Considering the above relations it is possible to derive the linear acoustic equations with source terms (LAE+S) for the fluctuations of the second type in the following form

$$\frac{\partial \mathbf{Q}'_{(2)}}{\partial t} + \frac{\partial \overline{\mathbf{A}_x}\mathbf{Q}'_{(2)}}{\partial x} + \frac{\partial \overline{\mathbf{A}_y}\mathbf{Q}'_{(2)}}{\partial y} = \mathbf{S}, \tag{19}$$

where

$$\mathbf{S} = \left(\overline{\mathbf{S}_{\mathrm{RANS}}} - \mathbf{S}^{\star(2)}_{(1)}\right) - \mathbf{S}'_{(1)} - \mathbf{S}'_{(1)(2)} \tag{20}$$

It should be noted that in the case of neglecting the term $\mathbf{S}'_{(1)(2)}$ the LAE+S (19) takes the form of Linear Euler Equations with Sources (LEE+S).[8]

In a close accordance with the Lighthill analogy principles[10] if the left hand side of LAE+S (19) describes the transport of fluctuations (2) through the nonuniform mean flow field (that is noise propagation) then the terms in the right hand side represent the source of noise generation.

If the fluctuations (1) and (2) relate to the turbulence and acoustics correspondingly the form of the terms lets us suppose that the first source component (term in the brackets) describes the interactions of "turbulence-turbulence" type while the term $\mathbf{S}'_{(1)(2)}$ can be relied to the "turbulence-acoustics" interactions.

Let us also note here that following to the Lilley's supposition only the quadratic terms consisting of velocity pulsation components can be considered as noise sources.[11] As applied to the LAE+S (19), only the term $\left(\overline{\mathbf{S}_{\mathrm{RANS}}} - \mathbf{S}^{\star(2)}_{(1)}\right)$ meets this supposition.

The case when $\mathbf{S}'_{(1)} = 0$ is put equal to zero corresponds to the situation if considered that the fluctuations of first and second types are respectively of the first and second order of smallness. Then writing sequentially the equations for the terms of the first, second and higher smallness order (by analogy with the method of successive approximations) one can derive the equation for the variables of the first smallness order in the form

$$\overline{\mathbf{P}}\left[\frac{\partial \mathbf{U}'_{(1)}}{\partial t} + \overline{\mathbf{A}^o_x}\frac{\partial \mathbf{U}'_{(1)}}{\partial x} + \overline{\mathbf{A}^o_y}\frac{\partial \mathbf{U}'_{(1)}}{\partial y}\right] = 0 \qquad (21)$$

(see the similar derivations for the fluctuations of second type (14)) or, in a way for conservative variables,

$$\frac{\partial \mathbf{Q}'_{(1)}}{\partial t} + \overline{\mathbf{A}_x\mathbf{P}}\frac{\partial \overline{\mathbf{P}^{-1}}\mathbf{Q}'_{(1)}}{\partial x} + \overline{\mathbf{A}_y\mathbf{P}}\frac{\partial \overline{\mathbf{P}^{-1}}\mathbf{Q}'_{(1)}}{\partial y} = 0 \qquad (22)$$

In this case, the term $\mathbf{S}'_{(1)}$ reduces up to the terms $H_{(1)}$ describing the refraction of turbulent fluctuations by the mean flow

$$H_{(1)} = \left(\frac{\partial \overline{\mathbf{P}\mathbf{A}^o_x}}{\partial x} + \frac{\partial \overline{\mathbf{P}\mathbf{A}^o_y}}{\partial y}\right)\mathbf{U}'_{(1)}$$

However it may be not always valid since in the general formulation used here the derivatives do not have to be of the same smallness order as the variables themselves. From the other side, this approach relates to the hypothesis on convecting noise sources. In the case of decomposition (10) the equation (21) transforms to simple convective equation for nonuniform field with the transport velocity $\mathbf{U}(\mathbf{x}) = (\tilde{\mathbf{u}}, \tilde{\mathbf{v}})$

$$\begin{aligned}\frac{\partial u''_{(1)}}{\partial t} + \tilde{u}\frac{\partial u''_{(1)}}{\partial x} + \tilde{v}\frac{\partial u''_{(1)}}{\partial x} &= 0\\ \frac{\partial v''_{(1)}}{\partial t} + \tilde{u}\frac{\partial v''_{(1)}}{\partial x} + \tilde{v}\frac{\partial v''_{(1)}}{\partial x} &= 0\end{aligned} \qquad (23)$$

So if the hypothesis on convecting noise sources is accepted it is necessary specially to satisfy the equation (21) at the stage of semi-determenistic modeling.

It is proposed to take the LAE+S as a base for further modeling of nonhomogeneous linear equations governing both the propagation and generation of acoustic noise. To use this equation for the acoustic noise prediction in practice, it is necessary find an exact form of the source term $\mathbf{S}$ (20).

It is possible to do if the turbulent pulsation components are somewhat preliminarily predicted on the base of some theoretical, numerical or empirical considerations. A variant of semi-deterministic modeling used is described below.

## Semi-Deterministic Modeling of Velocity Turbulent Fluctuations

### Description of Stochastic Models

Let us simulate turbulent fluctuations of velocity components (that is the fluctuations of the first type) as stochastic time series with the specified statistical properties.

The experimental observations show that in the case of free turbulent flows the spectra of fluctuations generated within shear layers have noticeable peaks. This fact provides the possibility of using the method of envelopes[12] for the statistical modeling of turbulent pulsation. Implementing this method to the problem under study let us predict $u''_{(1)}$ and $v''_{(1)}$ as

$$\begin{aligned}u''_{(1)}(\mathbf{x},t) &= A(\mathbf{x},t)\cos\left(\Psi(\mathbf{x},t)\right)\\ v''_{(1)}(\mathbf{x},t) &= B(\mathbf{x},t)\cos\left(\Psi(\mathbf{x},t) + \varphi(\mathbf{x})\right)\end{aligned} \qquad (24)$$

where $\varphi(\mathbf{x})$ is a deterministic space dependent variable, $A$, $B$ and $\Psi$ - independent random variables with the following probability densities, mathematical expectations and dispersions

$$\begin{aligned}&p(A) = \frac{A}{\sigma_u{}^2}\exp\left(-\frac{A^2}{2\sigma_u{}^2}\right),\\ &E[A] = \sqrt{\frac{\pi}{2}}\sigma_u, \quad D[A] = \frac{4-\pi}{2}\sigma_u{}^2;\\ &p(B) = \frac{B}{\sigma_v{}^2}\exp\left(-\frac{B^2}{2\sigma_v{}^2}\right),\\ &E[B] = \sqrt{\frac{\pi}{2}}\sigma_v, \quad D[B] = \frac{4-\pi}{2}\sigma_v{}^2;\\ &p(\Psi) = \frac{1}{2\pi}, \quad E[\Psi] = \pi, \quad D[\Psi] = \frac{\pi^2}{3}.\end{aligned} \qquad (25)$$

In so doing, the amplitudes $A$ and $B$ are the random variables with the Rayleigh probability distribution. The parameters $\sigma_u$ and $\sigma_v$ used in the definition of probability density and two central moments of variables $A$ and $B$ have a meaning of standard deviation of stochastic variables $u''_{(1)}$ and $v''_{(1)}$ respectively. The phase $\Psi$ is a variable uniformly distributed in the segment $[-\pi, \pi]$.

To describe a stochastic series, the information on time correlation have to be introduced. For this purpose let us derive the time gradients of the variables $u''_{(1)}$ and $v''_{(1)}$ as

$$\begin{aligned} \frac{\partial u''_{(1)}}{\partial t} &= \dot{A}\cos(\Psi) - A\dot{\Psi}\sin(\Psi) \\ \frac{\partial v''_{(1)}}{\partial t} &= \dot{B}\cos(\Psi+\varphi) - B\dot{\Psi}\sin(\Psi+\varphi) \end{aligned} \tag{26}$$

where $\dot{A}$ and $\dot{B}$ are independent random variables normally distributed as

$$\begin{aligned} p(\dot{A}) &= \frac{1}{\sigma_u \Delta\sqrt{2\pi}} \exp\left(-\frac{\dot{A}^2}{2{\sigma_u}^2\Delta^2}\right), \\ E[\dot{A}] &= 0, \quad D[\dot{A}] = \sigma_u\Delta; \\ p(\dot{B}) &= \frac{1}{\sigma_v \Delta\sqrt{2\pi}} \exp\left(-\frac{\dot{B}^2}{2{\sigma_v}^2\Delta^2}\right), \\ E[\dot{B}] &= 0, \quad D[\dot{B}] = \sigma_v\Delta \end{aligned} \tag{27}$$

and the random variable $\dot{\Psi}$ is distributed with the probability density

$$p(\dot{\Psi}) = \frac{\Delta^2}{2\left[\left(\dot{\Psi}-\omega_*\right)^2 + \Delta^2\right]^{3/2}}, \quad E[\dot{\Psi}] = \omega_*. \tag{28}$$

Meaning the peak circular frequency and mean width of the corresponding normalized spectrum, the parameters $\omega_*$ and $\Delta$ characterize spectral and hence correlation properties of the time series.

To simulate the evolution of turbulent fluctuations of velocity components in time, it is proposed to arrange the following stochastic process having the form of regression with the initial values equal to the mathematical expectations of modeled variables (that is equal to zero since $E[u''_{(1)}] = E[v''_{(1)}] = 0$)

$$\begin{aligned} & u''_{(1)}(\mathbf{x},0) = 0, \quad v''_{(1)}(\mathbf{x},0) = 0 \\ & u''_{(1)}(\mathbf{x},t+\tau) = u''_{(1)}(\mathbf{x},t) + \tau\frac{\partial u''_{(1)}}{\partial t}(\mathbf{x},t+\tau) \\ & v''_{(1)}(\mathbf{x},t+\tau) = v''_{(1)}(\mathbf{x},t) + \tau\frac{\partial v''_{(1)}}{\partial t}(\mathbf{x},t+\tau) \end{aligned} \tag{29}$$

So prescribed process provides homogeneous unsteadiness and its stability. The typical example of this stochastic process realization is given in Fig. 1.

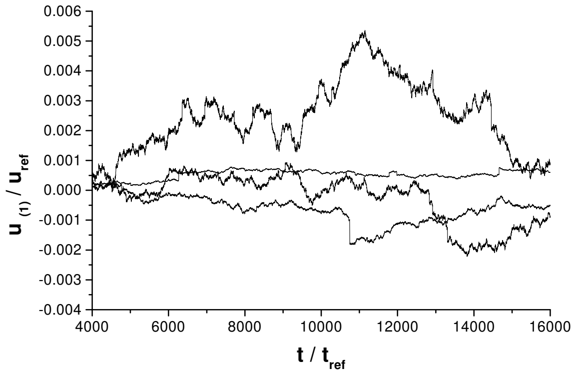


**Fig. 1 Evolution of fluctuations of streamwise turbulent velocity**

In the case of sharply defined spectra, the turbulent velocity components $u''_{(1)}$ and $v''_{(1)}$ can be modeled also as

$$\begin{aligned} u''_{(1)}(\mathbf{x},t) &= A(\mathbf{x},t)\cos(\omega_* t + \Theta(\mathbf{x},t)) \\ v''_{(1)}(\mathbf{x},t) &= B(\mathbf{x},t)\cos(\omega_* t + \Theta(\mathbf{x},t) + \varphi(\mathbf{x})) \end{aligned} \tag{30}$$

where the gradient of random variable $\Theta$ is distributed with the probability density

$$p(\dot{\Theta}) = \frac{\Delta^2}{2\left[\dot{\Theta}^2 + \Delta^2\right]^{3/2}}, \quad E[\dot{\Theta}] = 0. \tag{31}$$

To provide the validity of equation (21), that is to consider the noise sources convection, let us note that the form of $u''_{(1)}$ and $v''_{(1)}$ modeled statistically transforms to a plane wave solution for the simple convection equation if we introduces a special time shift in the representations (24) or (30). In so doing, the model (24) takes the form

$$\begin{aligned} u''_{(1)} &= A(\mathbf{x},t-t_{conv})\cos(\Psi(\mathbf{x},t-t_{conv})) \\ v''_{(1)} &= B(\mathbf{x},t-t_{conv})\cos(\Psi(\mathbf{x},t-t_{conv}) + \varphi) \end{aligned} \tag{32}$$

where time shift $t_{\mathrm{conv}}$ is defined as

$$t_{\mathrm{conv}} = \frac{\mathbf{Ux}}{|\mathbf{U}|^2}$$

The time shift is applied to the model (30) in a similar, even more convenient way. So statistically defined turbulent velocity pulsation components are the solutions of the equation (21) in the sense of mathematical expectation.

Thus, to generate the time series $u''_{(1)}(\mathbf{x},t)$ and $v''_{(1)}(\mathbf{x},t)$, that is to simulate the amplitudes $A(\mathbf{x})$, $B(\mathbf{x})$, phase $\Psi$ or $\Theta$ and their gradients, it is necessary to determine the following parameters of a problem

- standard deviations $\sigma_u(\mathbf{x})$ and $\sigma_v(\mathbf{x})$;
- phase shift $\varphi(\mathbf{x})$;
- peak circular frequency $\omega_*(\mathbf{x})$ and mean spectrum width $\Delta(\mathbf{x})$.

**Standard Deviations $\sigma_u$ and $\sigma_v$. Phase Shift $\varphi$**

The standard deviations $\sigma_u$ and $\sigma_v$ as well as the phase shift $\varphi$ needed for constructing the above stochastic process can be predicted on the base of the mean turbulence characteristics calculated previously by steady RANS equations. For this purpose, let us take the fields Reynolds stresses (equal to $-R_{xx}$, $-R_{xy}$ and $-R_{yy}$ respectively (16)). The diagonal stresses carry the information on parameters $\sigma_u$ and $\sigma_v$. To reveal it, one can use the ergodic hypothesis and find that the time averaged square of velocity fluctuations presents the dispersion of these fluctuations

$$\begin{aligned} \overline{{u''_{(1)}}^2} &= \overline{A^2\cos^2\Psi} = \langle A^2\cos^2(\Psi)\rangle = \sigma_u^2 \\ \overline{{v''_{(1)}}^2} &= \overline{B^2\cos^2(\Psi+\varphi)} = \langle B^2\cos^2(\Psi+\varphi)\rangle = \sigma_v^2 \end{aligned} \tag{33}$$

where the notation $\langle\cdot\rangle$ means ensemble averaging.
The phase shift $\varphi$ between axial and radial components of turbulent velocity fluctuations can be revealed in a similar way as

$$\begin{aligned} \overline{u''_{(1)}v''_{(1)}} &= \overline{AB\cos\Psi\cos(\Psi+\varphi)} \\ &= \langle AB\cos\Psi\cos(\Psi+\varphi)\rangle = \frac{\pi}{4}\sigma_u\sigma_v\cos\varphi. \end{aligned} \tag{34}$$

Thus, under the assumption that $\overline{\rho {u''_{(1)}}^2} \approx \bar\rho\overline{{u''_{(1)}}^2}$, $\overline{\rho {v''_{(1)}}^2} \approx \bar\rho\overline{{v''_{(1)}}^2}$, $\overline{\rho u''_{(1)}v''_{(1)}} \approx \bar\rho\overline{u''_{(1)}v''_{(1)}}$ one can approximately find the required parameters of a problem

$$\sigma_u^2 = \frac{R_{xx}}{\bar\rho}, \quad \sigma_v^2 = \frac{R_{yy}}{\bar\rho}, \quad \sigma_u\sigma_v\cos\varphi = \frac{4}{\pi}\frac{R_{xy}}{\bar\rho}. \tag{35}$$

The information on standard deviations $\sigma_u$ and $\sigma_v$ automatically describes the location of sources distributed within a computational domain.
It should be noted that $\cos\varphi$ appears directly connected with the coefficient of correlation between $u''_{(1)}$ and $v''_{(1)}$[13]

$$\left|K(u''_{(1)},v''_{(1)})\right| = \frac{\left|\overline{u''_{(1)}v''_{(1)}}\right|}{\sqrt{\overline{{u''_{(1)}}^2}}\sqrt{\overline{{v''_{(1)}}^2}}} = \frac{\pi}{4}\left|\cos\varphi\right| < 1. \tag{36}$$

**Peak Frequency $\omega_*$. Spectrum Width $\Delta$**

The introduction of knowledge on such spectral properties as peak frequencies and mean spectrum width completes the proposed stochastic process of turbulent velocity field generation. The main supposition here is the presence of the sole peak in the pulsation spectra in shear layers. It is also assumed that this peak corresponds to the most unstable frequency and can be somewhat predicted in every space point. In particular, the unstable frequencies can be found with the help of stability analysis. However in the general case, for instance in the case of complicated configurations, it can be not so easy. As an alternative, it is proposed to implement numerically a special experiment on the flow exposure to acoustic noise radiation from artificial sources. Such numerical experiments were used before for predicting the characteristic frequencies in gas flows around bodies.[9] It concludes in the flow exposure to acoustic radiation and detection of the most amplified modes in the signal responses in different space points.

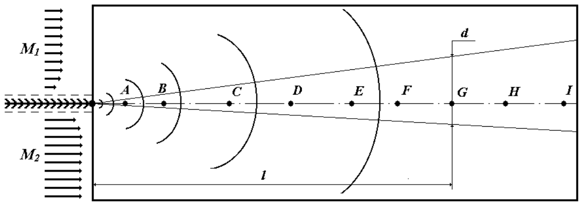


**Fig. 2 Scheme of flow exposure to noise radiation in mixing layer**

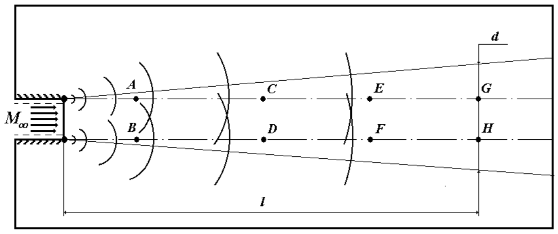


**Fig. 3 Scheme of flow exposure to noise radiation in jet**

For the problems on jets and mixing layers under consideration the law of acoustic source emission can be preset, for example, as

$$\xi'_{rand}(y,t) = a\,\bar\xi\sin(2\pi f_{rand}(t)\exp\left(-\left(\frac{y}{\delta}\right)^2\right) \tag{37}$$

where $f_{\mathrm{rand}}$ is picked uniformly from the preassigned segment, $\xi$ is the mean value of the excited parameter $\bar\xi$, $a$ characterizes the excitement level (we used $a = 0.005$), $\delta$ is a boundary layer width in the case of locating the artificial sources within boundary layers inside a nozzle exit (for jets) or at both sides of the splitting plate (for mixing layers).
At analysing the responses in a few space points distributed within the shear layers one can neglect the impact of the transverse direction and suppose that each cross-section has the same characteristic frequency. In

doing so, we find that the amplified frequencies detected by the means of numerical experiment perfectly satisfy to the known proportionality relation for plane mixing layers[8]

$$\omega_* = 2\pi\,\mathrm{St}\frac{U_M}{x} \tag{38}$$

where $U_M$ is a half-sum of free-stream speeds.

Here the co-ordinate $x$ means a longitudinal size that is a distance from the splitting plate tip to a point within the shear layer.

Under the above considerations, the knowledge of amplified frequencies in some points provides a way of predicting the Strouchal number St for the whole shear layer problem and, therefore, the frequency characteristics for the whole computational domain.

In order to predict the mean spectrum width $\Delta$, we are forced to advance one assumption more. Let us suppose that the normalized spectrum is approximately the same both in the cases of artificially excited and real turbulent flows, and the principal difference remains only in the power of the corresponding spectra which is characterized by the dispersion. If this supposition is accepted one can analyse the fluctuations' spectra for the excited flow and calculate the mean spectrum width using the formula

$$\Delta^2 = 2\int_0^\infty (\omega-\omega_*)^2\, s(\omega)\, d\omega \tag{39}$$

where $s(\omega) = \dfrac{S(\omega)}{\sigma^2}$ is a spectrum normalized by the dispersion $\sigma^2$, so that $\int_0^\infty s(\omega)\, d\omega = 1$.

In the case of plane mixing layers it was found that the mean spectrum width $\Delta$ also well satisfy to the similar back proportionality law as (38) with the replacement of Strouchal number $St$ by the introduced coefficient $C_\Delta$. So following the same way as in the case of peak frequencies, one can predict the spectrum width $\Delta$ in every shear layer point.

In the plane jet problems, when considering the control points within both shear layers around the jet we have not found a unified similarity law. This fact is connected with the difference in shear layer development in the region upstream from the end of jet core and in the region of fully expanded jet. To avoid the necessity of conjunction of different similarity laws, the interpolation both of peak frequencies $\omega_*$ and mean spectrum width $\Delta$ found in a set of control points was used.

In general, it is supposed that the acoustic noise of real flows is mostly produced on the unstable frequencies predicted with the proposed numerical experiment. The wide band noise radiation is provided by the stochastic sound nature modeled as a result of stochastic description of turbulent velocity fluctuations.

## Numerical Implementation

The proposed way of acoustic noise prediction is numerically implemented in the form of 3 consecutive in time computational stages. At first, the steady mean flow parameters are computed with the help of RANS equations (5). The output result of the first stage is the nonuniform mean fields of density, velocity, pressure and Reynolds stresses (16). The second stage concludes in the numerical experiment on flow exposure to white noise irradiation from artificial sources. The goal of this computation is the prediction of peak frequencies and mean spectrum widths. The third stage consists in generating the turbulent velocity pulsation field as (29) and solving the LAE+S (19) which are carried out in parallel.

At the first stage, the RANS equations with standard $k-\varepsilon$ turbulence model are computed. In some problems, the turbulent viscosity coefficient $C_\mu$ is taken equal to 0.05 in contrast with the usual value 0.09, in order to decrease the impact of turbulent viscosity. The Roe scheme or kinetically consistent finite difference (KCFD) schemes of the second accuracy order[14] with the use of MUSCL-approximation for the space derivatives and the Jameson time stepping of the second or third order[15] are implemented.

At solving the LAE+S, the second order MacCormack scheme or second order linearised KCFD schemes have been used. The treatment of free output boundaries are carried out with using either the Thompson characteristic boundary conditions[16] or the perfect match layer.[17] In the last case, the volume of buffer zone is experimentally determined as about a half-size of real computational domain.

## Numerical Results

The results of implementing the above way of noise field modeling are demonstrated here on the example of plane mixing layer problems.

As a mixing layer problem the supersonic and subsonic test cases are taken from the paper.[8] The main parameters of the problems are the following $M_a = 3.1$, $M_b = 1.7$, $M_c = 0.33$, $\delta_{\theta 0} = 0.556$ mm (in the case of supersonic flows) and $M_a = 0.7$, $M_b = 0.3$, $M_c = 0.19$, $\delta_{\theta 0} = 0.107$ mm (in the subsonic case), $Re_{\delta_{\theta 0}} = U_a \delta_{\theta 0}/\mu_a = 2000$, where $M_c$ is the convective Mach number, $\delta_{\theta 0}$ – the momentum thickness. The instantaneous acoustic fields for pressure and transverse velocity pulsation for both problems are given respectively in Figures 4, 5, 7, 8. The dilatation profiles for the supersonic mixing layer are demonstrated in Figure 6.

## Concluding Remarks

A way to modeling acoustic noise generated in turbulent gas flows is developed. Following the proposed strategy the acoustic noise generation and propagation

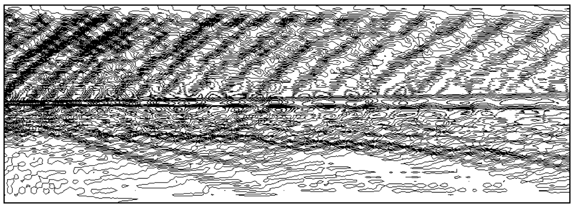

**Fig. 4 Pressure pulsation field in supersonic mixing layer**

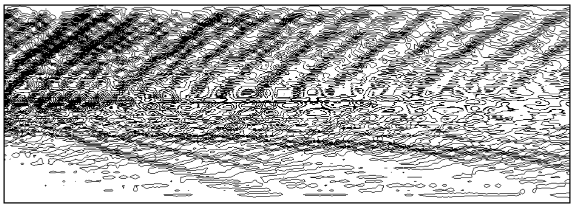

**Fig. 5 Transverse velocity pulsation field in supersonic mixing layer**

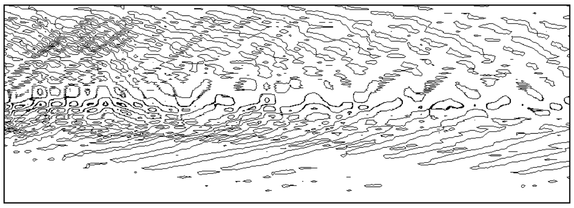

**Fig. 6 Dilatation contours in supersonic mixing layer**

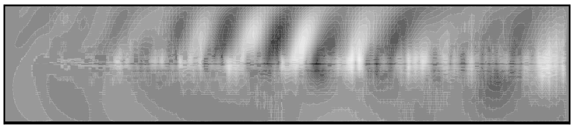

**Fig. 7 Pressure pulsation field in subsonic mixing layer**

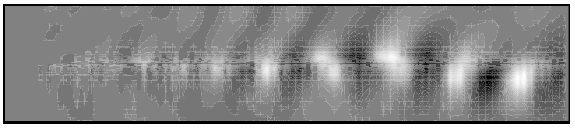

**Fig. 8 Transverse velocity pulsation field in subsonic mixing layer**

are directly simulated in a whole computational domain (including shear layers themselves and the fields around).

At the same time, the authors recognize the fact that there is no rigorous substantiation of the decomposition of fluctuations and this problem still needs a deep study. We also admit that a careful investigation may result in a significant narrowing of applicability limits. Taking this in mind, we tried to conduct all the derivations and considerations in the most general form. From this point of view, it seems useful to study the models of sound field simulation basing directly on the NLDE that is without applying the decomposition of disturbances. Another promising way not requiring the decomposition of fluctuations may be the usage of LES results as input mean flow fields at solving the equations for pulsation components. In this case, the LES equations and NLDE (or its linear analogue) have to be solved in parallel (probably, on the meshes of different scales) with the consideration of mutual influence of filtered flow field and noise dynamics.

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## References

[1]Struminskii, V., "On One New Direction of Studying Turbulence Problems," *Turbulent flows*, edited by V. Struminskii, Moscow, Nauka, 1977, pp. 20–24, in Russian.

[2]Bailly, C. and Juve, D., "Numerical Solution of Acoustic Propagation Problems Using linearized Euler's Equations," AIAA Paper 98–2267, 1998.

[3]Mankbadi, R., Hixon, R., Shih, S.-H., and Povinelli, L., "Use of Linearized Euler Equations for Supersonic Jet Noise Prediction," *AIAA Journal*, Vol. 36, 1998, pp. 140–147.

[4]Viswanathan, K. and Sankar, L.-N., "Toward the direct calculation of noise: Fluid/acoustic coupled simulation," *AIAA Journal*, Vol. 33, 1995, pp. 2271–2279.

[5]Bailly, C. and Juve, D., "A Stochastic Approach To Compute Subsonic Noise Using Linearized Euler's Equations," AIAA Paper 99–1872, 1999.

[6]Bechara, W., Bailly, C., Lafon, P., and Candel, S., "Stochastic Approach to Noise Modeling for Free Turbulent Flows," *AIAA Journal*, Vol. 32, No. 3, 1994, pp. 455–463.

[7]Colonius, T., Lele, S., and Moin, P., "The Sound Generated by a Two-Dimensional Shear Layer: The Far Field Directivity from Computations and Acoustic Analogies," *Comput. Aeroacoustics ASME*, Vol. 219, 1995, pp. 47–52.

[8]Bastin, F., Lafon, P., and Candel, S., "Computation of jet mixing noise due to coherent structures," *J. Fluid Mech.*, Vol. 335, 1997, pp. 261–304.

[9]Alexandrov, A., Chetverushkin, B., and Kozubskaya, T., "Numerical Investigation of Viscous Compressible Gas Flows by Means of Flow Field Exposure to Acoustic Radiation," *Proceedings of Parallel CFD 2000 conference, Trondheim, Norway*, North Holland, Elservier, May 2000.

[10]Lighthill, M., "On sound generated aerodynamically: 1. general theory," *Proc. Roy. Soc. London*, No. 211 in A, 1952, pp. 564–587.

[11]Lilley, G., "The Radiated Noise from Isotropic Turbulence," *Theoret. Comput. Fluid Dynamics*, Vol. 6, 1994, pp. 281–301.

[12]Bunimovich, V., *Fluctuational process in radio receiving devices*, Moscow, Sovetskoe radio, 1951, in Russian.

[13]Schlichting, H., *Grenzshicht theorie*, Verlag G. Braun, Karlsruhe, 1965.

[14]Elizarova, T. and Chetverushkin, B., "On one numerical algorithm for calculation of gasdynamic flows," *Dokladi Akademii Nauk SSSR*, Vol. 279, No. 1, 1984, pp. 80–83, in Russian.

[15]Jameson, A., "Numerical Solution of the Euler Equations for Commpessible inviscid Fluids," *Numerical Methods for the Euler Equations of Fluid Dynamics*, SIAM,Philadelphia, 1985, pp. 199–231.

[16]Thompson, K., "Time-Dependent Boundary Conditions for Hyperbolic Systems, II," *J. Comput. Phys.*, Vol. 89, 1990, pp. 439–461.

[17]Hayder, M., Hu, F., and Hussaini, M., "Towards Perfectly Absorbing Boundary Conditions for Euler Equations," ICASE Report 97-25, 1997.