Contribution I3M-Montpellier

Bruno Koobus*

(*)I3M, Université Montpellier II France

Kick off MAIDESC

Role of I3M

- Space-time error estimations for MUSCL
- Multi-rate time-advancing
- Curved CENO and error

Role of I3M
Research

1. Space-time error estimations for MUSCL

Montpellier will develop a space-time error model for a second-order ALE formulation. The input is an unsteady metric, a mesh velocity and a time-step all defined along time-interval. The error model will rely on time interpolation errors, the space component being estimated as in [LDA] for the MUSCL scheme. This work will be performed in the parallel scalable CFD platform AIRONUM. AIRONUM was demonstrated in adaptive mode in cooperation with Rocquencourt during ANR-ECINADS.

T3-D5 (Montpellier, M18): Space-time error for a second-order ALE scheme for 3D Navier-Stokes, Theoretical report.

Research (2)

2. Multi-rate time-advancing

Many physical phenomena show multiple scales to which mesh adaption will apply successfully, but the time-step for smallest details should not be applied to larger details. Montpellier will introduce a multi- rate time-advancing model in the complexity analysis (determining the optimal discretization) and will develop a multi-rate scheme in the computer solution algorithm.
Montpellier will implement a multi-rate scheme into the parallel scalable LES solver AIRONUM.
Parallel time advancing algorithms will be developped which will achieve optimal load balancing between processors.

T5-D2 (Montpellier): Multi-rate time-advancing for high-Reynolds LES, theoretical report (M12). Implementation and numerical results, test case ATC2 (M42).
T5-D5 (Montpellier): Synthesis on mesh/PDE global adaption algorithms (M45)

Research (3)

3. Curved CENO and error

Contribution of INRIA and Montpellier:
- Errors for order 3 and curved elements.
- Development of a third space-accurate sheme in curved boundaries, implementation in the parallel CFD solver AIRONUM, and application to test case ATC2.

Test cases :

ATC1: Falling water column (Sophia: resp., Cemef, Lemma, Montpellier): This simple test case shows three scales (small: interface motion, medium: water velocity, large: gas velocity) and will be used for measuring the progress in algorithms.
ATC2: Unsteady (LES) flow around a circular cylinder at different Reynolds numbers (Montpellier, Sophia): This test case will focus on the evaluation of the efficiency of the multirate time advancing.

T2-D2 (Montpellier): ATC2 specification and first computation, M9.

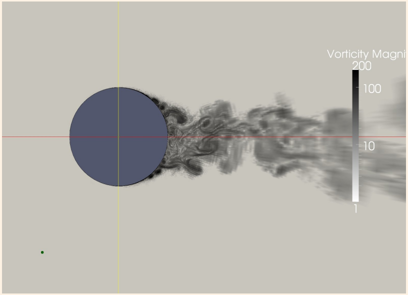
Thesis

Error estimations for ALE formulation and MUSCL scheme, third order accurate schemes with curved boundaries, parallel multi-rate time-advancing scheme.

Motivations

An example

Circular cylinder at Reynolds 1M : thin boundary layers and unsteady separated shear layers, with vortex shedding.



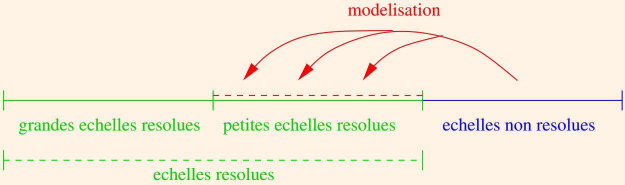
Circular cylinder (1.2M nodes), iso-contours of the vorticity magnitude.
Reynolds number = 1M, Mach number = 0.1, RANS/VMS-LES hybrid model.

Turbulence modeling and applications

VMS-LES approach

Main features :

- Approach based on variational projections of the Navier-Stokes equations $\Rightarrow$ equations governing different scales of the solution (large resolved scales, small resoved scales, unresolved scales),
- Effects of the unresolved scales only modeled in the equations governing the small resolved scales :



Turbulence modeling and applications

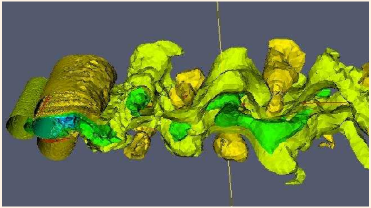
VMS-LES approach (2)

The VMS-LES option chosen allows to take into account :

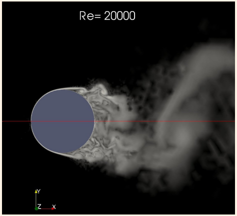
- the 3D compressible Navier-Stokes equations,
- unstructured meshes,
- a finite element/finite volume formulation,
- the scales separation with a simple and efficient procedure obtained from averaging on macro-cells,
- bluff body flows with vortex sheding.

Turbulence modeling and applications

Circular cylinder



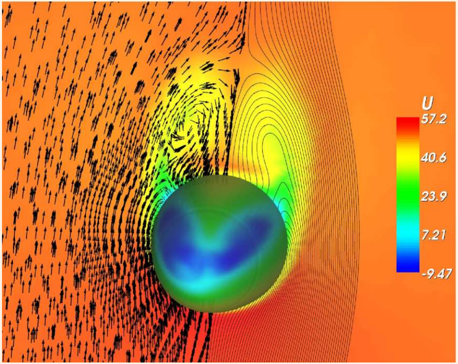
Circular cylinder, Reynolds 3900 : instantaneous streamwise velocity.



Circular cylinder, Reynolds 20000 : magnitude of vorticity.

Turbulence modeling and applications

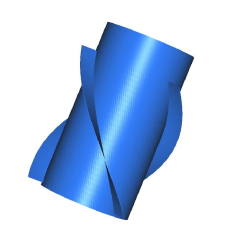
Prolate spheroid



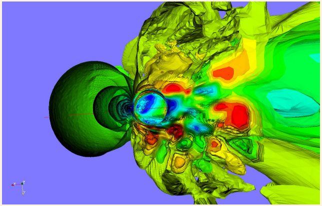
Prolate spheroid : velocity and streamlines (Reynolds 40000).

Turbulence modeling and applications

Spar

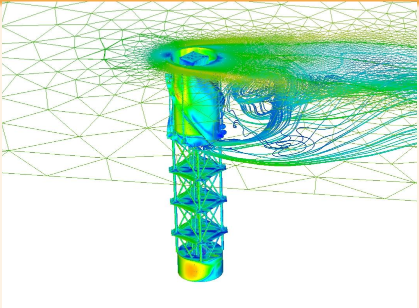


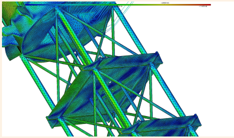
Simplified geometry of the spar.

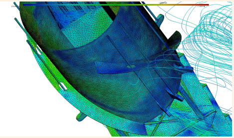


Flow around the spar (Reynolds 300000).

Spar (2)



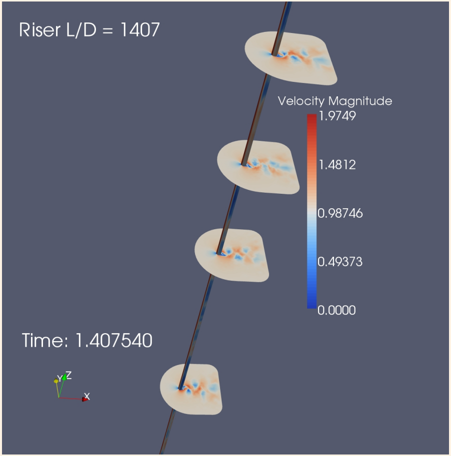




Simulation of flow around a spar with lower tubular metal structure.

Turbulence modeling and applications

Riser



Simulation of the flow around a riser.

RANS/VMS-LES hybrid model

- Central idea of this hybrid approach :
  - Solve the RANS equations in the whole domain,
  - Correct the mean flow field by adding fluctuations provided by a VMS-LES model in regions where the grid resolution is fine enough for VMS-LES.
- Basic ingredients of this hybrid approach :
  - a RANS model,
  - a VMS-LES model,
  - a blending function.

RANS/VMS-LES hybrid model (2)

$$\left(\frac{\partial W_h}{\partial t}, \mathscr{X}_i\right) + \left(\nabla \cdot F(W_h), \mathscr{X}_i, \Phi_i\right) = -\theta\left(\tau^{RANS}\left(W_h\right), \Phi_i\right) - (1-\theta)\left(\tau^{LES}\left(W'_h\right), \Phi'_i\right)$$

where $\theta = tanh\left[\left(\frac{\Delta}{l_{RANS}}\right)^2\right]$

with $l_{RANS} = \dfrac{k^{3/2}}{\varepsilon}$ et Δ = local mesh size.

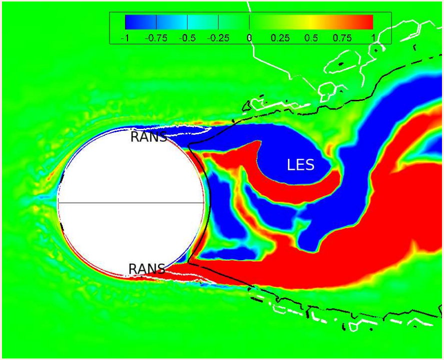
$$\Downarrow$$

$\Delta \ll l_{RANS}$: $\theta \rightarrow 0$ (VMS-LES mode)

$\Delta \gg l_{RANS}$: $\theta \rightarrow 1$ (RANS mode)

Turbulence modeling and applications

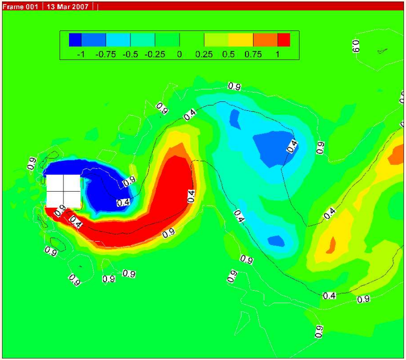
Circular cylinder, Reynolds number 140000



Instantaneous isocontours of spanwise vorticity (simulation GR1-LRS). The lines are the isolines of the blending function: $\theta = 0.1$ (black) and $\theta = 0.9$ (white)

Turbulence modeling and applications

Square cylinder, Reynolds number 22000



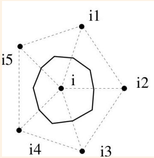
Isocontours of spanwise vorticity. The lines are the isolines of the blending function: $\theta = 0.4$ (black) and $\theta = 0.9$ (white).

Transition to order 3

Definition of the scheme (CENO)

Finite Volume approach (2D) :

$$\frac{d}{dt}\int_{C_i} u(x,y,t)dxdy + \int_{\partial C_i} \vec{f}(u(x,y,t)).\vec{n}ds = 0$$



$$\frac{d}{dt}\int_{C_i} u(x,y,t)dxdy + \sum_{k\in V(i)} \int_{\partial C_i \cap \partial C_k} \vec{f}(u(x,y,t)).\vec{n}ds = 0$$

Transition to order 3 (2)

Definition of the scheme (CENO)

Polynomial reconstruction :

Average of a function g over cell C_k : $\overline{g}^k = \frac{1}{area(C_k)} \int_{C_k} g(x,y)dxdy$

We define $$P_i^n = \overline{u^n}^i + \sum_{\alpha \in I} c_{i,\alpha}^n \left[(X - X_{0,i})^\alpha - \overline{(X - X_{0,i})^\alpha}^i \right]$$

$\overline{P_i^n}^i = \overline{u^n}^i$ is satisfied.

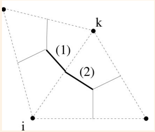
$c_{i,\alpha}^n$ chosen to minimize $H_i = \sum_{k \in N(i)} (\overline{P_i^n}^k - \overline{u^n}^k)^2$

$\Rightarrow$ Linear system with unknowns $c_{i,\alpha}^n$ (5 in 2D)

Transition to order 3 (3)

Definition of the scheme (CENO)

Flux evaluation :



Interfaces $C_i \cap \partial C_k$ between C_i and C_k, (1) : $\partial C_{ik}^{(1)}$ and (2) : $\partial C_{ik}^{(2)}$

$$\begin{aligned}
\int_{\partial C_i \cap \partial C_k} \vec{f}(u(x,y,t)).\vec{n}ds &= \sum_{l=1,2} \int_{\partial C_{ik}^{(l)}} \vec{f}(u(x,y,t)).\vec{n}ds \\
&= \sum_{l=1,2} \int_{\partial C_{ik}^{(l)}} \vec{f}(P_i(x,y,t)).\vec{n}ds \\
&= \sum_{m=1,2} \omega_m \vec{f}(P_i(x_{g_m,ik}^{(l)}, y_{g_m,ik}^{(l)}, t)) \vec{\nu_{ik}}^{(l)}
\end{aligned}$$

Transition to order 3 (4)

Definition of the scheme (CENO)

Flux evaluation (2) :

$$\vec{f}(P_i(x^{(l)}_{g_m,ik}, y^{(l)}_{g_m,ik}, t)).\vec{\nu}_{ik} = \Phi(P_i(x^{(l)}_{g_m,ik}, y^{(l)}_{g_m,ik}, t), P_k(x^{(l)}_{g_m,ik}, y^{(l)}_{g_m,ik}, t), \vec{\nu}_{ik})$$

where Roe's scheme is used as approximate Riemann solver :

$$\Phi(u_1, u_2, \vec{\nu}) = \frac{\vec{f}(u_1) + \vec{f}(u_2)}{2}.\vec{\nu} - \frac{\gamma}{2}\left|\frac{\partial \vec{f}}{\partial u}\left(\frac{u_1 + u_2}{2}\right).\vec{\nu}\right|(u_2 - u_1)$$

Application with mesh adaption

Scramjet, thesis of A. Carabias (INRIA Sophia and Rocquencourt)

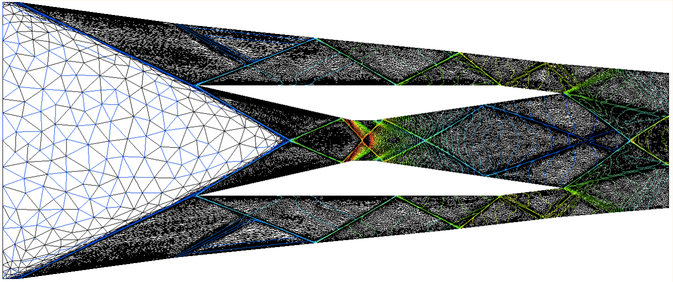
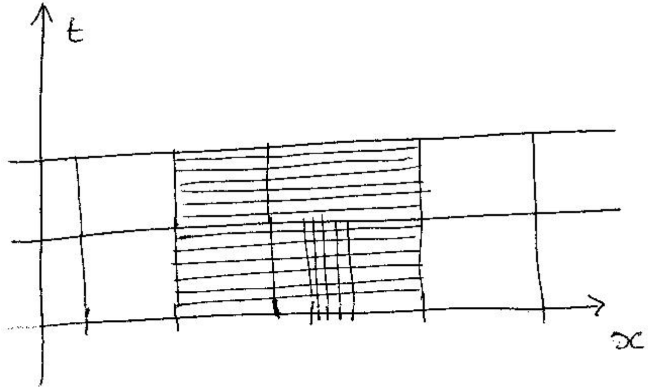


Figure: 2D anisotropic mesh adaption (31460 nodes), iso-contours of Mach number. Inlet Mach number = 3, CENO scheme.

Schémas “multirate”

Multi-rate:

Avancer en temps de manière précise en utilisant des plus grands pas de temps sur les zones grossières.



Schémas “multirate”(2)

De nombreuses méthodes :

Gear, Wells, Engstler, Lubich Lohner, Morgan, Zienkiewicz, Berger Collino, Fouquet, Joly, Piperno, Diaz, Grote, Dawson, Du, Dupont, Blum, Lisky, Rannacher, Ewing, Lazarov, Vassilev, Faille, Nataf, Wolf, Hulbert, Hughes, Erickson, Guoy, Sullivan, Ungor, Cailleau, Fedorenko, Barnier, Blayo, Debreu, Halpern, Nataf,...

Une première direction

Prédicteur sur une grille grossière, interpolation sur la grille fine et en temps, correcteur sur la grille fine (cf. Blayo *et al.*).

Merci pour votre attention.