Adaptation de maillage pour des géométries mobiles

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Motivations

Objectives

- Update the ALE Hessian-metrics for mesh adaptation with moving geometries.
- Combine goal-oriented mesh adaptation and simulations with moving bodies.

1. Introduction

2. Time-accurate Hessian-based Mesh Adaptation
   - State of the art
   - ALE metric Update

3. Anisotropic Goal-oriented mesh adaptation
   - State of the art
   - Adjoint Resolution ALE
   - Mesh adaptation

Mesh Adaptation

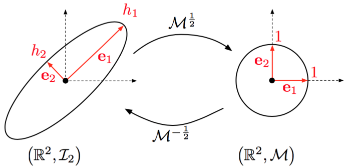
Main idea : introduce the use of metrics field, and notion of unit mesh.

[George, Hecht and Vallet., Adv Eng. Software 1991]

- Riemannian metric space: $\mathcal{M}$: $d \times d$ symmetric definite positive matrix

$$\langle u, v \rangle_{\mathcal{M}} = {}^t\, \mathbf{u}\mathcal{M}\mathbf{v} \Rightarrow \ell_{\mathcal{M}}(\mathbf{a}, \mathbf{b}) = \int_0^1 \sqrt{{}^t\mathbf{ab}\ \mathcal{M}(\mathbf{a} + \mathbf{t}\mathbf{ab})\ \mathbf{ab}}\ \mathrm{d}\mathbf{t}$$

$$|K|_{\mathcal{M}} = \int_K \sqrt{\det \mathcal{M}}\ \mathrm{d}|K|$$



continuous **Metric Field** → discrete **Mesh**.

Time-accurate Feature-based Mesh Adaptation

Deriving the best mesh to compute the characteristics of a given solution w in space and time

[Tam et al.,CMAME 2000], [Picasso, SIAMJSC 2003], [Formaggia et al, ANM 2004], [Frey and Alauzet CMAME 2005], [Gruau and Coupez, CMAME 2005], [Huang, JCP 2005], [Compere et al., 2007]

- **Discrete** space-time mesh adaptation problem :

Find $\mathcal{H}_{L^p}^{opt}$ having N_{st} vertices such that

$$\mathcal{H}_{L^p}^{opt} = \mathrm{Argmin}_{\mathcal{H}} ||u - \Pi_h u||_{\mathcal{H},\mathbf{L}^p(\Omega\times[0,T])}$$

- Well-posed **Continuous** space-time mesh adaptation problem :

Find $\mathcal{M}_{\mathbf{L^p}}^{\mathbf{opt}}$ of complexity N_{st} such that

$$E_{L^p}(\mathcal{M}_{L^p}^{opt}) = \min_{\mathcal{M}} \left(\int_0^T \int_\Omega \mathrm{Trace}(\mathcal{M}(\mathbf{x},t)^{-\frac{1}{2}} |H_u(\mathbf{x},t)| \mathcal{M}(\mathbf{x},t)^{-\frac{1}{2}})^p \mathrm{d}\mathbf{x}\mathrm{d}t \right)^{\frac{1}{p}}$$

$\Rightarrow$ Solved by variational calculus

Optimal Mesh

$$\mathcal{M}_{L^p}^{opt} = N_{st}^{\frac{2}{3}} \left(\int_0^T \tau(t)^{\frac{-2p}{2p+3}} \mathcal{K}(t) \mathrm{d}t \right)^{-\frac{2}{3}} \tau(t)^{\frac{2}{2p+3}} \left(\det|H_u(\mathbf{x},t)|\right)^{\frac{-1}{2p+3}} |H_u(\mathbf{x},t)|$$

with $\mathcal{K}(t) = \left(\int_\Omega \left(\det|H_u(\mathbf{x},t)|\right)^{\frac{p}{2p+3}} \mathrm{d}\mathbf{x} \right)$

Global normalization term requires the whole computation

- A global fixed-point algorithm
  - $\Rightarrow$ to compute the space-time metric complexity
  - $\Rightarrow$ to converge the **non-linear** mesh adaptation problem
  - $\Rightarrow$ to predict the solution evolution
- Split the simulation into several time sub-intervals and set an adapted mesh for each sub-interval
  - $\Rightarrow$ to limit the number of meshes

$$[0, T] = [t_0 = 0, t_1] \cup [t_1, t_2] \cup \ldots [t_{kmax}, T]$$

Unsteady Feature-based Mesh Adaptation : Algorithm

```
For j=1,nptfx
 For i=1,nadap
```

- $\mathcal{S}^j_{0,i} = \texttt{InterpolateSolution}(\mathcal{H}^j_{i-1}, \mathcal{S}^j_{i-1}, \mathcal{H}^j_i)$
- $\mathcal{S}^j_i = \texttt{SolveState}(\mathcal{S}^j_{0,i}, \mathcal{H}^j_i)$
- $|H_{\max}|^j_i = \texttt{ComputeHessianMetric}(\mathcal{H}^j_i, \{\mathcal{S}^j_i(k)\}_{k=1,nk})$

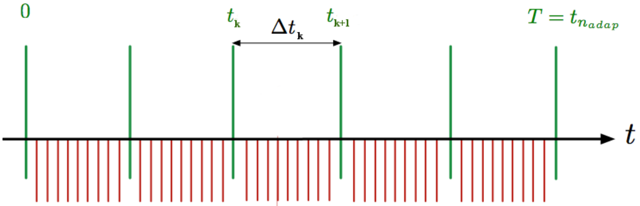
End for $\quad \mathcal{C}^j = \texttt{ComputeSpaceTimeComplexity}(\{|H_{\max}|^j_i\}_{i=1,nadap})$

$\mathcal{M}^{j-1}_i = \texttt{ComputeUnsteadyLpMetrics}(\mathcal{C}^{j-1}, |H_{\max}|^{j-1}_i)$

$\mathcal{H}^j_i = \texttt{GenerateAdaptedMeshes}(\mathcal{H}^{j-1}_i, \mathcal{M}^{j-1}_i)$

End for

$$[0, T] = [t_0 = 0, t_1] \cup [t_1, t_2] \cup \ldots [t_{kmax}, T]$$



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End for

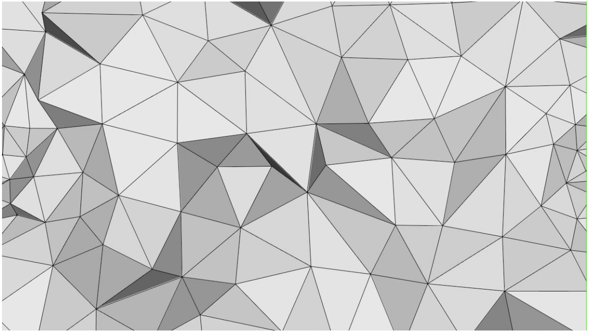
$$[0, T] = [t_0 = 0, t_1] \cup [t_1, t_2] \cup \ldots [t_{kmax}, T]$$

It is then possible to take into account the mesh motion inside the error estimate $\Longrightarrow$ optimal space-time adapted mesh in ALE framework

Mesh optimization for moving meshes

As mesh quality tends to decrease while the mesh is moving
$\Rightarrow$ regular **optimization phases** must be performed : smoothing and edge swapping

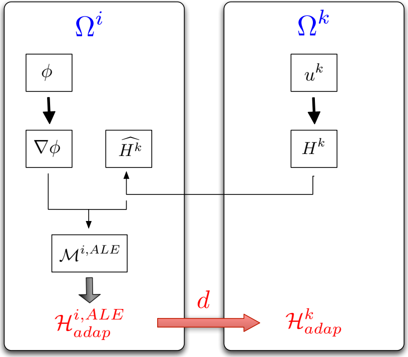
- Two planes moved at Mach 0.4 inside an inert air.
- The planes are translated and rotating
- 50 sub intervals and 3 adaptation loops
- Total space time complexity: $36,000,000$ vertices, average mesh size: $732,000$ vertices, $80,000$ timesteps



Moving the ALE metric

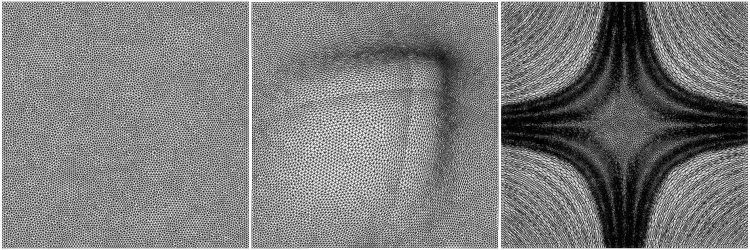
Let t^k be a time into the subset $[t^i, t^{i+1}]$,

$$\begin{array}{llll} \phi: & \Omega^i & \rightarrow & \Omega^k \\ & \mathbf{x}^i & \mapsto & \mathbf{x}^k = \mathbf{x}^i + \mathbf{d}(\mathbf{x}^i). \end{array}$$

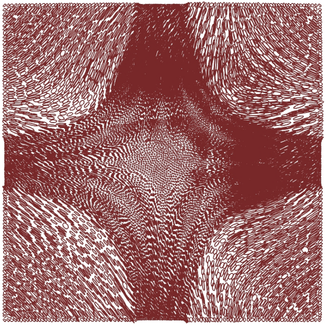


ALE Metric at t^k

Moving ALE metric



(From left to right) Mesh at t^i, Mesh at t^k, Hessian metric computed at t^k



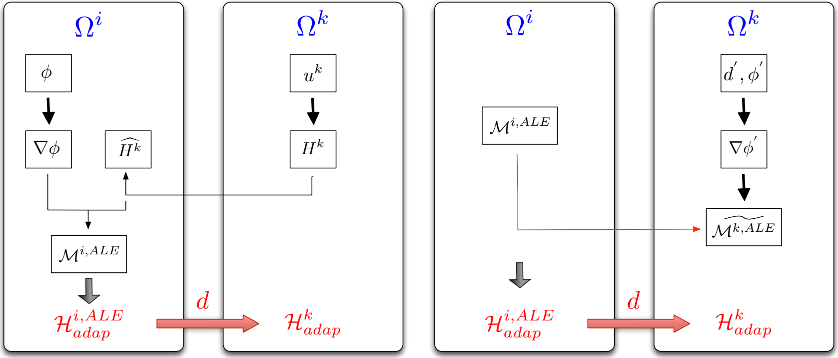
Metric without update, Updated metric

ALE metric update

Let t^k be a time into the subset $[t^i, t^{i+1}]$,

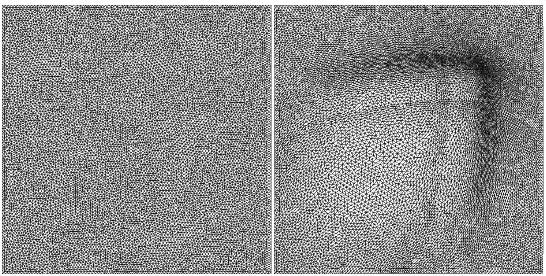
$$\phi' : \begin{array}{ccc} \Omega^k & \rightarrow & \Omega^i \\ \mathbf{x}^k & \mapsto & \mathbf{x}^i = \mathbf{x}^k + \mathbf{d}'(\mathbf{x}^k). \end{array}$$

$$\begin{aligned}
1 &= \left(\mathbf{e}^i(\mathbf{x}^i)\right)^T \ \mathcal{M}_{Lp}^{i,\mathrm{ALE}}\left(\mathbf{x}^i\right) \ \left(\mathbf{e}^i(\mathbf{x}^i)\right) \\
&= \left(\mathbf{e}^i(\,\boldsymbol{\phi}'(\mathbf{x}^k)\,)\right)^T \ \mathcal{M}_{Lp}^{i,\mathrm{ALE}}\left(\,\boldsymbol{\phi}'(\mathbf{x}^k)\,\right) \ \left(\mathbf{e}^i(\,\boldsymbol{\phi}'(\mathbf{x}^k)\,)\right) \\
&= \left(\left[\nabla^k \boldsymbol{\phi}'(\mathbf{x}^k)\right]^T \ \mathbf{e}^k(\mathbf{x}^k)\right)^T \ \mathcal{M}_{Lp}^{i,\mathrm{ALE}}(\mathbf{x}^i) \ \left(\left[\nabla^k \boldsymbol{\phi}'(\mathbf{x}^k)\right]^T \ \mathbf{e}^k(\mathbf{x}^k)\right) \\
&= \left(\mathbf{e}^k(\mathbf{x}^k)\right)^T \ \left\{\nabla^k \boldsymbol{\phi}'(\mathbf{x}^k) \ \mathcal{M}_{Lp}^{i,\mathrm{ALE}}(\mathbf{x}^i) \ \left[\nabla^k \boldsymbol{\phi}'(\mathbf{x}^k)\right]^T\right\} \ \left(\mathbf{e}^k(\mathbf{x}^k)\right) .
\end{aligned}$$

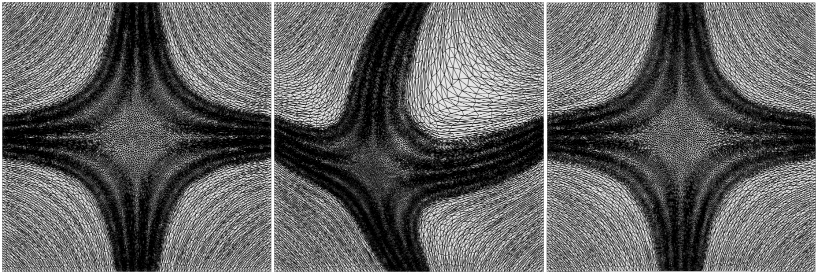


(From left to right) Metric without update, Updated metric

Current study : ALE metric Update

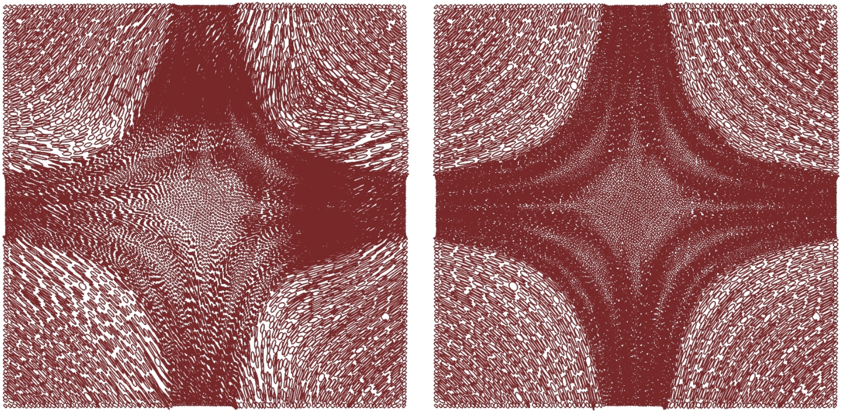


(From left to right) Mesh at t^i, Mesh at t^k

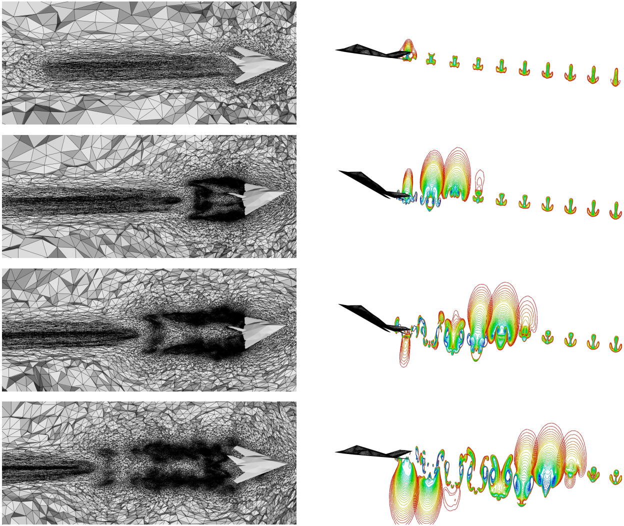


Hessian metric at t^k, Updated ALE Hessian metric at t^i, Moved and Updated ALE Hessian metric at t^k

ALE metric Update



(From left to right) Metric without update, Updated metric



Adapted meshes (view from the top) and Local Mach number isolines at different time steps for the nosing-up F117 test case.

Deriving the best mesh to observe a given output scalar functional

$$j(w) = \langle g, w \rangle$$

[Venditti and Darmofal, JCP 2003], [Jones et al., AIAA 2006], [Power et al, CMA 2006], [Wintzer et al., AIAA 2008], [Leicht and Hartmann, JCP 2010]

- Let W be the solution of the state equation $\Psi(W) = 0$
- Choose a scalar functional j.
- Minimize $\delta j_h = |j - j_h| = |(g, W) - (g, W_h)|$
- Introduce the adjoint state W^*

$$\left(\frac{\partial \Psi_h}{\partial W_h} \varphi_h, W_h^* \right) = (g, \varphi_h)$$

  to estimate the error $\delta j_h \;\approx\; (W^*, \Psi_h(W) - \Psi(W))$
- Minimize δj_h with an *a priori* error estimate

Unsteady Adjoint Resolution (Euler Equation)

The **continuous** state model on $\Omega \times [0, T]$ obeys to :

$$\Psi(W) = 0$$

The **discrete** state model writes :

$$W_h{}^{n} = W_h{}^{n-1} + \delta t^n \Phi_h(W_h^{n-1})$$

Consider a time-dependent functional :

$$j(W) = \int_0^T \int_\Gamma j_\Gamma(W(\mathbf{x}, t)\mathrm{d}\mathbf{x}\mathrm{d}t$$

The **continuous** adjoint state on $\Omega \times [0, T]$ obeys to :

$$\Psi^*(W, W^*) = 0 \quad \text{or} \quad -\frac{\partial W^*}{\partial t} - \left(\frac{\partial \mathcal{F}}{\partial W}\right) \nabla W^* = g \qquad (1)$$

The **discrete** adjoint state writes :

$$W_h{}^{*,n-1} = W_h{}^{*,n} + \delta t^n \frac{\partial j_h^{n-1}}{\partial W_h^{n-1}}(W_h^{n-1}) - \delta t^n (W_h{}^{*,n})^T \frac{\partial \Phi_h}{\partial W_h^{n-1}}(W_h^{n-1})$$

Time accurate Goal-oriented mesh adaptation

Difficulties

Computing $W^{*,n-1}$ at time t^{n-1} requires the knowledge of state W^{n-1} and adjoint state $W^{*,n}$

$\Rightarrow$ the knowledge of all states $W^n, n = 1 \cdots N$ is needed

Solve state foreward: $\Psi(W) = 0$

Solve adjoint state backward: $\Psi^*(W, W^*) = 0$

$\Rightarrow$ Large memory storage effort in 3D
(10^6 vertices & 10^3 iterations request 37.25 Gb)

Adopted solution

Solve state once to get checkpoints
State interpolation between two memory storage

Unsteady Feature-based mesh adaptation : Algorithm

For j=1,nptfx

For i=1,nadap

- $\mathcal{S}^j_{0,i} =$ ConservativeSolutionTransfer$(\mathcal{H}^j_{i-1}, \mathcal{S}^j_{i-1}, \mathcal{H}^j_i)$
- $\mathcal{S}^j_i =$ SolveState$(\mathcal{S}^j_{0,i}, \mathcal{H}^j_i)$

End for

For i=nadap,1

- $(\mathcal{S}^*)^j_i =$ AdjointStateTransfer$(\mathcal{H}^j_{i+1}, (\mathcal{S}^*_0)^j_{i+1}, \mathcal{H}^j_i)$
- $\{\mathcal{S}^j_i(k), (\mathcal{S}^*)^j_i(k)\} =$ SolveStateAndAdjointBackward$(\mathcal{S}^j_{0,i}, (\mathcal{S}^*)^j_i, \mathcal{H}^j_i)$
- $|H_{\max}|^j_i =$ ComputeGoalOrientedHessianMetric$(\mathcal{H}^j_i, \{\mathcal{S}^j_i(k), (\mathcal{S}^*)^j_i(k)\})$

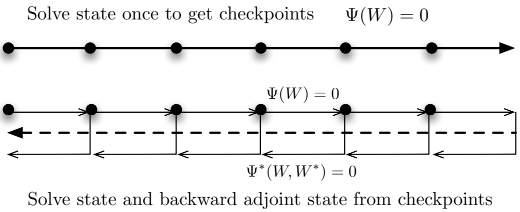
End for

$\mathcal{C}^j =$ ComputeSpaceTimeComplexity$(\{|H_{\max}|^j_i\}_{i=1,nadap})$

$\mathcal{M}^j_i =$ ComputeUnsteadyLpMetrics$(\mathcal{C}^{j-1}, |H_{\max}|^{j-1}_i)$

$\mathcal{H}^{j+1}_i =$ GenerateAdaptedMeshes$(\mathcal{H}^j_i, \mathcal{M}^j_i)$

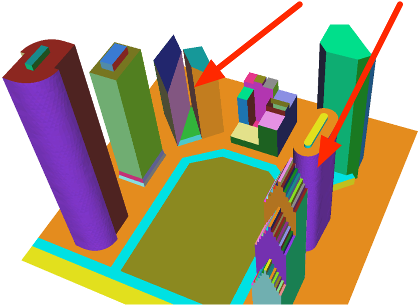
End for



Blast in the city

- Blast initialization : high density (10,0,0,0,25) and air (1,0,0,0,2.5)
- 3D town geometry (85 x 70 x 70 m^3) : 4187548 vertices
- cost function j : $j(W) = \int_0^T \int_\Gamma \frac{1}{2}(p - p_{air})^2 \mathrm{d}\Gamma \mathrm{d}t$.

The observation Γ are these 2 buildings



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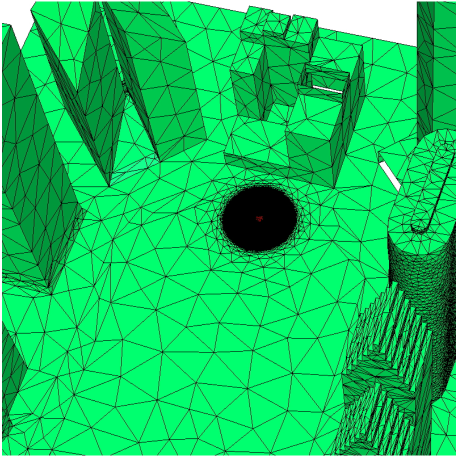


Unsteady Adjoint Resolution in ALE framework

The **discrete** unsteady state model in ALE writes :

$$W_h^n = W_h^{n-1} + \delta t^n \Phi_{h,ale}(W_h^{n-1})$$

Consider a time-dependent functional :

$$j(W) = \int_0^T \int_\Gamma j_\Gamma(W(\mathbf{x},t)\mathrm{d}\mathbf{x}\mathrm{d}t$$

The **continuous** unsteady adjoint state on $(\Omega, [0,T])$ in ALE obeys to :

$$\Psi^*(W, W^*) = 0 \quad \text{or} \quad -\frac{\partial W^*}{\partial t} - \left(\frac{\partial \mathcal{F}_{ale}}{\partial W}\right) \nabla W^* = g \qquad (2)$$

The **discrete** unsteady adjoint state in ALE writes :

$$W_h{}^{*,n-1} = W_h^{*,n} + \delta t^n \frac{\partial j_h^{n-1}}{\partial W_h^{n-1}}(W_h^{n-1}) - \delta t^n (W_h^{*,n})^T \frac{\partial \Phi_{h,ale}}{\partial W_h^{n-1}}(W_h^{n-1})$$

Unsteady Adjoint Resolution in ALE : Technical Difficulties

Mesh displacement σ

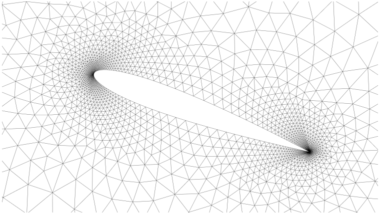
- Mesh displacement σ for the adjoint resolution in backward is the same as it was in the forward state resolution
- Jacobian of fluxes : $\dfrac{\partial \Phi_{h,ale}}{\partial W_h^{n-1}}$
- Boundary conditions : Boundary fluxes take also into account displacement of geometry

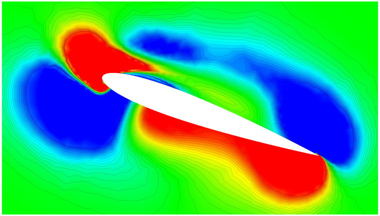
Deal with connectivities

Backward Mesh : store the positions but not necessary the connectivities

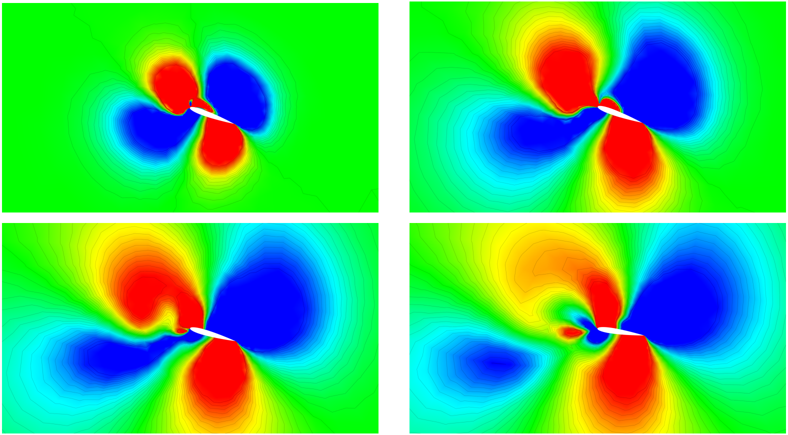
$\Rightarrow$ The adjoint state W^* verify the DGCL property :

$$(|C^{n+1}| - |C^n|) - \int_n^{n+1} \int_{\partial C(t)} (\mathbf{w}^* \cdot \mathbf{n}) \mathrm{d}\mathbf{s} \mathrm{d}t = 0$$





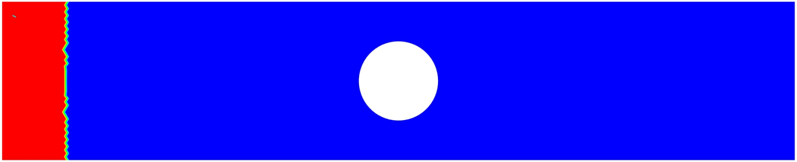
Mesh and Adjoint of the solution based on the drag functional output for the nosing-up NACA0012 test case at first adjoint iteration in backward (i.e. $t = 0.29$).



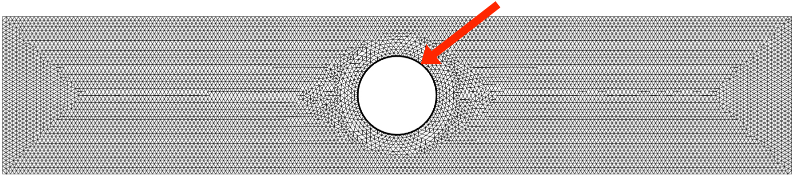
Adjoint of the solution isolines based on the drag functional output at different time steps for the nosing-up NACA0012 test case (*From left to right, top to bottom* $t = 2.5, 2., 1.5, 1.$).

FSI case

- Lift-off of a circle by a Mach 3 Shock Wave
- 2D geometry : $(1 \times 0.2\ m^2)$: 9064 Vertices
- cost function j : $j : j(W) = \int_0^T \int_\Gamma \frac{1}{2}(p - p_{air})^2 \mathrm{d}\Gamma \mathrm{d}t.$

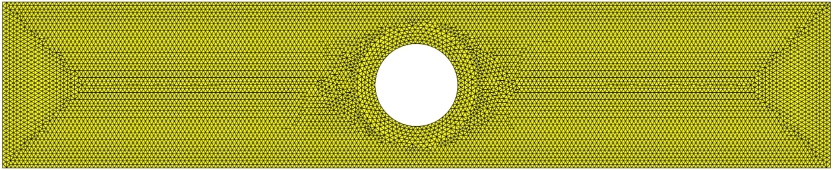


The observation is the pressure on this circle

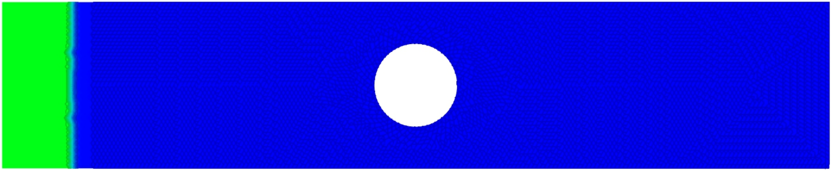


FSI Case

Mesh generated in forward

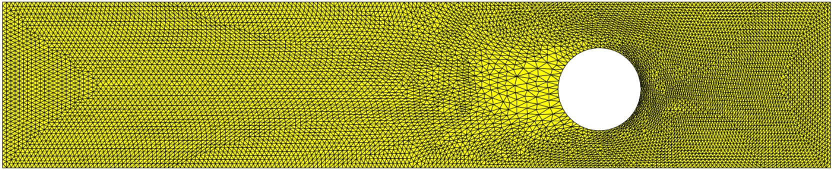


Solve State

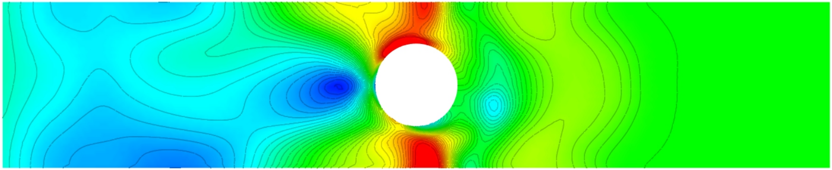


FSI Case

Mesh generated in backward

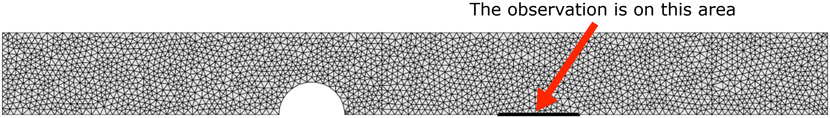


Adjoint state in backward



2D Pilot case Mesh Adaptation

- Blast initialization : high density (10,**0**,25) and air (1,**0**,2.5)
- 2D geometry : (5 x 0.5 m^2) : 2395 Vertices
- cost function j : $j : j(W) = \int_0^T \int_\Gamma \frac{1}{2}(p - p_{air})^2 \mathrm{d}\Gamma \mathrm{d}t.$



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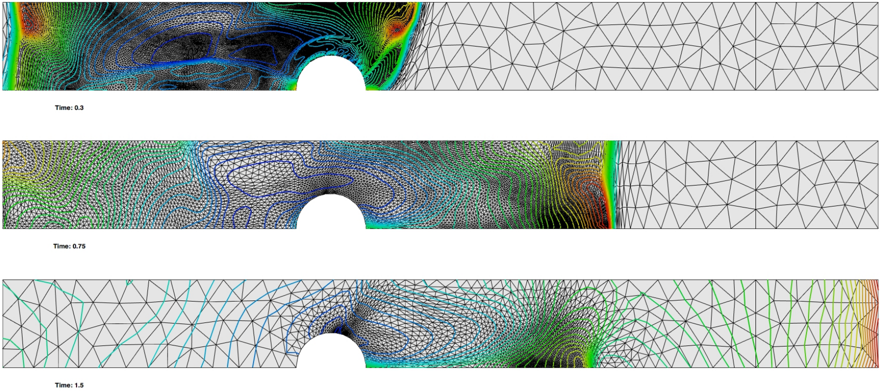
- cost function j : $j : j(W) = \int_0^T \int_\Gamma \frac{1}{2}(p - p_{air})^2 \mathrm{d}\Gamma \mathrm{d}t.$

From Top to Down : Time 0.3 - 0.75 - 1.5



Conclusions Perspectives

Conclusions

- ALE metric update
- Time accurate feature-based mesh adaptation in ALE framework
- Time accurate goal-oriented mesh adaptation
- Evaluation of Unsteady Adjoint on moving meshes

Perspectives

- AIAA (5^{th} June)
- Interpolation Adjoint solutions in 3D
- 3D Time accurate goal oriented mesh adaptation in ALE framework
- Convergence en maillage

Thank you for your attention