MULTIRATE METHOD AND TURBULENCE MODELLING

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Introduction

Work context

- Implementation and development of a simulation tool whose major ingredients are :
  - A numerical model suited to **industrial problems**
  - Turbulence models suited to the simulation of **turbulent flows** with massive separation and vortex shedding

Turbulence modelling

In the present work

- Evaluation of DDES and VMS-LES models for the prediction of the flow around a **circular cylinder** for **two subcritical regimes**
- A comparison of DDES and hybrid VMS-LES model for supercritical flow
- Hybrid simulation of a **supercritical flow** around a **tandem cylinder** :
  - Contain many features and difficulties encountered in industrial problems
  - Well documented benchmarks
  - First step before the computation of array of cylinders (offshore oil and gas industries, civil engineering, aeronautics)

Hybrid RANS/VMS-LES model

Motivations

- Computation of **massively separated flows at high Reynolds number** on unstructured mesh
- **RANS** : accuracy problems in flow regions with massive separation (as the flow around bluff-bodies)
- **VMS-LES** : more expensive than RANS, very fine resolution requirements in boundary layers at high Reynolds number
- **Hybrid : combines RANS and VMS-LES** in order to exploit the advantages of the two approaches :
  - **less computationally** expensive compared with VMS-LES
  - **better accuracy** than RANS for flows dominated by large unsteady structures
- Desired feature for the hybridation strategy : automatic and progressive switch from RANS to VMS-LES and vice versa

Hybrid RANS/VMS-LES model

- **Central idea of the proposed hybrid VMS model** :
  Correction of the mean flow field obtained with a RANS model by adding fluctuations given by a VMS-LES approach wherever the grid resolution is adequate

- **Decomposition of the flow variables** :

$$W = \underbrace{< W >}_{RANS} + \underbrace{W^c}_{correction} + W^{SGS}$$

$< W > \; = \;$ RANS flow variables
$W^c \; = \;$ remaining resolved fluctuations obtained with VMS-LES
(i.e. $< W > + W^c = W_h \; = \;$ VMS-LES flow variables)
$W^{SGS} \; = \;$ subgrid scale fluctuations

Hybrid RANS/VMS-LES model

Hybrid RANS/VMS-LES governing equations

- **Semi-discretization of the RANS equations** ($k-\epsilon$ Goldberg model + Menter correction) :

$$\left(\frac{\partial\langle W\rangle}{\partial t},\Phi_i\right)+\left(\nabla\cdot F(\langle W\rangle),\Phi_i\right)=-\left(\tau^{RANS}(\langle W\rangle),\Phi_i\right).$$

- **Final hybrid VMS model equations** (RANS eq. + modified governing eq. for reconstructed fluctuations) :

$$\left(\frac{\partial W_h}{\partial t},\Phi_i\right)+\left(\nabla\cdot F(W_h),\Phi_i\right)=$$
$$-\theta\left(\tau^{RANS}(\langle W\rangle),\Phi_i\right)-(1-\theta)\left(\tau^{LES}(W_h'),\Phi_i'\right)$$

  where W_h denotes the hybrid variables

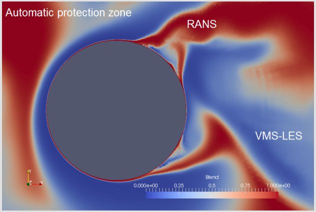
- $\theta\in[0,1]$ **is the blending function**

Hybrid RANS/VMS-LES model

Hybridization strategy

$$\theta = 1 - f_d(1 - \bar{\theta})$$

$f_d \simeq 0$ in the boundary layer
$f_d \simeq 1$ outside the boundary layer



where $\bar{\theta} = tanh(\xi^2)$ with $\xi = \dfrac{\Delta}{l_{RANS}}$ or $\xi = \dfrac{\mu_{SGS}}{\mu_{RANS}}$,

$$f_d = 1 - tanh((8r_d)^3) \text{ and } r_d = \frac{\nu_t + \nu}{max(\sqrt{u_{i,j}u_{i,j}}, 10^{-10})K^2 d_w^2}$$

RANS-DDES

- Based on the low Reynolds $k - \epsilon$ model proposed by **Goldberg and Ota**
- The turbulent viscosity is limited by the **Bradshaw's law** in a similar way to Menter's SST model

DDES/$k - \varepsilon$/Menter formulation

The dissipation term $D_k^{RANS} = \rho\varepsilon$ in the RHS of the $k - \epsilon$ equations is replaced by:

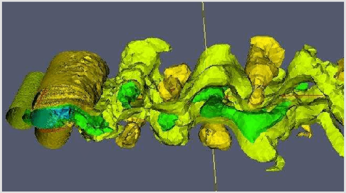
$$D_k^{DDES} = \rho \frac{k^{3/2}}{l_{DDES}}$$

with $l_{DDES} = \frac{k^{3/2}}{\epsilon} - f_d max\left(0, \frac{k^{3/2}}{\epsilon} - C_{DDES}\Delta\right)$ where $C_{DDES} = 0.65$ and Δ is a measure of local mesh size

Circular cylinder at Re= 3900 ∽ Subcritical regime

Test case definition

- **Flow parameters:**
  Mach = 0.1
  Reynolds = 3900
- **Computational grids:**
  1.46M nodes
  8.4M elements



Circular cylinder at Re= 3900 $\backsim$ Subcritical regime

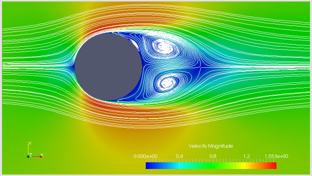
	Mesh	$\overline{C}_d$	$-\overline{C}_{p_b}$
Experiment			
Norberg		[0.94-1.04]	[0.83-0.93]
Present simulations			
No model	1.4M	0.88	0.73
DDES/$k-\varepsilon$/Menter	1.4M	0.88	0.73
VMS dyn.	1.4M	0.90	0.83
Other simulations, LES			
Lee	7.7M	[0.99-1/04]	[0.89-0.94]
Kravchenko	[0.5M-2.4M]	[1.04-1.38]	[0.93-1.23]

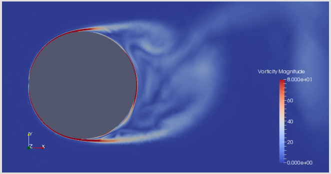
Table 1: **Circular cylinder: bulk flow parameters at Re=3900**

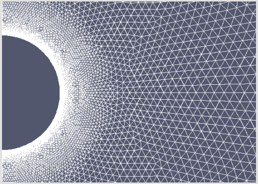
Circular cylinder at Re $= 20 \times 10^3$ ∽ Subcritical regime

Test case definition

- **Flow parameters** :
  Reynolds $= 20 \times 10^3$
  Mach=0.1
- **Computational grid** :
  1.8M nodes
  9.4M elements







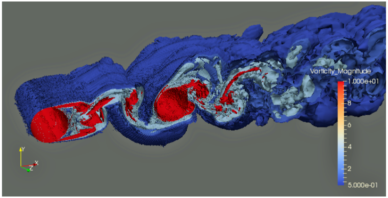
Circular cylinder at Re $= 20 \times 10^3$ ∽ Subcritical regime

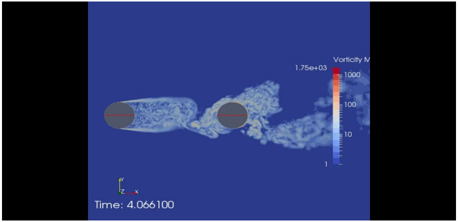
	$\overline{C_d}$	C_{Lrms}	-$\overline{Cp_b}$	Θ_{sep}	Lr	Lv	lmax	St
Experiment								
Norberg	1.16	0.47	1.16	78	1.03			0.194
Lim-Lee	1.16					1.0	37.0	
Present simulations								
no-model	1.27	0.61	1.35	82	0.96	0.94	38.7	0.201
URANS/Menter	1.27	0.71	1.25	85	0.64	0.53	33.2	0.231
DDES/$k - \varepsilon$/Menter	1.16	0.36	1.12	82	0.83	0.77	35.9	0.213
VmsWale-Dyn	1.18	0.46	1.20	81	0.96	0.71	35.8	0.201
Hybrid-VmsWale-dyn	1.15	0.48	1.15	86	0.88	0.83	35.2	0.210

Table 2: **Circular cylinder : bulk flow parameters at Re=20000**

Tandem Cylinders

- **Flow parameters** :
  Reynolds $= 1.66 \times 10^5$
  Mach=0.1





- **Computational grid** :
  2.71M nodes
  15.7M elements

Tandem Cylinders

	Mesh	$\overline{C}_d$ Cyl. 1	$\overline{C}_d$ Cyl. 2
Experiments			
BART		0.64	0.31
Present simulations			
DDES/$k-\varepsilon$/Menter	2.71M	0.65	0.44
Hybrid VMS dyn.	2.59M	0.64	0.38
Other simulations			
DES Aybay (2010)	6.7M	0.64	0.44
HRLES Vatsa (2010)	8.7M	0.64	0.45

Table 3: Tandem cylinder: Mean drag coefficients

Tandem Cylinders

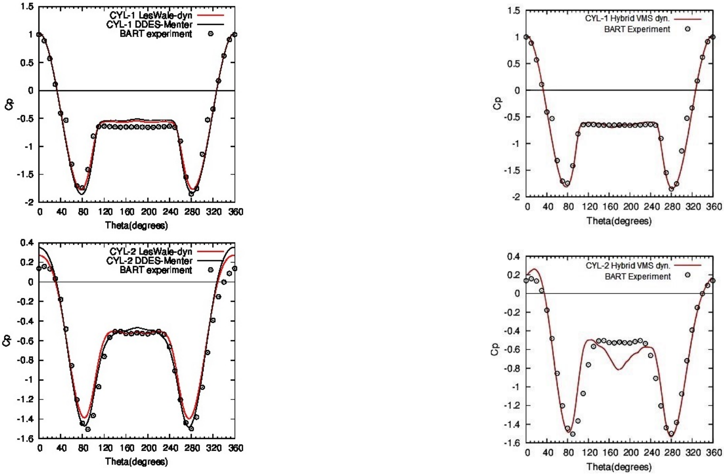


Figure 1: Mean pressure coefficient distribution DDES versus Hybrid LES dyn.(left) and Hybrid VMS dyn. (right)

Multirate time advancing by volume agglomeration

Work context

- Development of a new **explicit multirate time advancing scheme** for the solution of the compressible Navier-Stokes equations :
    - based on control volume agglomeration
    - well suited to our numerical framework using a mixed finite volume/finite element formulation
    - developed in a parallel numerical framework

Multirate time advancing by volume agglomeration

Multirate based on agglomeration - Objectives

A frequent configuration in CFD calculations combines :

- An explicit time advancing scheme for accuracy purpose
- A computational grid with a very small portion of much smaller elements than in the remaining mesh

Examples:

- *Isolated traveling shock*
- *Boundary layer at high Reynolds number (few tens of microns thick) in LES computations where vortices around one centimeter are captured*

Explicit time advancing schemes with global time stepping are too costly
$\rightarrow$ the **multirate time stepping approach** is an interesting alternative

Multirate time advancing by volume agglomeration

Inner and outer zones - Definition

- **Inner and outer zones** :
  - Let Δt be the global time step over the computational domain
  - Define the **outer zone** as the set of cells for which the explicit scheme is stable for a time step $K\Delta t$, and the **inner zone** as its complement
  - Definition of these zones through the local time steps

Multirate time advancing by volume agglomeration

Inner and outer zones - Definition

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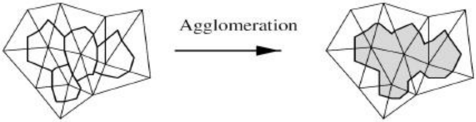
Coarse grid

- **Objective** :
  - Advancement in time with time step $K\Delta t$
  - Advancement in time preserving accuracy in the outer zone (space order of 3, RK4)
  - Advancement in time consistent in the inner zone
- Define the **coarse grid** as the macro cells in the inner zone + the fine cells in the outer zone

Multirate time advancing by volume agglomeration



fine cells i | macro cell I

Multirate based on agglomeration - Definition

- Flux on the coarse grid :
  - Assembling of the nodal flux Ψ_i on the fine cells (as usual)
  - Fluxes sum on the macro cells I (inner zone) :

$$\Psi^I = \sum_{k \in I} \Psi_k$$

- Volumes sum on the macro cells I (inner zone) : $Vol^I = \sum_{k \in I} Vol_k$ where Vol_k is the volume of $cell_k$

Multirate time advancing by volume agglomeration

Multirate based on agglomeration - Algorithm

Step 1 (predictor) :

Advancement in time with Runge-Kutta (for example) on the macro cells in the inner zone and on the fine cells in the outer zone, with time step $K\Delta t$:

For $\alpha = 1, RKstep$

$$\text{outer zone :} \quad vol_i w_i^{(\alpha)} = vol_i w_i^{(0)} + b_\alpha K\Delta t \; \Psi_i^{(\alpha-1)}$$

$$\text{inner zone :} \quad vol^I w^{I,(\alpha)} = vol^I w^{I,(0)} + b_\alpha K\Delta t \; \Psi^{I,(\alpha-1)}$$

$$w_i^{(\alpha)} = w^{I,(\alpha)} \quad \text{for } i \in I$$

EndFor α.

Multirate time advancing by volume agglomeration

Multirate based on agglomeration - Algorithm

Step 2 (corrector) :

- Unknowns frozen in the outer zone
- Time interpolation of these unknowns (those useful for the next point)
- In the inner zone : using these interpolated values, advancement in time with the chosen explicit scheme and time step Δt
- Complexity mastered (proportional to the number of points in the inner zone)

  $\rightarrow$ Complexity : CostMultirate(K) = CostExplicit(N)*(1/K + Ninnernodes(K)/N)

ALE calculation of a traveling contact discontinuity

Test case definition

- **Simulation** :
  - Compressible Euler equations are solved in a rectangular parallelepiped
  - Density is initially discontinuous at the middle of the domain
  - Velocity and pressure are uniform

- **A deforming mesh** :

Nodes	Elements	Subdomains
25K	92K	2

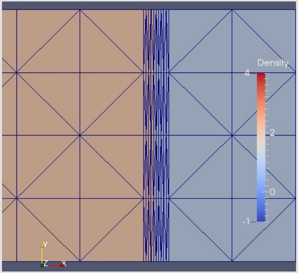


Figure 2: Instantaneous mesh with mesh concentration in the middle of zoom

ALE calculation of a traveling contact discontinuity

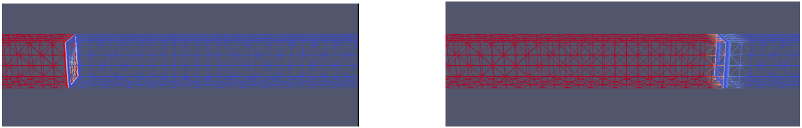


Figure 3: Zooms on the moving contact discontinuity using a load balancing procedure at two different time steps

K	CPU expl. (s/dt/node)	N	$N^{small}(K)$	Expected gain (scalar)	CPU expli phase (s/Kdt)	CPU inner phase (s/Kdt)	Measured gain (parallel)
5	$2.49.10^{-5}$	25000	325	4.7	0.124	0.244	1.7
10	$4.88.10^{-5}$	25000	325	8.8	0.122	0.482	2.0
15	$7.70.10^{-5}$	25000	325	12.5	0.126	0.729	3.0

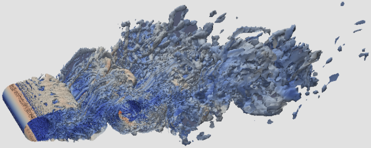
Table 4: **ALE propagation of a contact discontinuity**: factor K between large and small time-step length, CPU parallel rate in seconds per explicit time-step per node, total number of nodes, number of nodes in inner region, theoretical gain in scalar mode, CPU of explicit phases in seconds per explicit time-step, CPU of inner phases in seconds per explicit time-step, and measured parallel gain.

Tandem Cylinders

Test case definition

- **Simulation** :
  Hybrid VMS-LES simulation combined with non-dynamic and dynamic versions of the WALE SGS model

- **Flow parameters** :
  Reynolds $= 1.66 \times 10^5$
  Mach=0.1
  $L/D = 3.7$



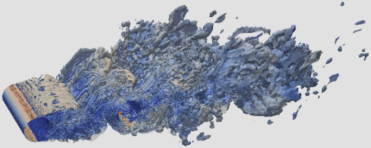
Tandem Cylinders

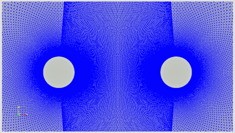
Test case definition

- **Simulation** :
  Hybrid VMS-LES simulation combined with non-dynamic and dynamic versions of the WALE SGS model
- **Computational grids** :

Nodes	Elements	Subdomains
16M	92M	768

- **Flow parameters** :
  Reynolds $= 1.66 \times 10^5$
  Mach=0.1
  $L/D = 3.7$





Tandem Cylinders

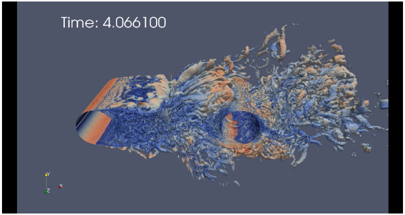


Figure 4: Animation of Q-criterion to observe the vortical structures in the flow

K	CPU expl. (s/dt/node)	N	$N^{small}(K)$ (%)	Expected gain (scalar)	CPU expli phase (s/Kdt)	CPU inner phase (s/Kdt)	Measured gain (parallel)
5	5.10^{-7}	15505671	18	2.63	1.55	6.93	1.25
10	$9.8.10^{-7}$	15505671	24	2.94	1.52	14.15	1.46
20	2.10^{-6}	15505671	35	2.50	1.53	28.94	2.05

Table 5: **Fine mesh - Tandem cylinder**

Cylinder

Test case definition

- **Simulation** :
  Hybrid VMS-LES simulation combined with non-dynamic version of the WALE SGS model

- **Flow parameters** :
  Reynolds $= 8.4 \times 10^6$
  Mach$=0.1$

Cylinder

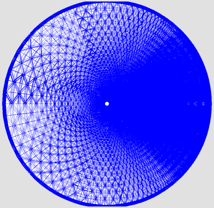
Test case definition

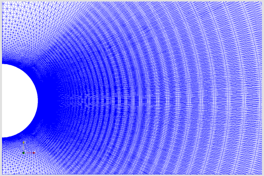
- **Simulation** :
  Hybrid VMS-LES simulation combined with non-dynamic version of the WALE SGS model

- **Computational grids** :

Nodes	Elements	Subdomains
4.3M	25M	768

- **Flow parameters** :
  Reynolds $= 8.4 \times 10^6$
  Mach$=0.1$





Cylinder

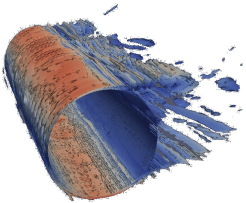


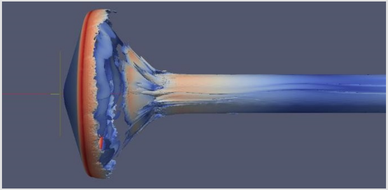
Figure 5: Instantaneous Q-criterion isosurfaces

K	CPU expl. s/dt/node	N	$N^{small}(K)$ (%)	Expected gain (scalar)	CPU expli phase (s/Kdt)	CPU inner phase (s/Kdt)	Measured gain (parallel)
5	$4.2.10^{-7}$	4290048	15	2.86	0.36	1.53	1.25
10	$8.5.10^{-7}$	4290048	19	3.45	0.36	3.12	1.46
20	$1.7.10^{-6}$	4290048	24	3.45	0.36	6.24	1.62

Table 6: **Circular cylinder**

Space probe model

Test case definition

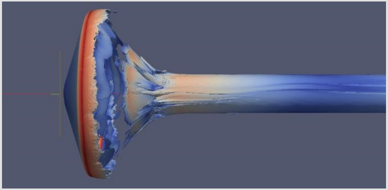


- **Simulation** :
  Hybrid VMS-LES simulation combined with non-dynamic version of the WALE SGS model

- **Flow parameters** :
  Reynolds $= 1 \times 10^6$
  Mach=2.0

Space probe model

Test case definition

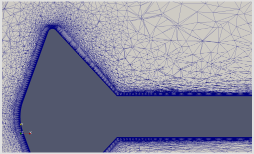


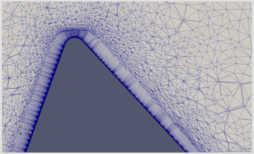
- **Simulation** :
  Hybrid VMS-LES simulation combined with non-dynamic version of the WALE SGS model

- **Flow parameters** :
  Reynolds $= 1 \times 10^6$
  Mach=2.0

- **Computational grids** :

Nodes	Elements	Subdomains
4.38M	25.8M	192





Space probe model

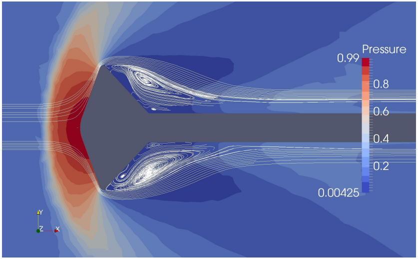


Figure 6: Instantaneous pressure with streaklines

K	CPU expl. (s/dt/node)	N	$N^{small}(K)$	Expected gain (scalar)	CPU expli phase (s/Kdt)	CPU inner phase (s/Kdt)	Measured gain (parallel)
10	$4.13.10^{-6}$	4381752	56	10	1.81	4.36	2.18
40	$4.17.10^{-6}$	4381752	151	40	1.83	17.35	2.89

Table 7: **Spatial probe**

Conclusion, perspectives

- Presentation of a new multirate scheme based on agglomeration and relying on a **prediction step** and a **correction step**
- The proposed multirate strategy has been applied in complex CFD problems such as the prediction of three-dimensional flows around bluff bodies with complex hybrid turbulence models
- Efficiency improvement for all investigated problems
- Still some progress to do to **adapt the domain decomposition** in such a way that the cores workload becomes shared equally for both steps of the multirate scheme
- Further efficiency can be gained in some cases if more than two zones can be considered (Inner zone - Medium zone - Outer Zone)

Thank you for your attention !