A VOMUME-AGGLOMERATION MULTIRATE TIME ADVANCING APPROACH

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Introduction

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- Development of a new **explicit multirate time advancing scheme** for the simulation of PDEs which is :

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    - well suited to our numerical framework using a mixed finite volume/finite element formulation

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- **Evaluation of the efficiency** of the present multirate time integration scheme on several benchmarks flows :

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  - flows around bluff bodies like a tandem cylinders

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  - a moving contact discontinuity
  - flows around bluff bodies like a tandem cylinders
  - a circular cylinder
  - a space probe model

Multirate time advancing by volume agglomeration

Multirate based on agglomeration - Definition

A frequent configuration in mesh adaptation combines :

- An explicit time advancing scheme for accuracy purpose
- A computational grid with a very small portion of much smaller elements than in the remaining mesh

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- *Isolated traveling shock*
- *Boundary layer at high Reynolds number (few tens of microns thick) in LES computations where vortices around one centimeter are captured*

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Explicit time advancing schemes with global time stepping too costly

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- **Inner and outer zones** :
  - Let Δt be the global time step over the computational domain
  - Define the **outer zone** as the set of cells for which the explicit scheme is stable for a time step $K\Delta t$, and the **inner zone** as its complement
  - Definition of these zones through the local time steps

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- **Coarse grid** :
  - **Objective** :
    - Advancement in time with time step $K\Delta t$
    - Advancement in time preserving accuracy in the outer zone (space order of 3, RK4)
    - Advancement in time consistent in the inner zone
  - Define the **coarse grid** as the macro cells in the inner zone + the fine cells in the outer zone

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  - Define the **coarse grid** as the macro cells in the inner zone + the fine cells in the outer zone
  - **Methods** :
    - Advancement in time of the chosen explicit scheme on the coarse grid with $K\Delta t$
    - Residual smoothing for stability purpose

Multirate time advancing by volume agglomeration

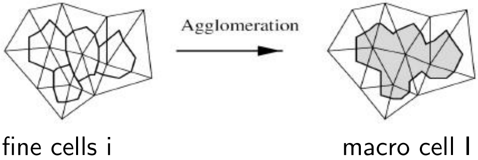
Multirate based on agglomeration - Definition

- Flux on the coarse grid :
  - Assembling of the nodal flux Ψ_i on the fine cells (as usual)
  - Fluxes sum on the macro cells I (inner zone) :

$$\Psi^I = \sum_{k \in I} \Psi_k$$

- Smoothing of the coarse flux (inner zone) :

$$\Psi^I = \Big(\sum_{K \in \mathcal{V}(I)} \Psi^K vol^K \Big) / \Big(\sum_{K \in \mathcal{V}(I)} vol^K \Big)$$



Multirate time advancing by volume agglomeration

Multirate based on agglomeration - Algorithm

Step 1 (predictor) :

Advancement in time with Runge-Kutta (for example) on the macro cells in the inner zone and on the fine cells in the outer zone, with time step $K\Delta t$:

Pour $\alpha = 1, RKstep$

$$\text{outer zone :} \quad vol_i w_i^{(\alpha)} = vol_i w_i^{(0)} + b_\alpha K\Delta t \; \Psi_i^{(\alpha-1)}$$

$$\text{inner zone :} \quad vol^I w^{I,(\alpha)} = vol^I w^{I,(0)} + b_\alpha K\Delta t \; \Psi^{I,(\alpha-1)}$$

$$w_i^{(\alpha)} = w^{I,(\alpha)} \quad \text{for } i \in I$$

Fin α.

Multirate time advancing by volume agglomeration

Multirate based on agglomeration - Algorithm

Step 2 (corrector) :

- Unknowns frozen in the outer zone
- Time interpolation of these unknowns (those useful for the next point)
- In the inner zone : using these interpolated values, advancement in time with the chosen explicit scheme and time step Δt
- Complexity mastered (proportional to the number of points in the inner zone)

ALE calculation of a traveling contact discontinuity

Test case definition

- **Simulation** :
  - Compressible Euler equations are solved in a rectangular parallelepiped
  - Density is initially discontinuous at the middle of the domain
  - Velocity and pressure are uniform

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- **A deforming mesh** :

Nodes	Elements	Subdomains
25K	92K	2

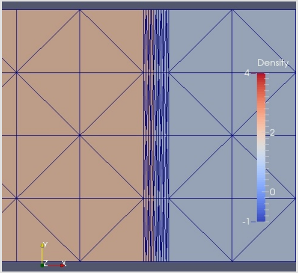

Figure 1: Instantaneous mesh with mesh concentration in the middle of zoom

ALE calculation of a traveling contact discontinuity

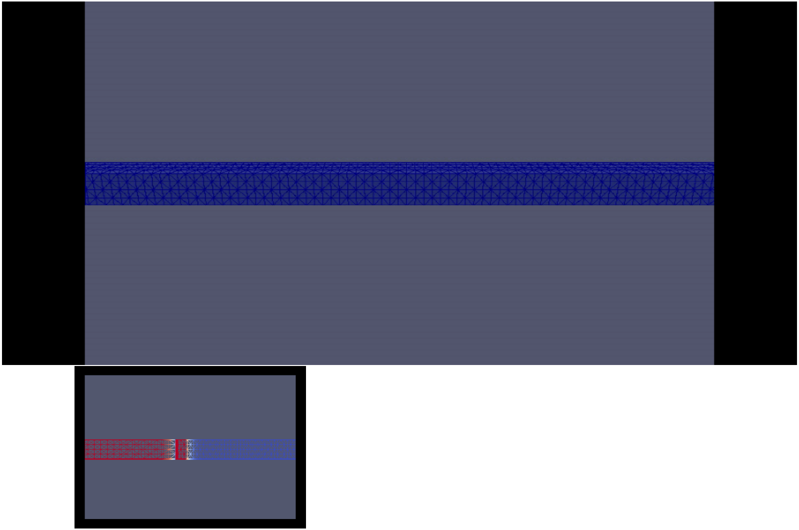

Figure 2: Animation of the moving contact discontinuity using a load balancing procedure

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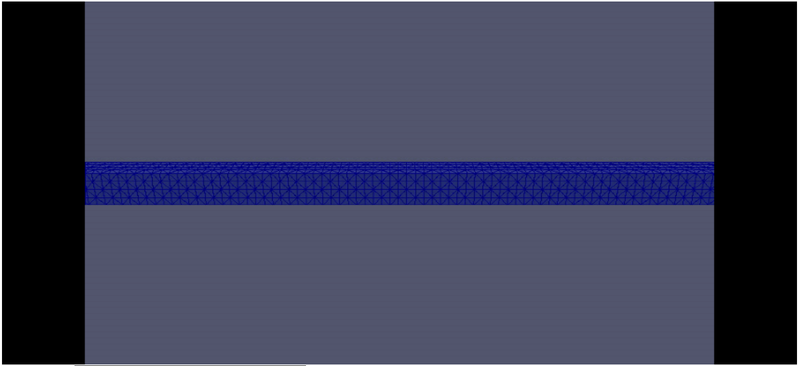


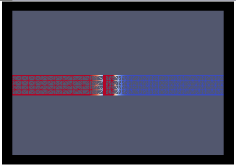


Figure 2: Animation of the moving contact discontinuity using a load balancing procedure

K	Nodes in the inner zone (%)	Gain in efficiency
5	1.3	1.15
10	1.3	1.58
15	**1.3**	**1.75**

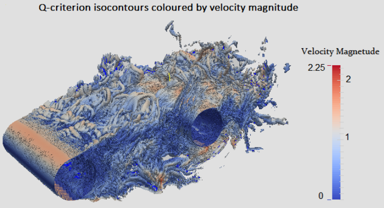
Table 1: Efficiency of the multirate scheme

Tandem Cylinders

Test case definition

- **Simulation** :
  Hybrid VMS-LES simulation combined with non-dynamic and dynamic versions of the WALE SGS model

- **Flow parameters** :
  Reynolds $= 1.66 \times 10^5$
  Mach=0.1
  $L/D = 3.7$



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Test case definition

- **Simulation** :
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- **Computational grids** :

Nodes	Elements	Subdomains
2.59M	15M	192
16M	92M	768

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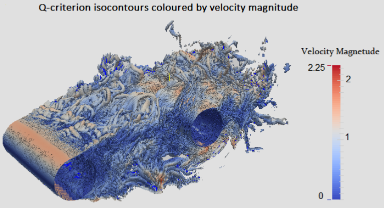

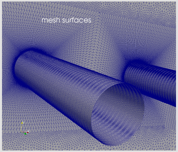

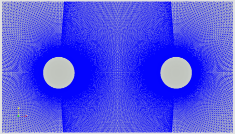

Figure 3: Coarse (left) and fine (right) unstructured grids

Tandem Cylinders - Coarse grid

K	Nodes in the inner zone (%)	Gain in efficiency
5	16	1.09
10	25	1.14

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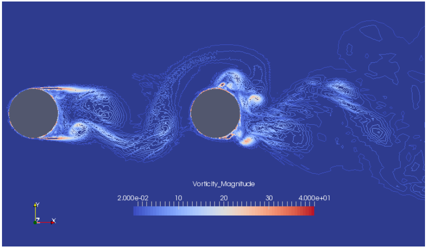

Figure 4: Instantaneous vorticity magnitude contours

Tandem Cylinders - Fine grid

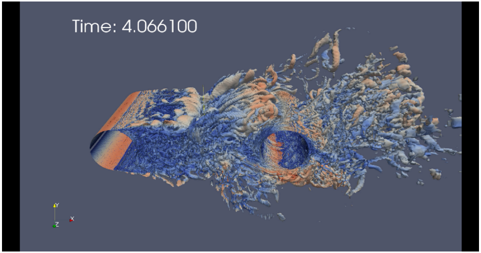

Figure 5: Animation of Q-criterion to observe the vortical structure in the flow

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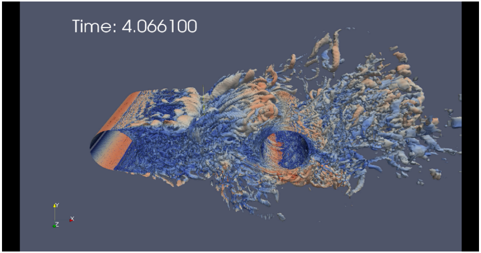

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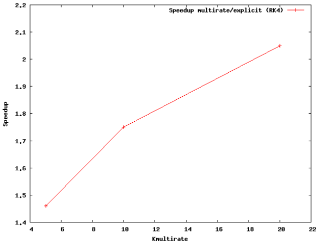

Figure 6: Speedup multirate versus explicit(RK4)

Cylinder

Test case definition

- **Simulation** :
  Hybrid VMS-LES simulation combined with non-dynamic version of the WALE SGS model

- **Flow parameters** :
  Reynolds $= 8.4 \times 10^6$
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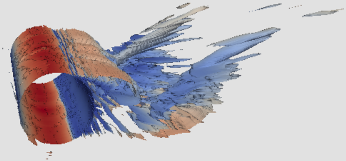

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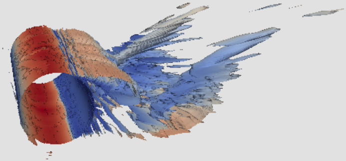
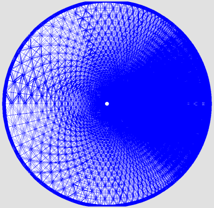
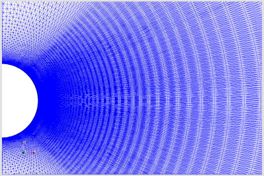
4.3M	25M	768







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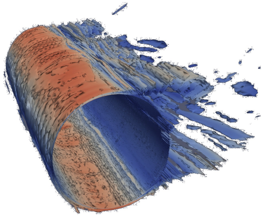

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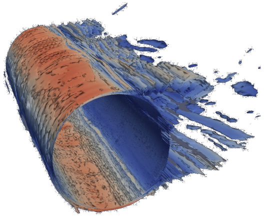
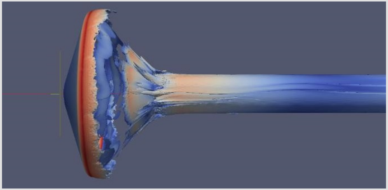


K	Nodes in the inner zone (%)	Gain in efficiency
5	15	1.25
10	19	1.46
20	**24**	**1.62**

Table 3: Efficiency of the multirate scheme

Spatial probe

Test case definition



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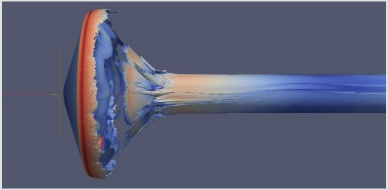

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- **Computational grids** :

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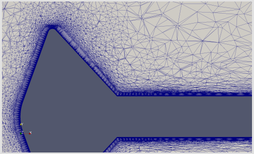

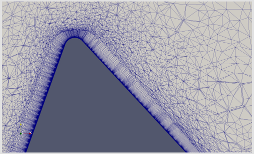

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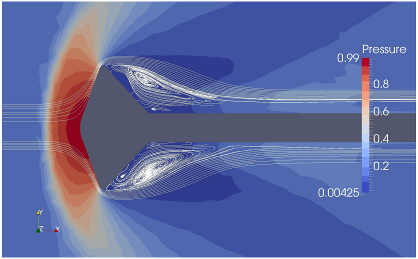

Figure 7: Instantaneous pressure with streaklines

K	Nodes in the inner zone	Gain in efficiency
10	56	2.18
40	**151**	**2.89**

Table 4: Efficiency of the multirate scheme

Conclusion, perspectives

- Presentation of a new multirate scheme based on agglomeration and relying on a **prediction step** and a **correction step**

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- Efficiency improvement for all investigated problems

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- The proposed multirate strategy has been applied in complex CFD problems such as the prediction of three-dimensional flows around bluff bodies with complex hybrid turbulence models
- Efficiency improvement for all investigated problems
- Still some progress to do to **adapt the domain decomposition** in such a way that the cores workload becomes shared equally for both steps of the multirate scheme

Thank you for your attention !