I3M-Montpellier - ANR MAIDESC

Emmanuelle Itam*, Bruno Koobus*

(∗) I3M, Université de Montpellier, France

Réunion MAIDESC - 8 avril 2015
INRIA Paris-Rocquencourt

Introduction and motivations

Work context

- Implementation and development of a simulation tool whose major ingredients are :
  - A numerical model suited to **industrial problems**
  - Turbulence models suited to the simulation of **turbulent flows** with massive separations and vortex shedding

In the present work

- Evaluation of **dynamic** and **hybrid VMS-LES models** on the prediction of vortex shedding flows
- Simulation of the flow around **circular** and **tandem cylinders** :
- Multirate time advancing

Introduction and motivations

Work context

- Implementation and development of a simulation tool whose major ingredients are :
  - A numerical model suited to **industrial problems**
  - Turbulence models suited to the simulation of **turbulent flows** with massive separations and vortex shedding

In the present work

- Evaluation of **dynamic** and **hybrid VMS-LES models** on the prediction of vortex shedding flows
- Simulation of the flow around **circular** and **tandem cylinders** :
- Multirate time advancing

Introduction and motivations

Work context

- Implementation and development of a simulation tool whose major ingredients are :
  - A numerical model suited to **industrial problems**
  - Turbulence models suited to the simulation of **turbulent flows** with massive separations and vortex shedding

In the present work

- Evaluation of **dynamic** and **hybrid VMS-LES models** on the prediction of vortex shedding flows
- Simulation of the flow around **circular** and **tandem cylinders** :
- Multirate time advancing

Introduction and motivations

Work context

- Implementation and development of a simulation tool whose major ingredients are :
  - A numerical model suited to **industrial problems**
  - Turbulence models suited to the simulation of **turbulent flows** with massive separations and vortex shedding

In the present work

- Evaluation of **dynamic** and **hybrid VMS-LES models** on the prediction of vortex shedding flows
- Simulation of the flow around **circular** and **tandem cylinders** :
  - Contain many features and difficulties encountered in industrial problems
  - Well documented benchmarks
  - First step before the computation of array of cylinders (offshore oil and gas industries, civil engineering, aeronautics)
- Multirate time advancing

Introduction and motivations

Work context

- Implementation and development of a simulation tool whose major ingredients are :
  - A numerical model suited to **industrial problems**
  - Turbulence models suited to the simulation of **turbulent flows** with massive separations and vortex shedding

In the present work

- Evaluation of **dynamic** and **hybrid VMS-LES models** on the prediction of vortex shedding flows
- Simulation of the flow around **circular** and **tandem cylinders** :
  - Contain many features and difficulties encountered in industrial problems
  - Well documented benchmarks
  - First step before the computation of array of cylinders (offshore oil and gas industries, civil engineering, aeronautics)
- Multirate time advancing

Introduction and motivations

Work context

- Implementation and development of a simulation tool whose major ingredients are :
  - A numerical model suited to **industrial problems**
  - Turbulence models suited to the simulation of **turbulent flows** with massive separations and vortex shedding

In the present work

- Evaluation of **dynamic** and **hybrid VMS-LES models** on the prediction of vortex shedding flows
- Simulation of the flow around **circular** and **tandem cylinders** :
  - Contain many features and difficulties encountered in industrial problems
  - Well documented benchmarks
  - First step before the computation of array of cylinders (offshore oil and gas industries, civil engineering, aeronautics)
- Multirate time advancing

Introduction and motivations

Work context

- Implementation and development of a simulation tool whose major ingredients are :
  - A numerical model suited to **industrial problems**
  - Turbulence models suited to the simulation of **turbulent flows** with massive separations and vortex shedding

In the present work

- Evaluation of **dynamic** and **hybrid VMS-LES models** on the prediction of vortex shedding flows
- Simulation of the flow around **circular** and **tandem cylinders** :
  - Contain many features and difficulties encountered in industrial problems
  - Well documented benchmarks
  - First step before the computation of array of cylinders (offshore oil and gas industries, civil engineering, aeronautics)
- Multirate time advancing

Introduction and motivations

Work context

- Implementation and development of a simulation tool whose major ingredients are :
  - A numerical model suited to **industrial problems**
  - Turbulence models suited to the simulation of **turbulent flows** with massive separations and vortex shedding

In the present work

- Evaluation of **dynamic** and **hybrid VMS-LES models** on the prediction of vortex shedding flows
- Simulation of the flow around **circular** and **tandem cylinders** :
  - Contain many features and difficulties encountered in industrial problems
  - Well documented benchmarks
  - First step before the computation of array of cylinders (offshore oil and gas industries, civil engineering, aeronautics)
- Multirate time advancing

Introduction and motivations

Work context

- Implementation and development of a simulation tool whose major ingredients are :
  - A numerical model suited to **industrial problems**
  - Turbulence models suited to the simulation of **turbulent flows** with massive separations and vortex shedding

In the present work

- Evaluation of **dynamic** and **hybrid VMS-LES models** on the prediction of vortex shedding flows
- Simulation of the flow around **circular** and **tandem cylinders** :
  - Contain many features and difficulties encountered in industrial problems
  - Well documented benchmarks
  - First step before the computation of array of cylinders (offshore oil and gas industries, civil engineering, aeronautics)
- Multirate time advancing

Mixed-element-volume method

Spatial discretization

- A mesh built from a non **structured tetrahedrization**
- Degrees of freedom located at nodes i
- **Mixed variational formulation**: Finite Element/Finite Volume method (FE/FV)

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathcal{F}(W) = \nabla \cdot \mathcal{R}(W) \tag{1}$$

- **Diffusive fluxes** $\mathcal{R}(W)$ evaluated by **FE method**
- **Convective fluxes** $\mathcal{F}(W)$ by **FV method**
- Variational formulation with 2 types of **test functions**: P1 FE functions Φ_i and characteristic functions χ_i

Time discretization

- Implicit time integration by a second order backward difference scheme

Mixed-element-volume method

Spatial discretization

- A mesh built from a non **structured tetrahedrization**
- Degrees of freedom located at nodes i
- **Mixed variational formulation**: Finite Element/Finite Volume method (FE/FV)

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathcal{F}(W) = \nabla \cdot \mathcal{R}(W) \tag{1}$$

  - **Diffusive fluxes** $\mathcal{R}(W)$ evaluated by **FE method**
  - **Convective fluxes** $\mathcal{F}(W)$ by **FV method**
  - Variational formulation with 2 types of **test functions**: P1 FE functions Φ_i and characteristic functions χ_i

Time discretization

- Implicit time integration by a second order backward difference scheme

Mixed-element-volume method

Spatial discretization

- A mesh built from a non **structured tetrahedrization**
- Degrees of freedom located at nodes i
- **Mixed variational formulation**: Finite Element/Finite Volume method (FE/FV)

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathcal{F}(W) = \nabla \cdot \mathcal{R}(W) \tag{1}$$

  - **Diffusive fluxes** $\mathcal{R}(W)$ evaluated by **FE method**
  - **Convective fluxes** $\mathcal{F}(W)$ by **FV method**
  - Variational formulation with 2 types of **test functions**: P1 FE functions Φ_i and characteristic functions χ_i

Time discretization

- Implicit time integration by a second order backward difference scheme

Mixed-element-volume method

Spatial discretization

- A mesh built from a non **structured tetrahedrization**
- Degrees of freedom located at nodes i
- **Mixed variational formulation**: Finite Element/Finite Volume method (FE/FV)

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathcal{F}(W) = \nabla \cdot \mathcal{R}(W) \qquad (1)$$

  - **Diffusive fluxes** $\mathcal{R}(W)$ evaluated by **FE method**
  - **Convective fluxes** $\mathcal{F}(W)$ by **FV method**
  - Variational formulation with 2 types of **test functions**: P1 FE functions Φ_i and characteristic functions χ_i

$$Vol(C_i)\frac{dW_i}{dt} + \sum_{j \in N(i)} \int_{\partial C_{ij}} \mathcal{F}(W) \cdot n d\sigma = -\int_{T, i \in T} \mathcal{R}(W) \cdot \nabla \Phi_i$$

Time discretization

- Implicit time integration by a second order backward difference scheme

Mixed-element-volume method

Spatial discretization

- A mesh built from a non **structured tetrahedrization**
- Degrees of freedom located at nodes i
- **Mixed variational formulation**: Finite Element/Finite Volume method (FE/FV)

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathcal{F}(W) = \nabla \cdot \mathcal{R}(W) \qquad (1)$$

  - **Diffusive fluxes** $\mathcal{R}(W)$ evaluated by **FE method**
  - **Convective fluxes** $\mathcal{F}(W)$ by **FV method**
  - Variational formulation with 2 types of **test functions**: P1 FE functions Φ_i and characteristic functions χ_i

Time discretization

- Implicit time integration by a second order backward difference scheme

Mixed-element-volume method

Evaluation of the convective terms by FV method :

- Convective fluxes integrated on a **dual mesh** built from medians ($2D$) and median plans ($3D$)

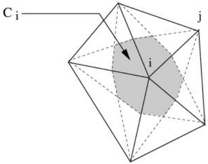


Figure 1: A control volume (the shaded area) constructed around each mesh node (in $2D$)

- A **high order** numerical dissipation scheme

VMS-LES

Important features of the VMS-LES approach

- No spatial filtering of the Navier-Stokes equations, but a **variational projection** of these equations
- *A priori* scale-separation
- The effects of the **unresolved structures modeled** only in the equations governing the **small resolved scales**

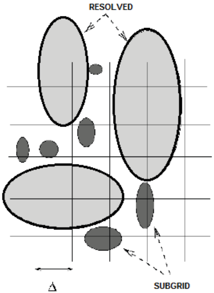


Figure 2: **LES** filtering operation

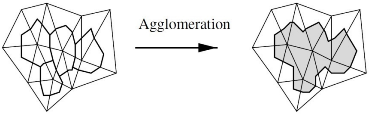


Figure 3: Building the **VMS** coarse level

VMS-LES

Important features of the VMS-LES approach

- No spatial filtering of the Navier-Stokes equations, but a **variational projection** of these equations
- *A priori* scale-separation
- The effects of the **unresolved structures modeled** only in the equations governing the **small resolved scales**

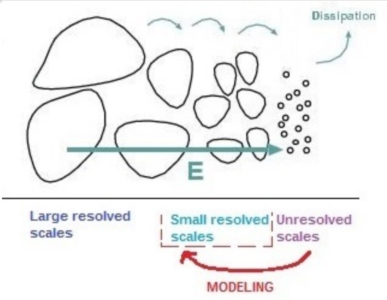


Figure 4: Modeling of the unresolved scales in VMS-LES

VMS-LES

Specificities of our VMS-LES approach

Development of a VMS model that can take into account :

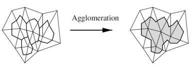
- the compressible Navier-Stokes equations ($3D$)
- a mixed element-volume framework
- unstructured meshes
- parallel computing
- vortex shedding flows

Turbulence model : VMS-LES

FE/FV semi-discretization of the Navier-Stokes equations $\dfrac{\partial W}{\partial t} + \nabla \cdot F(W) = 0$

$+$

Variational projection operators based on spatial average on agglomerated cells



$+$

Modeling of the effects of the unresolved scales only on the small resolved scales by analogy with a subgrid scale (SGS) eddy viscosity model

$\Downarrow$

VMS-LES governing equations :

$$\left(\frac{\partial W_h}{\partial t}, \chi_i\right) + (\nabla \cdot F(W_h), \chi_i, \Phi_i) = -\left(\tau^{LES}({W_h}'), {\Phi_i}'\right)$$

where " $'$ " $=$ small resolved scales

SGS models : Smagorinsky or WALE, with possibly dynamic procedure (Germano-Lilly, $C_{model} \rightarrow C_{model}(x,t)$)

RANS/VMS-LES hybrid model

Motivations

- Computation of **massively separated flows at high Reynolds number** on unstructured mesh
- RANS : accuracy problems in flow regions with massive separation (as the flow around bluff-bodies)
- VMS-LES : more expensive than RANS, very fine resolution requirements in boundary layers at high Reynolds number
- **Hybrid : combines RANS and VMS-LES** in order to exploit the advantages of the two approaches :
  - **less computationally** expensive compared with VMS-LES
  - **better accuracy** than RANS for flows dominated by large unsteady structures
- Desired feature for the hybridation strategy : automatic and progressive switch from RANS to VMS-LES and vice versa

RANS/VMS-LES hybrid model

Motivations

- Computation of **massively separated flows at high Reynolds number** on unstructured mesh
- RANS : accuracy problems in flow regions with massive separation (as the flow around bluff-bodies)
- VMS-LES : more expensive than RANS, very fine resolution requirements in boundary layers at high Reynolds number
- **Hybrid : combines RANS and VMS-LES** in order to exploit the advantages of the two approaches :
  - **less computationally** expensive compared with VMS-LES
  - **better accuracy** than RANS for flows dominated by large unsteady structures
- Desired feature for the hybridation strategy : automatic and progressive switch from RANS to VMS-LES and vice versa

RANS/VMS-LES hybrid model

Motivations

- Computation of **massively separated flows at high Reynolds number** on unstructured mesh
- RANS : accuracy problems in flow regions with massive separation (as the flow around bluff-bodies)
- VMS-LES : more expensive than RANS, very fine resolution requirements in boundary layers at high Reynolds number
- **Hybrid : combines RANS and VMS-LES** in order to exploit the advantages of the two approaches :
  - **less computationally** expensive compared with VMS-LES
  - **better accuracy** than RANS for flows dominated by large unsteady structures
- Desired feature for the hybridation strategy : automatic and progressive switch from RANS to VMS-LES and vice versa

RANS/VMS-LES hybrid model

Motivations

- Computation of **massively separated flows at high Reynolds number** on unstructured mesh
- RANS : accuracy problems in flow regions with massive separation (as the flow around bluff-bodies)
- VMS-LES : more expensive than RANS, very fine resolution requirements in boundary layers at high Reynolds number
- **Hybrid : combines RANS and VMS-LES** in order to exploit the advantages of the two approaches :
    - **less computationally** expensive compared with VMS-LES
    - **better accuracy** than RANS for flows dominated by large unsteady structures
- Desired feature for the hybridation strategy : automatic and progressive switch from RANS to VMS-LES and vice versa

RANS/VMS-LES hybrid model

Motivations

- Computation of **massively separated flows at high Reynolds number** on unstructured mesh
- RANS : accuracy problems in flow regions with massive separation (as the flow around bluff-bodies)
- VMS-LES : more expensive than RANS, very fine resolution requirements in boundary layers at high Reynolds number
- **Hybrid : combines RANS and VMS-LES** in order to exploit the advantages of the two approaches :
  - **less computationally** expensive compared with VMS-LES
  - **better accuracy** than RANS for flows dominated by large unsteady structures
- Desired feature for the hybridation strategy : automatic and progressive switch from RANS to VMS-LES and vice versa

RANS/VMS-LES hybrid model

- **Central idea of the proposed hybrid VMS model** :
  Correction of the mean flow field obtained with a RANS model by adding fluctuations given by a VMS-LES approach wherever the grid resolution is adequate
- **Decomposition of the flow variables** :

$$W = \underbrace{< W >}_{RANS} + \underbrace{W^c}_{correction} + W^{SGS}$$

$< W >$ = RANS flow variables
W^c = remaining resolved fluctuations obtained with VMS-LES
(i.e. $< W > + W^c = W_h$ = VMS-LES flow variables)
W^{SGS} = subgrid scale fluctuations

RANS/VMS-LES hybrid model

- **Central idea of the proposed hybrid VMS model** :
  Correction of the mean flow field obtained with a RANS model by adding fluctuations given by a VMS-LES approach wherever the grid resolution is adequate
- **Decomposition of the flow variables** :

$$W = \underbrace{< W >}_{RANS} + \underbrace{W^c}_{correction} + W^{SGS}$$

$< W >$ = RANS flow variables
W^c = remaining resolved fluctuations obtained with VMS-LES
(i.e. $< W > + W^c = W_h$ = VMS-LES flow variables)
W^{SGS} = subgrid scale fluctuations

RANS/VMS-LES hybrid model

- **Navier-Stokes equations :**

$$\left(\frac{\partial W}{\partial t}, \chi_i\right) + (\nabla \cdot F(W), \chi_i, \Phi_i) = 0$$

- **RANS equations** :

$$\left(\frac{\partial \langle W \rangle}{\partial t}, \chi_i\right) + (\nabla \cdot F(\langle W \rangle), \chi_i, \Phi_i) = -\left(\tau^{RANS}(\langle W \rangle), \Phi_i\right)$$

- **VMS-LES equations** :

$$\left(\frac{\partial \langle W \rangle + W^c}{\partial t}, \chi_i\right) + (\nabla \cdot F(\langle W \rangle + W^c), \chi_i, \Phi_i) = -\left(\tau^{LES}([\langle W \rangle + W^c]'), \Phi_i'\right)$$

RANS/VMS-LES hybrid model

⇒ **governing equations for the reconstructed fluctuations (VMS-LES eq. - RANS eq.) :**

$$\left(\frac{\partial W^c}{\partial t}, \chi_i\right) + (\nabla \cdot F(\langle W\rangle + W^c), \chi_i, \Phi_i) - (\nabla \cdot F(\langle W\rangle), \chi_i, \Phi_i)$$

$$= \left(\tau^{RANS}(\langle W\rangle), \Phi_i\right) - \left(\tau^{LES}([\langle W\rangle + W^c]'), \Phi_i'\right)$$

⇒ **modified governing equations for the reconstructed fluctuations :**

$$\left(\frac{\partial W^c}{\partial t}, \chi_i\right) + (\nabla \cdot F(\langle W\rangle + W^c), \chi_i, \Phi_i) - (\nabla \cdot F(\langle W\rangle), \chi_i, \Phi_i)$$

$$= (1-\theta)\left\{\left(\tau^{RANS}(\langle W\rangle), \Phi_i\right) - \left(\tau^{LES}([\langle W\rangle + W^c]'), \Phi_i'\right)\right\}$$

$\theta \in [0,1]$ = blending function
$\theta \to 1 \Rightarrow$ RANS is recovered
$\theta \to 0 \Rightarrow$ VMS-LES is recovered

$\theta = \tanh(\xi^2)$
$\xi = \mu_{SGS}/\mu_{RANS}$ or $\xi = \Delta/l_{RANS}$

RANS/VMS-LES hybrid model

⇒ **governing equations for the reconstructed fluctuations (VMS-LES eq. - RANS eq.) :**

$$\left(\frac{\partial W^c}{\partial t}, \chi_i\right) + \left(\nabla \cdot F(\langle W \rangle + W^c), \chi_i, \Phi_i\right) - \left(\nabla \cdot F(\langle W \rangle), \chi_i, \Phi_i\right)$$

$$= \left(\tau^{RANS}(\langle W \rangle), \Phi_i\right) - \left(\tau^{LES}([\langle W \rangle + W^c]'), \Phi_i'\right)$$

⇒ **modified governing equations for the reconstructed fluctuations :**

$$\left(\frac{\partial W^c}{\partial t}, \chi_i\right) + \left(\nabla \cdot F(\langle W \rangle + W^c), \chi_i, \Phi_i\right) - \left(\nabla \cdot F(\langle W \rangle), \chi_i, \Phi_i\right)$$

$$= (1-\theta)\left\{\left(\tau^{RANS}(\langle W \rangle), \Phi_i\right) - \left(\tau^{LES}([\langle W \rangle + W^c]'), \Phi_i'\right)\right\}$$

$\theta \in [0,1]$ = blending function
$\theta \to 1 \Rightarrow$ RANS is recovered
$\theta \to 0 \Rightarrow$ VMS-LES is recovered

$\theta = \tanh(\xi^2)$
$\xi = \mu_{SGS}/\mu_{RANS}$ or $\xi = \Delta/l_{RANS}$

RANS/VMS-LES hybrid model

- **Final hybrid VMS model equations** (RANS eq. + modified governing eq. for reconstructed fluctuations) :

$$\left(\frac{\partial W_h}{\partial t},\chi_i\right)+(\nabla\cdot F(W_h),\chi_i,\Phi_i)=\theta\left(\tau^{RANS}(\langle W\rangle),\Phi_i\right)-(1-\theta)\left(\tau^{LES}(W_h'),\Phi_i'\right)$$

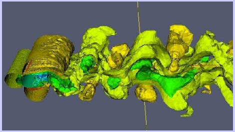
W_h = RANS solution corrected with VMS-LES reconstructed fluctuations

in regions of sufficiently fine grid resolution

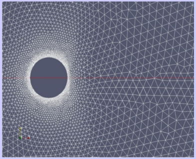
Circular cylinder - subcritical regime

Test case definition

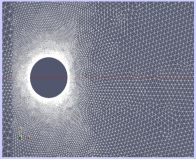
- **Flow parameters:**
  Reynolds=3900
  Mach=0.1



- **Computational grids:** :



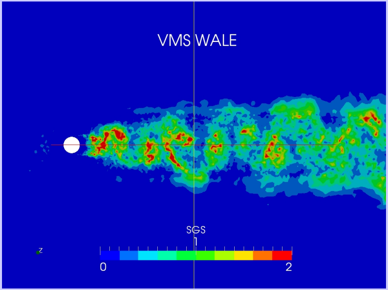
270K nodes, 1.6M elements

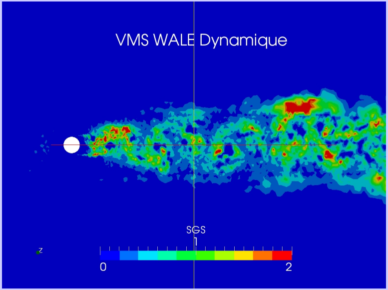


1.4M nodes, 8.4M elements

Circular cylinder - subcritical regime

Viscosity ratio μ_{SGS}/μ - non-dynamic VMS versus dynamic VMS





Circular cylinder - subcritical regime

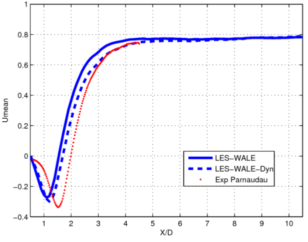
Bulk coefficients

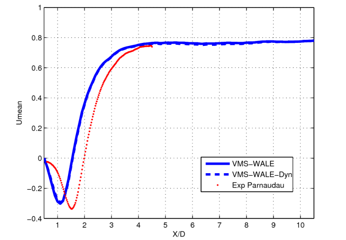
	Mesh size	$\overline{C}_d$	l_r/D	$-\overline{C}_{p_b}$	St
LES WALE	270K	0.96	1.06	0.96	0.22
LES dyn. WALE	270K	0.96	1.20	0.87	0.22
VMS WALE	270K	0.96	1.06	0.94	0.22
VMS dyn. WALE	270K	0.97	1.08	0.93	0.22
VMS WALE	1.4M	0.94	1.47	0.81	0.22
VMS dyn. WALE	1.4M	0.94	1.47	0.85	0.22
LES Lee et al. (2006)	7.7M	0.99-1.04	1.35-1.37	0.89-0.94	0.209-0.212
LES Kravchenko et al. (2000)	500K-2.4M	1.04-1.38	1-1.35	0.93-1.23	0.193-0.21
LES Parnaudeau et al. (2008)	44M	-	1.56	-	0.207-0.209
Experiments					
Parnaudeau et al. (2008)		-	1.51	-	0.206-0.210
Norberg (1987)		0.94-1.04	-	0.83-0.93	
Ong et al. (1996)		-	-	-	0.205-0.215

Circular cylinder - subcritical regime

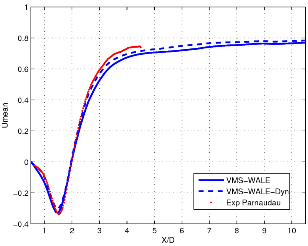
Mean streamwise velocity profile along the wake centerline

Coarse grid :



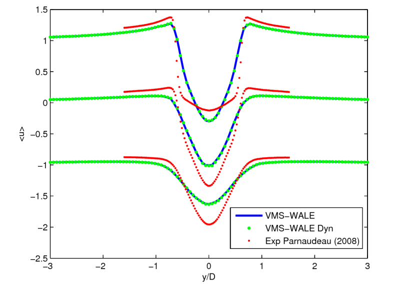


Fine grid :

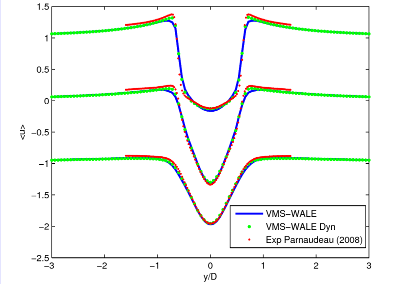


Circular cylinder - subcritical regime

Mean streamwise velocity profile at $x/D = 1.06, 1.54$ and 2.02



Coarse grid



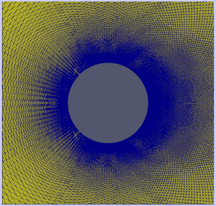
Fine grid

Zoom : 

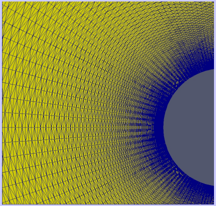
Circular cylinder - supercritical regime

Test case definition

- **Flow parameters:**
  $D = 0.057$m
  Mach $= 0.1$
  Reynolds $= 1$M



- **Computational grids:**
  1.2M nodes
  15M elements



Circular cylinder - supercritical regime

Bulk coefficients

	Mesh size	$\overline{C}_d$	C_l'	$-\overline{C}_{p_b}$	St
URANS	1.2M	0.24	0.06	0.25	0.46
VMS WALE	1.2M	0.36	0.22	0.22	0.10
Hybrid VMS WALE	1.2M	0.24	0.17	0.28	0.17
URANS of Catalano et al. (2003)	2.3M	0.40	-	0.41	0.31
LES of Catalano et al. (2003)	2.3M	0.31	-	0.32	0.35
LES of Kim at al. (2005)	6.8M	0.27	0.12	0.28	-
Experiments					
Shih et al. (1993)		0.24	-	0.33	-
Gölling (2006)		-	-	-	0.10
Zdravkovich (1997)		0.2-0.4	0.1-0.15	0.2-0.34	0.18

Circular cylinder - supercritical regime

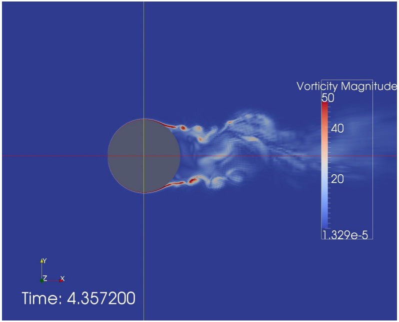
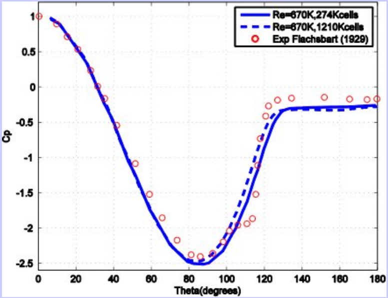


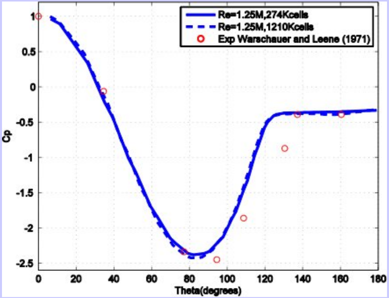
Figure 5: Vorticity magnitude - Hybride RANS/VMS-LES

Circular cylinder - supercritical regime

Mean pressure coefficient distribution



Reynolds 6.7×10^5

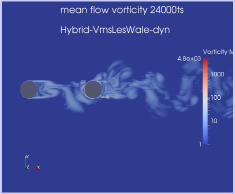


Reynolds 1.25×10^6

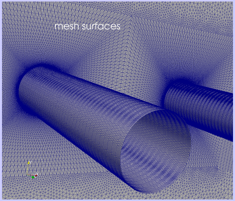
Tandem Cylinders

Test case definition

- **Flow parameters** :
  Reynolds $= 1.66 \times 10^5$
  Mach=0.1



- **Computational grids** :
  2.59M nodes
  15M elements



Tandem Cylinders

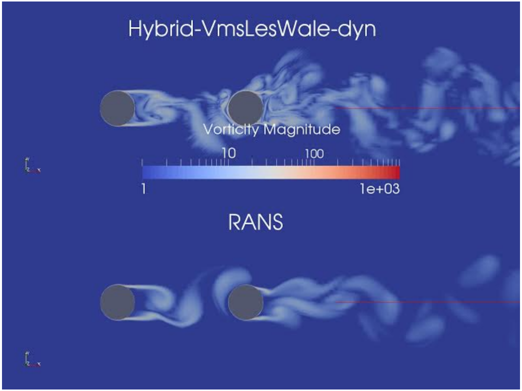
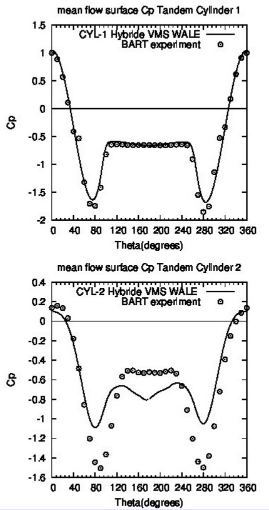


Figure 6: Vorticity magnitude - Hybride RANS/VMS-LES WALE dynamique

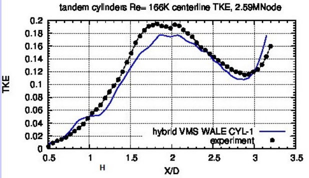
Tandem Cylinders

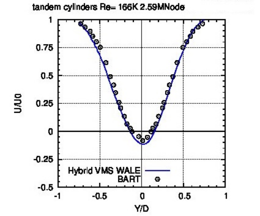
Mean pressure coefficient distribution



Tandem Cylinders

Mean centerline TKE between the two cylinders, mean C_d and mean spanwise velocity profile at $x/D = 4.45$ (preliminary results)





	Mesh size	$\overline{C}_d$ Cyl. 1	$\overline{C}_d$ Cyl. 2
Hybrid VMS dyn.	2.59M	0.64	0.38
Numerical results			
Lockard (2011)	2M-133M	0.33-0.80	0.29-0.52
DES Aybay (2010)	6.7M	0.64	0.44
HRLES Vatsa (2010)	8.7M	0.64	0.45
Experiments			
Neuhart et al. (2009)		0.64	0.31

Multirate time scheme

Multirate based on agglomeration

A frequent configuration in mesh adaptation combines :

- An explicit time advancing scheme for accuracy purpose
- A computational grid with a very small portion of much smaller elements than in the remaining mesh

Examples:

- Isolated traveling shock
- Boundary layer at high Reynolds number (few tens of microns thick) in LES computations where vortices around one centimeter are captured

Explicit time advancing schemes with global time stepping too costly

Multirate time scheme

Multirate based on agglomeration

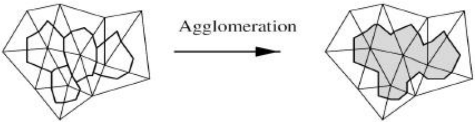
- **Inner and outer zones** :
  - Let Δt be the global time step over the computational domain
  - Define the **outer zone** as the set of cells for which the explicit scheme is stable for a time step $K\Delta t$, and the **inner zone** as its complement
  - Definition of these zones through the local time steps
- **Coarse grid** :
  - **Objective** :
    - Advancement in time with time step $K\Delta t$
    - Advancement in time preserving accuracy in the outer zone (space order of 3, RK4)
    - Advancement in time consistent in the inner zone
  - Define the **coarse grid** as the macro cells in the inner zone + the fine cells in the outer zone
  - **Methods** :
    - Advancement in time of the chosen explicit scheme on the coarse grid with $K\Delta t$

Multirate time scheme

Multirate based on agglomeration

- **Evaluation of the fluxes on the coarse grid** :
  - Assembling of the nodal fluxes Ψ_i on the fine cells (as usual)
  - Sum of the fluxes on each macro cell I located in the inner zone :

$$\Psi^I = \sum_{k \in I} \Psi_k$$



fine cells i macro cell I

Multirate time scheme

Multirate based on agglomeration

Step 1 (predictor) :

Advancement in time with Runge-Kutta (for example) on the macro cells in the inner zone and on the fine cells in the outer zone, with time step $K\Delta t$:

Pour $\alpha = 1$, *RKstep*

$$\text{outer zone :} \quad vol_i w_i^{(\alpha)} = vol_i w_i^{(0)} + b_\alpha K\Delta t \; \Psi_i^{(\alpha-1)}$$

$$\text{inner zone :} \quad vol^I w^{I,(\alpha)} = vol^I w^{I,(0)} + b_\alpha K\Delta t \; \Psi^{I,(\alpha-1)}$$

$$w_i^{(\alpha)} = w^{I,(\alpha)} \quad \text{for } i \in I$$

Fin α.

Multirate time scheme

Multirate based on agglomeration

Step 2 (corrector) :

- Unknowns frozen in the outer zone
- Time interpolation of these unknowns (those useful for the next point)
- In the inner zone : using these interpolated values, advancement in time with the chosen explicit scheme and time step Δt
- Complexity mastered (proportional to the number of points in the inner zone)

Conclusion, perspectives

Test cases (Circular cylinder, tandem cylinders)

- Significant reduction of the amount of SGS viscosity by the **dynamic procedure in VMS-LES**
- Less impact of the dynamic procedure in VMS-LES than in LES
- Nevertheless, in some cases, **sensible improvement** of the agreement with reference data made by the dynamic procedure in VMS-LES
- Presentation of a strategy for **blending RANS and VMS-LES**
- The supercritical flow around a circular cylinder **reasonably well predicted** by the proposed VMS hybrid model
- Still some progress to do in order to improve the hybridation strategy
- **Bulk and pressure predictions** of the tandem cylinder are promising

Conclusion, perspectives

Test cases (Circular cylinder, tandem cylinders)

- Significant reduction of the amount of SGS viscosity by the **dynamic procedure in VMS-LES**
- Less impact of the dynamic procedure in VMS-LES than in LES
- Nevertheless, in some cases, **sensible improvement** of the agreement with reference data made by the dynamic procedure in VMS-LES
- Presentation of a strategy for **blending RANS and VMS-LES**
- The supercritical flow around a circular cylinder **reasonably well predicted** by the proposed VMS hybrid model
- Still some progress to do in order to improve the hybridation strategy
- **Bulk and pressure predictions** of the tandem cylinder are promising

Conclusion, perspectives

Test cases (Circular cylinder, tandem cylinders)

- Significant reduction of the amount of SGS viscosity by the **dynamic procedure in VMS-LES**
- Less impact of the dynamic procedure in VMS-LES than in LES
- Nevertheless, in some cases, **sensible improvement** of the agreement with reference data made by the dynamic procedure in VMS-LES
- Presentation of a strategy for **blending RANS and VMS-LES**
- The supercritical flow around a circular cylinder **reasonably well predicted** by the proposed VMS hybrid model
- Still some progress to do in order to improve the hybridation strategy
- **Bulk and pressure predictions** of the tandem cylinder are promising

Conclusion, perspectives

Test cases (Circular cylinder, tandem cylinders)

- Significant reduction of the amount of SGS viscosity by the **dynamic procedure in VMS-LES**
- Less impact of the dynamic procedure in VMS-LES than in LES
- Nevertheless, in some cases, **sensible improvement** of the agreement with reference data made by the dynamic procedure in VMS-LES
- Presentation of a strategy for **blending RANS and VMS-LES**
- The supercritical flow around a circular cylinder **reasonably well predicted** by the proposed VMS hybrid model
- Still some progress to do in order to improve the hybridation strategy
- **Bulk and pressure predictions** of the tandem cylinder are promising

Conclusion, perspectives

Test cases (Circular cylinder, tandem cylinders)

- Significant reduction of the amount of SGS viscosity by the **dynamic procedure in VMS-LES**
- Less impact of the dynamic procedure in VMS-LES than in LES
- Nevertheless, in some cases, **sensible improvement** of the agreement with reference data made by the dynamic procedure in VMS-LES
- Presentation of a strategy for **blending RANS and VMS-LES**
- The supercritical flow around a circular cylinder **reasonably well predicted** by the proposed VMS hybrid model
- Still some progress to do in order to improve the hybridation strategy
- **Bulk and pressure predictions** of the tandem cylinder are promising

Conclusion, perspectives

Multitrate time advancing

- Implementation of the **multirate algorithm** in the parallel CFD code **AIRONUM** shared by INRIA Sophia-Antipolis, LEMMA company and university of Montpellier.
- Special attention was paid to issues related to **parallelism**, and in particular to the evaluation of the fluxes on the macro cells located at the boundary between neighboring subdomains.
- Preliminary runs with our multirate method for the tandem cylinders benchmark : results currently being processed... but the current domain decomposition limits the efficiency of this multirate approach.
- Need to improve the **domain decomposition** into subdomains, at the present time designed to minimize the inter-core communications, so that workload becomes (almost) equally shared by each computer core.
  For example, and for the tandem cylinders mesh (15 M tetrahedra) decomposed into 192 subdomains, some of them contain only inner nodes, and others only outer nodes.

End

Thank you for your attention!