Vers l'adaptation de maillage goal-oriented pour une interaction fluide structure

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Plan de la présentation

Formulation ALE

Système d'Euler en ALE

$$\frac{1}{\mathscr{J}}\frac{\partial \mathscr{J} W}{\partial t} + \nabla_x \mathscr{F}(W) = 0 \quad (+BC \text{ and } IC) \tag{1}$$

où $\mathscr{J}(\xi,t) = det\frac{\partial \phi}{\partial \xi}(\xi,t)$ avec ϕ la transformation.

Modèle totalement continu variationnel

$$\text{Find } W \in \mathscr{H}^1(Q_T) \text{ such that } \forall \varphi \in \mathscr{H}^1(Q_T), \quad (\Psi(W), \varphi) = 0$$

$$\begin{aligned}
\text{with } \; (\Psi(W), \varphi) &= \int_{\Omega_0} \varphi(0)(W_0 - W(0))\, \mathrm{d}\Omega \\
&+ \int_0^T \frac{\partial}{\partial t} \int_{\Omega_t} \varphi W \,\mathrm{d}\Omega\,\mathrm{d}t - \int_0^T \int_{\Omega_0} \tilde{W} \mathscr{J} \frac{d\tilde{\varphi}}{dt}\,\mathrm{d}\Omega\,\mathrm{d}t \\
&+ \int_0^T \int_{\Omega_t} \varphi \nabla \cdot \mathscr{F}(W)\,\mathrm{d}\Omega\,\mathrm{d}t - \int_0^T \int_{\partial\Omega_t} \varphi\, \breve{\mathscr{F}}(W).\mathbf{n}\,\mathrm{d}\Gamma\,\mathrm{d}t \qquad (2)
\end{aligned}$$

Modèle variationnel semi-discret en temps

Find $W \in \mathscr{H}^1(Q_T)$ such that $\forall \varphi \in \mathscr{H}^1(Q_T)$, $\quad (\Psi(W), \varphi) = 0$ with

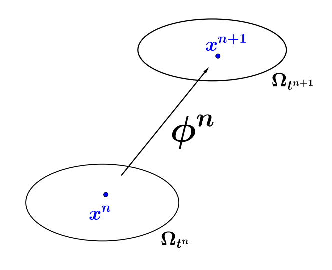
$$
\begin{aligned}
(\Psi(W), \varphi) = & \int_{\Omega_{t^1}} \varphi(t^1)(W_0 - W(.,t^1))\,\mathrm{d}\Omega \\
+ & \sum_{n=1}^{nmax} (t^{n+1} - t^n) \boxed{\frac{1}{t^{n+1} - t^n}\left[\int_{\Omega_{t^{n+1}}} \varphi(.,t^{n+1}) W(.,t^{n+1})\,\mathrm{d}\Omega - \int_{\Omega_{t^n}} \varphi(.,t^n) W(.,t^n)\,\mathrm{d}\Omega\right]} \\
+ & \sum_{n=1}^{nmax} (t^{n+1} - t^n) \boxed{\int_{\Omega_0} \frac{1}{t^{n+1} - t^n}\, \tilde{W}(.,t^n)\, \mathscr{J}^n \left(\tilde{\varphi}(.,t^{n+1}) - \tilde{\varphi}(.,t^n)\right) \mathrm{d}\Omega} \\
+ & \sum_{n=1}^{nmax} (t^{n+1} - t^n) \int_{\Omega_{t^n}} \varphi(.,t^n)\, \nabla \cdot \mathscr{F}(W(.,t^n))\,\mathrm{d}\Omega \\
- & \sum_{n=1}^{nmax} (t^{n+1} - t^n) \int_{\partial\Omega_{t^n}} \varphi(.,t^n)\, \breve{\mathscr{F}}(W(.,t^n)).\mathbf{n}\,\mathrm{d}\Gamma.
\end{aligned}
$$

Terme $\int_{\Omega_0} \frac{1}{t^{n+1} - t^n} \tilde{W}(.,t^n) \mathscr{J}^n \left(\tilde{\varphi}(.,t^{n+1}) - \tilde{\varphi}(.,t^n)\right) \mathrm{d}\Omega$

on considère $\varphi \in \mathscr{H}^1_{cst}(Q_T)$

Notations

Pour la suite on a besoin d'introduire : ϕ^n définie la transformation d'un niveau n au niveau $n+1$:



et

$$\varphi^n \text{ et } \hat{\varphi}^n \text{ telles que } \varphi^n(\mathbf{x}^n) = \varphi(\mathbf{x}^n, t^n), \quad \hat{\varphi}(\mathbf{x}^n, t^{n+1}) = \varphi^n(\phi^n(\mathbf{x}^n)) = \hat{\varphi}^n(\mathbf{x}^n).$$

Le pas de temps s'écrit alors :

$$0 = \frac{1}{t^{n+1} - t^{n}} \left[\int_{\Omega_{t^n}} \hat{\varphi}^{n}(\mathbf{x}^{n}) \hat{W}(\mathbf{x}^{n}, t^{n+1}) \ \mathscr{J}^{n}(\mathbf{x}^{n}) \, \mathrm{d}\Omega - \int_{\Omega_{t^n}} \varphi^{n}(\mathbf{x}^{n}) W(\mathbf{x}^{n}, t^{n}) \, \mathrm{d}\Omega \right] + \int_{\Omega_{t^n}} \varphi^{n}(\mathbf{x}^{n}) \nabla \cdot \mathscr{F}(W(\mathbf{x}^{n}, t^{n})) \, \mathrm{d}\Omega - \int_{\partial\Omega_{t^n}} \varphi^{n}(\mathbf{x}^{n}) \breve{\mathscr{F}}(W(\mathbf{x}^{n}, t^{n})).\mathbf{n} \, \mathrm{d}\Gamma.$$

Modèle variationnel discret

$$
\begin{aligned}
&\forall W \in \mathscr{H}^1(Q_T), \forall \varphi \in \mathscr{H}^1_{cst}(Q_T)\,,\\
&(\Psi_h(W)\,,\,\varphi) = \int_{\Omega_0} \varphi(.,t^0)(\Pi_h W_0 - \Pi_h W(.,t^1))\,\mathrm{d}\Omega\\
+\; & \sum_{n=1}^{nmax} \frac{1}{t^{n+1}-t^n}\left[\int_{\Omega_{t^n}} \widehat{\Pi_h\varphi}(.,t^n)\widehat{\Pi_h W}(.,t^{n+1})\,\mathscr{J}^n\,\mathrm{d}\Omega - \int_{\Omega_{t^n}} \Pi_h\varphi(.,t^n)\Pi_h W(.,t^n))\,\mathrm{d}\Omega\right]\\
+\; & \sum_{n=1}^{nmax} \int_{\Omega_{t^n}} \Pi_h\varphi(.,t^n))\,\nabla\cdot\Pi_h\mathscr{F}(W(.,t^n)))\,\mathrm{d}\Omega\\
-\; & \sum_{n=1}^{nmax} \int_{\partial\Omega_{t^n}} \Pi_h\varphi(.,t^n))\,\Pi_h\breve{\mathscr{F}}(W(.,t^n)).\mathbf{n}\,\mathrm{d}\Gamma.
\end{aligned}
\tag{4}
$$

Adjoint discret

Find $W^* \in \mathscr{H}^1_{cst}(Q_T)$ such that $\forall \psi \in \mathscr{H}^1(Q_T)$, $\left(\dfrac{\partial \Psi_h}{\partial W}(W)\psi\,,\,W^*\right) = 0$ with

$$\begin{aligned}
&\left(\frac{\partial \Psi_h}{\partial W}(W)\psi\,,\,W^*\right) = \int_{\Omega_0} W^*(.,t^0)(-\Pi_h\psi(.,t^1))\,\mathrm{d}\Omega \\
+\;& \sum_{n=1}^{nmax} \frac{1}{t^{n+1}-t^n}\left[\int_{\Omega_{t^n}} \left(\widehat{\Pi_h W^*}(.,t^n)\widehat{\Pi_h \psi}(.,t^{n+1})\;\mathscr{J}^n\,\mathrm{d}\Omega - \Pi_h W^*(.,t^n)\Pi_h\psi(.,t^n)\right)\mathrm{d}\Omega\right] \\
+\;& \sum_{n=1}^{nmax} \int_{\Omega_{t^n}} \Pi_h W^*(.,t^n)\,\nabla\cdot\Pi_h \frac{\partial \mathscr{F}}{\partial W}(\psi(.,t^n))\,\mathrm{d}\Omega \\
-\;& \sum_{n=1}^{nmax} \int_{\partial\Omega_{t^n}} \Pi_h W^*(.,t^n)\,\Pi_h \frac{\partial \breve{\mathscr{F}}}{\partial W}(\psi(.,t^n))\cdot\mathbf{n}\,\mathrm{d}\Gamma. \qquad (5)
\end{aligned}$$

Pour trouver l'équation de l'adjoint

On considère 2 niveaux $n = n$ et $n = n-1$:

$$\frac{1}{t^{n+1}-t^{n}}\left[\int_{\Omega_{t^n}}\left(\widehat{\Pi_h W^*}(.,t^n)\widehat{\Pi_h \psi}(.,t^{n+1})\ \mathscr{J}^{n}\,\mathrm{d}\Omega - \Pi_h W^*(.,t^n)\Pi_h\psi(.,t^n))\right)\mathrm{d}\Omega\right]$$
$$+\quad \frac{1}{t^{n}-t^{n-1}}\left[\int_{\Omega_{t^{n-1}}}\widehat{\Pi_h W^*}(.,t^{n-1})\widehat{\Pi_h \psi}(.,t^{n})\ \mathscr{J}^{n-1}\,\mathrm{d}\Omega - \Pi_h W^*(.,t^{n-1})\Pi_h\psi(.,t^{n-1}))\,\mathrm{d}\Omega\right]$$