Hybrid simulation of high-Reynolds number flows relying on a variational multiscale model

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Abstract We are interested in the study and improvement of the LES component in hybrid RANS-LES formulations. These models are not designed for blunt-body flows with laminar boundary layers, but it is interesting to examine how they behave in that case. A DDES model is compared with a dynamic Variational multi-scale (DVMS) LES model for two subcritical flows past a cylinder. We then propose a hybridation restricting to DVMS in LES regions. The performances of the different options are compared for subcritical flows and for a flow around a tandem cylinder.

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1 Nomenclature

τ^{LES}	SGS stress tensor
τ^{RANS}	Reynolds stress tensor
l_{DDES}	DDES characteristic length
Δ	local mesh size
$\bar{C}_d$	Mean drag
C_{Lrms}	Lift rms fluctuations
$\bar{C}_p$	Mean pressure coefficient
$\bar{C}_{p_b}$	Mean base pressure coefficient
St	Strouhal number
l_r	Recirculation length
D	Cylinder diameter

2 Introduction and motivations

This work takes place in a study of numerical simulation methods suited to industrial problems, equipped with turbulence models adapted to the simulation of turbulent flows with massive separations and vortex shedding. Recent contributions concerning this study can be found in [15, 14]. The numerical approximation is second-order accurate, applicable to unstructured tetrahedral meshes, and involve a very low numerical dissipation.

In order to address high Reynolds number flows, we consider a RANS-LES hybridization similar to the Detached Eddy Simulation (DES) [24, 23]. While LES is a natural approach for computing subcritical blunt body flows, DES is not designed for this regime but can yet offer reasonable predictions [2]. This can be due to the low impact of the RANS component on the boundary layers at these regimes. In the wake, it remains important to have a LES mode as accurate as possible. In [21], a formulation closer to the standard Smagorinsky model is introduced in a DES formulation by a modification of the energy production term. However, the shear layer between main flow and wake can suffer from the tendancy of a Smagorinsky-like model to reinforce the filtering in these regions. In the present study, we consider a DVMS LES model in which (1) the space-time dependent strength of the filter is controlled by a dynamic process, and (2) the width of the filter is numerically controlled by using a variational multiscale (VMS) formulation.

The present work focuses on the following issues:

- the evaluation of DDES and VMS models for the prediction of subcritical blunt body flows,
- the definition of a hybrid model inheriting the accuracy of DVMS,
- the comparison of these models for a subcritical flow and for a well documented supercritical flow, the flow around a tandem cylinder.

3 Numerical model

The Navier-Stokes system for compressible fluid flow is solved. The spatial discretization is based on a mixed finite-volume/finite-element formulation, with degrees of freedom located at vertices i of the tetrahedrization. The finite volume part is integrated on a dual mesh built in 2D from median (Figure 1), in 3D from median plans. The diffusive fluxes are evaluated by a finite-element method, whereas

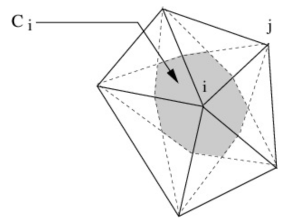

Fig. 1 Dual cell in 2D

a finite-volume method is used for the convective fluxes. The numerical approximation of the convective fluxes at the interface of neighboring cells is based on the Roe scheme [22]. In order to obtain second-order accuracy in space, the Monotone Upwind Scheme for Conservation Laws reconstruction method (MUSCL) [10] is used, in which the Roe flux is expressed as a function of reconstructed values of the discrete flow variable W_h at each side of the interface between two cells. We refer to [1] for details on the definition of these reconstructed values.

We just emphasize that particular attention has been paid to the dissipative properties of the resulting scheme since this is a key point for its successful use in LES. The numerical (spatial) dissipation provided by this scheme is made of sixth-order space derivatives [1] and is then concentrated on a narrow-band of the highest resolved frequencies. This is expected to limit undesirable damping by numerical dissipation of the large scales. Moreover, this dissipation is $O(\Delta x^5)$ and a parameter γ_S directly controls the amount of introduced viscosity and can be explicitly tuned in order to control the influence of numerical dissipation and, when necessary, reduce it to the minimal amount needed to stabilize the simulation. Time integration uses an implicit second-order backward differencing scheme. In [1], it is shown that when used in combination with moderate time steps, the time dissipation is also quite small.

4 Turbulence model: VMS-LES

The Variational Multiscale (VMS) model for the large eddy simulation (LES) of turbulent flows has been introduced in [6] in combination with spectral methods. In [7], an extension to unstructured finite volumes is defined. That method is adapted in the present work. Let us explain this VMS-LES approach in a *simplified context*. Assume the mesh is made of two embedded meshes, corresponding to a P^1-continuous finite-element approximation space V_h with the usual basis functions Φ_i vanishing on all vertices but vertex i. Let be V_{2h} its embedded coarse subspace V_{2h}. Let V_h' be the complementary space: $V_h = V_{2h} \oplus V_h'$. The space of *small scales* V_h' is spanned by only the fine basis functions Φ_i' related to vertices which are not vertices of V_{2h}. We write the compressible Navier-Stokes equations as follows: $\dfrac{\partial W}{\partial t} + \nabla \cdot F(W) = 0$ where $W = (\rho, \rho\mathbf{u}, \rho E)$. The VMS-LES discretization writes for $W_h = \sum W_i \Phi_i$:

$$\left(\frac{\partial W_h}{\partial t}, \Phi_i\right) + (\nabla \cdot F(W_h), \Phi_i) = -\left(\tau^{LES}({W_h}'), {\Phi_i}'\right) \quad (1)\ .$$

For a test function related to a vertex of V_{2h}, the RHS vanishes, which limits the action of the LES term to small scales. *In practice*, embedding two unstructured meshes V_h and V_{2h} is a constraint that we want to avoid. The coarse level is then built from the agglomeration of vertices/cells as sketched in Figure 2. It remains to

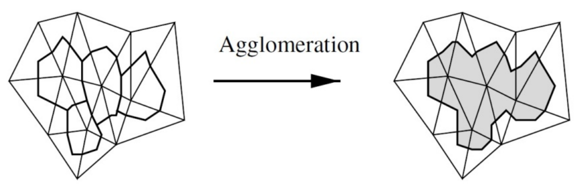

Fig. 2 Building the VMS coarse level

define the model term $\tau^{LES}(W_h')$. This term represents the SGS stress term, acting only on small scales W_h', and computed from the small scale component of the flow field by applying either a Smagorinsky or a WALE SGS model, [17], the constants of these models being evaluated by the Germano-Lilly dynamic procedure [4, 11, 15]. The main property of the VMS formulation is that the modeling of the unresolved structures is influencing only the small resolved scales, as described in Figure 3, in contrast with, for example the usual Smagorinsky model. This implies two main properties. First, the backscatter transfer of energy to large scales is not damped by the model. Second, the model is naturally unable to produce an artificial viscous layer at a no-slip boundary, or in shear layers.

5 Turbulence model: RANS-DDES

The RANS component used in the present study is the $k-\varepsilon$ model proposed by Goldberg [5]. The turbulent viscosity is limited by the Bradshaw's law in a similar way to Menter's SST model. In what we call DDES/$k-\varepsilon$/Menter, the above RANS

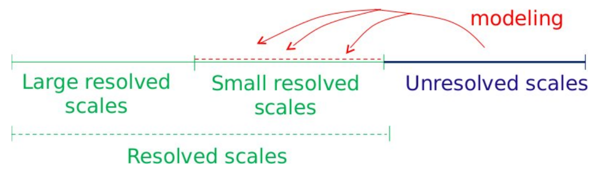

Fig. 3 VMS principle.

model is introduced in a DDES formulation [23] by replacing in the RHS of the k equation the $D_k^{RANS} = \rho\varepsilon$ dissipation term by:

$$D_k^{DDES} = \rho \frac{k^{3/2}}{l_{DDES}} \quad \text{where} \quad l_{DDES} = \frac{k^{3/2}}{\varepsilon} - f_d * max(0, \frac{k^{3/2}}{\varepsilon} - C_{DDES}\Delta)$$

$$\text{with} \quad f_d = 1 - tanh((8r_d)^3) \quad \text{and} \quad r_d = \frac{\nu_t + \nu}{max(\sqrt{u_{i,j}u_{i,j}}, 10^{-10})K^2 d_w^2}.$$

K denotes the von Karman constant ($K = 0.41$), d_w the wall-normal distance, $u_{i,j}$ the x_j-derivative of the ith-component of the velocity u, and the model constant C_{DDES} is set to the standard value 0.65 (ν_t and ν are the turbulent kinematic viscosity and the kinematic viscosity of the fluid, respectively).

6 Turbulence model: hybrid RANS/VMS-LES

The LES part of the above RANS-DDES is a kind of Smagorinsky model, but not exactly as remarked in [21]. The central idea of the hybrid VMS model which we propose is to combine the mean flow field obtained with the RANS component with the application of the DVMS model wherever the grid resolution is adequate. First, let us write the semi-discretization of the RANS equations :

$$\left(\frac{\partial \langle W_h \rangle}{\partial t}, \Phi_i\right) + (\nabla \cdot F(\langle W_h \rangle), \Phi_i) = -\left(\tau^{RANS}(\langle W_h \rangle), \Phi_i\right).$$

A natural hybridation writes:

$$\left(\frac{\partial W_h}{\partial t}, \Phi_i\right) + (\nabla \cdot F(W_h), \Phi_i) = -\theta \left(\tau^{RANS}(\langle W_h \rangle), \Phi_i\right) - (1-\theta)\left(\tau^{LES}(W_h'), \Phi_i'\right) \quad (5)$$

where W_h denotes now the hybrid variables and θ is defined by:

$$\theta = 1 - f_d(1 - tanh(\xi^2)) \quad \text{with} \quad \xi = \frac{\Delta}{l_{RANS}} \quad \text{or} \quad \xi = \frac{\mu_{SGS}}{\mu_{RANS}}$$

where f_d is defined as in Section 5.

7 Results

We consider three flows around one or two cylinders. The flow around a cylinder is strongly dependent on the turbulence which exists in the inflow and the turbulence which is created along the wall boundary layer after the stagnation point. In the case where the inflow involves no turbulence, turbulence is created on the wall at a sufficiently high critical Reynolds number, corresponding to the *drag crisis*. This Reynolds number is between $300,000$ and $500,000$. Under this number, the flow is subcritical and, in short, the boundary layer flow is laminar. Then in principle neither a RANS nor a hybrid model should be applied, but it is interesting to be able to use a unique model for various regimes.

7.1 Circular cylinder 3900 - subcritical

	Mesh	$\overline{C}_d$	$-\overline{C}_{p_b}$	C_{Lrms}	L_r	S_t
Experiments						
Norberg [18]		[0.94-1.04]	[0.83-0.93]	-	-	-
Parnaudeau [20]		-	-	-	1.51	0.210
Present simulations						
No model	1.4M	0.87	0.73	0.04	2.11	0.209
DDES/$k-\varepsilon$/Menter	1.4M	0.88	0.74	0.03	2.07	0.218
VMS dyn.	1.4M	0.96	0.84	0.12	1.54	0.215
Other simulations						
Lee (LES) [9]	7.7M	[0.99-1.04]	[0.89-0.94]		[1.35-1.37]	[0.209-0.212]
Kravchenko (LES) [8]	[0.5M-2.4M]	[1.04-1.38]	[0.93-1.23]		[1.-1.35]	[0.193-0.21]
SA-IDDES [2]	3.9M	0.98	0.83	0.109	1.67	0.214
$\bar{v}^2-f$ DES [2]	3.9M	1.02	0.87	0.14	1.42	0.222

Table 1 Bulk quantities for Re=3900 flow around a cylinder. $\overline{C}_d$ holds for the mean drag coefficient, $\overline{C}_{p_b}$ for the mean pressure coefficient at cylinder basis, C_{Lrms} for the root mean square of lift time fluctuations, Lr is the recirculation length, and St the Strouhal number.

A first series of calculations with a Reynolds number of $Rey = 3900$, is compared with other LES and DES computations [9, 8, 2] and with measurements [20, 18]. Three model options are compared. One can notice that DDES behaves very similarly to the *no model* variant (that is to say without any turbulence model), due to a too low level of turbulence introduced by this model. This behavior is reported in the literature, see e.g. [2]. In contrast, the VMS-based calculation is closer to experiments, as illustrated by the bulk coefficients shown in Table 1.

7.2 Circular cylinder 20000 - subcritical

A second example is the flow around the circular cylinder with a Reynolds number of $20,000$, more than five times larger.

	$\overline{C_d}$	C_{Lrms}	-$\overline{Cp_b}$	Θ_{sep}	Lr	Lv	Imax	St
Experiments								
Norberg [19]	1.16	0.47	1.16	78	1.03			0.194
Lim-Lee [12]	1.16					1.0	37.0	
Present simulations								
No model	1.27	0.61	1.35	82	0.96	0.94	38.7	0.201
URANS/Menter	1.27	0.71	1.25	85	0.64	0.53	33.2	0.216
DDES/$k-\varepsilon$/Menter	1.16	0.36	1.12	82	0.83	0.77	35.9	0.213
VmsWale-Dyn	1.18	0.46	1.20	81	0.96	0.71	35.8	0.201
Hybrid-VmsWale-dyn	1.15	0.46	1.15	86	0.88	0.83	35.2	0.210

Table 2 Bulk flow parameters at Re=20000. Θ_{sep} is the separation angle, Lv denotes the x-location of the maximum in the turbulent intensity distribution along the wake axis, and Imax holds for the maximum turbulence intensity along the wake axis. The other symbols are the same as in Table 1.

We use a mesh of 1.8M nodes. Comparisons are done with a rather recent set of measures by Norberg [19] and Lim-Lee [12]. We first verify that a computation without any turbulence model produces a fair result with a nearly 10% error on mean drag, an overprediction of rms of lift coefficient of nearly 30%. The application of our URANS gives an even less accurate of bulk coefficients, except pressure at rear stagnation point C_{P_b}. Our DDES model, built with the Golberg/Menter RANS model, is not so bad but underpredicts several bulk coefficients (rms of lift coefficient, recirculation length and x-location of the maximum turbulent intensity along the wake axis). In contrast, a LES calculation with the VMS-Wale-dynamic options gives reasonable predictions. As for our hybrid VMS model, a good overall agreement with the experimental data is observed. The accuracy of the prediction is lower than for the LES for the recirculation length and the Strouhal number, but higher (in particular for C_{Lrms} and $\overline{Cp_b}$) than for DDES.

The calculation of these two subcritical flows tends to show that, for this Reynolds number interval, DDES (1) behaves in a way very close to a no model formulation for $Re = 3900$, (2) is less predictive than LES models, and in particular the VMS dynamic model which we consider, (3) is less accurate than the proposed Hybrid model for $Re = 20000$.

Comparisons for subcritical high Reynolds numbers like 140000 introduce a second difficulty for DES/DDES computations since they predict a supercritical (or close to supercritical) flow, see e.g. [25]. We retain that these results tend to confirm the interest to re-introduce a sophisticated LES model in hybrid formulations.

7.3 Tandem cylinder

The last case concerns the flow around two cylinders in tandem, at Re=1.66×10^5. This test case was studied in an AIAA workshop, see [13]. Among the conclusions of the workshop, addressing this case with LES was considered as a too difficult task with existing computers. Results of DES-based computations were much closer to measurements. We have computed the case applying our DDES version (with a mesh of 2.7M nodes) and applying the hybrid VMS-LES-Smagorinsky-dynamic combination (with a mesh of 2.59M nodes) . Some comparison of bulk coefficients is given in Table 3. Experimental outputs are taken from [16]. The drag coefficient for the first cylinder shows a small scatter with the various hybrid calculations. The simulation around the second cylinder is more challenging as also illustrated by the pressure and the resolved turbulent kinetic distributions (Figure 4). One can observe, first, that the drag prediction obtained by the DDES model for the second cylinder is in accordance with that of Aybay [13], who used a DES/$k-\omega$/SST model, and second, that this prediction is improved by our hybrid VMS-LES model with a deviation to measurements which is two times smaller.

	Mesh	$\overline{C}_d^{①}$	$\overline{C}_d^{②}$	$C_{Lrms}^{①}$	$C_{Lrms}^{②}$	$St^{①}$
Experiment						
Neuhart [16] and Lockard [13]						
(BART facility)		0.64	0.31			0.231
Present simulations						
DDES/$k-\varepsilon$/Menter	2.7M	0.65	0.44	0.156	0.59	0.214
Hybrid dyn.	2.59M	0.64	0.38	0.077	0.79	0.214
Other simulations						
DES $k-\omega$ SST, Aybay [26]	6.7M	0.64	0.44			0.223
HRLES, Vatsa[26]	8.7M	0.64	0.45			0.208
S-A MDDES, Lockard (2014)	16M	0.50	0.45	0.072	0.643	0.229
S-A DDES, Garbaruk [3]	11M	0.48	0.42	0.078	0.612	0.244

Table 3 Tandem cylinder: Mean drag coefficient (experimental coefficients are computed by integrating experimental pressure). Circled superscripts hold for cylinder 1 and cylinder 2.

8 A short provisional synthesis

Applying a hybrid model on subcritical cylinder flows provokes two kind of prediction losses. First, the turbulent treatment of the boundary layers deviates from the real flow characteristics. It is known that close to critical Reynolds number, this deteriorates the prediction. For lower Reynolds number (3900) this effect is rather small and the model remains a predictive one. Second, the treatment of wake by the hybrid model is of lower accuracy than a good LES model. We propose a way to

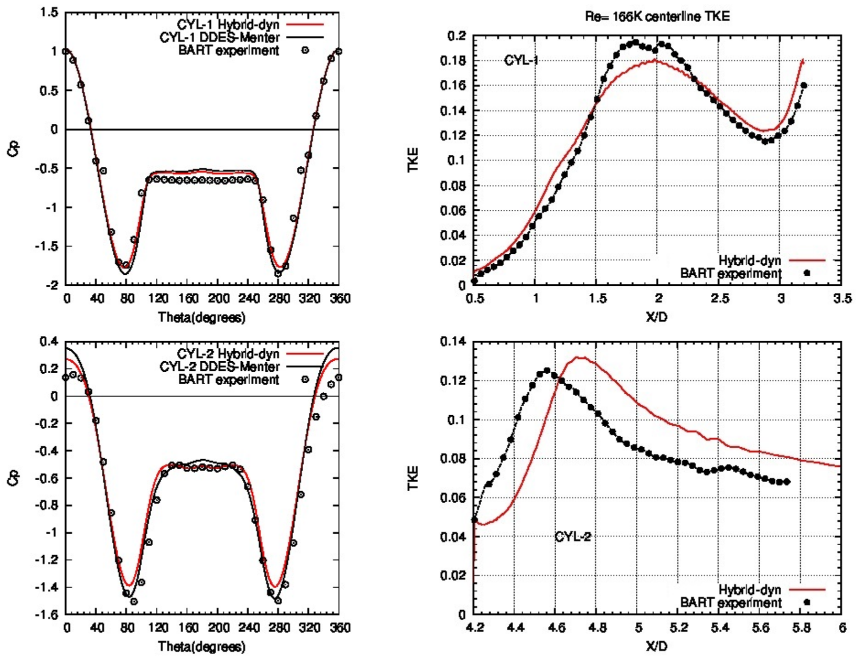

Fig. 4 Tandem cylinder - Left : Mean pressure coefficient distribution on the upstream cylinder (top frame) and on the downstream cylinder (bottom frame) - Right : Resolved turbulent kinetic energy along the centerline between the cylinders (top frame) and behind the second cylinder (bottom frame).

reintroduce a better LES capturing which tends to improve the results for the two last flows of our study.

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References

1. Camarri, S., Koobus, B., Salvetti, M., Dervieux, A.: A low-diffusion MUSCL scheme for LES on unstructured grids. Computers and Fluids **33**, 1101–1129 (2004)
2. D'Alessandro, V., Montelpare, S., Ricci, R.: Detached eddy simulations of the flow over a cylinder at Re = 3900 using OpenFOAM a low-diffusion MUSCL scheme for LES on unstructured grids. Computers and Fluids **136**, 152–169 (2016)
3. Garbaruk, A., Shur M., S., M., Spalart, P.R., Balakrishnan, R.: DDES and IDDES of tandem cylinders. In: Proceedings of the Benchmark problems for Airframe Noise Computations BANC, vol. anl-10/26. Argonne National Laboratory (2010)

4. Germano, M., Piomelli, U., Moin, P., Cabot, W.: A dynamic subgrid-scale eddy viscosity model. Phys. Fluids A **3**(7), 1760–1765 (1991)
5. Goldberg, U., Peroomian, O., Chakravarthy, S.: A wall-distance-free $k-\varepsilon$ model with enhanced near-wall treatment. Journal of Fluids Engineering **120**, 457–462 (1998)
6. Hughes, T., Mazzei, L., Jansen, K.: Large eddy simulation and the variational multiscale method. Comput. Vis. Sci. **3**, 47–59 (2000)
7. Koobus, B., Farhat, C.: A variational multiscale method for the large eddy simulation of compressible turbulent flows on unstructured meshes-application to vortex shedding. Comput. Methods Appl. Mech. Eng. **193**, 1367–1383 (2004)
8. Kravchenko, A., Moin, P.: Numerical studies of flow over a circular cylinder at re=3900. Phys. Fluids **12**(2), 403–417 (1999)
9. Lee, J., Park, N., Lee, S., Choi, H.: A dynamical subgrid-scale eddy viscosity model with a global model coefficient. Phys. Fluids **18**(12) (2006)
10. Leer, B.V.: Towards the ultimate conservative scheme. iv : a new approach to numerical convection. J. Comp. Phys. **23**, 276–299 (1977)
11. Lilly, D.: A proposed modification of the Germano subgrid-scale closure method. Phys. Fluids **A4**, 633 (1992)
12. Lim, H., Lee, S.: Flow control of circular cylinders with longitudinal grooved surfaces. AIAA Journal **40**(10), 2027–2035 (2002)
13. Lockard, D.: Summary of the tandem cylinder solutions from the benchmark problems for airframe noise computations-i. In: Proceedings of Workshop AIAA-2011-353 (2011)
14. Moussaed, C., Salvetti, M., Wornom, S., Koobus, B., Dervieux, A.: Simulation of the flow past a circular cylinder in the supercritical regime by blending RANS and variational-multiscale LES models. Journal of Fluids and Structures **47**, 114–123 (2014)
15. Moussaed, C., Wornom, S., Salvetti, M., Koobus, B., Dervieux, A.: Impact of dynamic subgrid-scale modeling in variational multiscale large-eddy simulation of bluff body flows. Acta Mechanica **12**, 3309–3323 (2014)
16. Neuhart, D., Jenkins, L., Choudhari, M., Khorrami, M.: Measurements of the flowfield interaction between tandem cylinders. AIAA Paper **2009-3275** (2009)
17. Nicoud, F., Ducros, F.: Subgrid-scale stress modelling based on the square of the velocity gradient tensor. Flow, Turbulence and Combustion **62**, 183–200 (1999)
18. Norberg, C.: Effects of Reynolds number and low-intensity free-sream turbulence on the flow around a circular cylinder. Publ. No. 87/2, Department of Applied Thermosc. and Fluid Mech., Chalmer University of Technology, Gothenburg, Sweden (1987)
19. Norberg, C.: Fluctuating lift on a circular cylinder: review and new measurements. J. Fluids Struct. **17**, 57–96 (2003)
20. Parneaudeau, P., Carlier, J., Heitz, D., Lamballais, E.: Experimental and numerical studies of the flow over a circular cylinder at Reynolds number 3900. Phys. Fluids **20**(085101) (2008)
21. Reddy, K., Ryon, J., Durbin, P.: A DDES model with a Smagorinsky-type eddy viscosity formulation and log-layer mismatch correction. International Journal of Heat and Fluid Flow **50**, 103–113 (2014)
22. Roe, P.: Approximate Riemann solvers parameters vectors and difference schemes. J. Comp. Phys. **43**, 357–372 (1981)
23. Spalart, P., Deck, S., Strelets, M., Shur, M., Travin, A., Squires, K.: A new version of detached-eddy simulation, resistant to ambiguous grid densities. Theor. Comput. Fluid Dyn. **20**, 181–195 (2006)
24. Spalart, P., Jou, W.H., Strelets, M., Allmaras, S.: Comments on the feasibility of LES for wings, and on a hybrid RANS/LES approach. In: C. Liu, Z. Liu (eds.) Advances in DNS/LES, 1st AFOESR Int. Conf. On DNS/LES,(1997), Ruston, LA, pp. 137–147. Greyden Press, Columbus, OH (1997)
25. Travin, A., Shur, M., Strelets, M., Spalart, P.: Detached-eddy simulations past a circular cylinder. Flow, Turbulence and Combustion **63**, 293–313 (1999)
26. Vatsa, V., Lockard, D.: Assessment of hybrid RANS/LES turbulence models for areoacoustics applications. AIAA Paper **2010-4001** (2010)