UNCERTAINTY MANAGEMENT FOR ROBUST INDUSTRIAL DESIGN IN AERONAUTICS

UMRIDA

INRIA Contribution to UMRIDA

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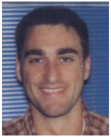


Inria informatics mathematics

- **WP 2** Perturbation methods 2.1.-2.2 (2.4PM):
  Large number of uncertain variables.
  Deliverable D2.1–06-11 (delivered).
- **WP 2** Perturbation methods 2.3 (21.5PM): numerical properties:

Scope of this talk

$\Longrightarrow$ error estimates and correctors
$\Longrightarrow$ new operators for the generation of adaptive meshes

- **WP 3-3.2** Test cases (8.5PM)
- **WP 5** Exploitation (1PM).

Problem statement

- A numerical output, "u",
- an error estimate.

Assumptions

- Numerical error control uses more and more frequently adapted meshes, which reinforce mesh convergence,
- error estimates depend in most case on asymptotic mesh convergence.

Proposed approach:

- Mesh adaptation for flow problems,
- use of correctors for proposing a probabilistic model, e.g.

$$X \sim \mathcal{N}(\mu, \sigma^2) = \mathcal{N}(u + corrector, (corrector)^2).$$

- a posteriori study of the validity of the probabilistic model.

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Status and (quick) review on error estimation for UQ and adaptivity

- Hessian-based (geometric)

$$\|R_h(W_h) - \Pi_h R_h(W_h)\|_{\mathbf{L}^p} \leq N^{-\frac{2}{3}} \left(\int_\Omega \det \left(|H_{R_h(W_h)}| \right)^{\frac{p}{2p+3}} \right)^{\frac{2p+3}{3p}}$$

  - key ingredients: (convergent) recovery of derivatives of W_h
    close-form of the optimal metric (natural anisotropy)

- Goal-oriented

$$|j(W) - j_h(W_h)|, \text{ with } j(W) = \int_\gamma \left(\frac{p - p_\infty}{p_\infty} \right)^2$$

  - key ingredients: PDE-dependent adjoint system $A_h^* W_h^* = \frac{\partial j_h}{\partial W}$
    close-form of the optimal metric (Euler and laminar NS)

- Norm-oriented

$$\|\Pi_h W - W_h\|_{\mathbf{L}^2(\Omega)}$$

  - key ingredients: a corrector is involved (basis for UQ)
    corrector used as second member for goal-oriented-estimate
    close-form of the optimal metric (Euler and laminar NS)

Proof of concept: An *a priori* corrector

$$-\Delta u = f \text{ in } \Omega = [0,1] \times [0,1] \quad u = 0 \text{ on } \partial\Omega$$

$$\int_\Omega \nabla(\Pi_h u - u_h) \cdot \nabla\phi_h \, \mathrm{d}\Omega = \int_\Omega \nabla\phi_h \nabla(\Pi_h u - u) \, \mathrm{d}\Omega$$
$$= \sum_{\partial T_{ij}} \nabla(\phi_h|_{T_i} - \phi_h|_{T_j}) \cdot \mathbf{n}_{ij} \int_{\partial T_{ij}} (\Pi_h u - u) \, \mathrm{d}\sigma$$

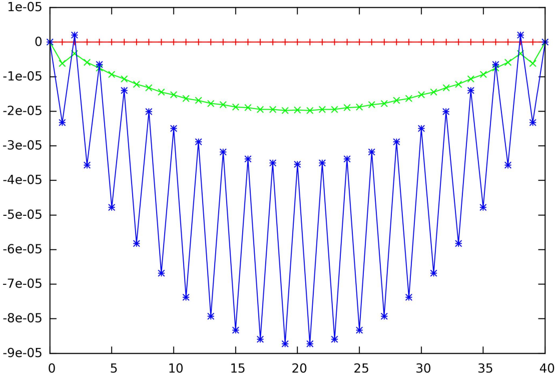
Approximate $\Pi_h u - u \approx H(u) \cdot \delta X \cdot \delta X$ with a superconvergent approximation of the Hessian of u_h, obtain $u'_h \approx \Pi_h u - u_h$:

$$\int_\Omega \nabla u'_h \cdot \nabla\phi_h \, \mathrm{d}\Omega = \sum_{\partial T_{ij}} \nabla(\phi_h|_{T_i} - \phi_h|_{T_j}) \cdot \mathbf{n}_{ij} \int_{\partial T_{ij}} H_h(u_h) \cdot \delta X_{\mathcal{M}} \cdot \delta X_{\mathcal{M}} \, \mathrm{d}\sigma.$$

[Brethes and Dervieux, 2013]

Proof of concept: An *a priori* corrector

Example: $u(x) = x(1-x)y(1-y)$, 1600 vertices



In red: $\Pi_h u - \Pi_h u = 0$, in blue: $\Pi_h u - u_h$, in green $\Pi_h u - (u_h + u'_h)$

The corrector performs 70% of its work (for a mesh with English flag topology).

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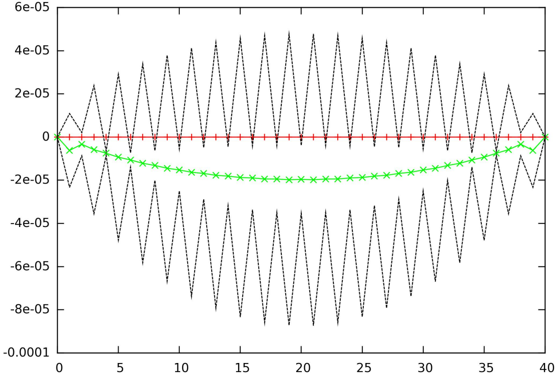


Proposed "uncertainty interval"

in red: exact $\Pi_h u - \Pi_h u$
in green corrected $\Pi_h u - (u_h + u_h'),$
in **black** $\Pi_h u - (u_h + u_h') \pm |u_h'|.$

Goal: Minimize $\|\Pi_h u - u_h\|_{\mathbf{L}^2}$

Parameterize the discretization "h" by a Riemannian metric $\mathcal{M}$.

Step 1: first solve the linearised error system:

$$\int\int \phi div \frac{\partial \mathcal{F}_h}{\partial W}(-\delta W) \; = \; \int\int \nabla \phi_h(\mathcal{F}_h(W_h) - \mathcal{F}(W)).$$

Step 2: then solve the adjoint system:

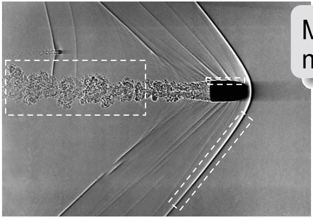
$$\bar{A}_h^* W_h^* = \frac{\partial \delta W}{\partial W},$$

Step 3: compute the adjoint metric with the corrector as functional:

$$\mathcal{M}^{opt} = \min \|\delta W\|$$

the three-step process being re-iterated until we get a fixed point.

Adaptivity is the core of our approach for UQ
Industrial problems are involved $\Longrightarrow$ complex geometries



Many phenomena $\Longrightarrow$ Many kinds of meshes

- Turbulent flow: isotropic,structured, ...
- Shock waves: anisotropic $O(1/100 - 1000)$
- Boundary-layers: quasi-structured $O(1/10^4 - 10^6)$

Frontal	High-Quality	Small Anisotropy
Delaunay	Robust	Anisotropy but Bad Quality
Octree-based	Robust	Surface mesh not constrained
Cartesian	Robust	Low Anisotropy, viscous effects
BL Extrusion		Closure of the domain, adaptivity
Local Refinement	Robust	Slow, High Anisotropy but Bad Quality

$\Longrightarrow$ No Unique Technology
$\Longrightarrow$ Robustness decreases with Geometry Complexity

Robustness is the primary concern

1. Local mesh modification operators
   - adaptivity is an iterative procedure
   - no mesh $\Longrightarrow$ no solution
   - always a valid mesh on output
   - use of simplicial meshes

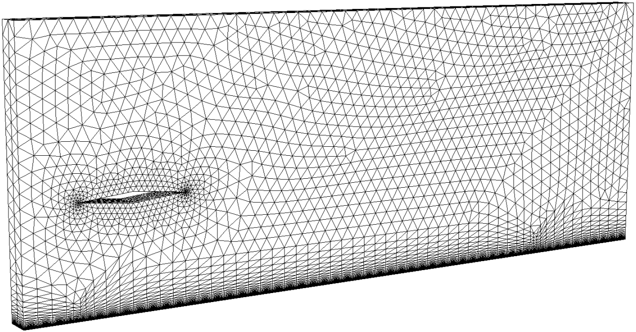
Handling all types of meshes is the secondary concern

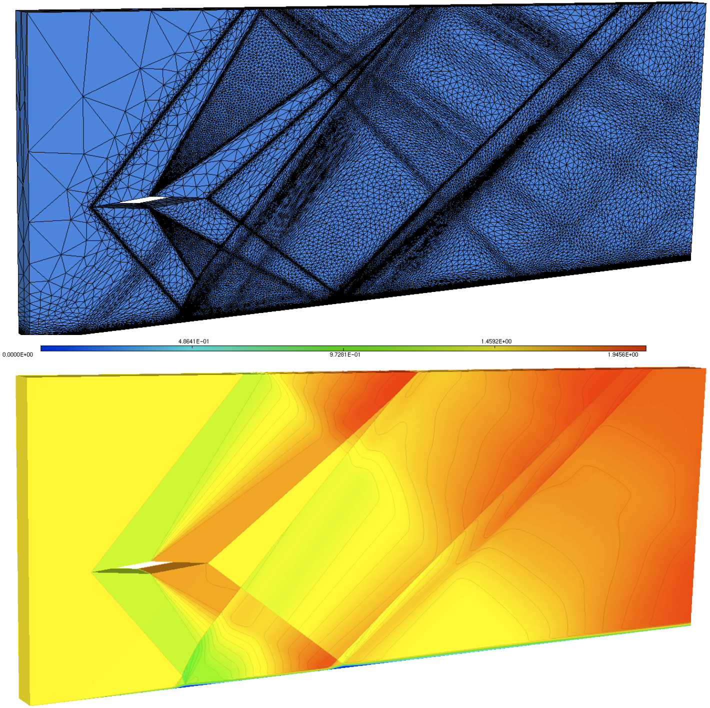
2. Unique operator
   - mesh adaptation : surface-volume
   - mesh optimization: edge-face swaps, point smoothing
   - boundary layer mesh generation: hybrid entities insertion

Example: boundary-layer shock interaction

- Mach 1.4, Re=2.7 10^7
- Viscous plate, RANS
- Adaptation on the Mach/Density, 20 iterations, Total cpu 1h (8 procs)
- 280 000 Vertices and 1.3 M tets.

- Initial mesh with boundary layer
- Adaptivity based on a boundary layer metric

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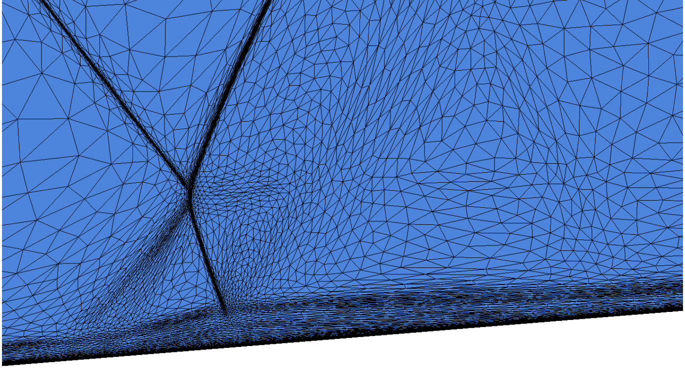

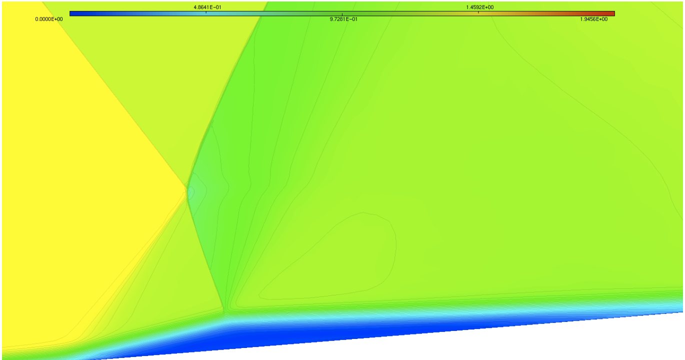
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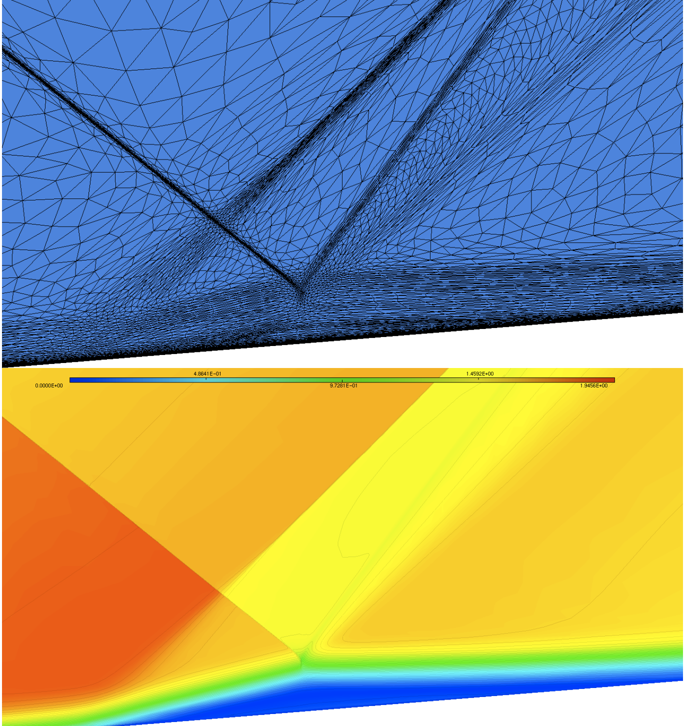
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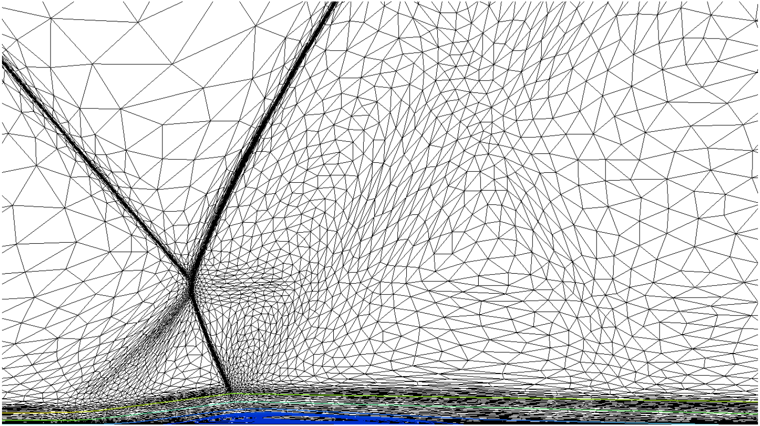

Anisotropy and alignment

Conclusions

Error estimations

- Model problem : Poisson equation
- Current work directed at Euler equations
- Ongoing validation for manufactured solution
- Corrector studied with MUSCL scheme

Mesh adaptation/generation

- Operator operational for anisotropic phenomena, complex surface
- Current work directed at full coupling adaptive BL