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Pattern Recognition Letters 26 (2005) 819–828

Pattern Recognition Letters

www.elsevier.com/locate/patrec

Fuzzy relations applied to minimize over segmentation in watershed algorithms

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Received 18 November 2003; received in revised form 24 August 2004

Abstract

This paper presents a novel method to minimise the over-segmentation that inherently results after applying a watershed algorithm. The proposed technique characterises each of the segmented regions and then employs the composition of fuzzy relations to group together similar regions.

Keywords: Image segmentation; Fuzzy C-means; Fuzzy relations; Watersheds

1. Introduction

Most image applications are based on the extraction and analysis of meaningful features, which may be hard to distinguish from all the information contained in the image. A simplification step is then needed to filter out the required features, which can then be further processed either by a computational algorithm or by a human operator. Segmentation is a widespread image simplification operation, essential in most content-based imaging systems. Tracking segmented objects in video, image retrieval from databases and image pattern recognition are just some examples. Many different approaches exist to achieve segmentation. Roughly classified, they can be divided into three categories (Bueno et al., 2001). Segmentation without modelisation of the image, where low-level image processing algorithms belong to, Segmentation with an available model, such as deformable models and knowledge-based models, and Hybrid segmentation models. As for the first type of algorithms, the method works well on images with high SNR, good contrast and strong homogeneity. The second category of algorithms is indeed more robust but its success still is limited by the difficulty to incorporate the knowledge about the shapes to segment; also, there is a loss of generality as the

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0167-8655/$ - see front matter
doi:10.1016/j.patrec.2004.09.036

implementation becomes application dependant, and finally, the intervention of an operator is frequently required to incorporate the a priori information. The same concerns remain valid for the approaches contained in the third category. Among the low-level algorithms, the watershed segmentation, initially proposed by Beucher and Lantuéjoul (1979), has been widely used and has provided satisfactory results when the SNR and homogeneity conditions, mentioned above, are met. Otherwise the result may exhibit over-segmentation, which is undeniably the huge inconvenience of this approach. On the other hand, watersheds have the valuable advantage of partitioning the image and always returning a set of closed contours even in low contrast conditions. A variation algorithm to the watersheds, the marker-based watershed (Meyer and Beucher, 1990) has been proposed to avoid the over-segmentation problem. In this technique there are as many final regions as markers in the image. The problem is that automatic placement of the markers is difficult and thus, is often done by an operator. Also, the accuracy on the contours obtained is diminished depending on the number of markers placed in the image. In this paper a novel method to minimise the inherently watersheds over-segmentation is proposed, delivering thus, a simplified image with meaningful closed contours. In order to reduce the partition complexity, the use of fuzzy relations to cluster similar regions is implemented. The algorithm has been applied on some generic images and MRI medical images for which ground-truth segmentations have been defined.

The rest of the paper is as follows: Section 2 explains the watershed implementation employed for the segmentation of images. In Section 3 the details of the fuzzy algorithm applied to minimise over-segmentation are presented. Section 4 shows meaningful results. Finally Section 5 presents the conclusions of this paper.

2. Watershed segmentation

The watershed segmentation is a technique developed from morphological algorithms, which follows a geological analogy. The image to be segmented can be considered as a topographical surface, S, where the grey levels or image intensities, $I(x,y) = I(s)$ correspond to altitude values. A minimum at an altitude value j, m^j, in this landscape, is a dip in the ground surrounded by strictly higher land. A catchment basin, $CB_i(m_i^j)$, is then the area around the minimum m_i^j in S where water falling on it would flow down into the minimum. At each pixel where two or more catchment basins meet, an imaginary 'dam' is built. At the end of a recursive process, each minimum is surrounded by dams, which delimit the associated catchment basins. These dams correspond to the watersheds of the topographical surface $WT(S)$. This type of morphological transform can also be seen as an edge detector as it can naturally identify boundaries of objects within an image. Though it does not necessarily depend on traditional edge detection techniques, it is applied on the gradient image, $G(I) = |\nabla I(S)|$, or the morphological gradient image, $G(I) = I \oplus SE - I \ominus SE$, where $\oplus$ is the dilation operator, $\ominus$ is the erosion operator and SE is the associated Structuring Element. The watershed segmentation determines all regional minima in the (gradient) image, and is implemented on ordered queues as described by Meyer (1994). Briefly explained, the algorithm can be divided into three phases: Firstly, all pixels in the gradient image $G(I)$ are scanned looking for regional minima. Let us define N, the set of neighbours, (x',y'), for a pixel (x,y) in $G(I)$. When 8-connectivity is used, $x' = \{x-1, x, x+1\}$; $y' = \{y-1, y, y+1\}$. IF $G(x,y) > G(x',y')$ $x',y' \in N(x,y)$ then $G(x,y)$ is labelled as non-regional-minima (NRM) and put into a first-input–first-output (FIFO) queue Q. Subsequently, while Q is not empty, its first element is popped out. Let $G(x',y')$ be the first output element of Q. IF the label of $G(x'',y'')$ is void, $x'',y'' \in N(x',y')$ and $G(x,y) = G(x'',y'')$ THEN the label $G(x'',y'')$ is set to NRM and $G(x'',y'')$ is put in Q.

In a second step the adjacent pixels of the minima found are put into an ordered queue (OQ). Starting from label $i = 1$ all pixels in $G(I)$ are scanned again. IF the label of $G(x,y)$ is void THEN $G(x,y) \in CB_i$ and $G(x,y)$ is put in a FIFO queue Q. Again, While Q is not empty, its first element is popped out. Let $G(x',y')$ be the first output

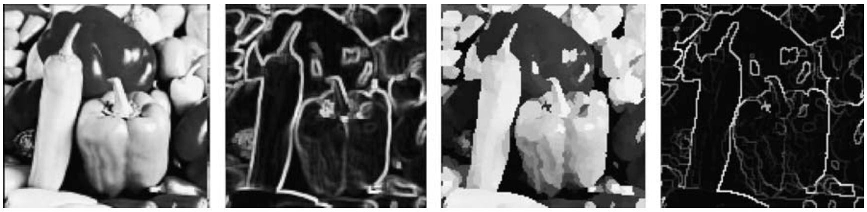

Fig. 1. Image results after application of the watershed algorithm. The following images are depicted from left to right. Original image, morphological gradient image, mosaic image with 682 regions and weighted watershed image.

element of Q. IF the label of $G(x'', y'')$ is void, $x'', y'' \in N(x', y')$ THEN $G(x'', y'') \in \mathrm{CB}_i$ and $G(x'', y'')$ is put in Q; otherwise $G(x'', y'')$ is labelled NRM and put in a grey value ordered queue OQ. In the final stage, pixels in the ordered queue with the lowest grey value are popped out. Let $G(x, y)$ be the first output element of OQ. IF label of $G(x', y')$ is void, $x', y' \in N(x, y)$, THEN $G(x, y) \in \mathrm{CB}_k$ if $G(x', y') \in \mathrm{CB}_k$ for $k = 1, \ldots, i$.

From the labels assigned to the different catchment basins, a mosaic image, as it has been described in the literature by Meyer and Beucher (1990), is subsequently produced where each mosaic region has the value of the minimum pixel intensity inside the corresponding catchment basin. The mosaic image is equivalent to the segmented image in computer vision terms. A weighted watershed, as defined by Razaz and Hagyard (2000), can be generated by drawing the separating dams with a grey level value proportional to the intensity difference between adjacent regions. This will have a visual effect of unifying the watershed image, as shown in Fig. 1, though the separation between regions still remains. In the following section we present an efficient algorithm to merge similar catchments and effectively diminish over-segmentation.

3. Over-segmentation minimisation

The proposed approach is based on the application of the Fuzzy C-means algorithm together with composition of Fuzzy relations.

The Fuzzy C-means algorithm is one of the most widely used clustering algorithms. It was initially proposed by Bezdek (1973). It is an unsupervised algorithm, based on the minimisation of a fuzzy objective function, which is based on the intra-class scatter of the given data. The algorithm performs a partition of the data into c clusters and c centres, one for each cluster, are generated. For a detailed description of the Fuzzy C-means algorithm, the reader can consult (Bezdek, 1981).

The Fuzzy C-means algorithm on its own could be, for instance, used to group together pixels having similar grey-level value. This will without doubt diminish the number of regions but because no information is taken into account about the connectivity between regions, this simple approach is more prone to errors. In order to construct a more robust technique, the use of Fuzzy Relations is introduced.

Let X and Y be two universes of discourse. Then,

$$R(X, Y) = \{((x, y), \mu_R(x, y))/(x, y) \in X \times Y\}$$

is a binary fuzzy relation in $X \times Y$, and the strength of the relation between elements $x \in X$ and $y \in Y$ is given by the fuzzy value $\mu_{\mathrm{R}}(x, y)$.

It is also possible to define a fuzzy relation that exists among the elements of a single set X. A binary relation of this type is denoted by $R(X, X)$ or $R(X^2)$.

Now let us consider two binary relations $P(X, Y)$ and $Q(Y, Z)$ defined with the common set Y. The composition of these two relations is denoted by

$$R(X, Z) = P(X, Y) \circ Q(Y, Z)$$

and is defined as a subset $R(X, Z)$ of $X \times Z$ such that $(x, z) \in R$ if and only if there exists at least one $y \in Y$ such that $(x, y) \in P$ and $(y, z) \in Q$.

The composition operation for fuzzy relations can take several forms just in the same way as union and interception fuzzy-set operators do. The most common form of the fuzzy relation composition is the max–min composition, defined by

$$\mu_{P \circ Q}(x,z) = \max_{y \in Y} \min(\mu_P(x,y), \mu_Q(y,z)).$$

If the composition between relations $P(X, Y)$ and $Q(Y, Z)$ is thought of as representing the existence of a relational chain between elements of X and Z, then the max–min composition for fuzzy relations can be interpreted as indicating the strength of such a relational chain. The strength of the relation between elements x and z is then the strength of the strongest chain between them.

The concepts of fuzzy relations can be applied to the problem of watershed over-segmentation. For this purpose let us define X as the set of m mosaic regions (or catchment basins) that we seek to simplify. Y is the set of n grey value levels from the histogram of the mosaic image. It is possible then to establish the two following relations:

R1: x is connected to x,
R2: x has grey value y.

The first relation is defined by the following membership function:

$$\mu_{\mathrm{R1}}(x_i, x_j) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } x_i \not\leftrightarrow x_j \\ |gv(x_i) - gv(x_j)| & \text{if } x_i \leftrightarrow x_j \end{cases}$$

where $gv(x)$ is a function giving the grey value level of region x, and $x_i \leftrightarrow x_j$ reads x_i is adjacent to x_j.

The matrix μ_{R1} is actually a compatibility-relation matrix as its elements verify the properties of reflexivity $\mu(x_i, x_i) = 1$ and symmetry $\mu(x_i, x_j) = \mu(x_j, x_i)$.

The second relation is actually a crisp relation where

$$\mu_{\mathrm{R2}}(x_i, y_j) = \begin{cases} 1 & \text{if } gv(x_i) = y_j \\ 0 & \text{elsewhere} \end{cases}$$

At this point, we can consider again the idea of the Fuzzy C-means algorithm employed to group together pixels having similar grey-level values. Only, this time we can apply the composition of the fuzzy relations defined above to constrain merging the pixels only between adjacent regions. Let Z be the set of c different clusters into which we want to simplify the mosaic region. We can define the following relation:

R3: y belongs to z.

In this case the notion of belonging is that of a typical fuzzy set with $\mu_{\mathrm{R3}}(y_j, z_k)$, evaluating the membership of grey value y_j into cluster z_k. The fuzzy matrix μ_{R3} is found by running the Fuzzy C-means algorithm. The choice of this particular clustering technique was made not only because it has already been implemented in a wide range of applications with successful results, but more particularly because it naturally leads to a fuzzy partition of the data whereas using other popular

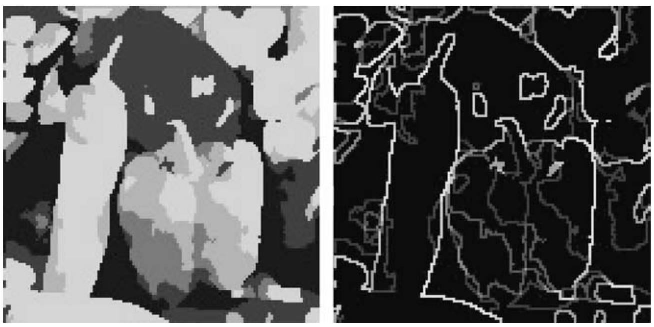

Fig. 2. Simplified mosaic and watershed 'peppers' images after connected regions are merged by the proposed approach. The number of classes required to run the Fuzzy C-means algorithm was set to 5.

clustering methods such as the Leader algorithm (Hartigan, 1975), Self Organizing Maps (Kohonen, 1998), Substractif clustering (Yager and Filev, 1994), etc., would then need a further fuzzification step.

The whole procedure of watershed simplification can be reduced to the application of the following composition rule:

R1 ∘ R2 ∘ R3: x_i is connected to x_j and x_j has grey value y and y belongs to cluster z.

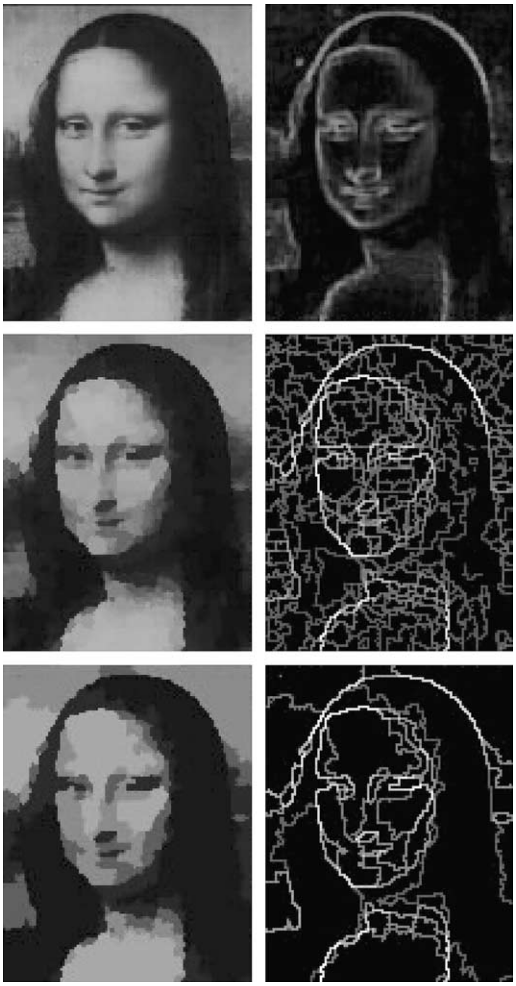

Fig. 3. Results obtained for the 'Monalisa' image. Top row: original and gradient images; middle row: mosaic and weighted watershed images after using the watershed algorithm; bottom row: simplified mosaic and weighted watershed images after using the proposed algorithm.

4. Results

In order to demonstrate the efficiency of the proposed method, some experiments were carried out on different types of images: Firstly the 'Peppers' image shown in Fig. 1 was processed. This is a 128×128 8-bits coded image (256 grey value levels). The resulting watershed image contains 682 basins. When the latter image is processed with the proposed algorithm, the number of basins is reduced to only 90. The resulting simplified mosaic and watershed image are shown in Fig. 2.

Then a second experiment was carried out on four other typical images that were also segmented with the watersheds algorithm and then simplified with our proposed technique. These images are

Fig. 4. Results obtained for the 'Cameraman' image. Top row: original and gradient images; middle row: mosaic and weighted watershed images after using the watershed algorithm; bottom row: simplified mosaic and weighted watershed images after using the proposed algorithm.

'Monalisa', 'Cameraman', 'House' and 'Lena'. The original images, together with the first watershed segmentation obtained running the Meyer algorithm, and the simplified segmented images, after connected regions are merged by the proposed approach, are respectively shown in Figs. 3–6. All original images are coded in 256 grey levels. For a better visualization, a constant value of 40 was added to the contours in the weighted watershed images. Table 1 indicates how the number of basins is greatly reduced by running our algorithm. In all cases the gradient of the image was obtained as stated in Section 2 and using a square structuring element of size 3×3.

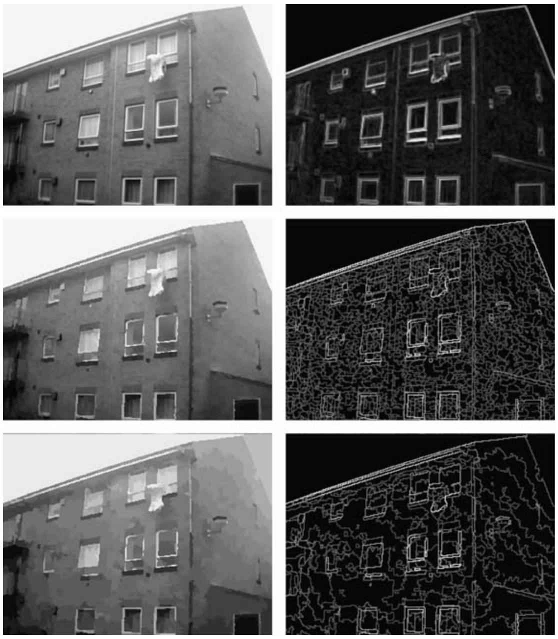

Fig. 5. Results obtained for the 'House' image. Top row: original and gradient images; middle row: mosaic and weighted watershed images after using the watershed algorithm; bottom row: simplified mosaic and weighted watershed images after using the proposed algorithm.

Fig. 6. Results obtained for the 'Lena' image. Top row: original and gradient images; middle row: mosaic and weighted watershed images after using the watershed algorithm; bottom row: simplified mosaic and weighted watershed images after using the proposed algorithm.

Table 1
Comparison of the number of regions in the segmented head images before and after application of the proposed approach

Image	Size of image (in pixels)	Original number of basins (regions) with Meyer (1994) watershed algorithm	No. of clusters when running Fuzzy C-means	Simplified number of basins (regions) with the proposed approach
Peppers	128×128	682	5	90
Monalisa	134×105	615	5	68
Cameraman	256×256	2975	5	136
House	320×240	2962	16	384
Lena	256×256	3063	16	464

In the last experiment, the proposed approach was applied on images corresponding to a simulated head MRI phantom, which is publicly available from the McConnell Brain Imaging centre at the Montreal Neurological Institute and Hospital, Montreal, Canada; see http://www.bic.mni.mcgill.ca/brainweb/. The simulated images on which the proposed algorithm was tested correspond to T1 images of a normal brain with voxel size = 1 mm, noise = 3%, num echoes = 1, flip angle = 10, inu = 20%, tr = 18. Together with the phantom, ground truth for some brain tissues such as white matter, grey matter, cerebrospinal fluid, skull, intra-cranial cavity and glial tissue are provided. A detailed description of the phantom can be found in (Collins et al., 1998). From the phantom volume, six slices were chosen, with no particular criteria, to apply the proposed algorithm and use the resulting simplified segmentation to differentiate the white matter and grey matter tissues of the brain. The classification of these two tissues is an important research field in medical imaging (Schnack et al., 2001). The selected slice images from the whole brain volume are shown in Fig. 7. The resulting mosaic images by the watershed procedure are shown in Fig. 8. Each slice contains about 1800 basins (or regions) in the segmented image. The proposed approach is applied then on each slice setting the number of classes in the Fuzzy C-means algorithm equal to 5 (Fig. 9). According to our experiments, this is actually the lowest number of clusters for which the separation of the white matter and grey matter tissues is possible. Setting down the system to a lower number of classes causes a merging of the two anatomical structures. Tuning the Fuzzy C-means algorithm for a partition with bigger number of classes has the effect of leaving some loose regions that will not be merged into the corresponding anatomical tissue and thus will diminish the accuracy to delimit the white matter and grey matter tissues. The mean compression ratio between the number of

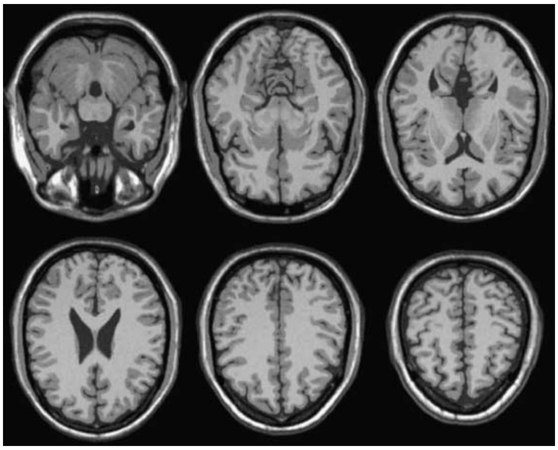

Fig. 7. Original brain phantom images.

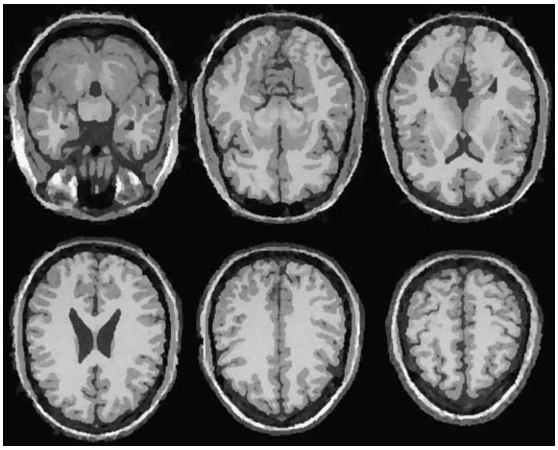

Fig. 8. Mosaic (segmented) images that were obtained after running the watershed algorithm.

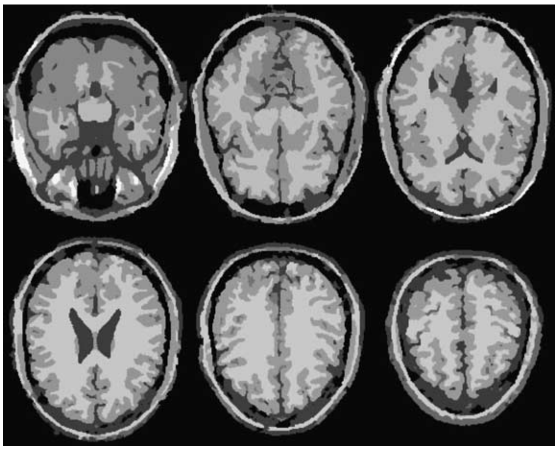

Fig. 9. Simplified mosaic (segmented) image after application of the proposed algorithm.

Table 2
Comparison of the number of regions in the segmented head images before and after application of the proposed approach

Image	Original number of basins (regions) with Meyer's (1994) watershed algorithm	Simplified number of basins (regions) with the proposed approach
Slice 1	1889	518
Slice 2	1896	644
Slice 3	1909	543
Slice 4	1891	505
Slice 5	1879	493
Slice 6	1883	418

regions in the image, after applying the proposed approach, and the original number of regions, for all the slices, is of 72.50 ± 3%. The details on the regions reduction are shown in Table 2.

The accuracy of the regions that have been merged can then be measured by comparing the simplified segmented images with the ground truth images. For this purpose, two separate threshold operations are performed to isolate firstly the white matter tissue, then the grey matter tissue. After applying the proposed approach, all regions, in each slice, are clustered into five classes (because the fuzzy membership matrix corresponding to relation R3 has been set up for five classes when running the Fuzzy C-means algorithm). Each class is associated with one grey value level, so when plotting the histogram of a simplified segmented image all pixels will be distributed into five grey level values, from which, the highest one can automatically be selected as the threshold value to isolate the white matter tissue. To isolate the white matter, the threshold operation has now one low and one high threshold value. The latter is still the highest grey level value in the histogram; the former is the second highest grey level value in the histogram. However, pixels outside the intracranial brain region, still remaining after the threshold operation, must be removed manually. Fig. 10 shows, for instance, the white matter tissue isolated after the latter process. The performance of the classification, for both, the grey and white matter, can be measured in terms of the overlap of pixels other than zero between the resulting thresholded images and the ground truth images for both tissues. The overlap measure has been employed consistently in the MR brain segmentation field (Bueno et al., 2001; Duta and Sonka, 1998; Rizzo et al., 1997) and is defined as follows:

Fig. 10. White matter tissue after threshold operation.

$$\text{overlap} = \frac{\text{GT} \cap \text{Seg}}{\text{GT} \cup \text{Seg}}$$

where GT is the ground truth image, and Seg is the thresholded segmented image.

The quantitative results in terms of the overlap measure are summarized in Table 3. The results obtained on the classification of both tissues are encouraging and consistent with those reported in the literature on the same data (Bueno et al., 2001).

The algorithm was implemented using the matlab programming language. When running on a PC equipped with a 730 MHz Intel processor and 256 Mbytes on RAM, the watershed segmentation was ready in 193.297 s on an image of size 217 × 181 pixels (one brain image slice), whereas

Table 3
Classification results on white and grey matter tissues when compared with the ground truth images given in (Collins et al., 1998)

Image	Pixels overlap (%)	
	White matter tissue	Grey matter tissue
Slice 1	74.0143	85.416
Slice 2	83.314	71.4337
Slice 3	84.3965	79.8118
Slice 4	88.1752	79.8066
Slice 5	88.1177	77.9472
Slice 6	80.4942	75.3821

minimisation of over-segmentation was done in 104.094 s.

5. Conclusions

Over-segmentation is an intrinsic problem appearing when the watershed transform is used. In this paper a new algorithm is proposed to merge similar regions to allow simplifying the segmented image. The composition of fuzzy relations is proposed to group together regions with similar grey value level under the constraint that only adjacent regions can be merged. Experimental results show to diminish the number of regions in the over-segmented image from 70% to 85%. Thus, by running the proposed algorithm, images with a higher degree of simplification in the number of regions and with a more homogeneous partition can be achieved in comparison to simply using the standard watershed segmentation obtained with the Meyer algorithm. The proposed approach was run on some generic images, but it could potentially be applied on some medical imaging applications. Particularly, it was shown to have good results at studying brain MRI images. The simplified images presented on this paper, and for this application, grouped reliably pixels associated to the white matter tissue as well as for the grey matter tissue. The highly homogenous regions resulting after running the proposed approach allowed an automatic selection of two thresholds that can reliably isolate the grey and white matter tissues. The results obtained are in high agreement with those provided by the ground truth images and are also consistent with other studies provided in the literature (Bueno et al., 2001). Should the segmentation images, that were obtained by running the standard Meyer watershed algorithm, be used directly for the separation of the above mentioned anatomical tissues, then the intervention of an operator would have been needed to select the thresholds and to recognize the different anatomical structures. In the present approach, if used in MRI brain analysis, the operator intervention is reduced to the selection of the intra-cranial region. One downside when implementing the proposed algorithm is that it has to be applied sequentially to each slice as the watershed transform works only in two dimensions. In a future work the watershed segmentation will be extended to three dimensions, and fuzzy composition rules will be used. This will lead to a more general system able to be applied to a larger number of applications including 3D medical imaging.

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