Domain decomposition methods for electromagnetic wave propagation problems involving heterogeneous media and complex domains

19th International Conference on Domain Decomposition Methods

Stéphane Lanteri
NACHOS project-team
INRIA Sophia Antipolis - Méditerranée research center, France

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Opening: scientific context

- Challenges with the simulation of ElectroMagnetic (EM) wave propagation
  - Geometrical characteristics of the propagation domain:
    - dimensions relatively to the wavelength,
    - irregularly shaped objects and singularities.
  - Physical characteristics of the propagation medium:
    - heterogeneity and anisotropy,
    - physical dispersion and dissipation.
  - Characteristics of the radiating sources and incident fields
- PDE model: the system of Maxwell equations

James Clerk Maxwell (1831-1879)

Opening: scientific context

- Modeling context
  - Time-harmonic regime
  - High frequency (F>100 MHz)
- Target applications
  - Interaction of EM fields with living tissues
    - Exposure of humans to EM fields from wireless communication systems
    - Medical applications (microwave imaging, microwave hyperthemia, etc.)
  - Microwave imaging for the detection of buried objects
- Numerical ingredients
  - Unstructured meshes (triangles in 2D, tetrahedra in 3D)
  - High order discontinuous finite element discretization method
    - Discontinuous Galerkin method with polynomial interpolation (DGTH-$\mathbb{P}_p$)
  - Hybrid iterative-direct domain decomposition based solution strategies

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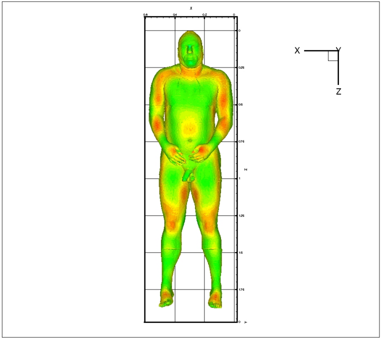
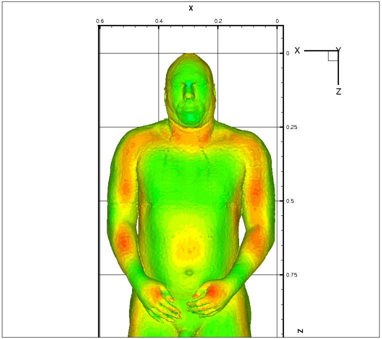
Opening: target applications

- Human exposure to electromagnetic fields
  - Multi-parametric studies, uncertainty quantification (source position, morphology, electromagnetic parameters)





- Plane wave exposure (F=2.14 GHz)
- Tetrahedral mesh: 899,872 vertices and 5,335,521 elements
- Discretization by a DG-$\mathbb{P}_2$ method: 320,131,260 d.o.f

Opening: objectives

- Goals of this study
  - Formulation and analysis of optimized Schwarz algorithms for the time-harmonic Maxwell equations
  - Design of hybrid iterative-direct domain decomposition based solvers for algebraic systems resulting from DG discretizations
- Collaborators
  - Victorita Dolean (Assistant Professor)
    Dieudonné Mathematics Laboratory (UMR 6621)
    University of Nice/Sophia Antipolis, France
  - Mohamed El Bouajaji (PhD student)
    NACHOS project-team
    INRIA Sophia Antipolis - Méditerranée research center, France
  - Martin Gander (Professor)
    Mathematics Section, University of Geneva, Switzerland
  - Ronan Perrussel (CNRS researcher)
    Ampère Laboratory (UMR 5005), Ecole Centrale de Lyon, France

Opening: related work

- Jin-Fa Lee *et al.*
  The Ohio State University, ElectroScience Laboratory, ECE Department
  - Non-overlapping DD method for modeling large finite antenna arrays
    S.-C. Lee, M.-N. Vouvakis and J.-F. Lee, J. Comput. Phys., Vol. 203 (2005)
  - DP-FETI like DD method for the solution of large electromagnetic problems
    M.-N. Vouvakis and J.-F. Lee, Copper Montain Conference on Iterative Methods (2004)
    - Second order vector wave equation for the electric field
    - Non-overlapping Schwarz algorithm
    - Study of zero-order (Robin) and second-order (with vector and scalar tangential rotational operators[1]) transmission conditions
    - Fourier analysis of formulations for TEz and TMz modes
    - Non-matching interface grids through the introduction of cement variables (surfacic electric current densities)
  - DD approach for non-conformal couplings between FEM and BEM
    M.-N. Vouvakis, K. Zhao, S.-M. Seo and J.-F. Lee
    J. Comput. Phys., Vol. 225 (2007)

[1] F. Collino, G. Delbue and P. Joly, CMAME, Vol. 148 (1997).

Opening: related work

- Jianming Jin *et al.*
  University of Illinois at Urbana-Champaign Department of Electrical and Computer Engineering
  - Dual-field time-domain finite element DD method (DFDD-TDFEM)
    Z. Lou and J-M Jin, J. Comput. Phys., Vol. 222 (2007)
    - Non-overlapping sudomains
    - Second order vector wave equations for the electric and magnetic fields in each subdomain
    - Staggered Newmark (leap-frog like) time integration
    - Implicit time integration within each subdomain and explicit time integration on subdomain interfaces
  - Dual-primal FETI algorithm for general 3D EM simulations (FETI-DPEM)
    Y-J Lin and J-M Jin, EEE Trans. Ant. Propag., Vol. 54, No. 10 (2006)
    Y-J Lin and J-M Jin, J. Comput. Phys., Vol. 228 (2009)
    - Extension of the FETI-DP method to the vector curl-curl wave equation
    - Dirichlet condition on tangential components of the electric field
    - Lagrange multipliers: Neumann-type condition (curl of the electric field)

Opening: related work

- Wei Hong *et al.*
  Southeast University, Nanjing
  State Key Laboratory of Millimeter Waves
  - Partial basic solution vector DD method (PBSV-DDM)
    for large-scale EM problems involving periodic structures
    Z.-Q. Lü, X. An and W. Hong
    IEEE Trans. Ant. Propag., Vol. 56, No. 8 (2008)
  - DD formulation inspired by the approach of Jin-Fa Lee *et al.*
    (Robin type interface condition and introduction of cement variables)
  - Formulation of a reduced (interface) system by means of the PBSV [2] method

- Jun Zou *et al.*
  The Chinese University of Hong Kong, Department of Mathematics
  - Nonoverlapping DD method for Maxwell's equations in 3D
    Q. Hu and J. Zou, SINUM, Vol. 41, No. 5 (2003)
  - Edge-element discretization
  - Study of a preconditioner for the Schur complement system involving
    coarse subspaces/solvers for curl-free and div-free functions

[2] X. An and Z.-Q. Lü, J. Comput. Phys., Vol. 219 (2006)

Opening: related work

- A. Schädle, F. Schmidt *et al.*
  Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB)
  Numerical Analysis and Modelling department
  - DD strategy for the computation of the EM field within periodic structures
    A. Schädle, L. Zschiedrich, S. Burger, R. Klosea, and F. Schmidt
    J. Comput. Phys., Vol. 226 (2007)
  - Schwarz algorithm with transparent boundary conditions at subdomain interfaces approximated by PML

Opening: related work

- Domain decomposition algorithms for time-harmonic Maxwell equations with damping
  A. Alonso Rodriguez and A. Valli, ESAIM: M2AN, Vol. 35, No. 4 (2001)
  - Non-overlapping DD method
  - Study of several preconditioning methods for the Steklov-Poincaré operator
- New non-overlapping domain decomposition methods for the time-harmonic Maxwell system
  A. Alonso Rodriguez and L. Gerardo-Giorda, SISC, Vol. 28, No. 1 (2006)
- Overlapping Schwarz preconditioners for indefinite time-harmonic Maxwell equations
  J. Gopalakrishnan and J.E. Pasciak, Math. Comp, Vol. 72, No. 241 (2001)
  - Study of Schwarz preconditioners for edge-element discretization
- Developments in overlapping Schwarz preconditioning of high order nodal discontinuous Galerkin discretizations
  J. S. Hesthaven, L. N. Olson, and L. C. Wilcox, LNCSE, Vol. 55 (2007)
  - Helmholtz equation, two-level Schwarz preconditioning for a Krylov method
  - Overlap improves convergence when using high order interpolation
  - Necessity of a sufficiently resolved coarse grid at higher frequencies

Opening: related work

- DD19 MS6 - August 17, 14:00pm-15:40pm, Room No. 1
  Domain decomposition methods for electromagnetic wave propagation problems
  - J. Jin and Z. Lou
    University of Illinois at Urbana-Champaign
    Department of Electrical and Computer Engineering
  - J.-F. Lee, V. Rawat and Z. Peng
    Ohio State University
    ElectroScience Laboratory, ECE Department
  - J. Zou and Q. Hu
    The Chinese University of Hong Kong
    Department of Mathematics
  - W. Hong, H.X. Zhou, W.D. Li and L.Y. Sun
    Southeast University, Nanjing
    State Key Laboratory of Millimeter Waves
  - M. Gander, V. Dolean, M. El Bouajaji and S. Lanteri
    University of Geneva, Mathematics Section and
    INRIA Sophia Antipolis-Méditerranée

Outline

Outline

The time-harmonic Maxwell equations

$$\varepsilon \mathbf{i}\omega \mathbf{E} - \text{rot}(\mathbf{H}) = -z_0 \mathbf{J} \quad , \quad \mu \mathbf{i}\omega \mathbf{H} + \text{rot}(\mathbf{E}) = 0$$

- $\mathbf{E} = \mathbf{E}(\mathbf{x})$ is the electric field and $\mathbf{H} = \mathbf{H}(\mathbf{x})$ is the magnetic field
- $\mathbf{J} = \mathbf{J}(\mathbf{x})$ is the conductive current : $\mathbf{J} = \sigma \mathbf{E} \quad (z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}})$
- $\varepsilon = \varepsilon(\mathbf{x})$: (relative) electric permittivity
- $\mu = \mu(\mathbf{x})$: (relative) magnetic permeability
- $\sigma = \sigma(\mathbf{x})$: electric conductivity
- Boundary conditions
  - PEC boundary : $\mathbf{n} \times \mathbf{E} = 0$
  - Absorbing boundary : $\mathbf{n} \times \mathbf{E} + z\mathbf{n} \times (\mathbf{n} \times \mathbf{H}) = \mathbf{n} \times \mathbf{E}^{\infty} + z\mathbf{n} \times (\mathbf{n} \times \mathbf{H}^{\infty})$

Pseudo-conservative system form

$$\mathbf{i}\omega Q\mathbf{W} + \nabla \cdot F(\mathbf{W}) = \mathbf{S} \quad \text{with} \quad \mathbf{W} = {}^t(\mathbf{E}, \mathbf{H}) \quad \text{and} \quad \mathbf{S} = {}^t(-z_0\mathbf{J}, 0_{3\times 1})$$

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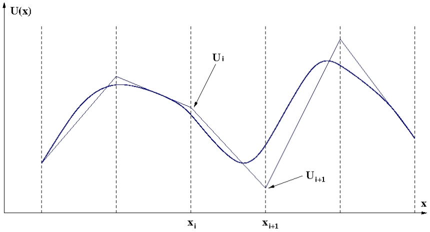
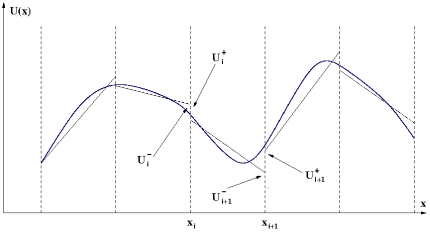
Outline



Continuous P1 interpolation



Discontinuous P1 interpolation

- Naturally adapted to heterogeneous media and discontinuous solutions
- Can easily deal with unstructured, possibly non-conforming meshes (h-adaptivity)
- High order with compact stencils and non-conforming approximations (p-adaptivity)
- Usually rely on polynomial interpolation but can also accomodate alternative functions (e.g plane waves)
- Amenable to efficient parallelization
- But leads to larger problems compared to continuous finite element methods

Discontinuous Galerkin discretization method

Basic properties

DG for electromagnetic wave propagation in heterogeneous media

- Heterogeneity is ideally treated at the element level
  - Discontinuities occur at material (i.e element) interfaces
  - Mesh generation process is simplified
- Wavelength varies with ϵ and μ
  - For a given mesh density, approximation order can be adapted at the element level in order to fit to the local wavelength

Discretization of irregularly shaped domains

- Unstructured simplicial meshes
- The basic support of the DG method is the element (triangle in 2D and tetrahedron in 3D)
- Local refinement is facilitated by allowing non-conformity
- Non-conformity opens the route to the coupling of different discretization methods (e.g structured/unstructured)

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- Non-conformity opens the route to the coupling of different discretization methods (e.g structured/unstructured)

- Triangulation: $\mathcal{T}_h = \bigcup_{i=1}^{N} \tau_i$
- Assume $\mathbf{J} = 0$ for simplicity of the presentation
- $\mathbf{W}_i(\mathbf{x}) \in \mathcal{P}_i = \mathbb{P}_m[\tau_i]$ and $\mathbf{W}_i(\mathbf{x}) = \sum_{j=1}^{d_i} \mathbf{W}_{ij}\varphi_{ij}(\mathbf{x})$ with $\mathbf{W}_{ij} \in \mathbb{C}^6$

$$\int_{\tau_i} \varphi \left(\mathbf{i}\omega Q\mathbf{W} + \nabla \cdot F(\mathbf{W}) \right) d\mathbf{x} = 0$$

$$\Leftrightarrow \quad \int_{\tau_i} \mathbf{i}\omega Q\mathbf{W}\varphi d\mathbf{x} - \int_{\tau_i} \nabla\varphi \cdot F(\mathbf{W}) d\mathbf{x} + \int_{\partial\tau_i} \left(F(\mathbf{W}) \cdot \mathbf{n} \right) \varphi d\sigma = 0$$

- Calculation of the boundary term on $\partial\tau_i$: centered or upwind numerical flux

Discontinuous Galerkin discretization method

Formulation

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Discontinuous Galerkin discretization method

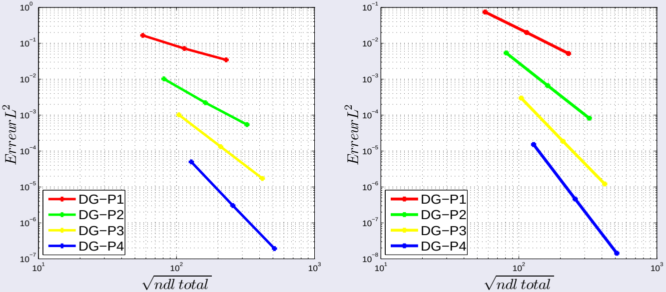
Numerical results in the 2D TMz case

$$\begin{cases} \mu \mathbf{i} \omega H_x + \dfrac{\partial E_z}{\partial y} = 0 \\ \mu \mathbf{i} \omega H_y - \dfrac{\partial E_z}{\partial x} = 0 \\ \varepsilon \mathbf{i} \omega E_z - \dfrac{\partial H_y}{\partial x} + \dfrac{\partial H_x}{\partial y} = 0 \end{cases}$$

- DGTH-$\mathbb{P}_p$ method based on Lagrange (nodal) interpolation
  - Triangular mesh
  - Sparse block matrix, $3n_p \times 3n_p$ (with $n_p = ((p+1)(p+2))/2$)
  - MUMPS multifrontal sparse matrix solver
    (P.R. Amestoy, I.S. Duff and J.-Y. L'Excellent, CMAME, Vol. 184, 2000)

Numerical convergence of the DGTH-$\mathbb{P}_p$ method

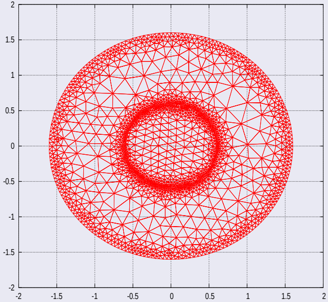
- Plane wave in vacuum, F=300 MHz
- Non-uniform triangular meshes



	$\mathbb{P}_1$	$\mathbb{P}_2$	$\mathbb{P}_3$	$\mathbb{P}_4$
Centered flux	1.1	2.1	2.9	4.0
Upwind flux	1.9	3.0	4.0	5.0

Scattering of a plane wave by a dielectric cylinder, F=300 MHz

- # vertices = 2078 and # elements = 3958
- Comparison between conforming DGTH-$\mathbb{P}_p$ and non-conforming DGTH-$\mathbb{P}_{p_i}$ methods



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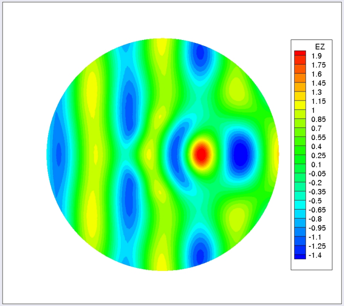
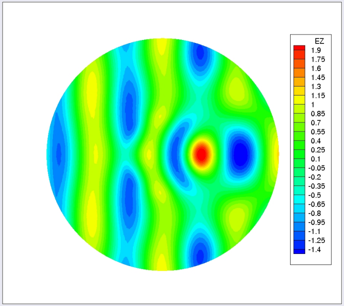


$\mathbb{P}_4$



$\mathbb{P}_{p_i}$ $(i = 1, 2, 3, 4)$

DGTH-$\mathbb{P}_4$ and non-conforming DGTH-$\mathbb{P}_{p_i}$ methods
Contour lines of E_z

Discontinuous Galerkin discretization method

Numerical results in the 2D TMz case

- Scattering of a plane wave by a dielectric cylinder, F=300 MHz
- Centered numerical flux

Conforming DGTH-$\mathbb{P}_p$ methods

n_z	Method	L2 error on E_z	CPU	RAM LU
390,274	DGTH-$\mathbb{P}_1$	0.37977	1.3 sec	97 MB
1,186,224	DGTH-$\mathbb{P}_2$	0.05830	4.1 sec	255 MB
3,225,808	DGTH-$\mathbb{P}_3$	0.05527	7.9 sec	547 MB
7,033,834	DGTH-$\mathbb{P}_4$	0.05522	15.7 sec	954 MB

Non-conforming DGTH-$\mathbb{P}_{p_i}$ method

n_z	Method	L2 error on E_z	CPU	RAM LU
1,267,878	DGTH-$\mathbb{P}_{1,4}$	0.05586	3.7 sec	252 MB

$\mathbb{P}_1$	$\mathbb{P}_2$	$\mathbb{P}_3$	$\mathbb{P}_4$
1495	2037	243	183

Local definition of p_i based on the value of a triangle area

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Outline

Time harmonic Maxwell system

$$\mathcal{L}\mathbf{W} = \mathrm{i}\omega G_0\mathbf{W} + G_x\partial_x\mathbf{W} + G_y\partial_y\mathbf{W} + G_z\partial_z\mathbf{W} - \mathbf{S} = 0$$

- Flux matrices

$$G_l = \begin{bmatrix} 0_{3\times 3} & N_l \\ -N_l & 0_{3\times 3} \end{bmatrix} \text{ for } \; l = x,y,z \; \text{ and with } \; {}^tN_l = -N_l$$

- Property : for any $\mathbf{n} = {}^t(n_x, n_y, n_z)$ with $\| \mathbf{n} \| = 1$,

$$C(\mathbf{n}) = G_0^{-1}\left(n_x G_x + n_y G_y + n_z G_z\right) \quad \text{is diagonalizable}$$

$$C(\mathbf{n}) = T(\mathbf{n})\Lambda(\mathbf{n})T^{-1}(\mathbf{n})$$

$$\text{Eigenvalues} \; : \; \lambda_{1,2} = -c \; , \; \lambda_{3,4} = 0 \; , \; \lambda_{5,6} = c \text{ with } c = \frac{1}{\sqrt{\varepsilon\mu}}$$

Domain decomposition solver

Formulation in the continuous case

Schwarz algorithm

- $\Omega = \bigcup_{j=1}^{N_s} \Omega_j$, $\mathbf{W}^j = \mathbf{W}|_{\Omega_j}$
- $\Gamma = \Gamma_a$ (for the presentation)
- Overlapping subdomains

$$\left\{ \begin{array}{rcl} \mathcal{L}\mathbf{W}^{j,p+1} & = & 0 \ \text{ in } \Omega_j \\ \mathcal{B}_{\mathbf{n}_{jl}}\mathbf{W}^{j,p+1} & = & \mathcal{B}_{\mathbf{n}_{jl}}\mathbf{W}^{l,p} \ \text{ on } \Gamma_{jl} = \partial\Omega_j \cap \bar{\Omega}_l \\ G_{\mathbf{n}}^{-}\mathbf{W}^{j,p+1} & = & G_{\mathbf{n}}^{-}\mathbf{W}_{\text{inc}} \ \text{ on } \Omega_j \cap \Gamma_a \end{array} \right.$$

Classical (natural) interface conditions

$$\mathcal{B}_{\mathbf{n}} \equiv G_{\mathbf{n}}^{-}$$

$$G_{\mathbf{n}}^{-}\mathbf{W} \iff \mathbf{n} \times \mathbf{E} + z\mathbf{n} \times (\mathbf{n} \times \mathbf{H}) \ \text{(impedance condition)}$$

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- Convergence result
  - V. Dolean, M.J. Gander and L. Gerardo-Giorda, SISC, Vol. 31, No. 3 (2009)
  - Fourier analysis (for constant ε and μ)
  - $\Omega_1 =]-\infty, b[\times\mathbb{R}^2$ and $\Omega_2 =]a, +\infty[\times\mathbb{R}^2$ with $a \leq b$

Convergence rate (non-conductive case)

$$\rho(\mathbf{k},\delta) = \left| \left(\frac{\sqrt{\mathbf{k}^2 - \tilde{\omega}^2} - \mathrm{i}\tilde{\omega}}{\sqrt{\mathbf{k}^2 - \tilde{\omega}^2} + \mathrm{i}\tilde{\omega}} \right) e^{-\delta\sqrt{\mathbf{k}^2 - \tilde{\omega}^2}} \right|$$

with $\delta = b - a$ and $\tilde{\omega} = \omega\sqrt{\varepsilon\mu}$

$$\rho(k,\delta) = \begin{cases} \left| \dfrac{\sqrt{\tilde{\omega}^2 - \mathbf{k}^2} - \tilde{\omega}}{\sqrt{\tilde{\omega}^2 - \mathbf{k}^2} + \tilde{\omega}} \right| & \text{if } |\mathbf{k}|^2 \leq \tilde{\omega}^2 \quad \text{(propagative modes)} \\ e^{-\delta\sqrt{\mathbf{k}^2 - \tilde{\omega}^2}} & \text{if } |\mathbf{k}|^2 > \tilde{\omega}^2 \quad \text{(evanescent modes)} \end{cases}$$

Schwarz algorithm with optimized interface conditions

- V. Dolean, M.J. Gander and L. Gerardo-Giorda, SISC, Vol. 31, No. 3 (2009)
- $\mathcal{S}_j$ for $j = 1, \cdots, N_s$: tangential operator

$$\text{Interface condition} \;:\; \left(\mathcal{B}_{\mathbf{n}_{jl}} + \mathcal{S}_j \mathcal{B}_{\mathbf{n}_{lj}}\right) \mathbf{W}^{j,p+1} = \left(\mathcal{B}_{\mathbf{n}_{jl}} + \mathcal{S}_j \mathcal{B}_{\mathbf{n}_{lj}}\right) \mathbf{W}^{l,p}$$

Optimal interface operators

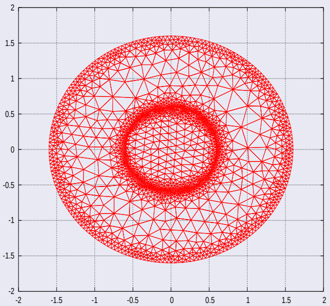
$$\mathcal{S}_j = \alpha_j = (\mathrm{i}\tilde{\omega})^{-1}(p_j - \mathrm{i}p_j) \quad \text{for} \quad j = 1, 2$$

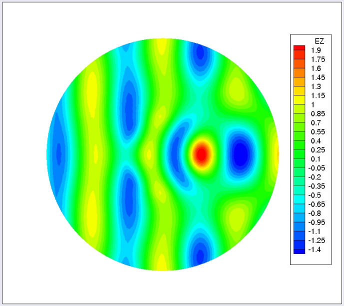
Case	p_1	p_2	Asymptotic ρ
1	0	0	1
2	$\dfrac{\sqrt{C}C_{\tilde{\omega}}^{\frac{1}{4}}}{\sqrt{2}\sqrt{h}}$	$\dfrac{\sqrt{C}C_{\tilde{\omega}}^{\frac{1}{4}}}{\sqrt{2}\sqrt{h}}$	$1 - \dfrac{\sqrt{2}C_{\tilde{\omega}}^{\frac{1}{4}}}{\sqrt{C}}\sqrt{h}$
3	$\dfrac{C^{\frac{1}{4}}C_{\tilde{\omega}}^{\frac{3}{8}}}{2h^{\frac{1}{4}}}$	$\dfrac{C^{\frac{3}{4}}C_{\tilde{\omega}}^{\frac{1}{8}}}{h^{\frac{3}{4}}}$	$1 - \dfrac{C_{\tilde{\omega}}^{\frac{1}{8}}}{C^{\frac{1}{4}}}h^{\frac{1}{4}}$

Conductive case $\Rightarrow$ talk of M. Gander in DD19 MS6

Domain decomposition solver

- Numerical results in 2D (TMz mode)
  - Scattering of a plane wave by a dielectric cylinder, F=300 MHz
  - # vertices = 2078 and # elements = 3958





Domain decomposition solver

- Numerical results in 2D (TMz mode)
  - Scattering of a plane wave by a dielectric cylinder, F=300 MHz
  - # vertices = 2078 and # elements = 3958 - Upwind flux

Classical Schwarz method

Method	L2 error on E_z	N_s	# iter BiCGStab ($\varepsilon = 10^{-6}$)
DGTH-$\mathbb{P}_1$	0.16400	4	317
-	0.16400	16	393
DGTH-$\mathbb{P}_2$	0.05701	4	650
-	0.05701	16	734
DGTH-$\mathbb{P}_3$	0.05519	4	1067
-	0.05519	16	1143
DGTH-$\mathbb{P}_4$	0.05428	4	1619
-	0.05427	16	1753
DGTH-$\mathbb{P}_i$	0.05487	4	352
-	0.05487	16	414

Domain decomposition solver

- Numerical results in 2D (TMz mode)
  - Scattering of a plane wave by a dielectric cylinder, F=300 MHz
  - # vertices = 2078 and # elements = 3958 - Upwind flux
  - DGTH-$\mathbb{P}_i$ ($N_s = 4$): 25.8 sec (classical) / 3.6 sec (optimized)

Optimized Schwarz method (case 1)

Method	L2 error on E_z	N_s	# iter BiCGStab ($\varepsilon = 10^{-6}$)
DGTH-$\mathbb{P}_1$	0.16457	4	52 (6.1) [a]
-	0.16467	16	83 (4.7)
DGTH-$\mathbb{P}_2$	0.05705	4	61 (10.7)
-	0.05706	16	109 (6.7)
DGTH-$\mathbb{P}_3$	0.05519	4	71 (15.0)
-	0.05519	16	139 (8.2)
DGTH-$\mathbb{P}_4$	0.05427	4	83 (19.5)
-	0.05527	16	170 (10.3)
DGTH-$\mathbb{P}_i$	0.05486	4	49 (7.2)
-	0.05491	16	81 (5.1)

[a] # iter classical/# iter optimized

Domain decomposition solver

- Numerical results in 2D (TMz mode)
  - Scattering of a plane wave by a dielectric cylinder, F=300 MHz
  - # vertices = 2078 and # elements = 3958
  - $N_s = 4$ subdomains

Optimized Schwarz method (case 1)

Method	Flux	L2 error on E_z	# iter BiCGStab ($\varepsilon = 10^{-6}$)	RAM LU (min/max)
DGTH-$\mathbb{P}_1$	Upwind	0.16457	52	26 MB/ 27 MB
-	Centered	0.35274	53	15 MB/ 15 MB
DGTH-$\mathbb{P}_2$	Upwind	0.05705	61	69 MB/ 71 MB
-	Centered	0.05823	61	39 MB/ 41 MB
DGTH-$\mathbb{P}_3$	Upwind	0.05519	71	140 MB/147 MB
-	Centered	0.05520	77	86 MB/ 90 MB
DGTH-$\mathbb{P}_4$	Upwind	0.05427	83	237 MB/249 MB
-	Centered	0.05527	85	156 MB/161 MB
DGTH-$\mathbb{P}_i$	Upwind	0.05486	49	54 MB/ 69 MB
-	Centered	0.05583	49	33 MB/ 42 MB

- Solution methods
  - Interface system
    - BiCGstab(ℓ) (G.L.G. Sleijpen and D.R. Fokkema, ETNA, Vol.1, 1993)
    - No preconditioner, $\ell = 6$
  - Local systems
    - MUMPS multifrontal sparse matrix solver
      (P.R. Amestoy, I.S. Duff and J.-Y. L'Excellent, CMAME, Vol. 184, 2000)
    - Mixed arithmetic strategy: LU in 32 bit + iterative refinement
- Hardware platform
  - Bull Novascale 3045 system of the CEA/CCRT center
    (Centre de Calcul Recherche et Technologie)
  - Intel Itanium 2/1.6 GHz, InfiniBand

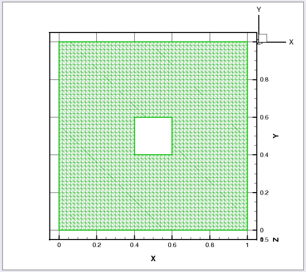
Domain decomposition solver

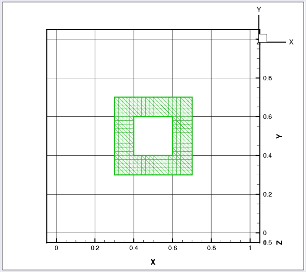
Numerical results in the 3D case

- Scattering of a plane wave by a coated perfectly conducting cube
- F=900 MHz, $\Omega = [0,1]^3$

Characteristics of the tetrahedral meshes

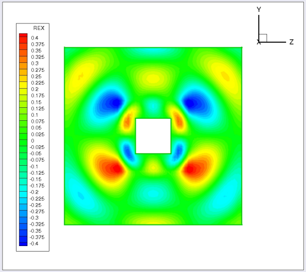
Mesh	# vertices	# tetrahedra
M1	131,922	744,000
M2	355,714	2,041,536

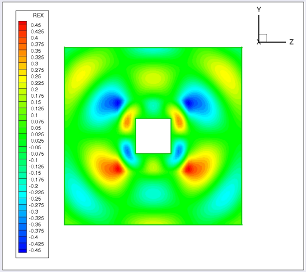




- Scattering of a plane wave by a coated perfectly conducting cube
- F=900 MHz, $\Omega = [0,1]^3$

Contour lines of E_x for $x = 0.5$, DGTH-$\mathbb{P}_1$ method



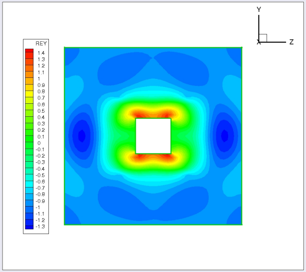


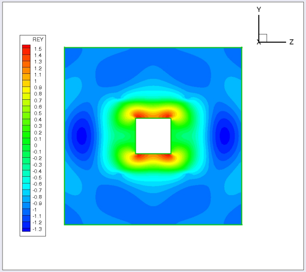
Mesh M1, # vertices = 131,922 Mesh M2, # vertices = 355,714

Centered flux

- Scattering of a plane wave by a coated perfectly conducting cube
- F=900 MHz, $\Omega = [0, 1]^3$

Contour lines of E_y for $x = 0.5$, DGTH-$\mathbb{P}_1$ method





Mesh M1, # vertices = 131,922   Mesh M2, # vertices = 355,714

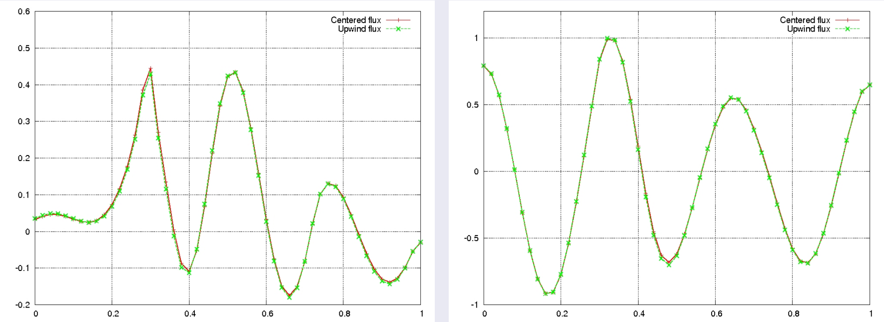
Centered flux

Domain decomposition solver

Numerical results in the 3D case

- Scattering of a plane wave by a coated perfectly conducting cube
- F=900 MHz, $\Omega = [0,1]^3$

x-wise distributions for $y = z = 0.3$, DGTH-$\mathbb{P}_1$ method

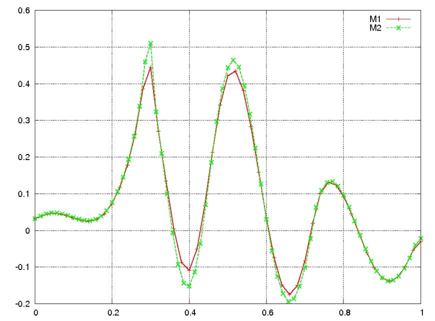


E_x component $\qquad$ E_y component

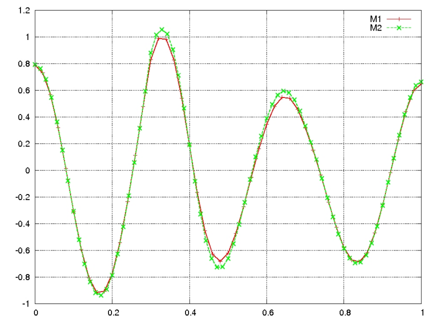
Mesh M1, # vertices = 131,922

- Scattering of a plane wave by a coated perfectly conducting cube
- F=900 MHz, $\Omega = [0,1]^3$

x-wise distributions for $y = z = 0.3$, DGTH-$\mathbb{P}_1$ method



E_x component

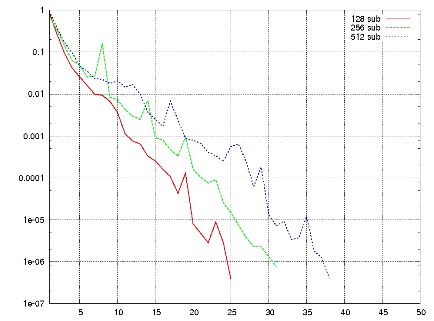


E_y component

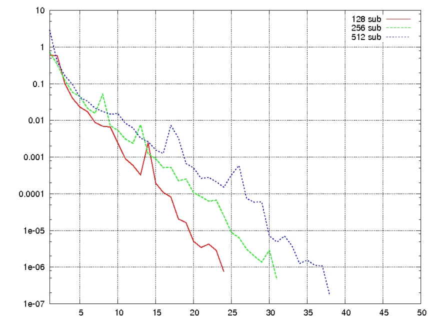
Centered flux

- Scattering of a plane wave by a coated perfectly conducting cube
- F=900 MHz, $\Omega = [0,1]^3$

Solution of the interface system, DGTH-$\mathbb{P}_1$ method



Centered flux



Upwind flux

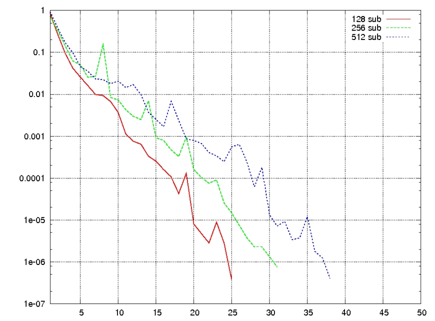
Mesh M1, # vertices = 131,922

Domain decomposition solver

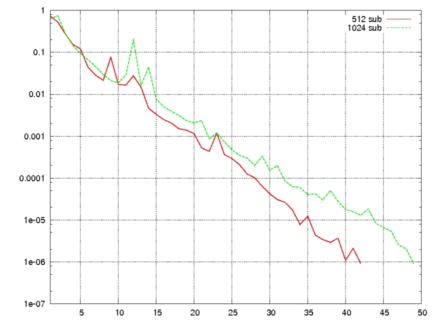
Numerical results in the 3D case

- Scattering of a plane wave by a coated perfectly conducting cube
- F=900 MHz, $\Omega = [0, 1]^3$

Solution of the interface system, DGTH-$\mathbb{P}_1$ method



Mesh M1, # vertices = 131,922



Mesh M2, # vertices = 355,714

Centered flux

- Scattering of a plane wave by a coated perfectly conducting cube
- F=900 MHz, $\Omega = [0,1]^3$
- Mesh M1, # vertices = 131,922

Performance results, DGTH-$\mathbb{P}_1$ method

Flux	# d.o.f	N_s	# it	CPU (min/max)	Elapsed time
Centered	17,856,000	128	25	650 sec/651 sec	652 sec
-	-	256	31	401 sec/402 sec	403 sec (1.60)[a]
-	-	512	38	180 sec/183 sec	184 sec (3.55)
Upwind	17,856,000	128	24	557 sec/558 sec	559 sec
-	-	256	31	318 sec/319 sec	320 sec (1.75)
-	-	512	38	142 sec/143 sec	144 sec (3.90)

[a] Parallel speedup

- Scattering of a plane wave by a coated perfectly conducting cube
- F=900 MHz, $\Omega = [0,1]^3$
- Mesh M1, # vertices = 131,922

Performance results, DGTH-$\mathbb{P}_1$ method

Flux	N_s	RAM LU (min/max)	CPU LU (min/max)	Elapsed time LU
Centered	128	1.17 GB/1.58 GB	180 sec/181 sec	182 sec
-	256	0.42 GB/0.64 GB	47 sec/ 48 sec	49 sec (3.7)[a]
-	512	0.16 GB/0.24 GB	12 sec/ 13 sec	14 sec (13.0)
Upwind	128	1.29 GB/1.77 GB	214 sec/215 sec	216 sec
-	256	0.46 GB/0.70 GB	55 sec/ 56 sec	57 sec (3.8)
-	512	0.18 GB/0.27 GB	14 sec/ 15 sec	16 sec (13.5)

[a] Parallel speedup

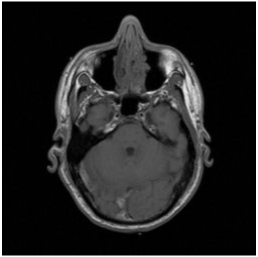
- Scattering of a plane wave by a coated perfectly conducting cube
- F=900 MHz, $\Omega = [0, 1]^3$
- Centered flux

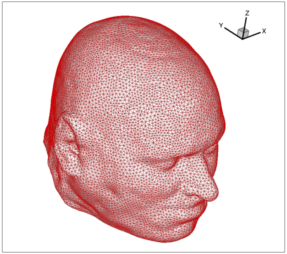
Performance results, DGTH-$\mathbb{P}_1$ method

Mesh	# d.o.f	N_s	# it	CPU (min/max)	Elapsed time
M1	17,856,000	128	25	650 sec/651 sec	652 sec
M2	48,996,864	512	42	705 sec/710 sec	711 sec
-	-	1024	49	380 sec/383 sec	384 sec (1.85)[a]

Mesh	N_s	RAM LU (min/max)	CPU LU (min/max)	Elapsed time LU
M1	128	1.17 GB/1.58 GB	180 sec/181 sec	182 sec
M2	512	0.61 GB/0.97 GB	92 sec/ 93 sec	93 sec
-	1024	0.23 GB/0.38 GB	23 sec/ 25 sec	27 sec (3.4)

[a] Parallel speedup



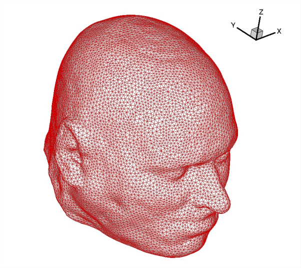


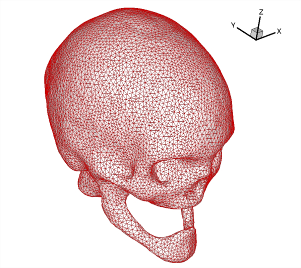
Geometric models

- Built from segmented medical images
- Extraction of surfacic (triangular) meshes of the tissue interfaces using specific tools
  - Marching cubes + adaptive isotropic surface remeshing
  - Delaunay refinement
- Generation of tetrahedral meshes using a Delaunay/Voronoi tool

- Plane wave exposure: F=1.8 GHz
- Characteristics of the tetrahedral meshes

Mesh	# vertices	# tetrahedra	L_{min} (mm)	L_{max} (mm)	L_{avg} (mm)
M1	188,101	1,118,952	9.04	23.86	9.09
M2	309,599	1,853,832	1.15	24.76	6.93

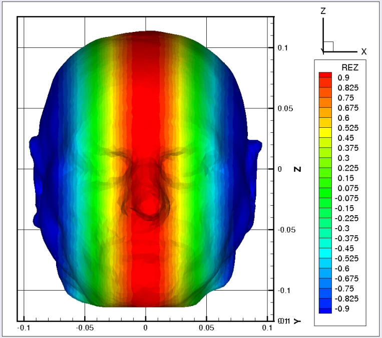


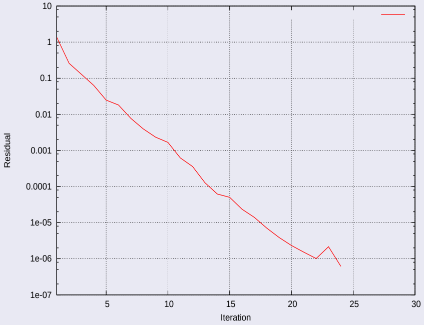


Domain decomposition solver

Numerical results in the 3D case: homogeneous propagation media

Performance results, DGTH-$\mathbb{P}_1$ method



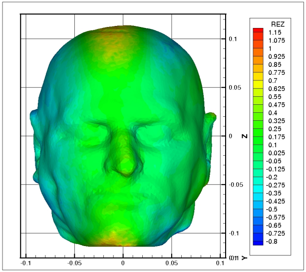


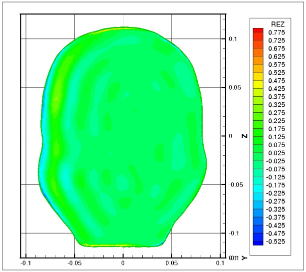
Mesh	# d.o.f	N_s	# it	CPU (min/max)	Elapsed time
M1	26,854,848	160	24	1204 sec/1209 sec	1210 sec

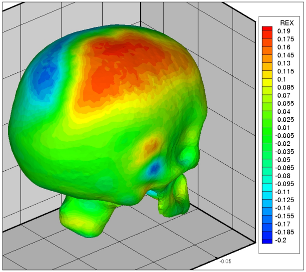
Mesh	RAM LU (min/max)	CPU LU (min/max)	Elapsed time LU
M1	2.1 GB/3.1 GB	493 sec/494 sec	495 sec

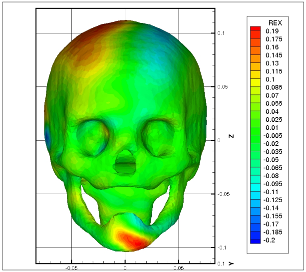
Domain decomposition solver

Numerical results in the 3D case: heterogeneous propagation media





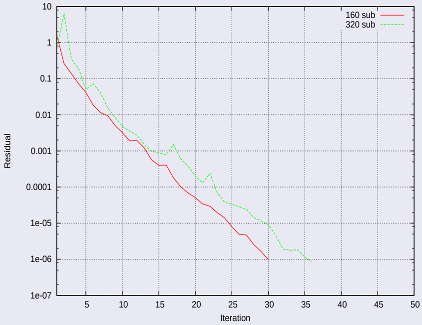


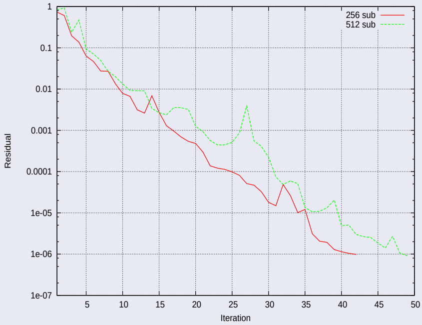


Domain decomposition solver

Numerical results in the 3D case: heterogeneous propagation media

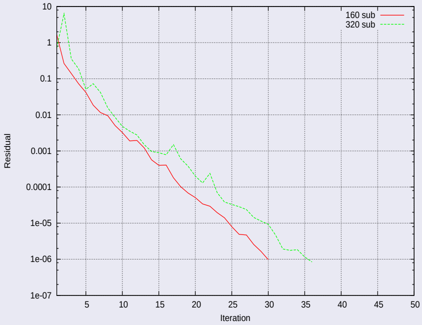
Performance results, DGTH-$\mathbb{P}_1$ method

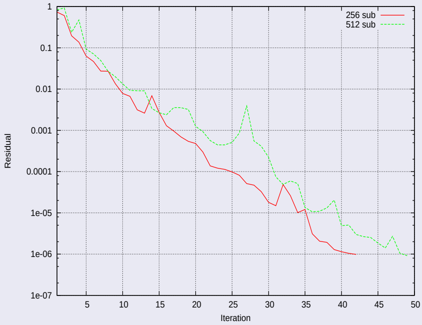




Mesh	# d.o.f	N_s	# it	CPU (min/max)	Elapsed time
M1	26,854,848	160	30	1311 sec/1313 sec	1314 sec
-	-	320	36	525 sec/ 527 sec	528 sec (2.5)
M2	44,491,968	256	42	1816 sec/1823 sec	1824 sec
-	-	512	49	782 sec/ 784 sec	785 sec (2.3)

Performance results, DGTH-$\mathbb{P}_1$ method





Mesh	N_s	RAM LU (min/max)	CPU LU (min/max)	Elapsed time LU
M1	160	2.1 GB/3.1 GB	490 sec/495 sec	496 sec
-	320	0.8 GB/1.2 GB	130 sec/131 sec	132 sec (3.8)
M2	256	2.2 GB/3.2 GB	525 sec/527 sec	528 sec
-	512	0.8 GB/1.3 GB	138 sec/140 sec	142 sec (3.7)

Outline

Closure: foreseen research directions

DGTH-$\mathbb{P}_p$ method

- Hierarchical basis expansions
- *hp*-adaptivity
- Hybridized DGTH formulation
  (B. Cockburn, J. Gopalakrishnan and R. Lazarov, SINUM, Vol. 47 (2009))

Domain decomposition methods

- Optimized Schwarz algorithms for conductive media
- Numerical treatment of second-order interface conditions with DGTH formulations in 2D and 3D
- Algebraic preconditioning of the interface system
  (in collaboration with L. Giraud and J. Roman, INRIA Bordeaux - Sud-Ouest)
- Subdomain solver
  - Shifted ILU preconditioned iterative solver
    (in collaboration with Y. Saad, University of Minnesota)
  - Block ILU preconditioned iterative solver
    (in collaboration with M. Bollhoefer, TU Braunschweig)

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Closure: scientific context

NACHOS project-team at INRIA Sophia Antipolis - Méditerranée

Scientific objectives

- Design, analysis and validation of numerical methods and high performance resolution algorithms for the computer simulation of evolution problems in complex domains and heterogeneous media
- Research directions
  - High-order finite element discretization methods on simplicial meshes
  - Hybrid explicit/implicit time integration strategies
  - Domain decomposition resolution algorithms
  - High performance computing related aspects

Computational electromagnetics

- System of Maxwell equations
- Interaction of EM fields with biological tissues
- Interaction of charged particles with EM fields (Vlasov/Maxwell equations)

Closure

Thank you for your attention!

Closure

Thank you for your attention!