Robust shape optimization for nano-photonics

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Seminar, team NACHOS (INRIA Sophia Antipolis) | July 2, 2019

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Silicon photonic

Nanophotonic components are devices allowing to manipulate light (see as an electromagnetic wave) at the nano/micro-metric scale.

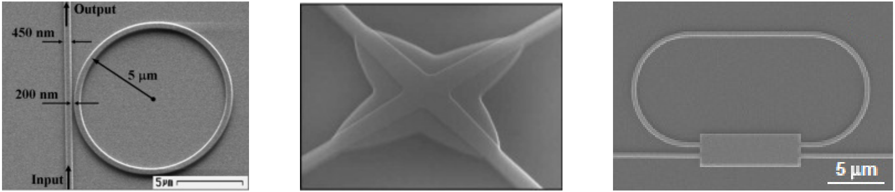
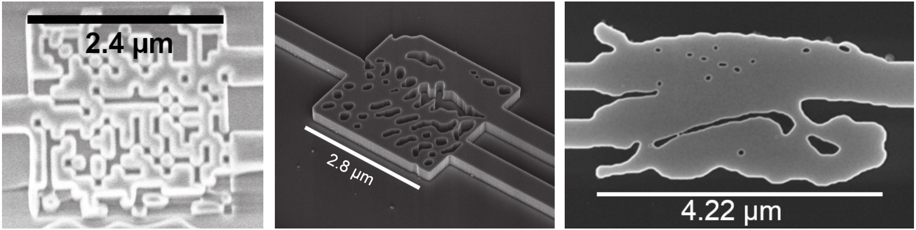


Fig. Examples of nanophotonics devices.

By connecting several components it is possible to complete circuit with complex optical properties.

Silicon photonic + topology optimization

Shape-optimization applied to nanophotonic: complex geometries.

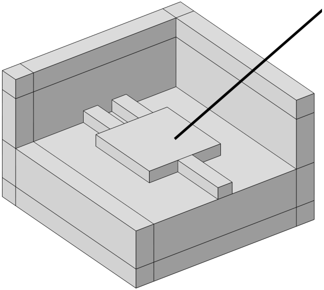


(a) *B. Shen et al., Nature photonics (2015)* **(b)** *A. Y. Piggot et al., Nature photonics (2015)* **(c)** *L. F. Frellsen et al., Optics Express (2016)*

Fig. MEB images of manufactured nanophotonic devices

Goal

Goal: automatically find the design of components that achieves an optical function as well as possible.



The optimizable domain $\mathcal{C}$ (that is to say the component).

We are looking for the repartition Ω of silicon inside the domain $\mathcal{C}$, that is:

$$\begin{aligned} \Omega &= \text{silicon part of the component,} \\ \mathcal{C}\backslash\Omega &= \text{air.} \end{aligned}$$

optimization of a shape in order to **maximize a physical objective**

1. Nanophotonic: description & modelization
    - Silicon photonic
    - PDE, Maxwell's equations
    - Waveguide modes

2. Shape optimization problem
    - Objective function
    - Algorithm
    - Results

3. Multi-objectives & robustness
    - Multi objectives
    - Worst-case robustness
    - Wavelength robustness

4. Geometrical robustness
    - Lithography and etching
    - Shape derivative for dilated/eroded shape
    - Application with lithography-etching

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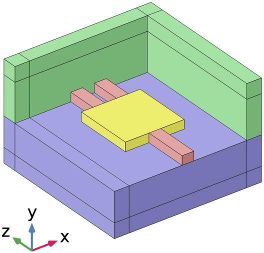
Maxwell's equations

(linear) **Maxwell's equations** allows to describe **the electric field E** : $\mathbb{R}^3 \to \mathbb{C}^3$ propagation at a given wavelength λ:

$$\nabla \times \nabla \times \mathbf{E} - k^2 \underset{\downarrow}{n}^2 \mathbf{E} \quad = \quad 0$$

where $k = 2\pi c/\lambda$ is the wavenumber, n the **the material's optical index** and c the speed of light.



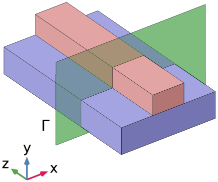
Optical indices:

- silicon in red ($n_{\text{silicon}} \simeq 3.5$),
- silica in blue ($n_{\text{silica}} \simeq 1.45$),
- air in green ($n_{\text{air}} = 1$),
- optimizable domain in yellow ($\mathcal{C}$),

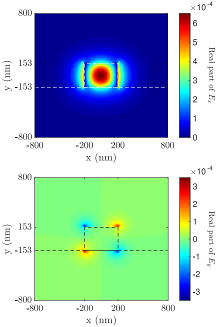
$$n_{\text{component}} = \begin{cases} n_{\text{silicon}} & \text{in} \quad \Omega \\ n_{\text{air}} & \text{in} \quad \mathcal{C}\backslash\Omega \end{cases} .$$

Waveguide modes 1/2

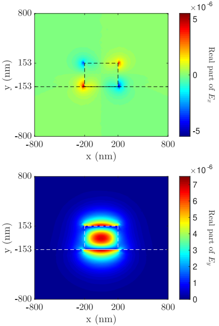
Components are linked through **waveguides**. Studying them shows that there exist several **propagation modes** (eigenvectors associated with the operator $\nabla \times \nabla \times -k^2 n^2 \mathrm{Id}$ such that $\mathbf{E}(x, y, z) = \mathbf{E}(x, y)e^{i\beta z}$).



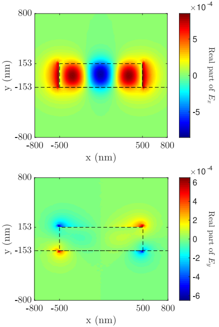
(a) A waveguide.



(b) Mode $\mathcal{E}_0$ TE$_0$



(c) Mode $\mathcal{E}_1$ TM$_0$



(d) Mode $\mathcal{E}_3$ TE$_2$

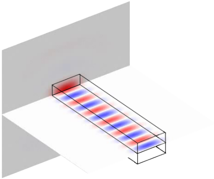
Waveguide modes 2/2

- Every electric field on a waveguide may be **decomposed on the $\mathcal{E}_j$**:

$$\mathbf{E}(x,y,z) = \underbrace{\sum_{j=1}^{N} \alpha_j \mathcal{E}_j(x,y) e^{i\beta_j z}}_{\text{modes along } z_+} + \underbrace{\sum_{j=1}^{N} \alpha_{-j} \mathcal{E}_{-j}(x,y) e^{-i\beta_{-j} z}}_{\text{modes along } z_-} + \underbrace{\int_{\mathbb{R}} \mathcal{E}^r_x(x,y) e^{i\beta^r_x z}\, dx}_{\substack{\text{essential spectrum} \\ \Leftrightarrow \text{ radiative modes}}}.$$

- **Orthogonality** for all $i \neq j$:

$$\int_\Gamma \left[\mathcal{E}_i \times \mathcal{H}_j^* + \mathcal{H}_i \times \mathcal{E}_j^*\right] \cdot \mathbf{n}\, ds = 0.$$



- Injection of the n-th mode ($\mathbf{E} = \mathcal{E}_n + \sum_{j=1}^{N} \alpha_{-j} \mathcal{E}_{-j}$):

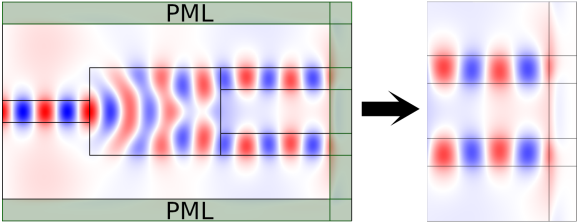
$$\boxed{\mathbf{n} \times \nabla \times \mathbf{E} + \frac{1}{2} i\omega\mu_0 \sum_{j=1}^{N} \mathbf{n} \times \mathcal{H}_j \int_\Gamma \left[\mathbf{E} \times \mathcal{H}_j^*\right] \cdot \mathbf{n}\, ds \;=\; 2i\omega\mu_0\, \mathbf{n} \times \mathcal{H}_n.}$$

Perfectly Matched Layer

To ensure the unicity of a solution in $\mathbb{R}^3$, the electric field must verify at infinity the **Silver-Müller radiation condition**

$$\lim_{|\mathbf{x}|\to\infty} |\mathbf{x}| \left(\nabla \times \mathbf{E} \times \frac{\mathbf{x}}{|\mathbf{x}|} - i\omega \mathbf{E} \right) = 0.$$

Numerically we use Perfectly Matched Layers (PML); areas around the component which **fully absorb electromagnetic waves** without causing **any reflections**.



$$\nabla \times \nabla \times \mathbf{E} - k^2 n^2 \mathbf{E} = 0$$

$$\downarrow$$

$$\nabla \times \Lambda^{-1} \nabla \times \mathbf{E} - k^2 n^2 \Lambda \mathbf{E} = 0$$

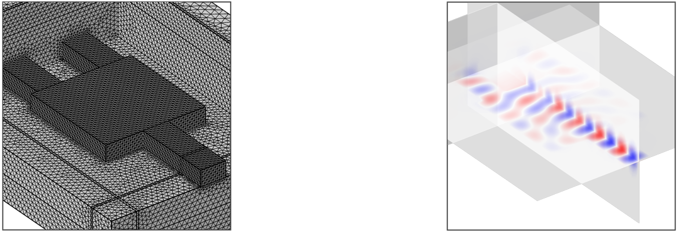
where Λ is a complex anisotrope matrix.

Full PDE

Variational formulation: $\mathbf{E} \in H(\mathbf{rot}, \mathcal{D})$ solution for all $\phi \in H(\mathbf{rot}, \mathcal{D})$ to

$$\int_{\mathcal{D}} \Lambda^{-1} \nabla \times \mathbf{E} \cdot \nabla \times \phi^* - k^2 n^2 \Lambda \mathbf{E} \cdot \phi^* \, d\mathbf{x} +$$

$$i\omega\mu_0 \int_\Gamma \mathbf{n} \times \left(2\,\mathcal{H}_n - \frac{1}{2} \sum_{j=1}^{N} \mathcal{H}_j \int_\Gamma \left[\mathbf{E} \times \mathcal{H}_j^* \right] \cdot \mathbf{n} \, dt \right) \cdot (\mathbf{n} \times \phi^* \times \mathbf{n}) \; ds = 0.$$



(a) Typical mesh ($\approx 5 \times 10^5$ tetras.) **(b)** FEM simulation (Nédélec el., order 2)

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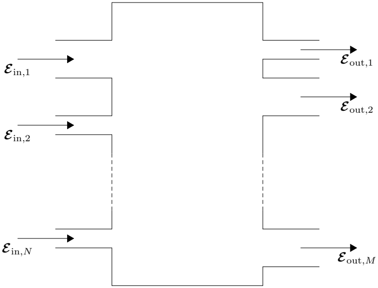
Objective function

We consider a nanophotonic component with N input waveguides and M output waveguides. For $j = 1, \dots, N$, we launch a mode $\mathcal{E}_{\text{in},j}$ into the j-th waveguide. We want to obtain other modes $\mathcal{E}_{\text{out},j}$ in the output waveguides.



The **power** through a surface Γ is physically defined as:

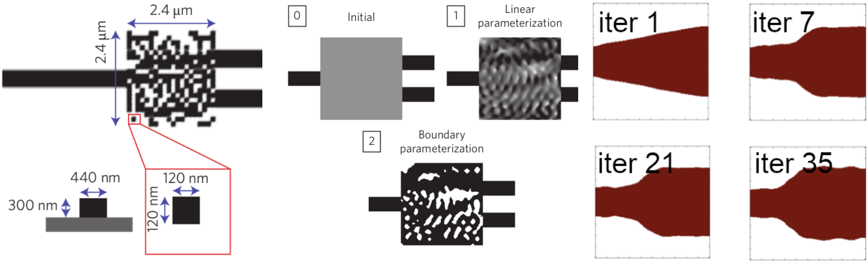
$$P^\Gamma = \frac{1}{2} \int_\Gamma \mathcal{R}e\left[\mathbf{E} \times \mathbf{H}^*\right] \cdot \mathbf{n}\, ds$$

We deduce the **power carried by a waveguide mode** $\mathcal{E}_n$ along Γ as:

$$\boxed{P_n^\Gamma = \left| \frac{1}{4} \int_\Gamma \left[\mathbf{E} \times \mathcal{H}_n^* + \mathbf{H} \times \mathcal{E}_n^*\right] \cdot \mathbf{n}\, ds \right|^2}$$

Shape optimization methods

Shape optimization (a.k.a. inverse design) is a set of methods allowing to **find an unparameterized structure which maximizes a given figure of merit**.



(a) Genetic algorithm, *B. Shen et al., Nature photonics (2015)*

(b) Density-based, *A. Y. Piggot et al., Nature photonics (2015)*

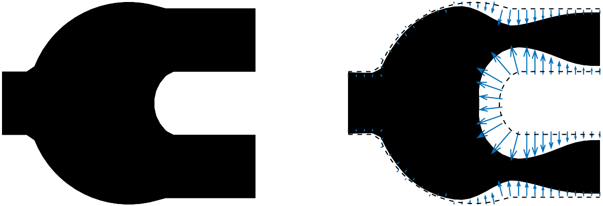
(c) Geometric (level-set), *C. M. Lalau-Keraly et al., Optics express (2013)*

Fig. Different shape optimization methods.

Shape optim. (Hadamard's method) 1/2

Goal: for an objective function $\mathcal{J}(\Omega)$, implement a gradient-based algorithm using the **shape derivative** of $\mathcal{J}(\Omega)$.

Difficulty: how to define what we meant by "small variation" of a shape ?



Let $\boldsymbol{\theta} : \mathbb{R}^3 \to \mathbb{R}^3$ a vector field such that Ω is modified into

$$(\mathrm{Id} + \boldsymbol{\theta})(\Omega) = \{\mathbf{x} + \boldsymbol{\theta}(\mathbf{x}), \mathbf{x} \in \Omega\}.$$

The following first order expansion holds

$$\mathcal{J}((\mathrm{Id} + \boldsymbol{\theta})(\Omega)) = \mathcal{J}(\Omega) + \mathcal{J}'(\Omega)(\boldsymbol{\theta}) + o(\boldsymbol{\theta}).$$

Shape optim. (Hadamard's method) 2/2

Maximization of the power carried by a mode on a waveguide Γ_{out}, that is:

$$\mathcal{J}(\Omega) \;=\; \left| \frac{1}{4\omega\mu_0} \int_{\Gamma_{\text{out}}} [\mathbf{E}_\Omega \times (\nabla \times \boldsymbol{\mathcal{E}}_j^*) + (\nabla \times \mathbf{E}_\Omega) \times \boldsymbol{\mathcal{E}}_j^*] \cdot \mathbf{n}\, ds \right|^2,$$

where $\mathbf{E}_\Omega$ is solution of the PDE with $n_{\text{component}} = n_{\text{air}} + (n_{\text{silicon}} - n_{\text{air}})\chi_\Omega$.

Using an adjoint-based method we find that:

$$\boxed{\mathcal{J}'(\Omega)(\boldsymbol{\theta}) = \int_{\partial\Omega} \boldsymbol{\theta} \cdot \mathbf{n}\, V(s)\, ds}$$

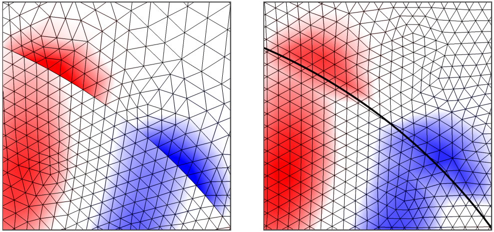
where $V(s) = k^2 \int_0^l \mathcal{R}e \left[\frac{\mathbf{n} \times \mathbf{E} \times \mathbf{n} \cdot \mathbf{n} \times \mathbf{A}^* \times \mathbf{n}}{(n_{\text{silicon}}^2 - n_{\text{air}}^2)^{-1}} - \frac{(n^2\mathbf{E}) \cdot \mathbf{n}\, (n^2\mathbf{A}^*) \cdot \mathbf{n}}{(n_{\text{silicon}}^{-2} - n_{\text{air}}^{-2})^{-1}} \right] dl,$

with **A** solution of a PDE close to the one of **E**.

$$\boldsymbol{\theta}_{\text{opt}} = tV(s)\,\mathbf{n} \text{ and } t \text{ small} \quad \Rightarrow \quad \mathcal{J}((\mathrm{Id} + \boldsymbol{\theta}_{\text{opt}})(\Omega)) > \mathcal{J}(\Omega).$$

Discontinuity at interfaces and regularity

The shape derivative use $(n^2\mathbf{E}) \cdot \mathbf{n}$. n^2 and $\mathbf{E} \cdot \mathbf{n}$ are **discontinuous on an interface** but numerically all the quantities are continuous on the same tetrahedron.



(a) E_y, *explicit* **(b)** E_y, *non-explicit*

Fig. An explicit mesh forces all the field's components to be continuous.

Optical index **regularization**:

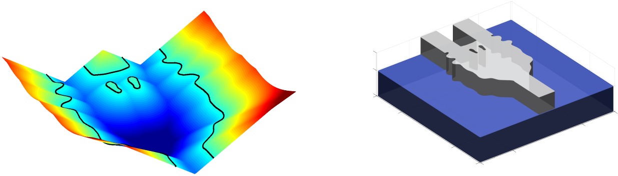
$$n_\eta^2 = n_{\text{air}}^2 + (n_{\text{silicon}}^2 - n_{\text{air}}^2) h_\eta(d_\Omega)$$

where h_η is a smooth approximation of the Heaviside function and d_Ω the signed distance to Ω.

let $\mathbf{E}_\eta$ be solution to Maxwell equation using n_η. We have

$$\lim_{\eta \to 0} \|\mathbf{E}_\eta - \mathbf{E}\|_{H(\mathbf{rot},\mathcal{D})} = 0.$$

Numerical representation of shapes



(a) Level-set function **(b)** 3D shape of the component

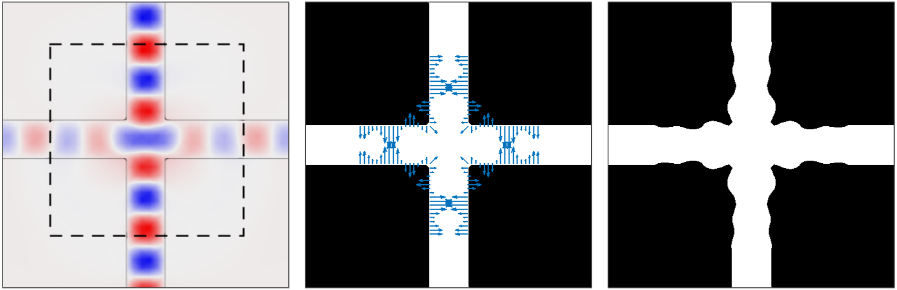
The shape Ω is represented by the 0 level set of the function $\phi : \mathbb{R}^2 \to \mathbb{R}$:

$$\Omega = \{\mathbf{x}, \phi(\mathbf{x}) < 0\}, \qquad \partial\Omega = \{\mathbf{x}, \phi(\mathbf{x}) = 0\}.$$

If Ω_0 is represented by $\phi_0 = \phi(\mathbf{x}, 0)$ then $\Omega_1 = (\mathrm{Id} + V\mathbf{n})(\Omega_0)$ is given by $\phi_1 = \phi(\mathbf{x}, 1)$ where ϕ is solution to the following Hamilton-Jacobi equation

$$\left\{ \begin{array}{rcl} \partial_t\phi(\mathbf{x}, t) + V\|\nabla_{\mathbf{x}}\phi(\mathbf{x}, t)\| & = & 0 \\ \phi(\mathbf{x}, 0) & = & \phi_0 \end{array} \right. .$$

Algorithm in practice



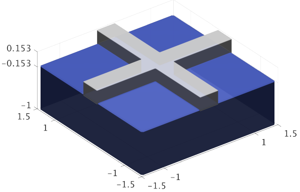
(a) *Computation of the electric field* $\mathbf{E}_\Omega$ *and the adjoint by FEM*

(b) *Computation of the vector field using the shape derivative*

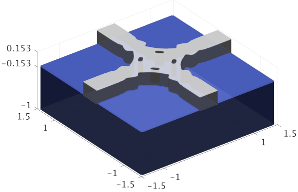
(c) *Modification of the shape solving the Hamilton-Jacobi*

Fig. One step of the shape optimization algorithm.

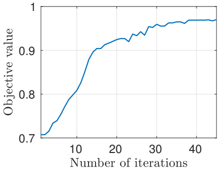
Example 1/3: crossing



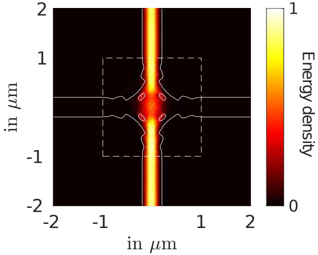
(a) *Initial shape*



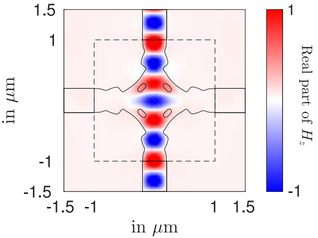
(b) *Optimized shape*



(c) *Conv. graph*



(d) *Energy density*



(e) *Electric field*

Example 1/3: crossing, multiple initializations

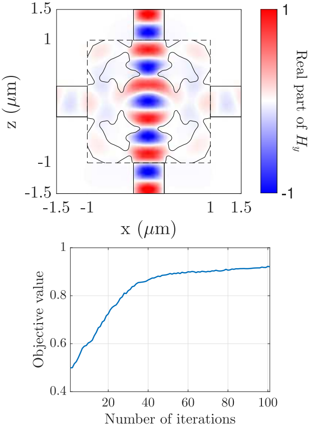


Fig. Initial shape as 2 × 2 holes.

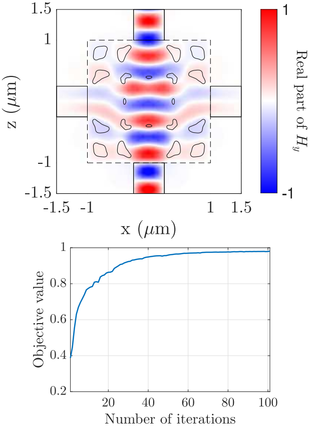


Fig. Initial shape as 5 × 5 holes.

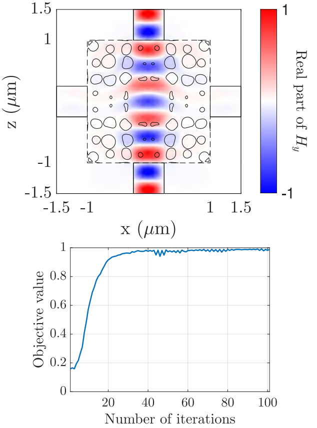
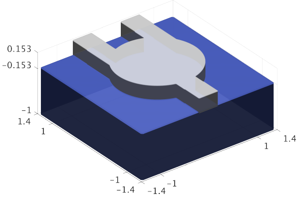
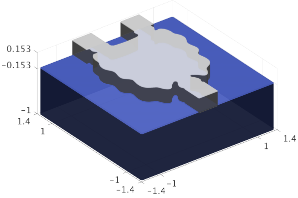


Fig. Initial shape as 8 × 8 holes.

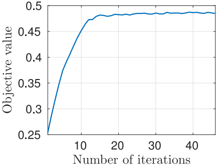
Example 2/3: power divider



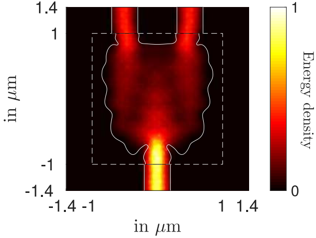
(a) *Initial shape*



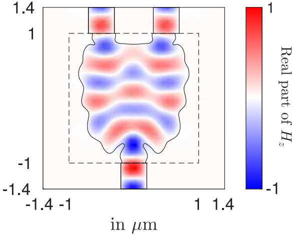
(b) *Optimized shape*



(c) Conv. graph

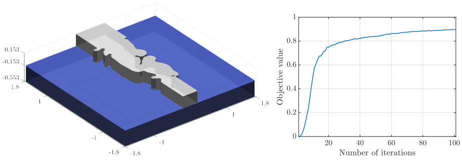


(d) *Energy density*

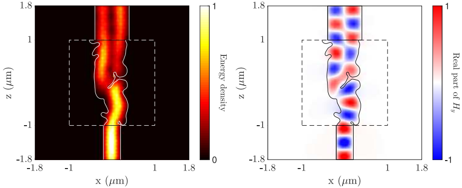


(e) *Electric field*

Example 3/3: modes converters

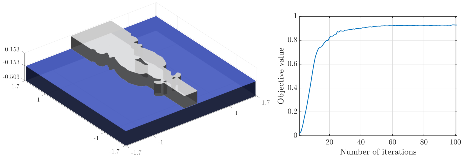


(a) *Optimized shape* **(b)** *Conv. graph*

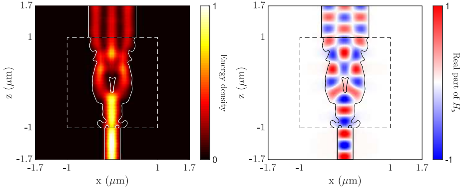


(c) *Energy density* **(d)** *Electric field*

Fig. $\mathcal{E}_0 \rightarrow \mathcal{E}_1$ converter.



(a) *Optimized shape* **(b)** *Conv. graph*



(c) *Energy density* **(d)** *Electric field*

Fig. $\mathcal{E}_0 \rightarrow \mathcal{E}_2$ converter.

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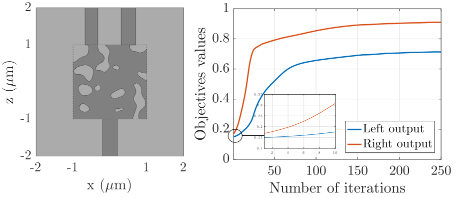
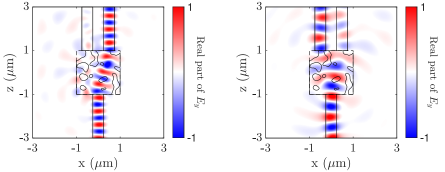
Multi-objectives: duplexer

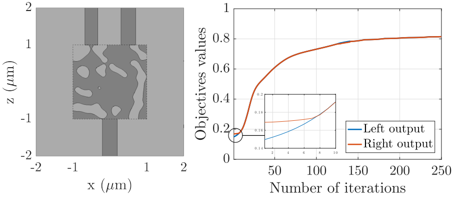


(a) *Optimized* **(b)** *Conv. graph*

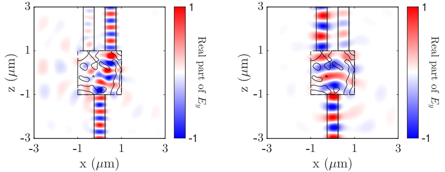


(c) $\lambda = 1.31\ \mu m$ **(d)** $\lambda = 1.55\ \mu m$

Fig. $\max_\Omega \mathcal{J}_{1.31}(\Omega) + \mathcal{J}_{1.55}(\Omega)$.



(a) *Optimized* **(b)** *Conv. graph*



(c) $\lambda = 1.31\ \mu m$ **(d)** $\lambda = 1.55\ \mu m$

Fig. $\max_\Omega \min[\mathcal{J}_{1.31}(\Omega), \mathcal{J}_{1.55}(\Omega)]$.

Note: on this slide (only) simulations are done in 2D.

Multi-objectives descent

We are interested in the case where **all the objectives $\mathcal{J}_i(\Omega)$ are of equal importance**, meaning that we maximize

$$\max_{\Omega} \min_{i=1,\ldots,N} \mathcal{J}_i(\Omega).$$

At each iteration we search for $\boldsymbol{\theta}$ such that

$$\max_{\boldsymbol{\theta}} \min_{i=1,\ldots,N} \mathcal{J}_i(\Omega) + \mathcal{J}_i'(\Omega)(\boldsymbol{\theta}).$$

We can find a **common** descent direction for each objectives by solving the following linear program:

$$\left\{\begin{array}{ll} \max\limits_{\boldsymbol{\alpha},\, r} & r \\ \textbf{s.t.} & \boldsymbol{\alpha} \in [0,1]^N,\ r \in \mathbb{R}, \\ & \sum\limits_{i=1}^{N} \alpha_i = 1, \mathcal{J}_i(\Omega) + \sum\limits_{j=1}^{N} \alpha_j \left\langle V_i, V_j \right\rangle < r,\ \ i = 1, \ldots, N \end{array}\right. .$$

Worst-case robustness

Nanophotonic components are subject to **several uncertain parameters**: wavelength, temperature, height of the silicon plate, edges rugosity, etching process etc. ...

Let X be the set of possibe parameters. For instance if we only consider λ then $X = [\lambda_0 - \varepsilon_\lambda, \lambda_0 + \varepsilon_\lambda]$ (more generally X is an hypercube).

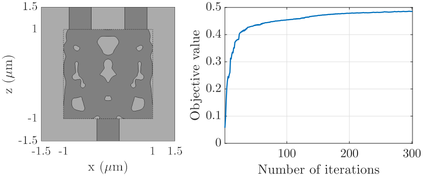
We are looking for a robust component (worst-case), that is

$$\max_{\Omega} \min_{\delta \in X} \mathcal{J}_\delta(\Omega) \simeq \max_{\Omega} \min_{\delta=\delta_1,\dots,\delta_n} \mathcal{J}_\delta(\Omega).$$

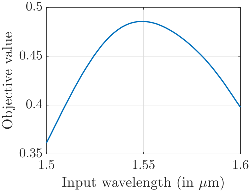
For a ±50 nm robustness we could maximize:

$$\max_{\Omega} \min \left[\mathcal{J}_{-50\text{ nm}}(\Omega), \mathcal{J}_{0\text{ nm}}(\Omega), \mathcal{J}_{+50\text{ nm}}(\Omega)\right].$$

Wavelength robustness 1/2: power divider

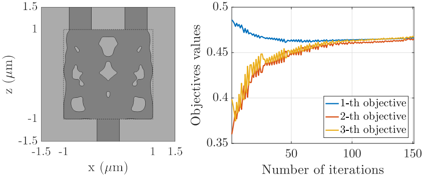


(a) *Optimized* **(b)** *Conv. graph*

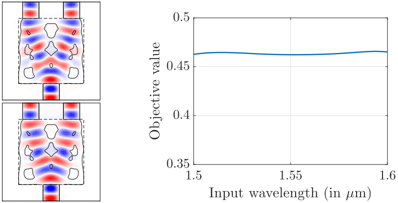


(c) *Spectrum*

Fig. Non-robust optimization.



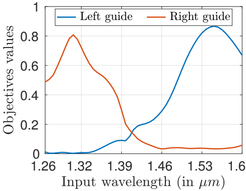
(a) *Optimized* **(b)** *Conv. graph*



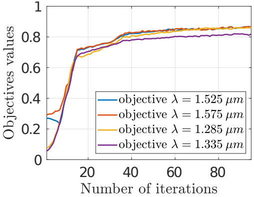
(c) *Sim. ±50 nm* **(d)** *Spectrum*

Fig. Robust ($\lambda \pm 50\,\text{nm}$) optimization.

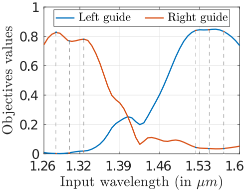
Wavelength robustness 2/2: duplexer



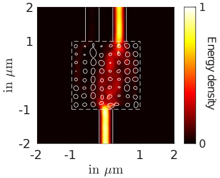
(a) *Initial spectrum*



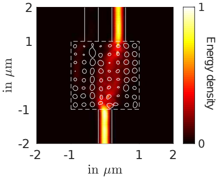
(b) *Conv. graph*



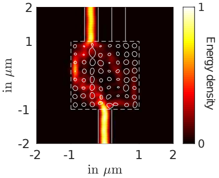
(c) *Optimized spectrum*



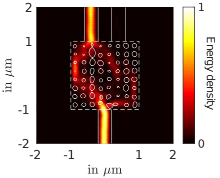
(d) $\lambda = 1.285\ \mu m$



(e) $\lambda = 1.335\ \mu m$



(f) $\lambda = 1.525\ \mu m$

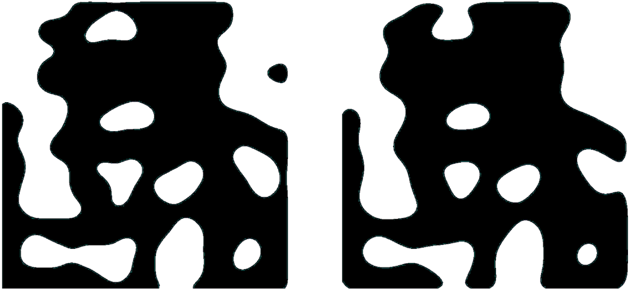


(g) $\lambda = 1.575\ \mu m$

1. Nanophotonic: description & modelization
   - Silicon photonic
   - PDE, Maxwell's equations
   - Waveguide modes
2. Shape optimization problem
   - Objective function
   - Algorithm
   - Results
3. Multi-objectives & robustness
   - Multi objectives
   - Worst-case robustness
   - Wavelength robustness
4. Geometrical robustness
   - Lithography and etching
   - Shape derivative for dilated/eroded shape
   - Application with lithography-etching

Physical problem

Because of the manufacturing process the produced shapes are not exactly the same as the ones we are asking for.

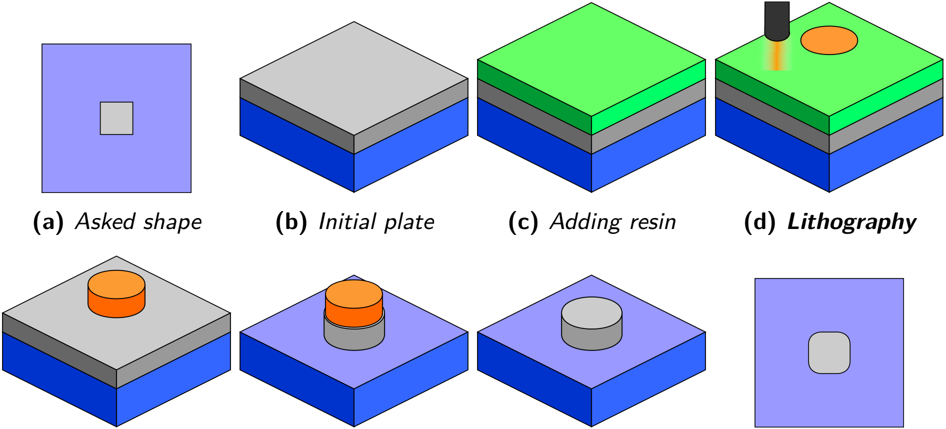


(a) *Optimal shape* Ω **(b)** *Procuced shape* $\Psi_\delta(\Omega)$

If we try to produce Ω we will get $\Psi_\delta(\Omega)$ with δ an uncertain parameter.

Question: What is the shape derivative of $\mathcal{J}_\delta(\Omega) = \mathcal{J}(\Psi_\delta(\Omega))$?

Lithography-etching process in practice



(a) *Asked shape* **(b)** *Initial plate* **(c)** *Adding resin* **(d)** ***Lithography***

(e) *Cleaning* **(f)** ***Etching*** **(g)** *Final plate* **(h)** *Obtained shape*

Fig. Main steps of the lithography-etching process.

Modelization

$$\boxed{\Omega \quad \rightarrow \quad \Omega_{\text{lithograhy}} \quad \rightarrow \quad \Omega_{\text{etching}}}$$

- **Lithography:** convolution-thresholding of the caracteristic function

$$\Omega_{\text{lithography}} = \{\mathbf{x}, (\chi_\Omega * G)(\mathbf{x}) > s\}$$

where G correspond to a centered gaussian with fixed variance and $s \in]0,1[$ the threshold to change the resin's state.

- **Etching:** dilation or erosion

$$\Omega_{\text{etching}} = (\mathrm{Id} + \delta \mathbf{n})(\Omega_{\text{lithography}})$$

where δ is a small uncertain parameter with values in the interval $[-\eta, \eta]$.

Lithography approximation

Reminder:

$$\Omega_{\text{lithography}} = \{\mathbf{x}, (\chi_\Omega * G)(\mathbf{x}) > s\}$$

Hardly differentiable ...

We locally approximate the shape up to the second order (locally convolution between a parabola and a gaussian). As such

$$\widetilde{\Omega}_{\text{lithography}} = (\text{Id} + f(\kappa)\mathbf{n})(\Omega)$$

where f is an analytic function.

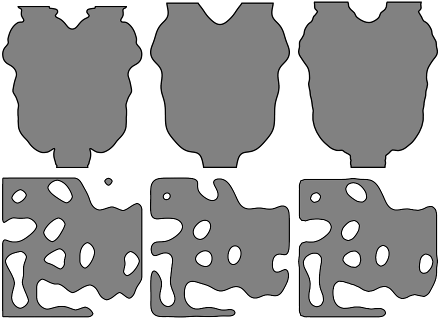


Fig. Optimized shape Ω, Ω after lithography, Ω after approximation.

In conclusion we have $\Omega_{\text{etching}} \simeq (\text{Id} + g_\delta(\mathbf{x})\mathbf{n})(\Omega)$.

Shape derivative for dilated/eroded shape

Theorem:

Let $\mathcal{J}_\delta(\Omega) = \mathcal{J}((\mathrm{Id} + \delta\mathbf{n})(\Omega))$ where $\mathcal{J}$ is defined as previously. If δ is sufficently small so that $(\mathrm{Id} + \delta\mathbf{n})$ is a diffeormorphism from Ω into the dilated shape $(\mathrm{Id} + \delta\mathbf{n})(\Omega)$ then we have the following shape derivative:

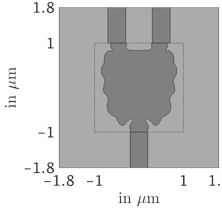
$$\mathcal{J}_\delta'(\Omega)(\boldsymbol{\theta}) = \int_{\partial\Omega} \boldsymbol{\theta} \cdot \mathbf{n} V_{(\mathrm{Id}+\delta\mathbf{n})(\Omega)} \circ (\mathrm{Id} + \delta\mathbf{n})(s) H \, ds$$

where $H = \left((\mathrm{Id} + \delta\nabla\mathbf{n})^{-1}\mathbf{n} \cdot \mathbf{n}\right) |\det(\mathrm{Id} + \delta\nabla\mathbf{n})|$.

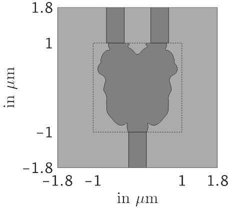
→ In other word, simulating Maxwell equations with $(\mathrm{Id} + \delta\mathbf{n})(\Omega)$ gives us information on how to modify Ω to optimize $\mathcal{J}_\delta(\Omega)$.

Solving $\max_\Omega \min[\mathcal{J}_{g_{-\eta}}(\Omega), \mathcal{J}(\Omega), \mathcal{J}_{g_{+\eta}}(\Omega)]$ should give us a shape which is robust to the (simplified) lithography-etching process with $\delta \in [-\eta, \eta]$.

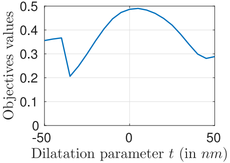
Example 1/4: power divider



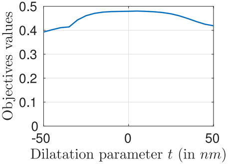
(a) *Non-robust shape Ω_{nr}*



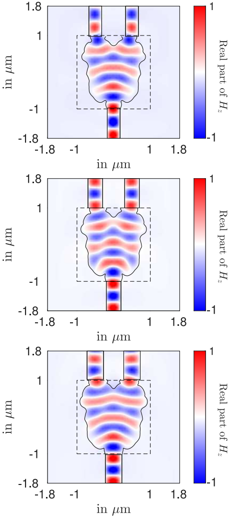
(b) *Robust shape Ω_r*



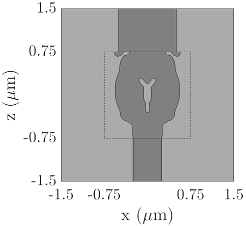
(c) *Objective variation for Ω_{nr}, $\delta \in [-50, 50]$ nm*



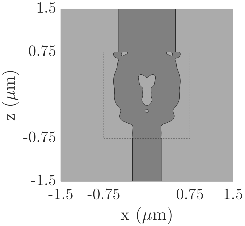
(d) *Objective variation for Ω_r, $\delta \in [-50, 50]$ nm*



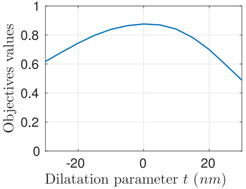
Example 2/4: mode converter $1 \rightarrow 3$



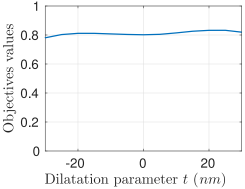
(a) *Non-robust* Ω_{nr}



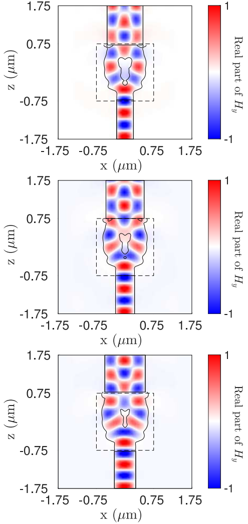
(b) *Robust* Ω_r



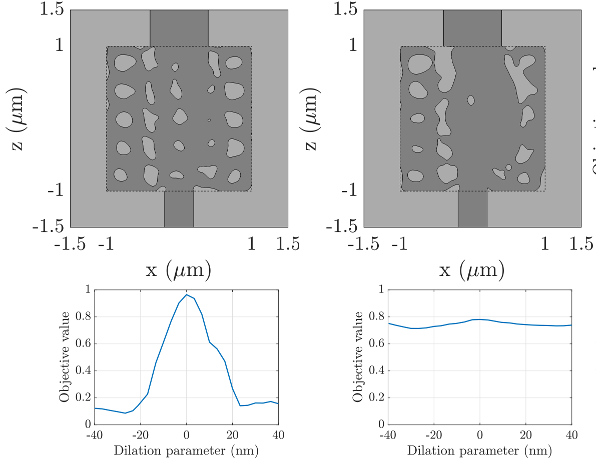
(c) *Objective for* Ω_{nr}, $\delta \in [-25, 25]$ *nm*



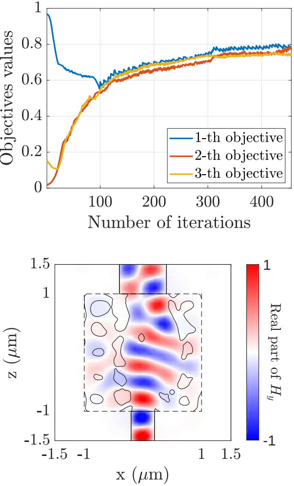
(d) *Objective for* Ω_r, $\delta \in [-25, 25]$ *nm*



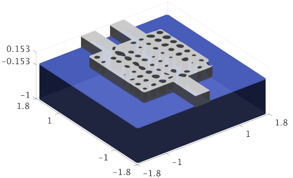
Example 3/4: mode converter $1 \to 2$



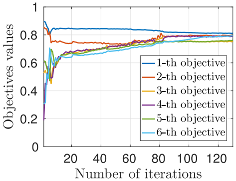
(a) Non-robust shape **(b)** Robust optimal shape



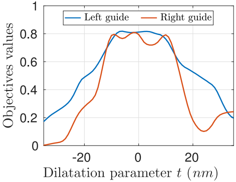
Example 4/4: duplexer



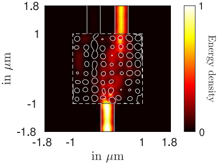
(a) *Optimal shape*



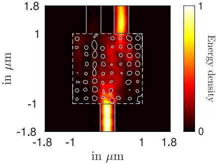
(b) *Conv. graph*



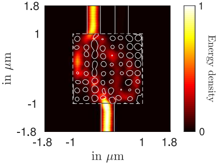
(c) *Robustness*



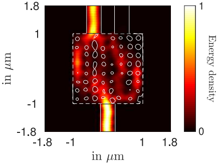
(d) $\lambda = 1.31\ \mu m$,
$\delta = +10\ nm$



(e) $\lambda = 1.31\ \mu m$,
$\delta = -10\ nm$



(f) $\lambda = 1.55\ \mu m$,
$\delta = +10\ nm$



(g) $\lambda = 1.55\ \mu m$,
$\delta = -10\ nm$

Thanks for your attention

- **Nicolas Lebbe**, Charles Dapogny, Édouard Oudet, Karim Hassan and Alain Glière, *Robust shape and topology optimization of nanophotonic devices using the level set method*, in press, Journal of Computational physics (2018).

- **Nicolas Lebbe**, Alain Glière and Karim Hassan, *High-efficiency and broadband photonic polarization rotator based on multilevel shape optimization*, published in Optics Letters 44, no. 8, 1960–1963 (2019).