Introduction to Model Order Reduction

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We discuss linear time-invariant systems (LTI systems) of type $\dot{x} = Ax + Bu, y = Cx + Du$, where u is an input function, x is the state variable and y is the output of the dynamical system. Often one is only interested in the input-output properties of this system, which are of small size, whereas the state variable is typically large. Model order reduction (MOR) methods deal with these kind of systems by reducing the number of state variables such that the error between the approximate output $\tilde{y}$ and the original output y is hopefully small. Although the computation of the reduced order model might be expensive, this is done only once (offline phase). In contrast to that, solving the reduced system many times with different inputs (online phase) is significantly accelerated.

We will present three different approaches to MOR. The first approach is called "proper orthogonal decomposition" (POD), where a low-rank singular value decomposition of a sequence of snapshots from the original dynamical systems is used to construct a reduced-order system. The second approach is based on "balanced truncation" which constructs the reduced-order model based the associated controllability and observability Gramians. Finally the third approach is based "moment matching methods", where the Taylor expansion of the underlying transfer function and the representation of its coefficients by Krylov subspaces is used to construct a reduced-order model.