FETI–DPEM2–full method as an efficient technic applied to 3D electromagnetic large-scale simulation

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Invited talk, Inria, 2014

Background and the aim of the PhD thesis

<table>
<tr><th>Time</th><th>Place</th></tr>
<tr><td>2005 – 2009
Bachelor</td><td rowspan="2">Novosibirsk State Technical University + SB RAS,
Faculty of Applied Mathematics and Computer science</td></tr>
<tr><td>2009 – 2011
Master 2</td></tr>
</table>

Time	Place
2005 – 2009 Bachelor	Novosibirsk State Technical University + SB RAS, Faculty of Applied Mathematics and Computer science
2009 – 2011 Master 2	
2011 – 2014 PhD	The Institut Fresnel, HIPE team (Hyperfrequency, Instrumentation, Processing, Experimentation)

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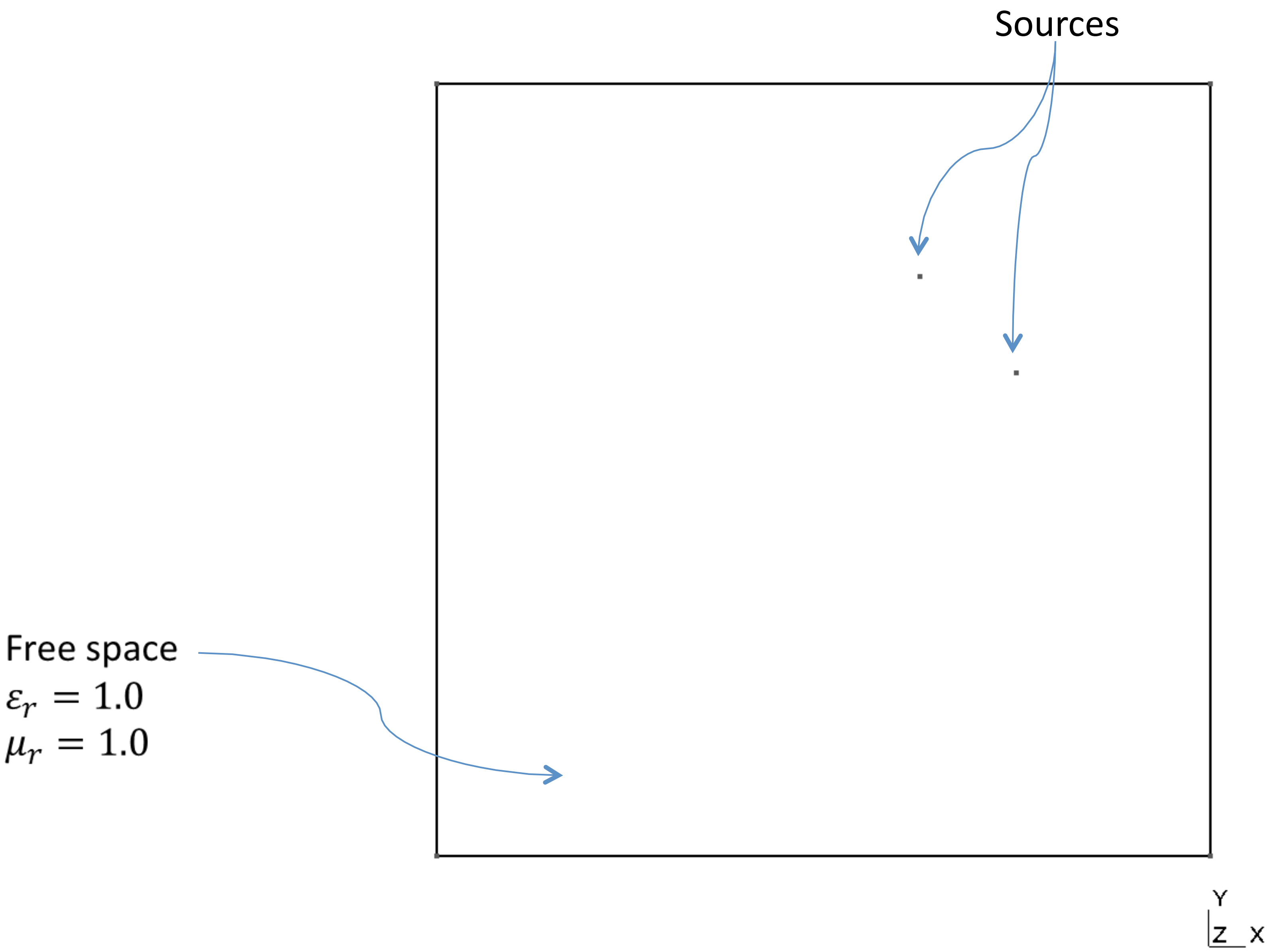
2011 – 2014 PhD	The Institut Fresnel, HIPE team (Hyperfrequency, Instrumentation, Processing, Experimentation)

Aim of PhD	**Create a powerful tool which would be able to solve Large-scale 2D & 3D electromagnetic problems for complex media**

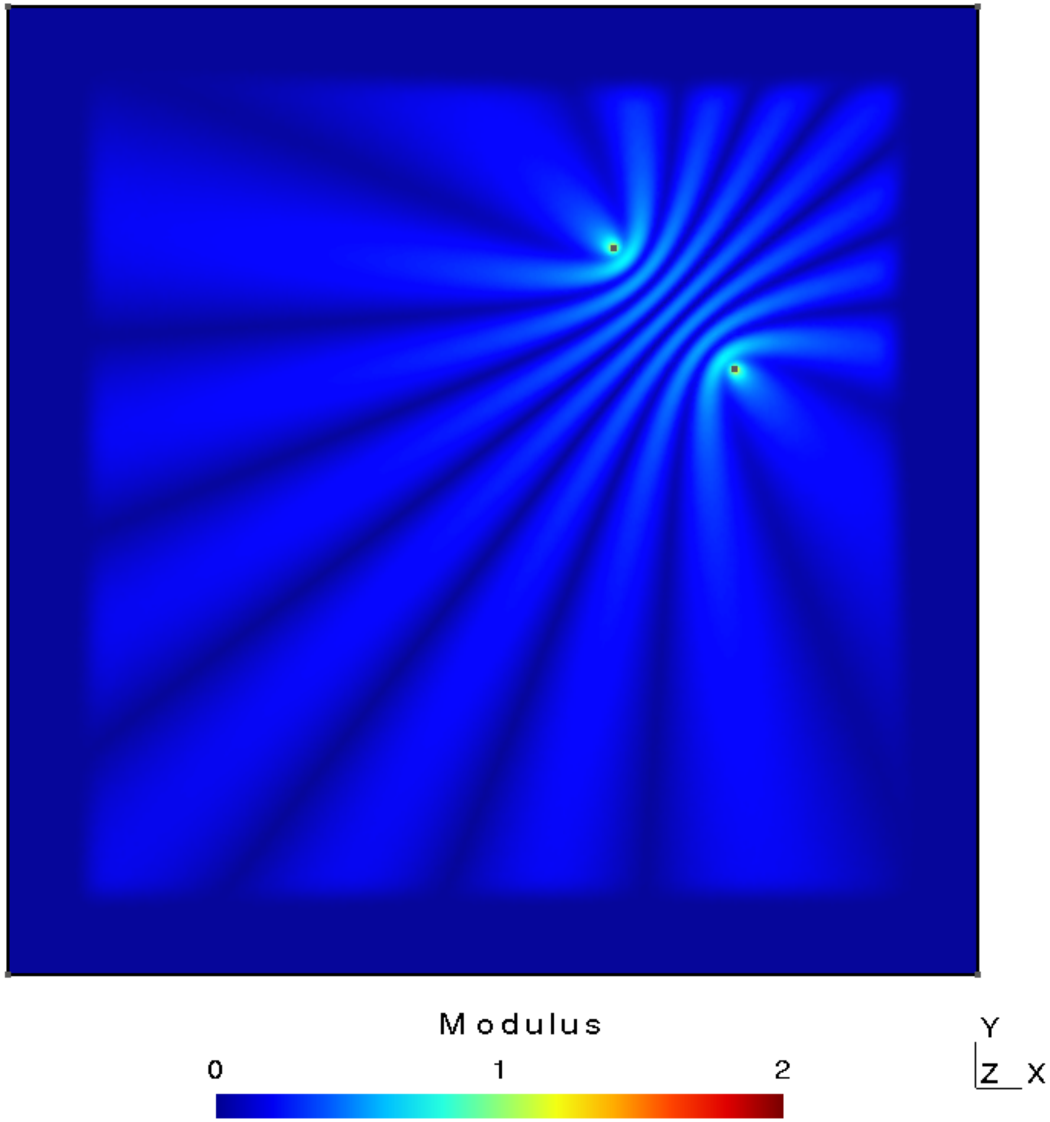
2D & 3D Direct Scattering problems
- Physical statement of problem
- Mathematical statement of problem

Numerical method
- FETI-DPEM2 classical approach
- Its modification (FETI-DPEM2-full method)
- Numerical results

3D quantitative Inverse problems
- Problem statement
- FETI Implementation
- Numerical results


Sources
Free space
$\varepsilon_r = 1.0$
$\mu_r = 1.0$
Y
Z
X

Incident field E^{inc}



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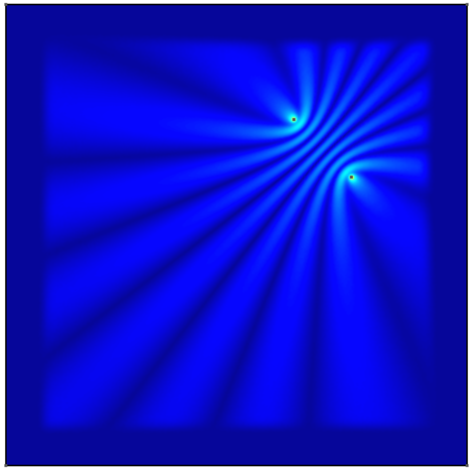

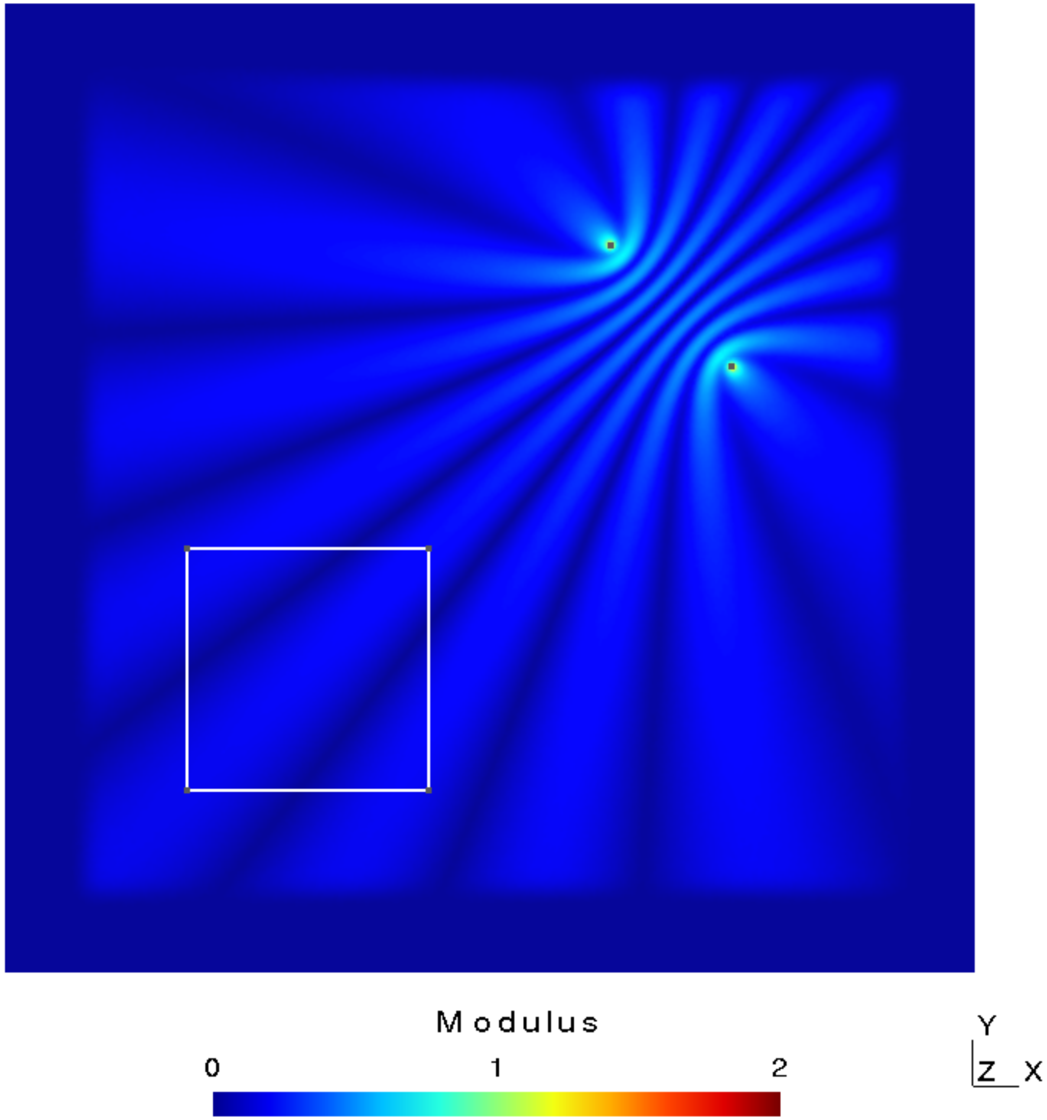

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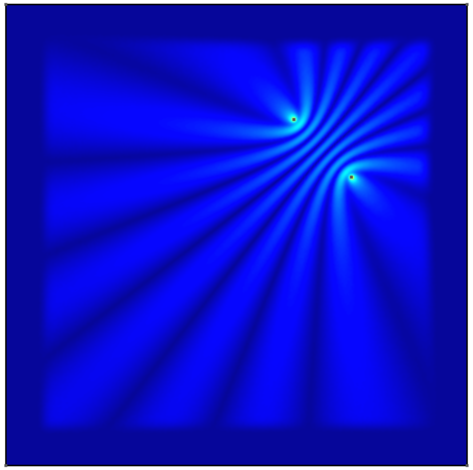
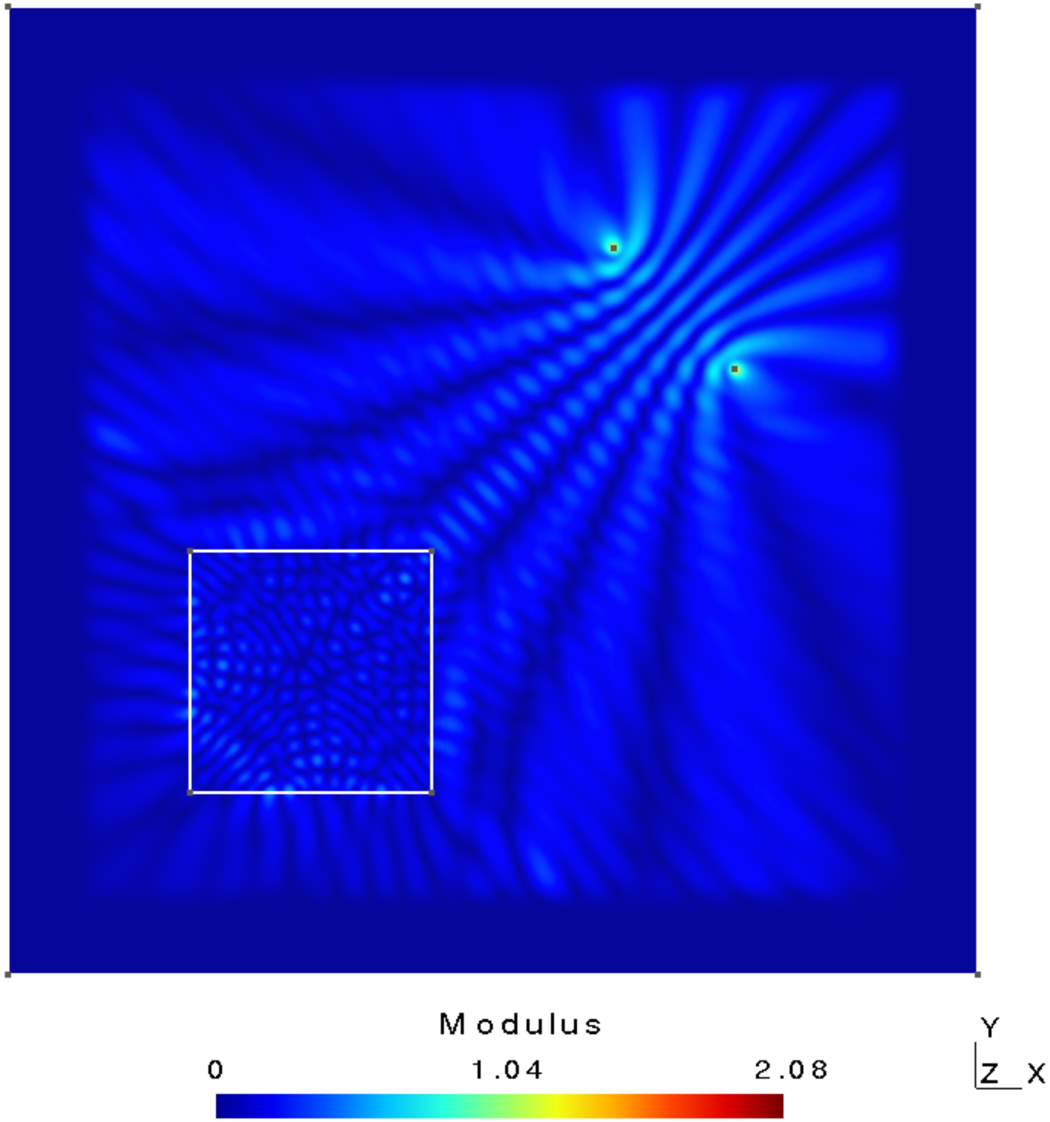


Total field E^{tot}



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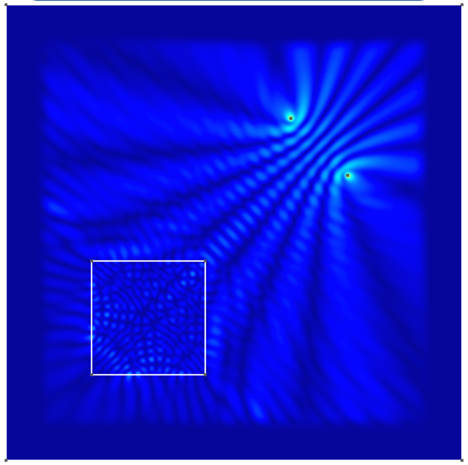

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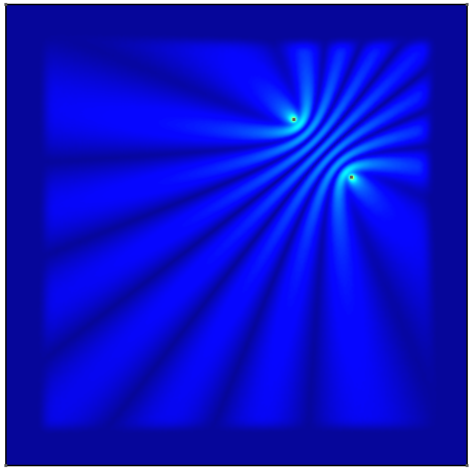

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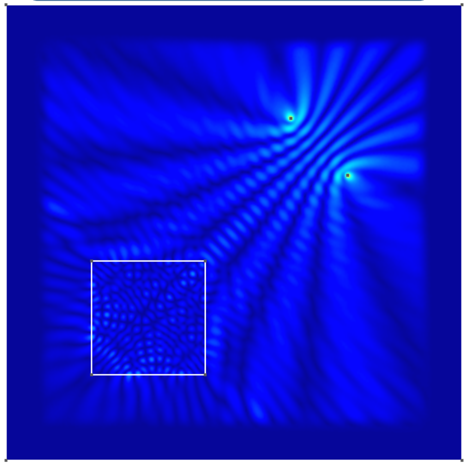

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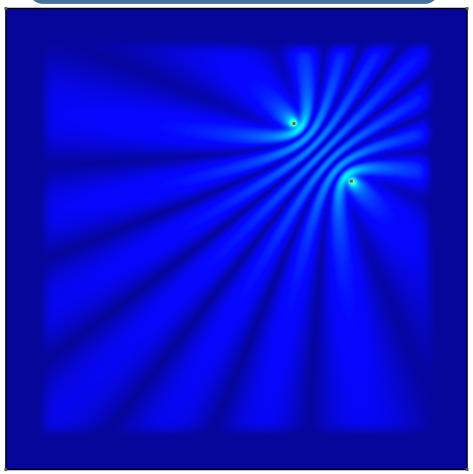

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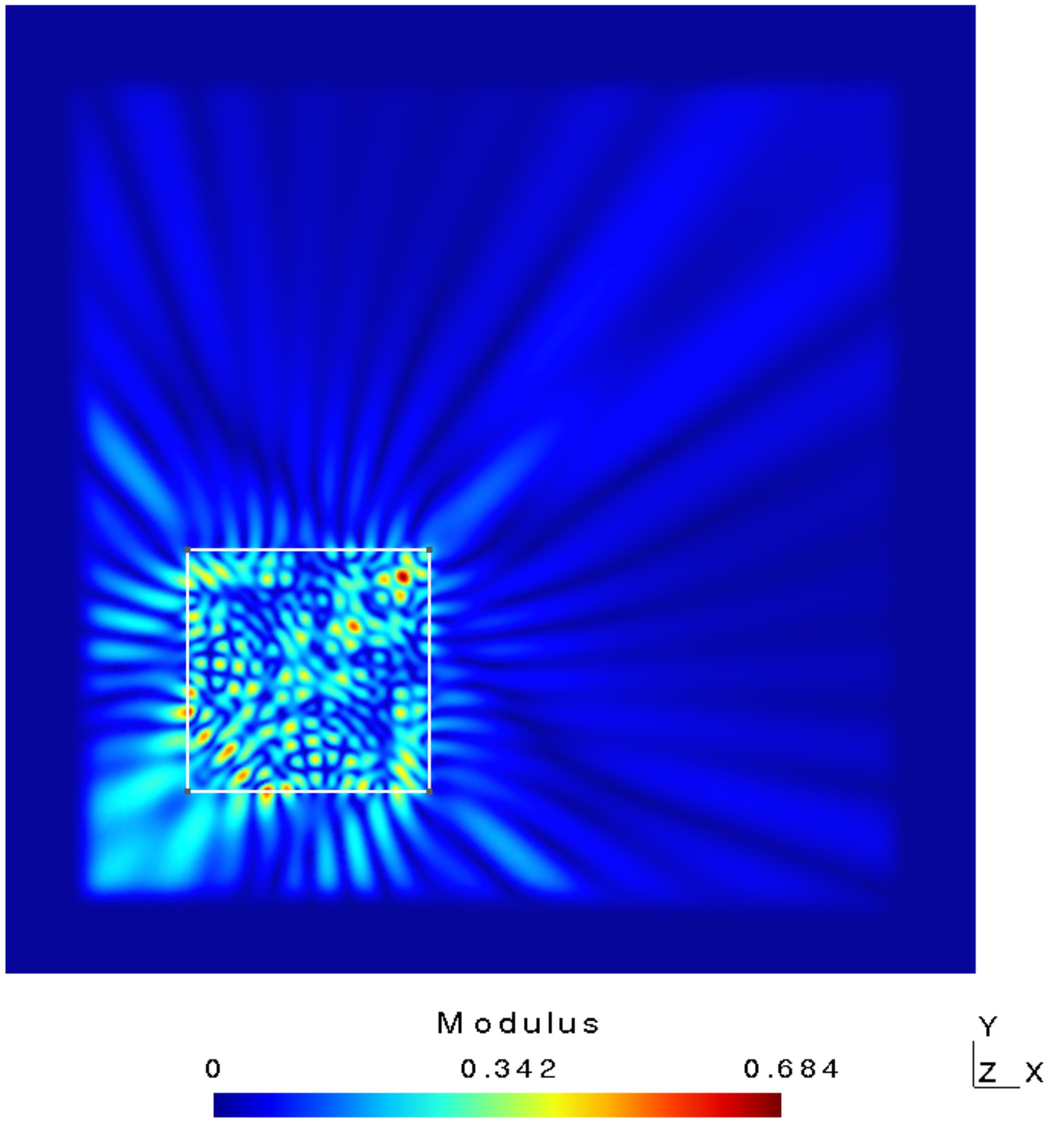
Scattered field $E^{sc} = E^{tot} - E^{inc}$

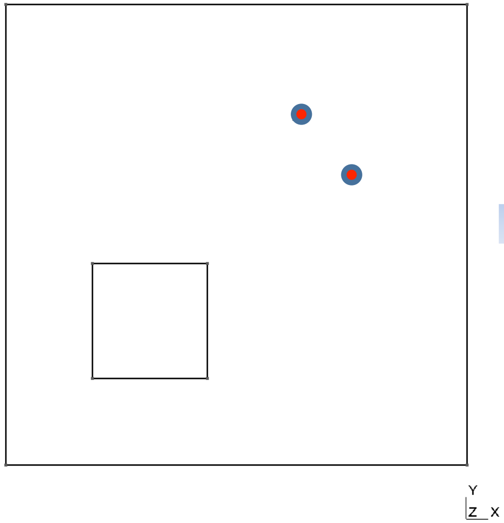


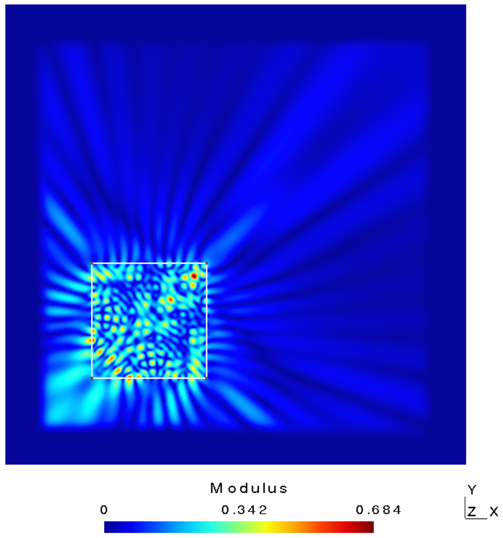
We know

To find

Information about
- Sources
- Objects (position, size, form, physical parameters)

Direct





Scattered field
E^{sc}

We know		To find	

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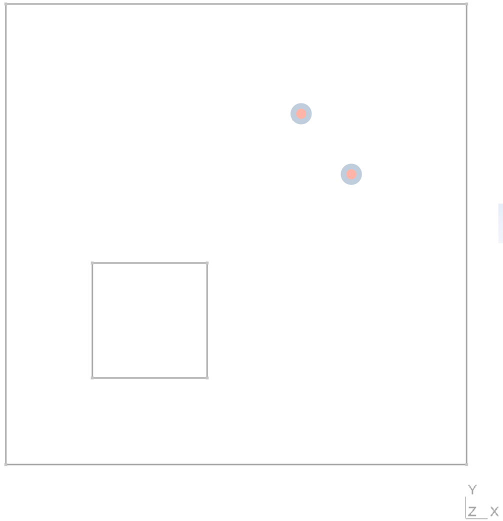
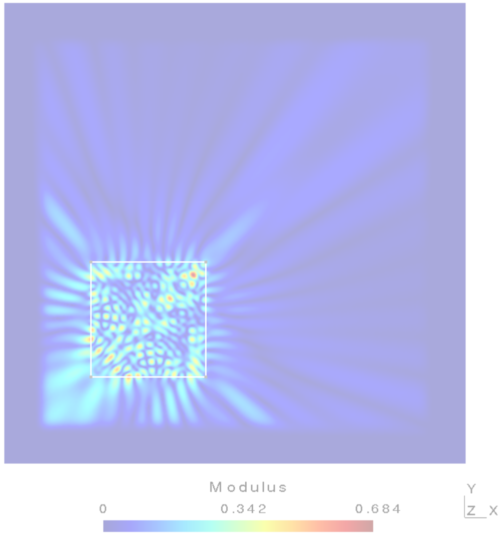


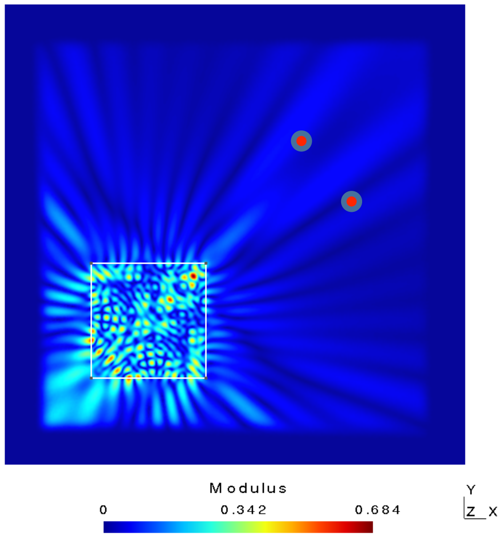


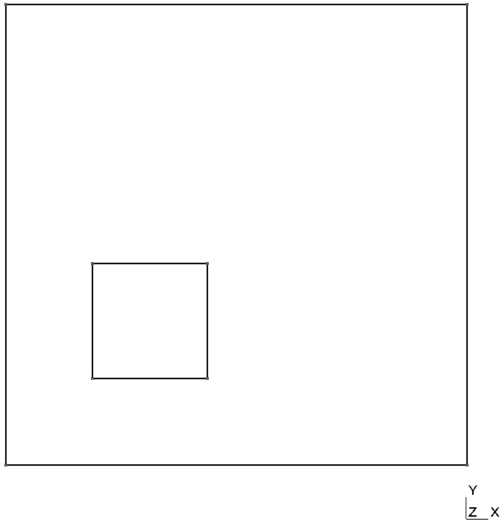
Scattered field E^{sc}

Information about
- Sources
- Scattered field E^{sc}

Inverse

Scattered field E^{sc}





Objects (position, size, form and physical parameters)

2D Helmholtz equation

$$\operatorname{div}\left(\frac{1}{\mu_r^{\text{tot}}}\operatorname{grad}\mathcal{E}^{\text{sc}}\right)+k_0^2\varepsilon_r^{\text{tot}}\mathcal{E}^{\text{sc}}=\mathcal{J}^{\text{sc}}\text{ in }\Omega$$

where:

$$\mathcal{J}^{\text{sc}}=-\operatorname{div}\left(\left[\frac{1}{\mu_r^{\text{tot}}}-\frac{1}{\mu_r^{\text{inc}}}\right]\operatorname{grad}\mathcal{E}^{\text{inc}}\right)-k_0^2\left[\varepsilon_r^{\text{tot}}-\varepsilon_r^{\text{inc}}\right]\mathcal{E}^{\text{inc}}$$

Radiation boundary condition

$$\frac{1}{\mu_r^{\text{tot}}}\frac{\partial\mathcal{E}^{\text{sc}}}{\partial n}-jk_0\mathcal{E}^{\text{sc}}=0\ \text{ on }\Sigma$$

3D Helmholtz equation

$$\nabla\times\left(\frac{1}{\mu_r^{\text{tot}}}\nabla\times\mathcal{E}^{\text{sc}}\right)-k_0^2\varepsilon_r^{\text{tot}}\mathcal{E}^{\text{sc}}=\mathcal{J}^{\text{sc}}\text{ in }\Omega$$

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Radiation boundary condition

$$\vec{n}\times\left(\frac{1}{\mu_r^{\text{tot}}}\nabla\times\mathcal{E}^{\text{sc}}\right)+jk_0\vec{n}\times\vec{n}\times\mathcal{E}^{\text{sc}}=0\text{ on }\Sigma$$

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Finite Element Method

Pros
✓ Well known
✓ Different media possible
Anisotropic
Inhomogeneous
✓ Arbitrary shaped objects

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○ Memory
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Domain Decomposition technique

- Domain Decomposition Method [1]
- FETI method [2]

References		
[1]	Després	1991
[2]	Farhat et al	2001

Finite Element Method

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✓ Well known	
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  - FETI-DPEM2 method [3]

References		
[1]	Després	1991
[2]	Farhat et al	2001
[3]	Li and Jin	2007

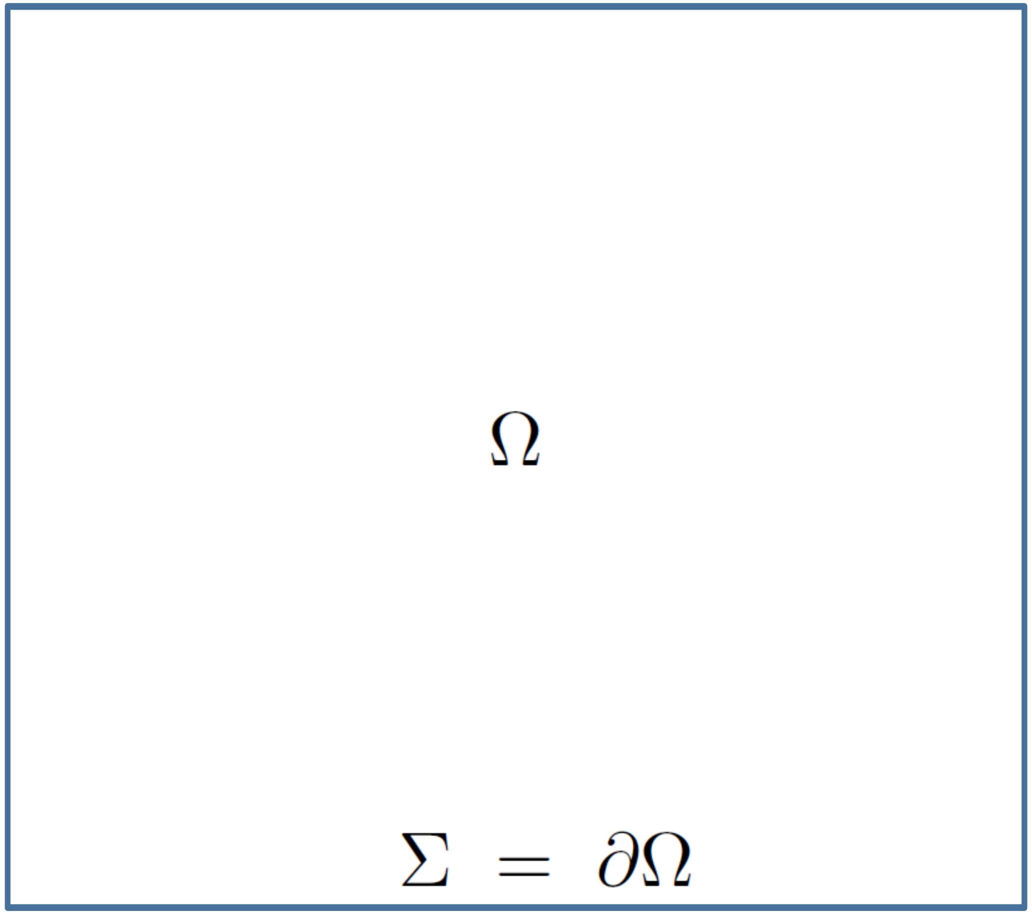
FEM statement of problem in 2D

Helmholtz equation in 2D case

$$-\mathrm{div}\left(\frac{1}{\mu_r}\mathrm{grad}\,\mathcal{E}\right) - k_0^2\varepsilon_r\mathcal{E} \;= jk_0Z_0\mathcal{J} \;\text{ in } \Omega$$

Radiation boundary condition

$$\frac{1}{\mu_r}\frac{\partial\mathcal{E}}{\partial n} - jk_0\mathcal{E} \;= 0 \;\text{ on } \Sigma$$



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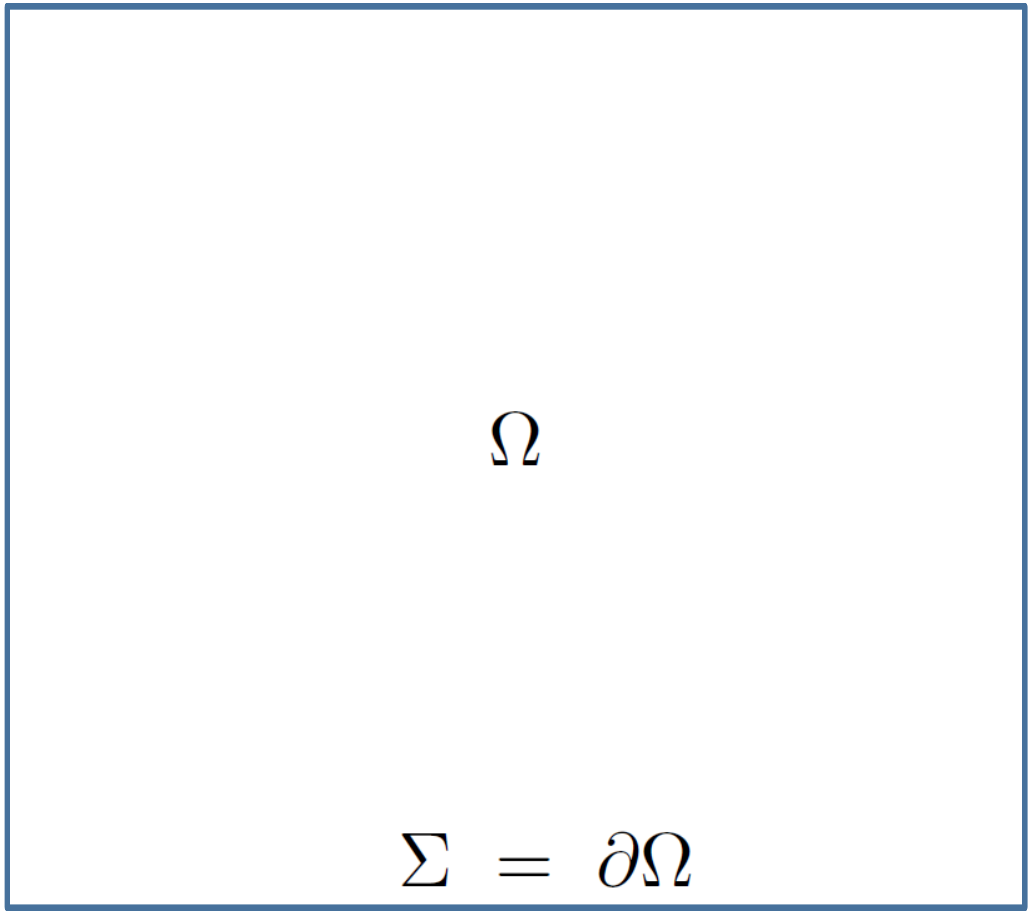
Finite-element discretization

$$\boxed{\mathbf{K}\;\mathbf{E} = \mathbf{f}}$$



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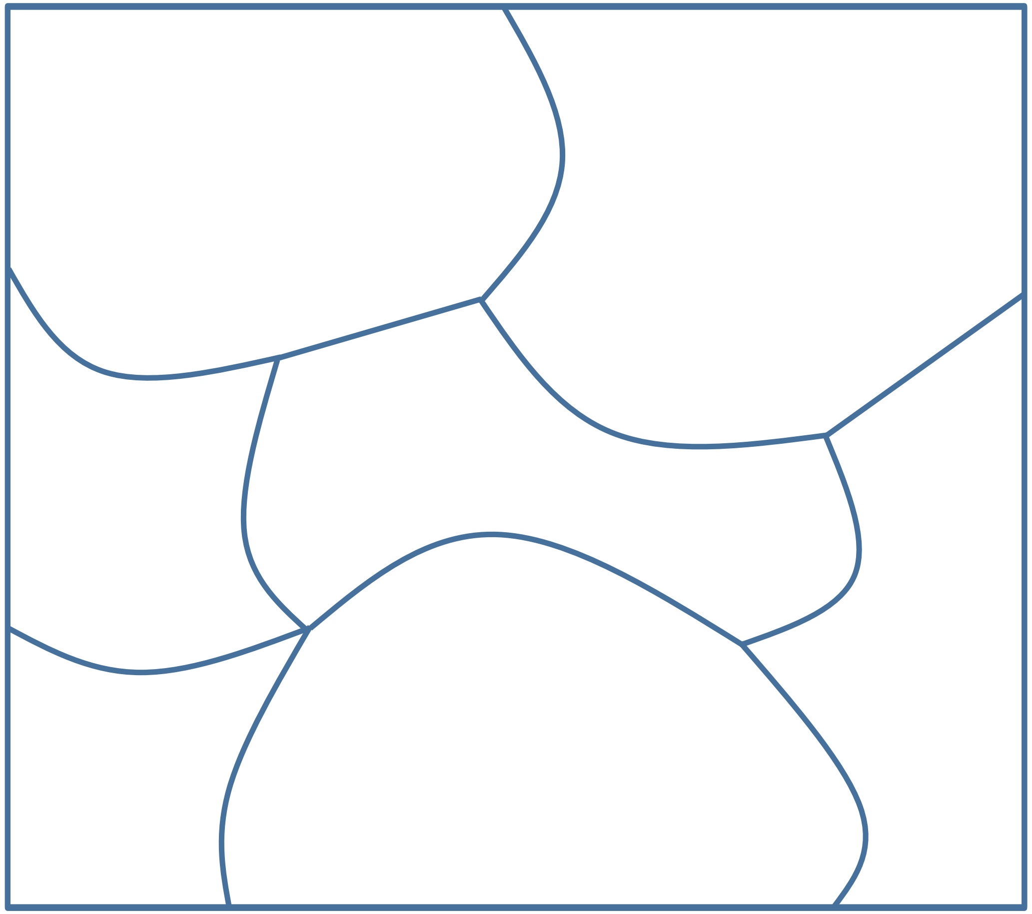

Helmholtz equation in 2D case

$$-\mathrm{div}\left(\frac{1}{\mu_r}\mathrm{grad}\,\mathcal{E}^i\right) - k_0^2\varepsilon_r\mathcal{E}^i = jk_0 Z_0 \mathcal{J}^i \text{ in } \Omega^i$$

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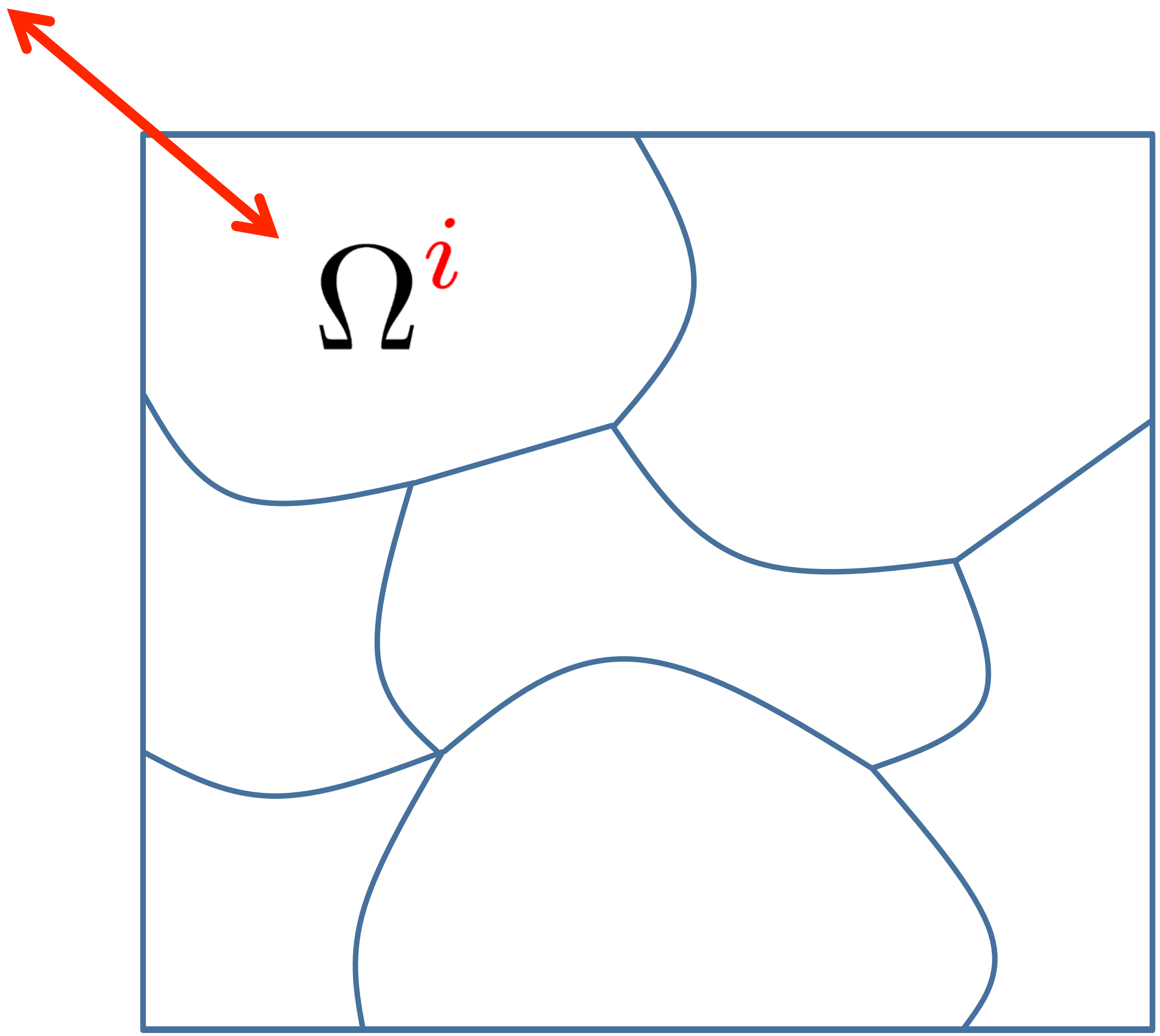

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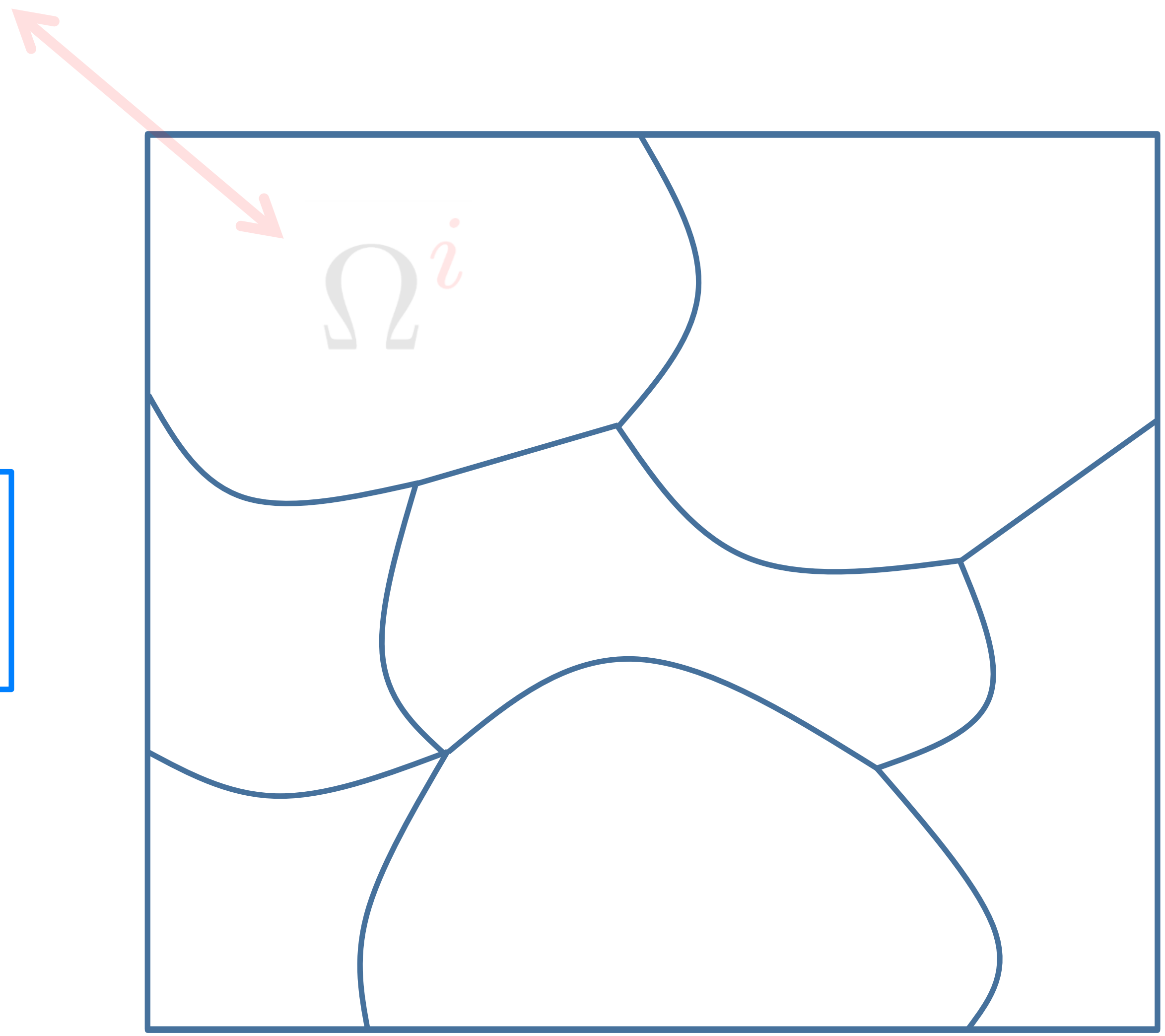

Finite-element discretization

$$\mathbf{K}\ \mathbf{E} = \mathbf{f}$$

$$K^i\ E^i = f^i - \int_{\Gamma^i} \frac{1}{\mu_r}\frac{\partial E^i}{\partial n}\,\Psi\ d\Gamma$$

Helmholtz equation in 2D case

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« **c** » - corner point

« **r** » - interface points - internal points

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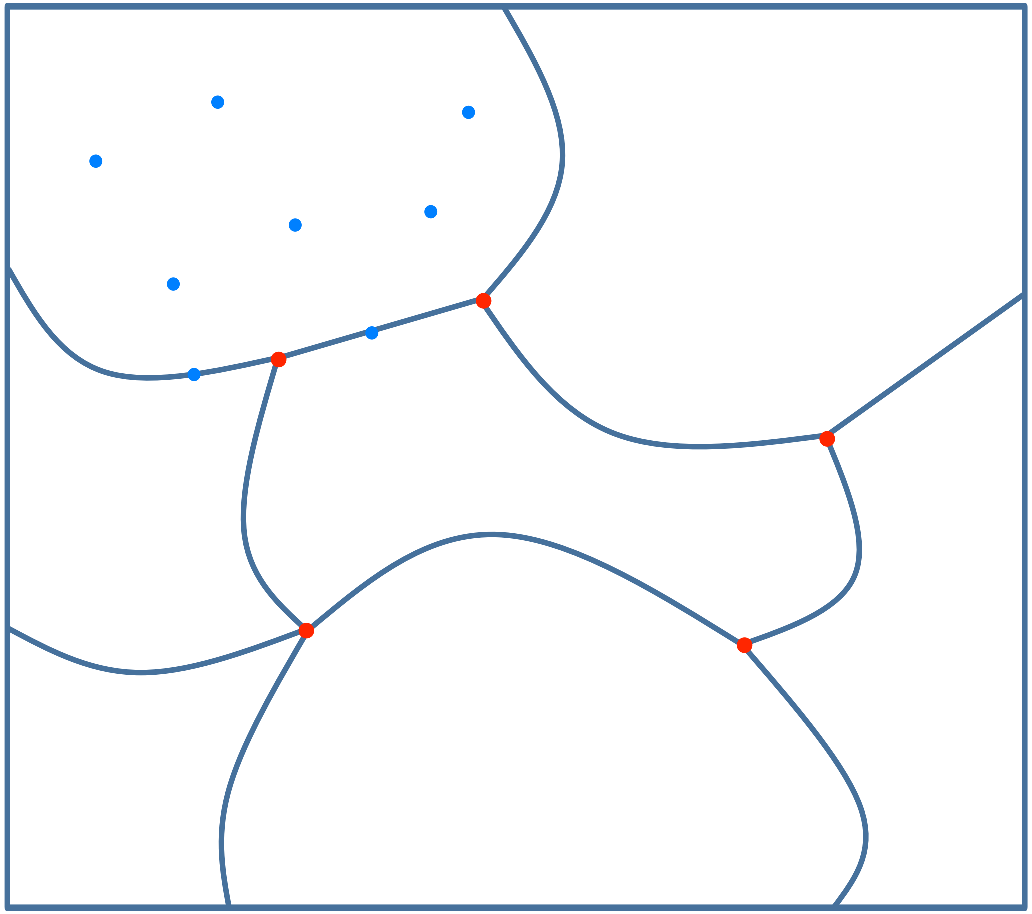

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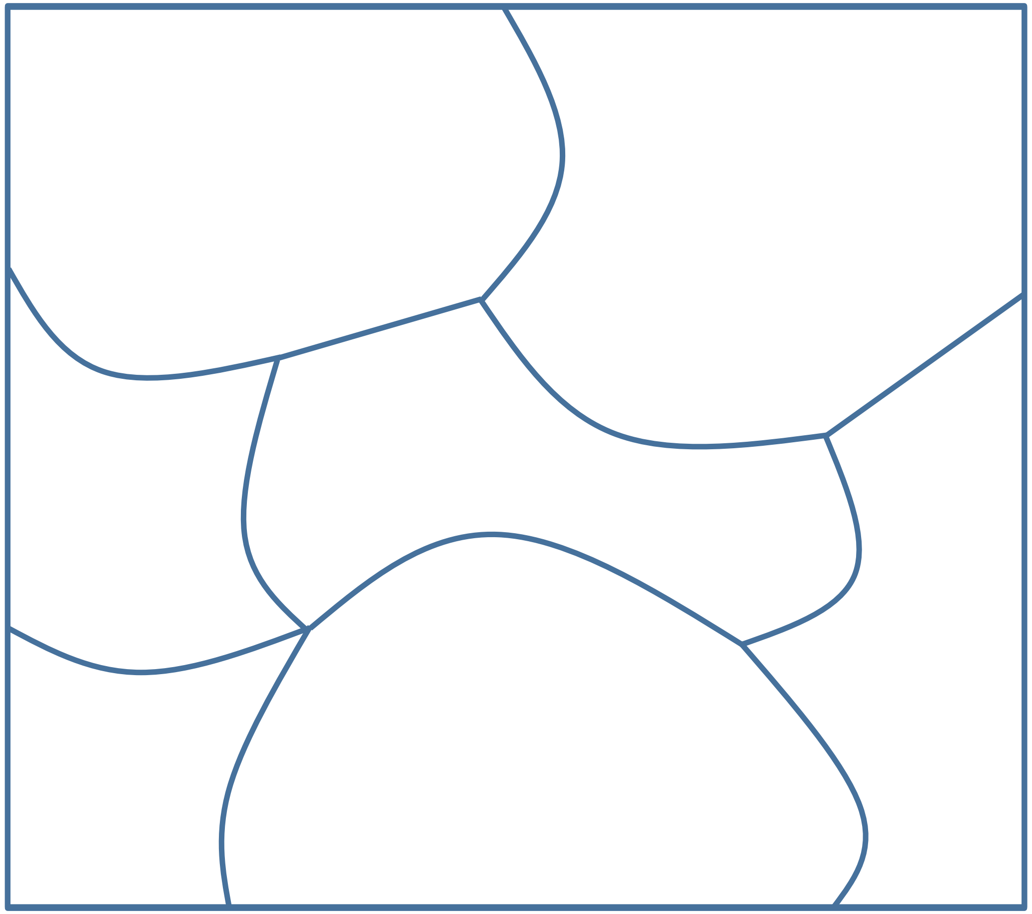
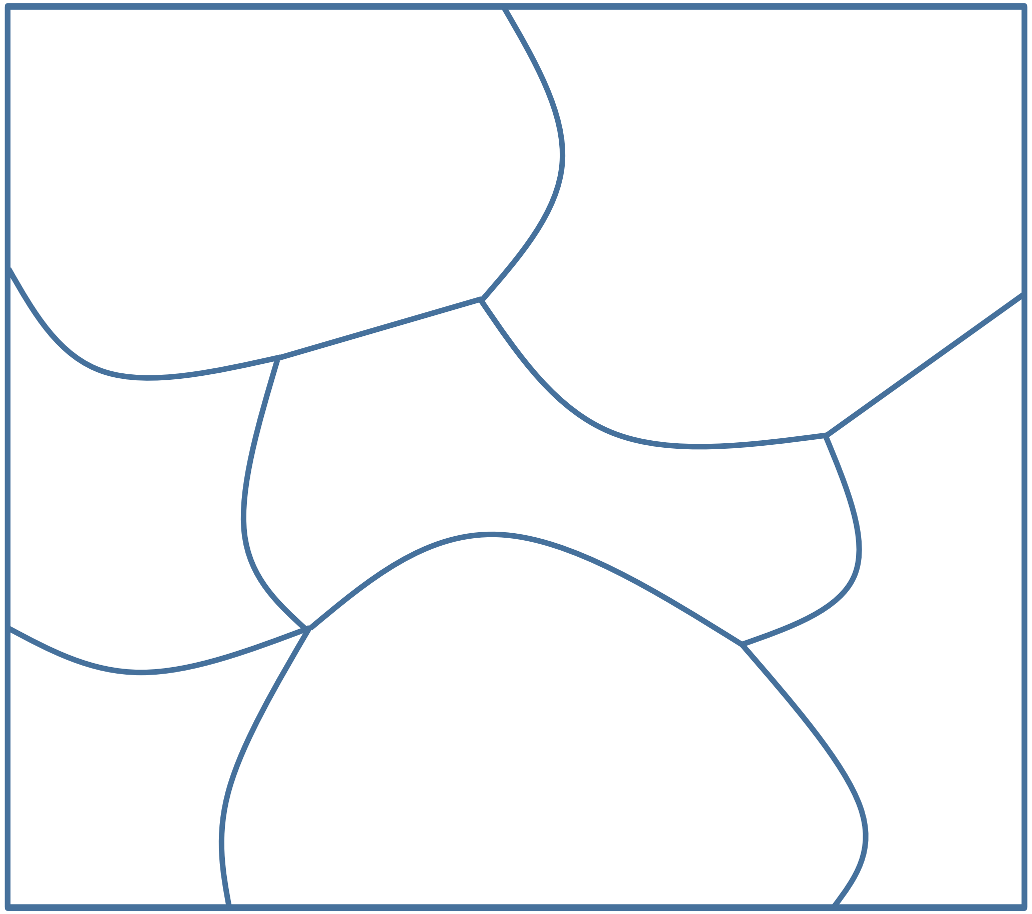
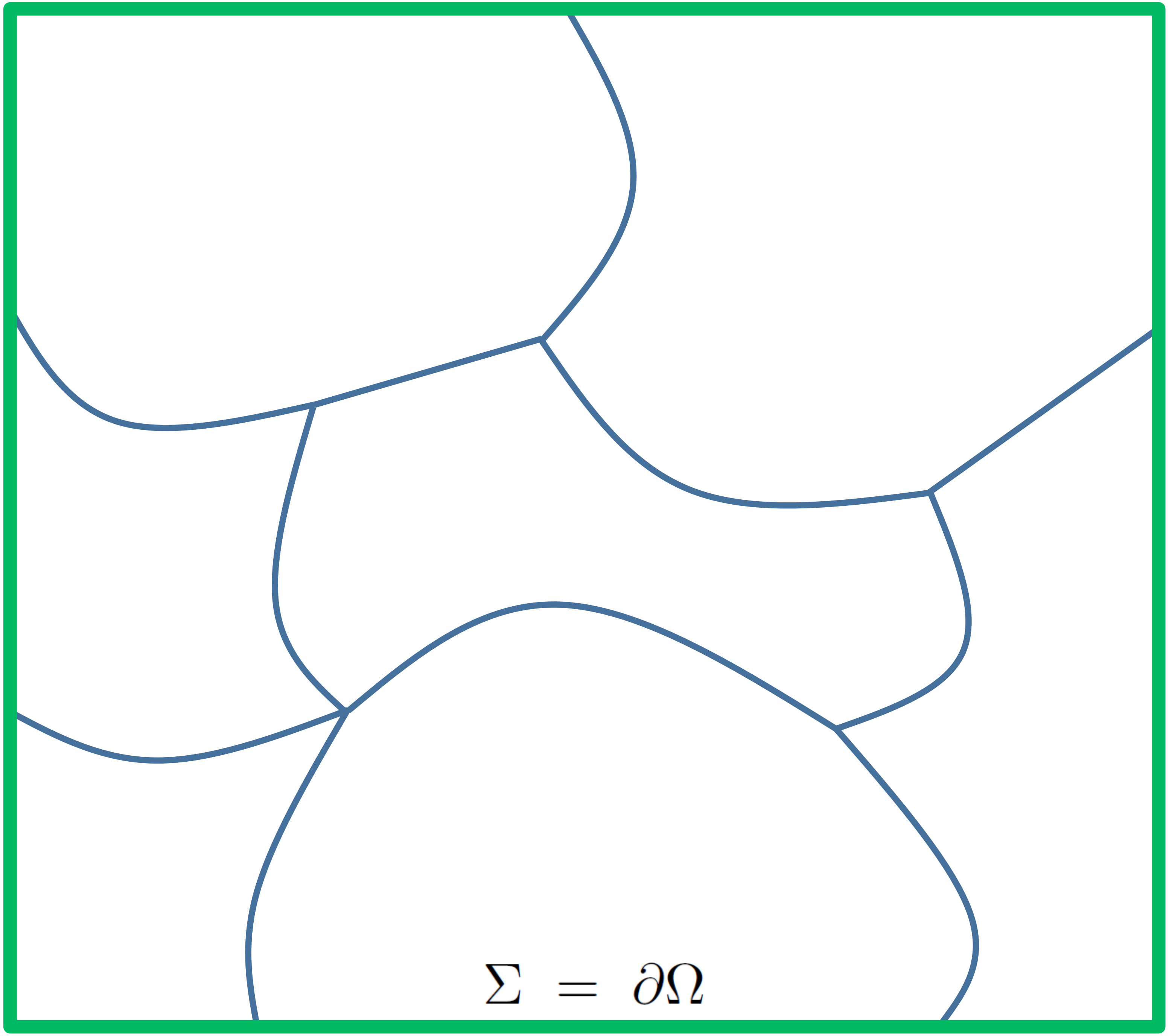
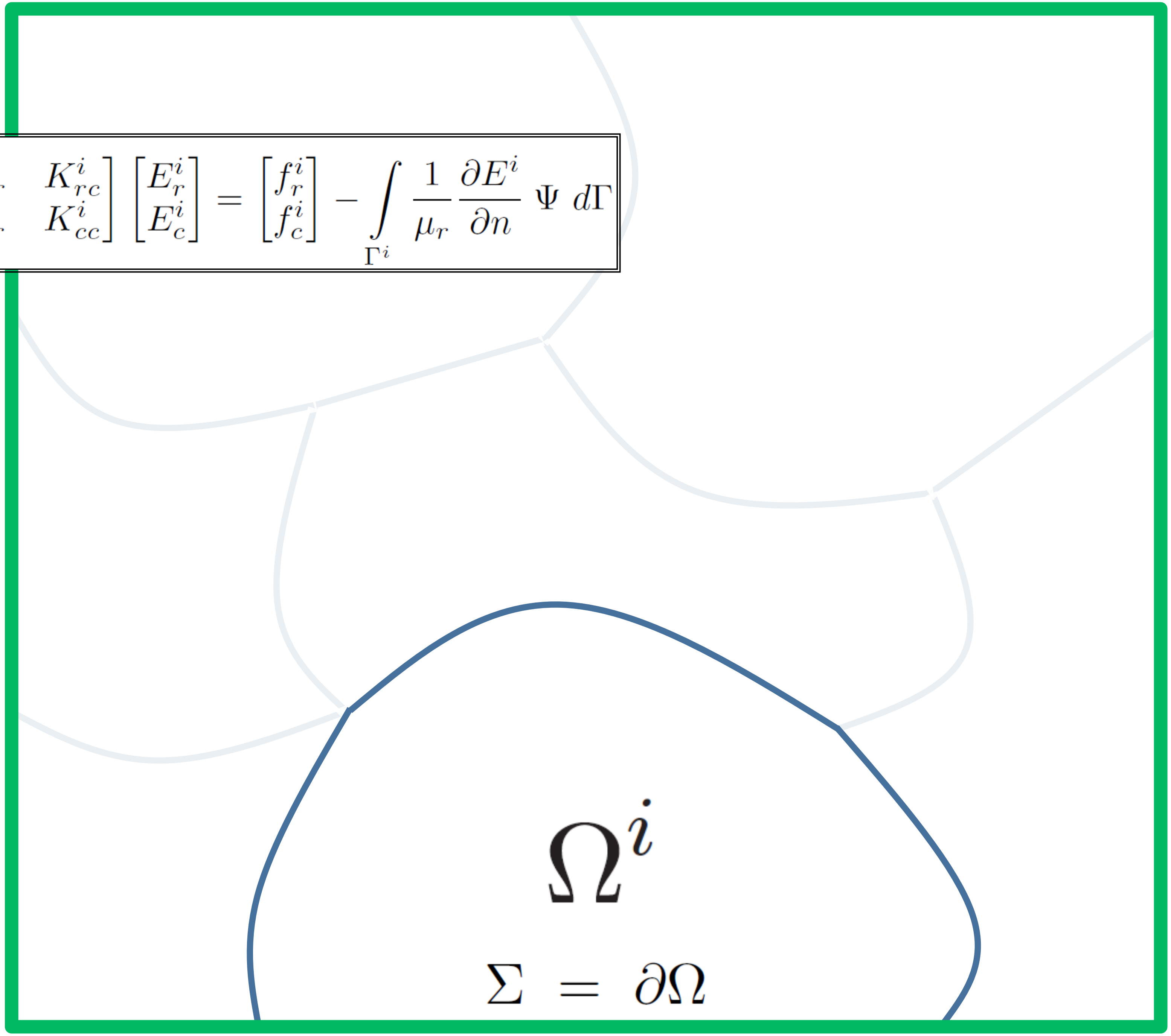
$$\begin{bmatrix} K_{rr}^i & K_{rc}^i \\ K_{cr}^i & K_{cc}^i \end{bmatrix} \begin{bmatrix} E_r^i \\ E_c^i \end{bmatrix} = \begin{bmatrix} f_r^i \\ f_c^i \end{bmatrix} - \int_{\Gamma^i} \frac{1}{\mu_r}\frac{\partial E^i}{\partial n}\Psi\ d\Gamma$$






$\Sigma = \partial\Omega$

$$\begin{bmatrix} K^i_{rr} & K^i_{rc} \\ K^i_{cr} & K^i_{cc} \end{bmatrix} \begin{bmatrix} E^i_r \\ E^i_c \end{bmatrix} = \begin{bmatrix} f^i_r \\ f^i_c \end{bmatrix} - \int_{\Gamma^i} \frac{1}{\mu_r} \frac{\partial E^i}{\partial n} \Psi \, d\Gamma$$



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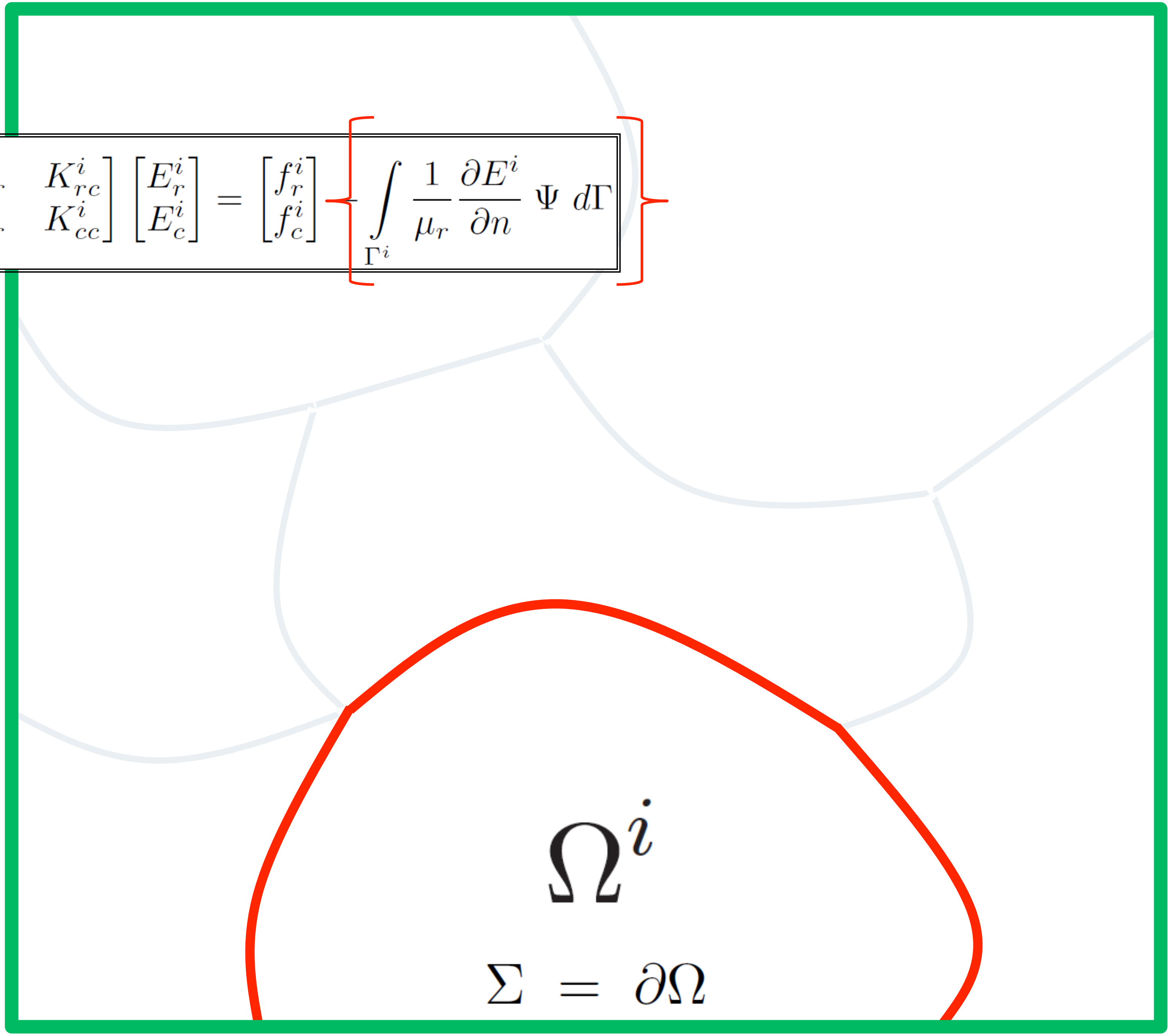

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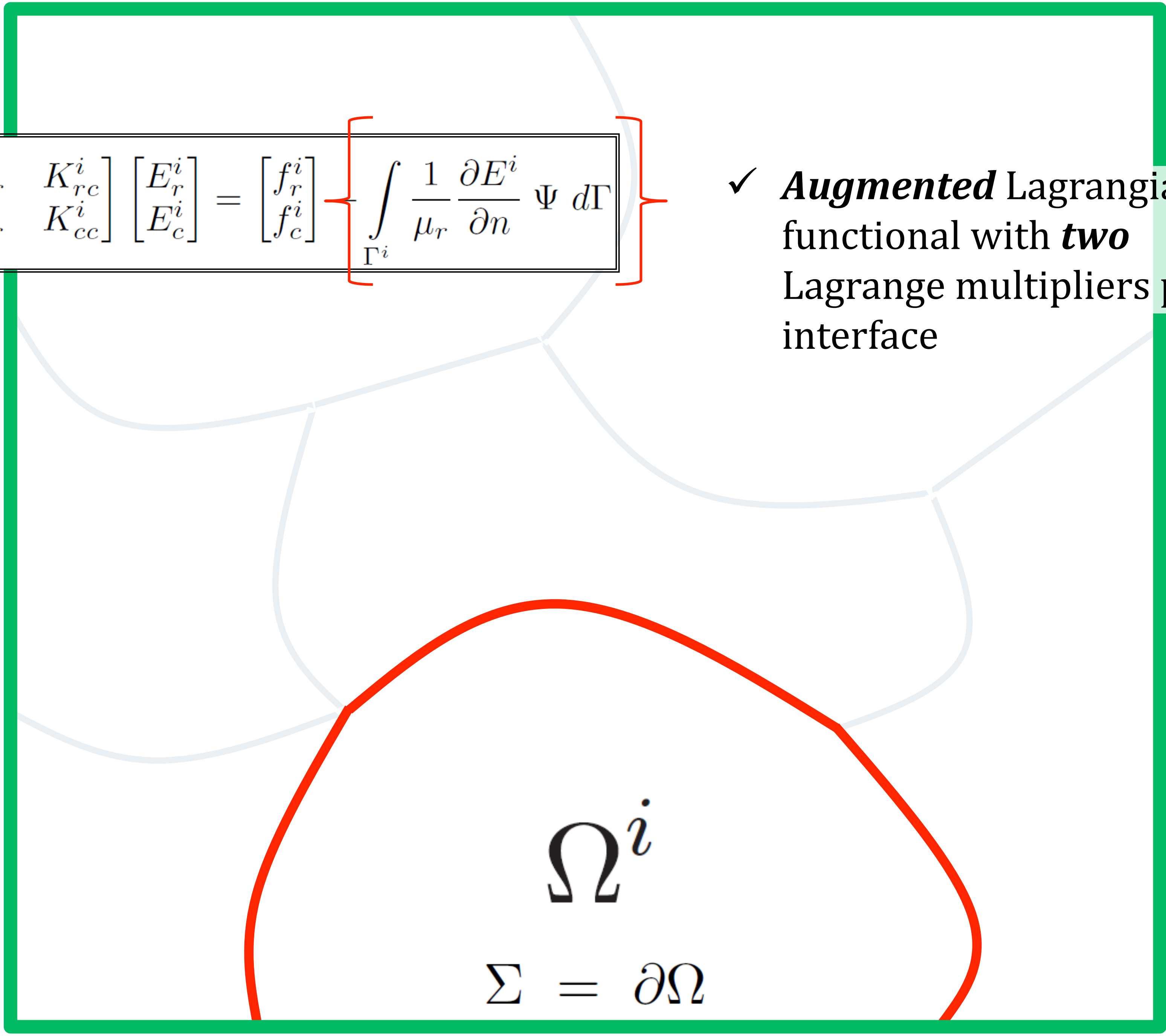


- ✓ ***Augmented*** Lagrangian functional with ***two*** Lagrange multipliers per interface

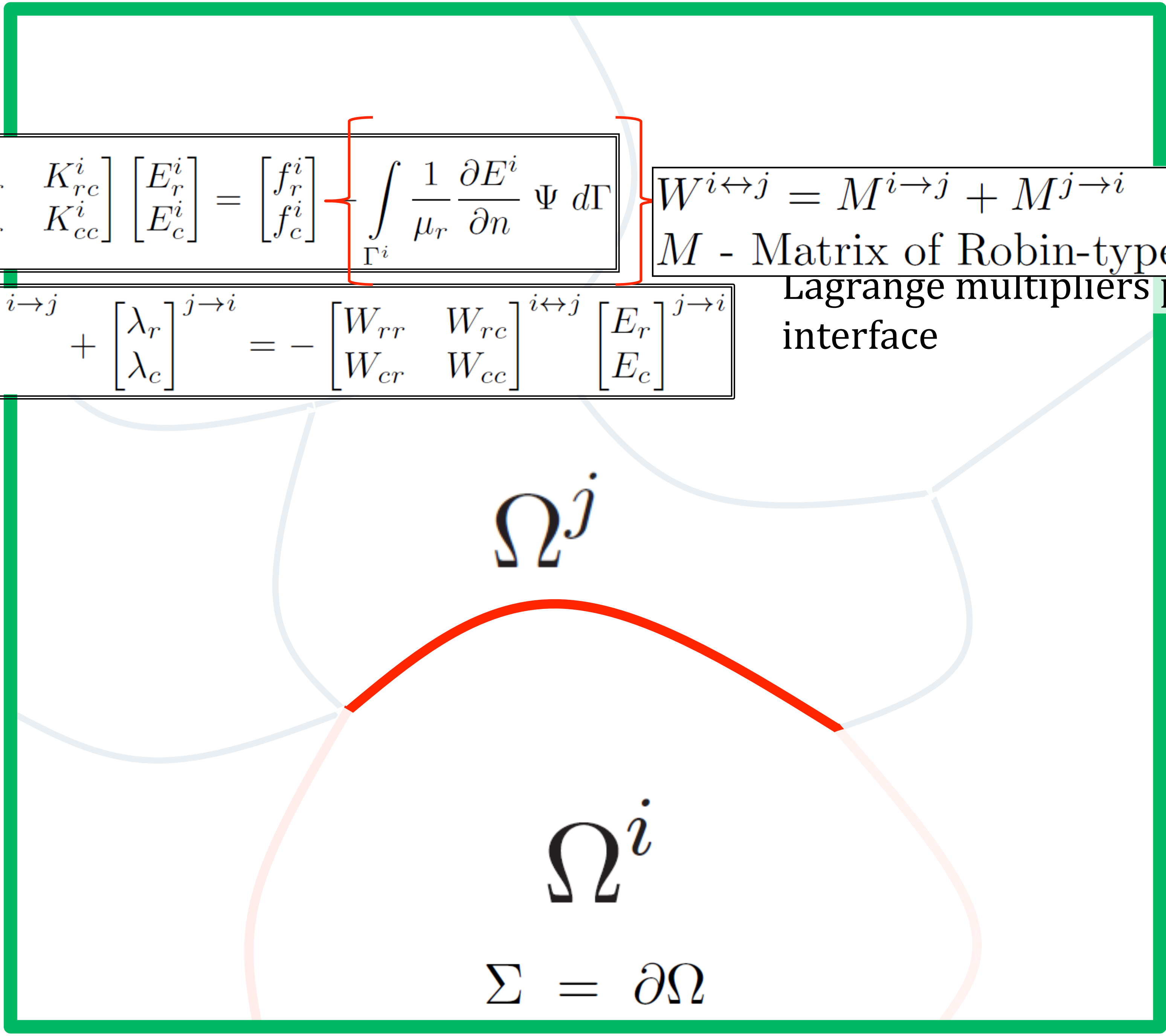
$$\begin{bmatrix} K_{rr}^i & K_{rc}^i \\ K_{cr}^i & K_{cc}^i \end{bmatrix} \begin{bmatrix} E_r^i \\ E_c^i \end{bmatrix} = \begin{bmatrix} f_r^i \\ f_c^i \end{bmatrix} - \left\{ \int_{\Gamma^i} \frac{1}{\mu_r} \frac{\partial E^i}{\partial n} \Psi \, d\Gamma \right\}$$

$$W^{i \leftrightarrow j} = M^{i \rightarrow j} + M^{j \rightarrow i}$$

M - Matrix of Robin-type B.C.

$$\begin{bmatrix} \lambda_r \\ \lambda_c \end{bmatrix}^{i \rightarrow j} + \begin{bmatrix} \lambda_r \\ \lambda_c \end{bmatrix}^{j \rightarrow i} = - \begin{bmatrix} W_{rr} & W_{rc} \\ W_{cr} & W_{cc} \end{bmatrix}^{i \leftrightarrow j} \begin{bmatrix} E_r \\ E_c \end{bmatrix}^{j \rightarrow i}$$

Lagrange multipliers per interface



Karush Kuhn Tucker conditions:

$$\begin{bmatrix} K_{rr}^i & K_{rc}^i \\ K_{cr}^i & K_{cc}^i \end{bmatrix} \begin{bmatrix} E_r^i \\ E_c^i \end{bmatrix} = \begin{bmatrix} f_r^i \\ f_c^i \end{bmatrix} - \begin{bmatrix} \lambda_r^i \\ \lambda_c^i \end{bmatrix}$$

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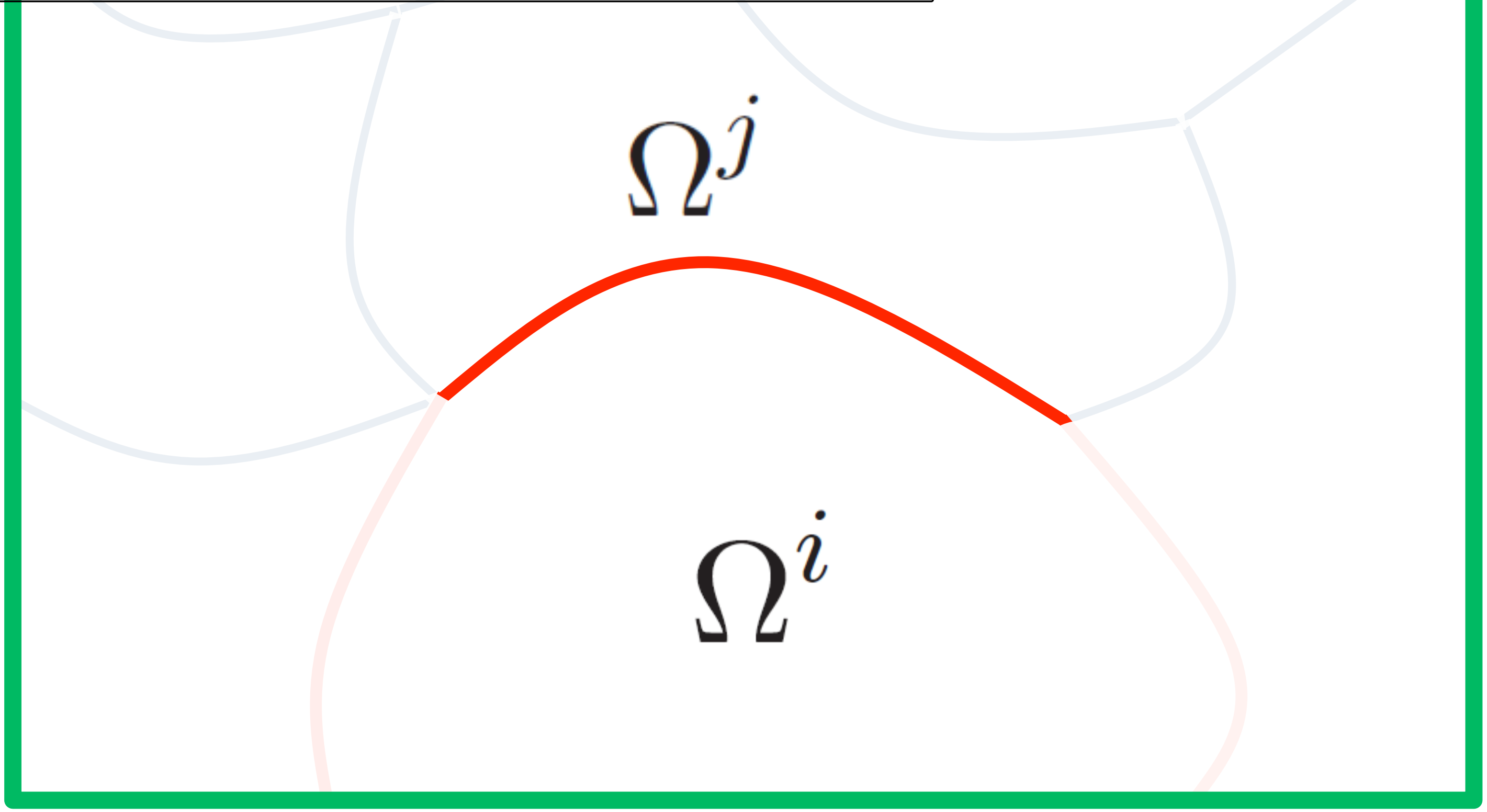

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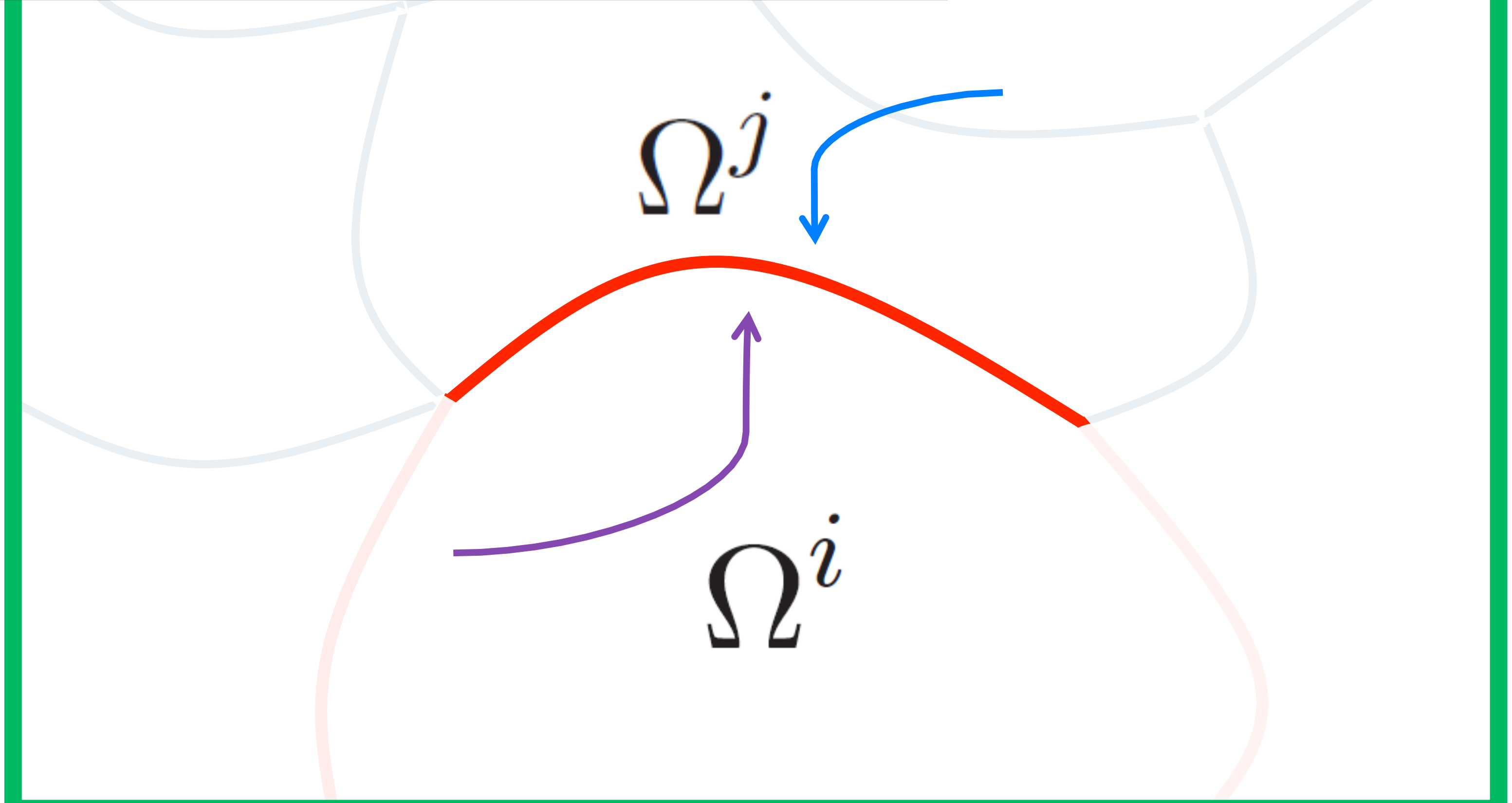
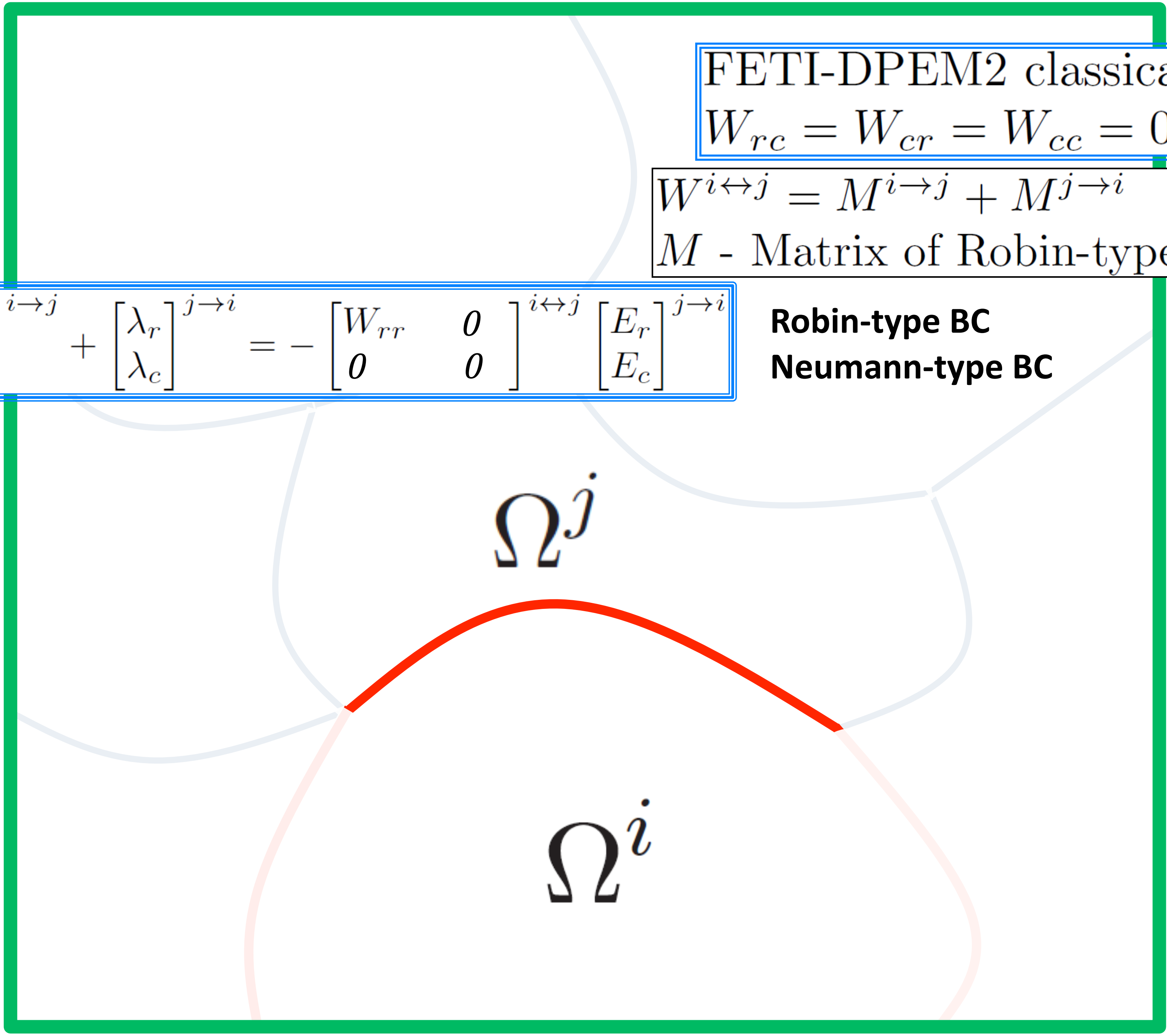


FETI-DPEM2 classical:
$W_{rc} = W_{cr} = W_{cc} = 0$

$W^{i\leftrightarrow j} = M^{i\rightarrow j} + M^{j\rightarrow i}$
M - Matrix of Robin-type B.C.

$$\begin{bmatrix}\lambda_r \\ \lambda_c\end{bmatrix}^{i\rightarrow j} + \begin{bmatrix}\lambda_r \\ \lambda_c\end{bmatrix}^{j\rightarrow i} = -\begin{bmatrix}W_{rr} & 0 \\ 0 & 0\end{bmatrix}^{i\leftrightarrow j}\begin{bmatrix}E_r \\ E_c\end{bmatrix}^{j\rightarrow i}$$

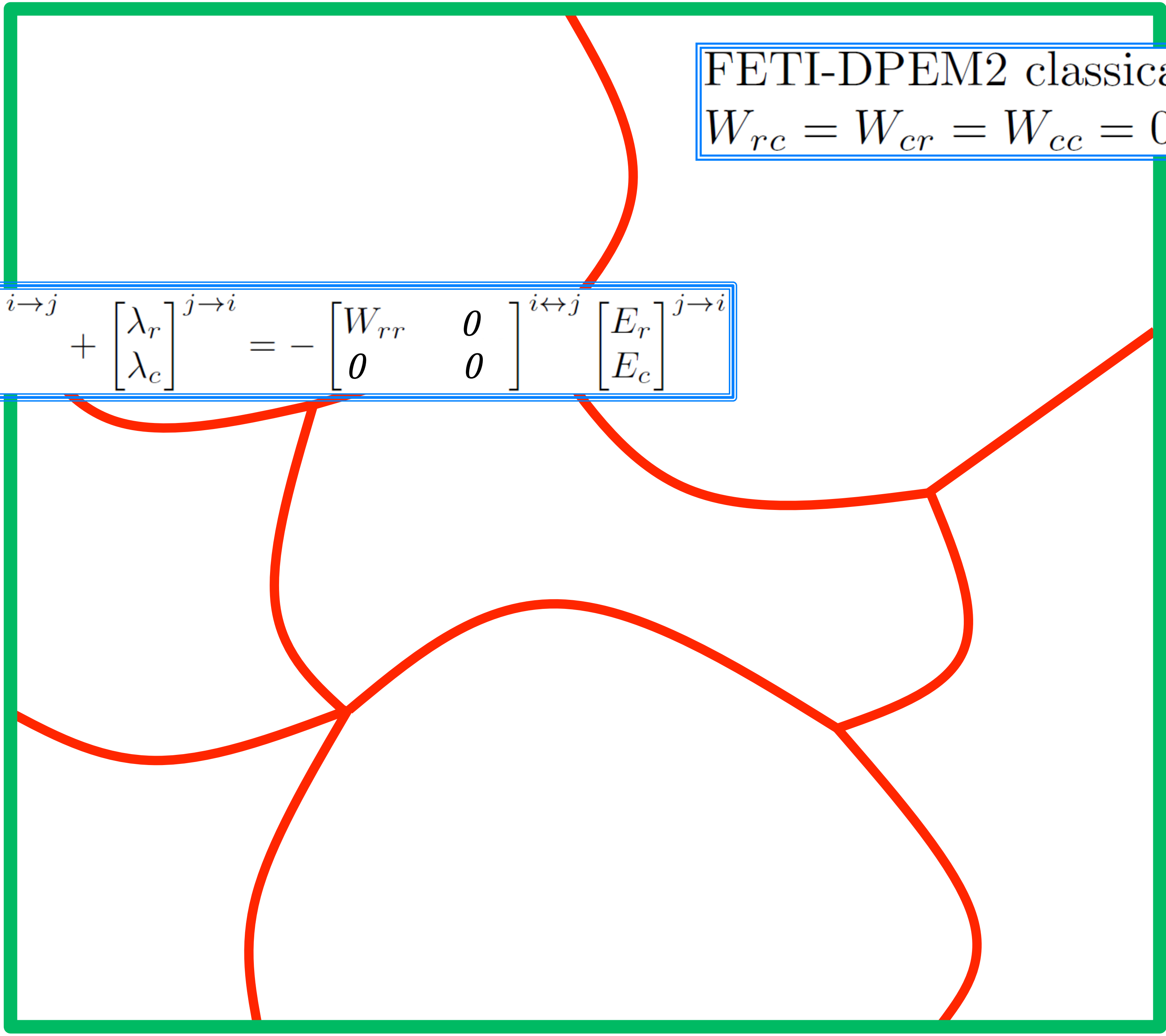
Robin-type BC
Neumann-type BC

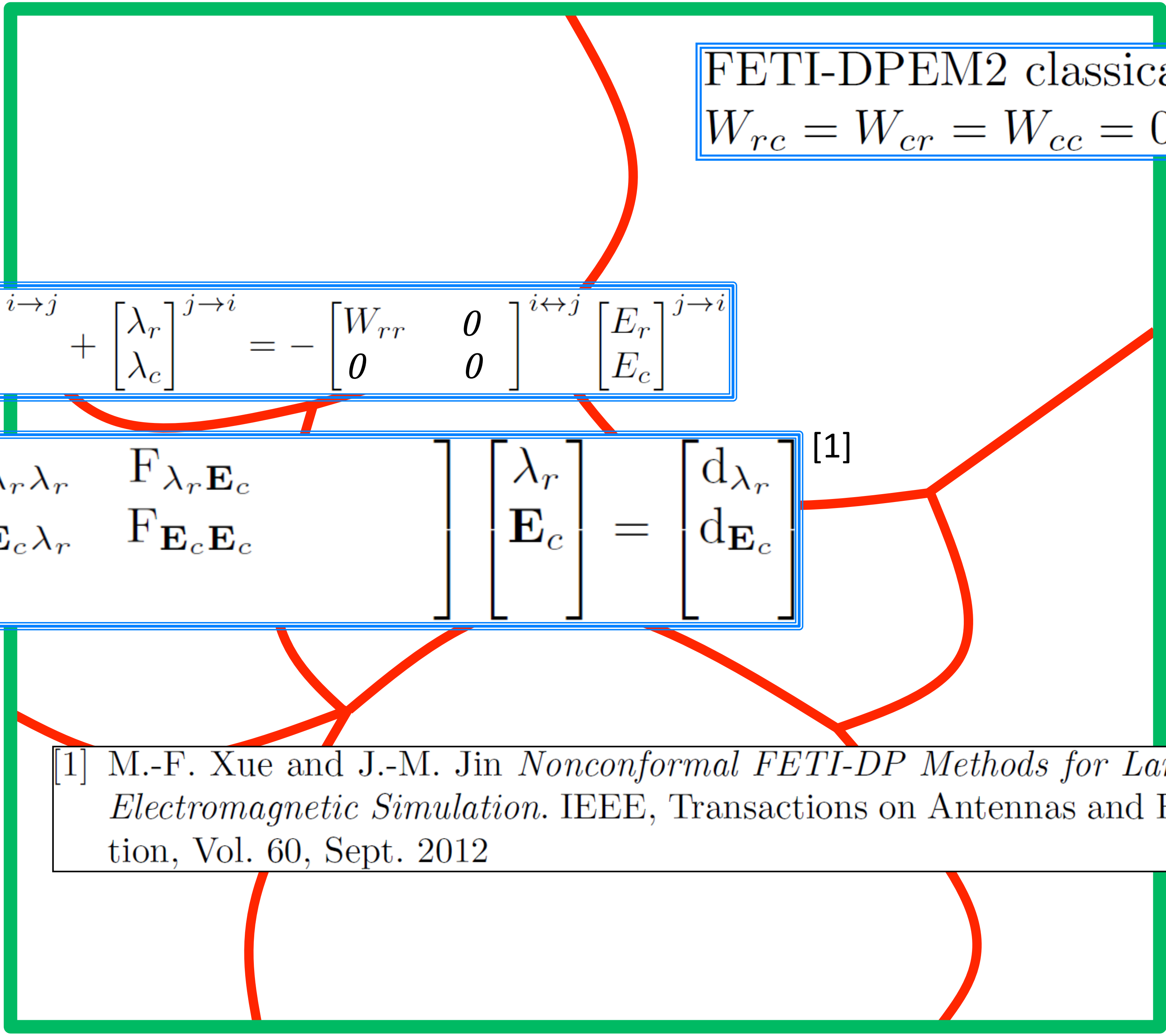


FETI-DPEM2 classical:

$$W_{rc} = W_{cr} = W_{cc} = 0$$

$$\begin{bmatrix}\lambda_r\\ \lambda_c\end{bmatrix}^{i\to j} + \begin{bmatrix}\lambda_r\\ \lambda_c\end{bmatrix}^{j\to i} = -\begin{bmatrix}W_{rr} & 0\\ 0 & 0\end{bmatrix}^{i\leftrightarrow j}\begin{bmatrix}E_r\\ E_c\end{bmatrix}^{j\to i}$$





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$$\begin{bmatrix} \lambda_r \\ \lambda_c \end{bmatrix}^{i\to j} + \begin{bmatrix} \lambda_r \\ \lambda_c \end{bmatrix}^{j\to i} = - \begin{bmatrix} W_{rr} & 0 \\ 0 & 0 \end{bmatrix}^{i\leftrightarrow j} \begin{bmatrix} E_r \\ E_c \end{bmatrix}^{j\to i}$$

$$\begin{bmatrix} \mathrm{F}_{\lambda_r\lambda_r} & \mathrm{F}_{\lambda_r\mathbf{E}_c} \\ \mathrm{F}_{\mathbf{E}_c\lambda_r} & \mathrm{F}_{\mathbf{E}_c\mathbf{E}_c} \end{bmatrix} \begin{bmatrix} \lambda_r \\ \mathbf{E}_c \end{bmatrix} = \begin{bmatrix} \mathrm{d}_{\lambda_r} \\ \mathrm{d}_{\mathbf{E}_c} \end{bmatrix}$$ [1]

[1] M.-F. Xue and J.-M. Jin *Nonconformal FETI-DP Methods for Large-Scale Electromagnetic Simulation*. IEEE, Transactions on Antennas and Propagation, Vol. 60, Sept. 2012

FETI-DPEM2 modificated:
$(W_{rc}, W_{cr}, W_{cc}) \neq 0$

FETI-DPEM2 classical:
$W_{rc} = W_{cr} = W_{cc} = 0$

$$\begin{bmatrix} \lambda_r \\ \lambda_c \end{bmatrix}^{i\to j} + \begin{bmatrix} \lambda_r \\ \lambda_c \end{bmatrix}^{j\to i} = -\begin{bmatrix} W_{rr} & 0 \\ 0 & 0 \end{bmatrix}^{i\leftrightarrow j} \begin{bmatrix} E_r \\ E_c \end{bmatrix}^{j\to i}$$ [2]

Robin-type BC
Robin-type BC

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[2] I. Voznyuk, H. Tortel and A. Litman *Scattered field computation with an extended FETI-DPEM2 method*. Progress In Electromagnetics Research, 2013 (to appear)

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$$\begin{bmatrix} \mathrm{F}_{\lambda_r\lambda_r} & \mathrm{F}_{\lambda_r\mathbf{E}_c} \\ \mathrm{F}_{\mathbf{E}_c\lambda_r} & \mathrm{F}_{\mathbf{E}_c\mathbf{E}_c} \end{bmatrix} \begin{bmatrix} \lambda_r \\ \mathbf{E}_c \end{bmatrix} = \begin{bmatrix} \mathrm{d}_{\lambda_r} \\ \mathrm{d}_{\mathbf{E}_c} \end{bmatrix}$$ [1] [2]

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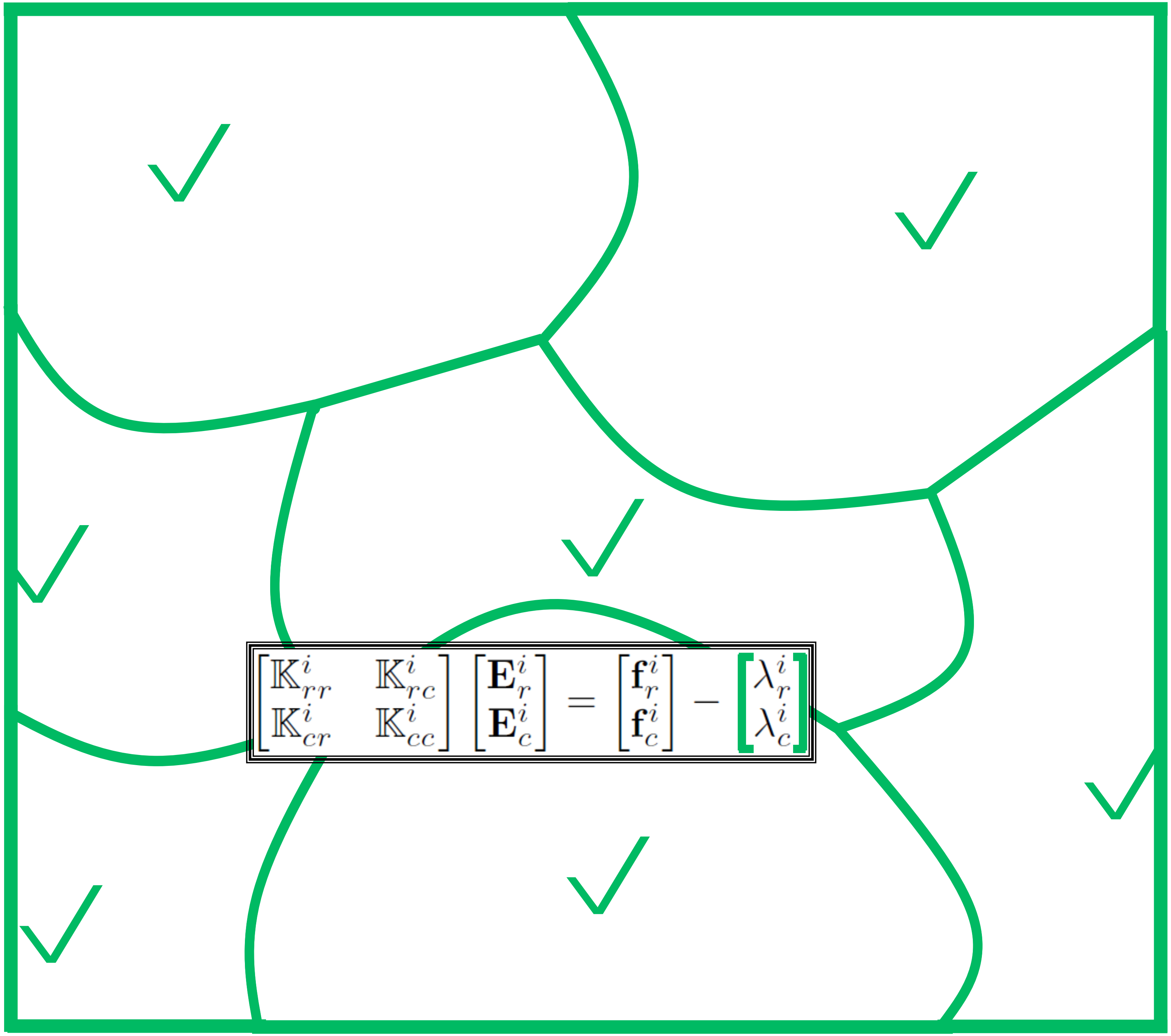
$$\begin{bmatrix}\mathrm{F}_{\lambda_r\lambda_r} & \mathrm{F}_{\lambda_r\mathbf{E}_c} & 0\\ \mathrm{F}_{\mathbf{E}_c\lambda_r} & \mathrm{F}_{\mathbf{E}_c\mathbf{E}_c} & \mathrm{F}_{\mathbf{E}_c\lambda_c}\\ \mathrm{F}_{\lambda_c\lambda_r} & \mathrm{F}_{\lambda_c\mathbf{E}_c} & \mathrm{F}_{\lambda_c\lambda_c}\end{bmatrix}\begin{bmatrix}\lambda_r\\ \mathbf{E}_c\\ \lambda_c\end{bmatrix} = \begin{bmatrix}\mathrm{d}_{\lambda_r}\\ \mathrm{d}_{\mathbf{E}_c}\\ \mathrm{d}_{\lambda_c}\end{bmatrix}$$ [2]

$$\begin{bmatrix}\mathbb{K}^i_{rr} & \mathbb{K}^i_{rc}\\ \mathbb{K}^i_{cr} & \mathbb{K}^i_{cc}\end{bmatrix}\begin{bmatrix}\mathbf{E}^i_r\\ \mathbf{E}^i_c\end{bmatrix} = \begin{bmatrix}\mathbf{f}^i_r\\ \mathbf{f}^i_c\end{bmatrix} - \begin{bmatrix}\lambda^i_r\\ \lambda^i_c\end{bmatrix}$$

[2] I. Voznyuk, H. Tortel and A. Litman *Scattered field computation with an extended FETI-DPEM2 method*. Progress In Electromagnetics Research, 2013 (to appear)



$$\begin{bmatrix} \mathbb{K}_{rr}^i & \mathbb{K}_{rc}^i \\ \mathbb{K}_{cr}^i & \mathbb{K}_{cc}^i \end{bmatrix} \begin{bmatrix} \mathbf{E}_r^i \\ \mathbf{E}_c^i \end{bmatrix} = \begin{bmatrix} \mathbf{f}_r^i \\ \mathbf{f}_c^i \end{bmatrix} - \begin{bmatrix} \lambda_r^i \\ \lambda_c^i \end{bmatrix}$$

Inside air

$\varepsilon_0 = 8.85 \cdot 10^{-12}\mathrm{F} \cdot \mathrm{m}^{-1}$

$\mu_0 = 4\pi \cdot 10^{-7}\mathrm{H} \cdot \mathrm{m}^{-1}$

$f_1 = f_2 = 800$ MHz

The wavelength $\lambda \approx 0.37$ m

Domain of $\approx 21\lambda \times 21\lambda$

Excitation

Sources located at $(1.5, 2.5, 0)$

$(2.5, 1.5, 0)$

Scatterers

3 squares

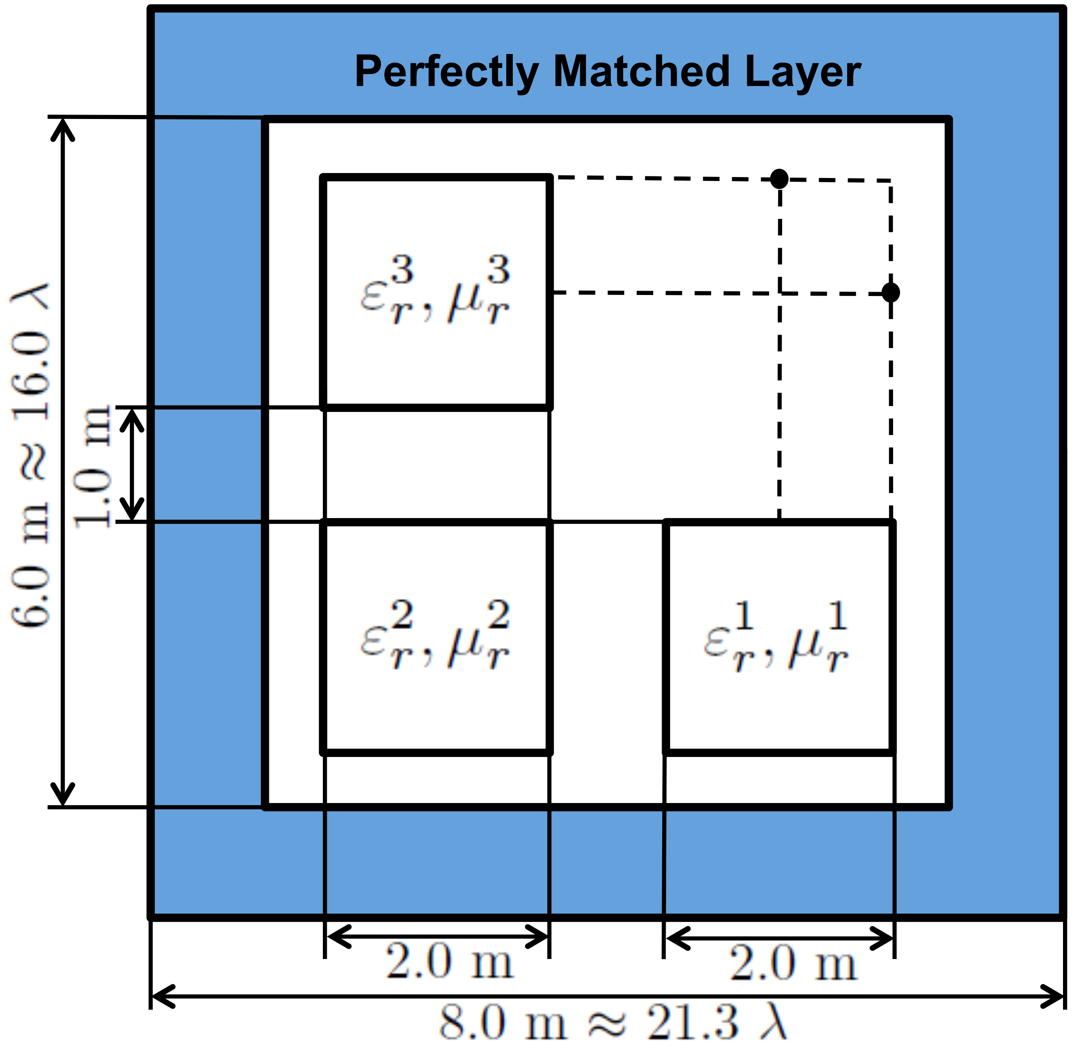
$\varepsilon_r^1 = 1.5$

$\varepsilon_r^2 = 3.0$

$\varepsilon_r^3 = 5.0$

$\mu_1 = \mu_2 = \mu_3 = 1.0$

$a_1 = a_2 = a_3 = 2\,\mathrm{m} \approx 5.4\lambda$

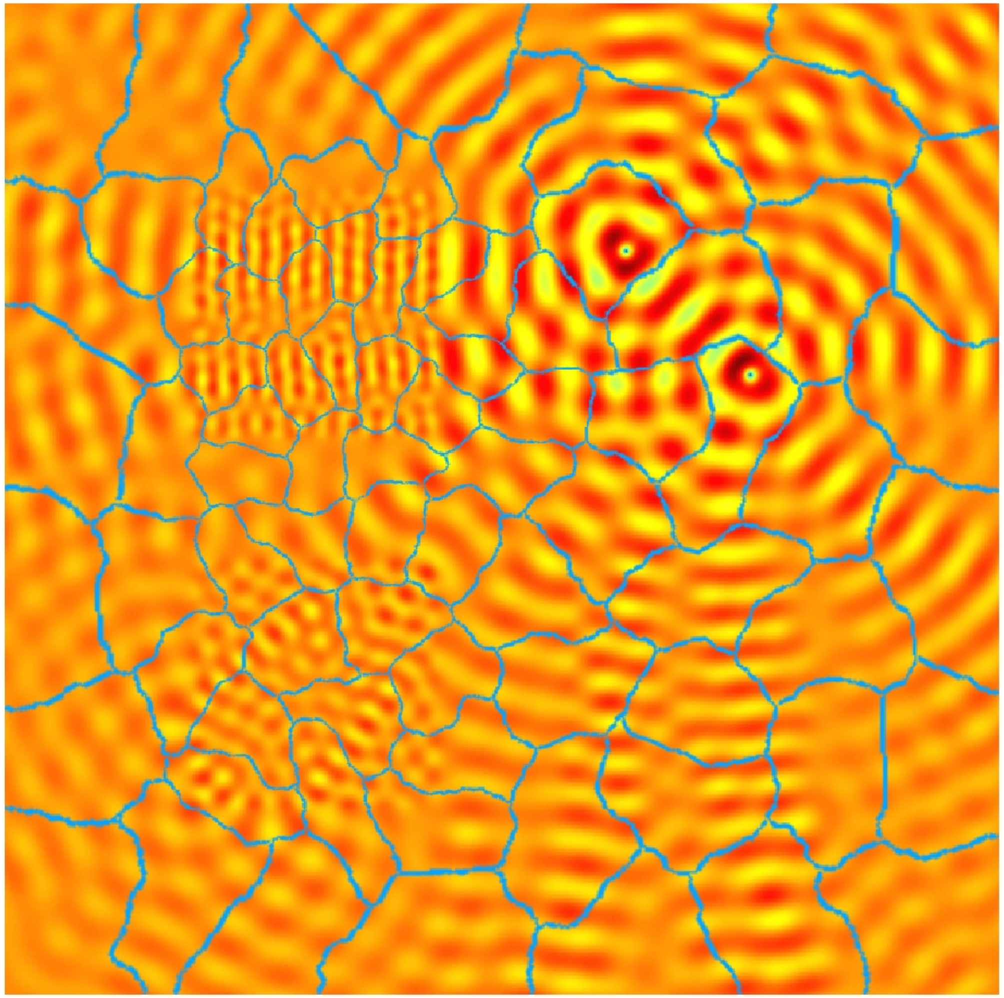


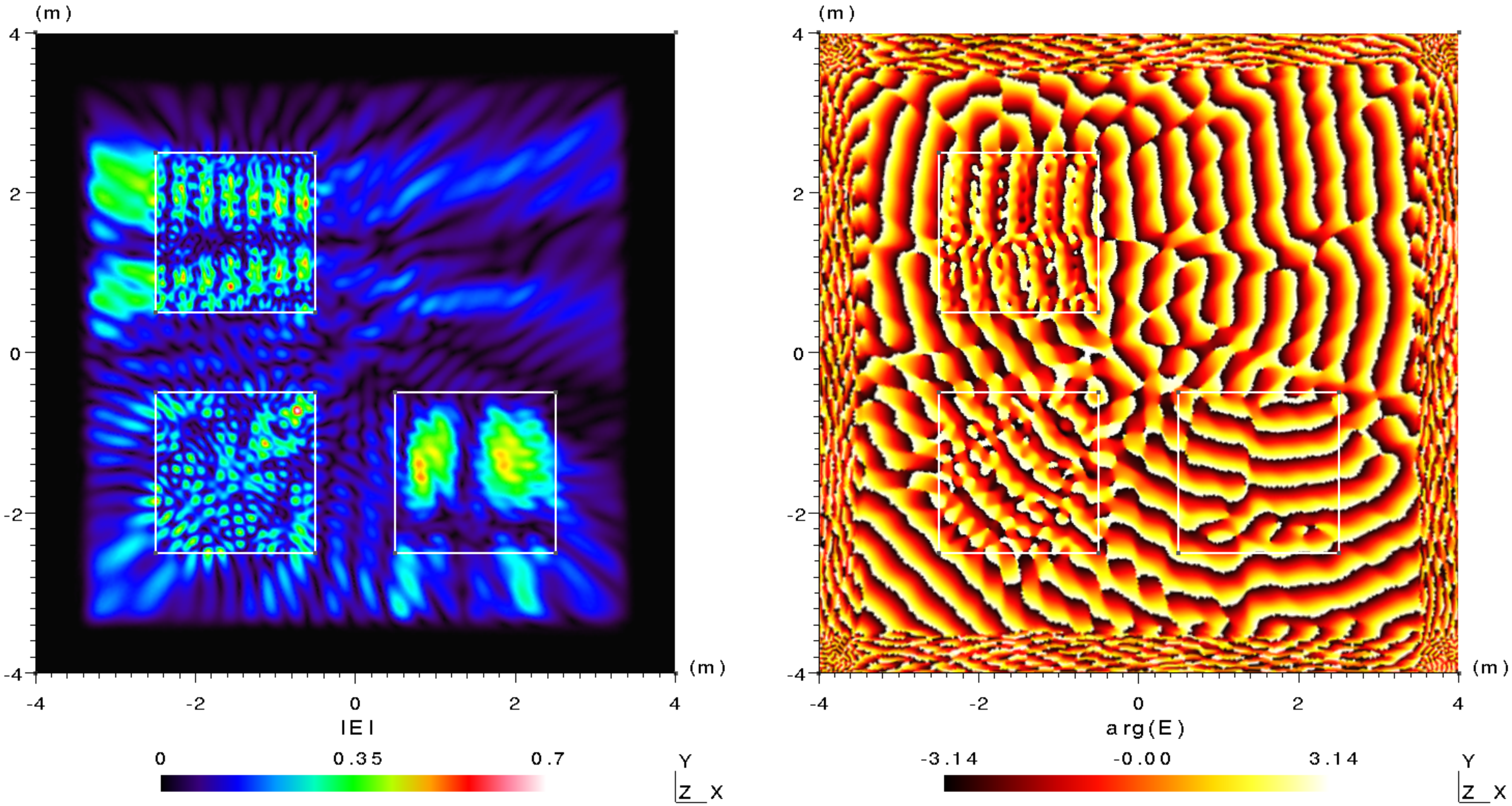
Total number of unknowns ($\mathbf{E}$): 426,574

80 subdomains
Size of interface problem (λ_r): 93,846 ≈ 22%
Total number of corner-points ($\mathbf{E}_c$): 138

Total number of equations for λ_c: 414 ≈ 0.1%

$\mathrm{Im}(\mathbf{E}^{\mathrm{tot}})$





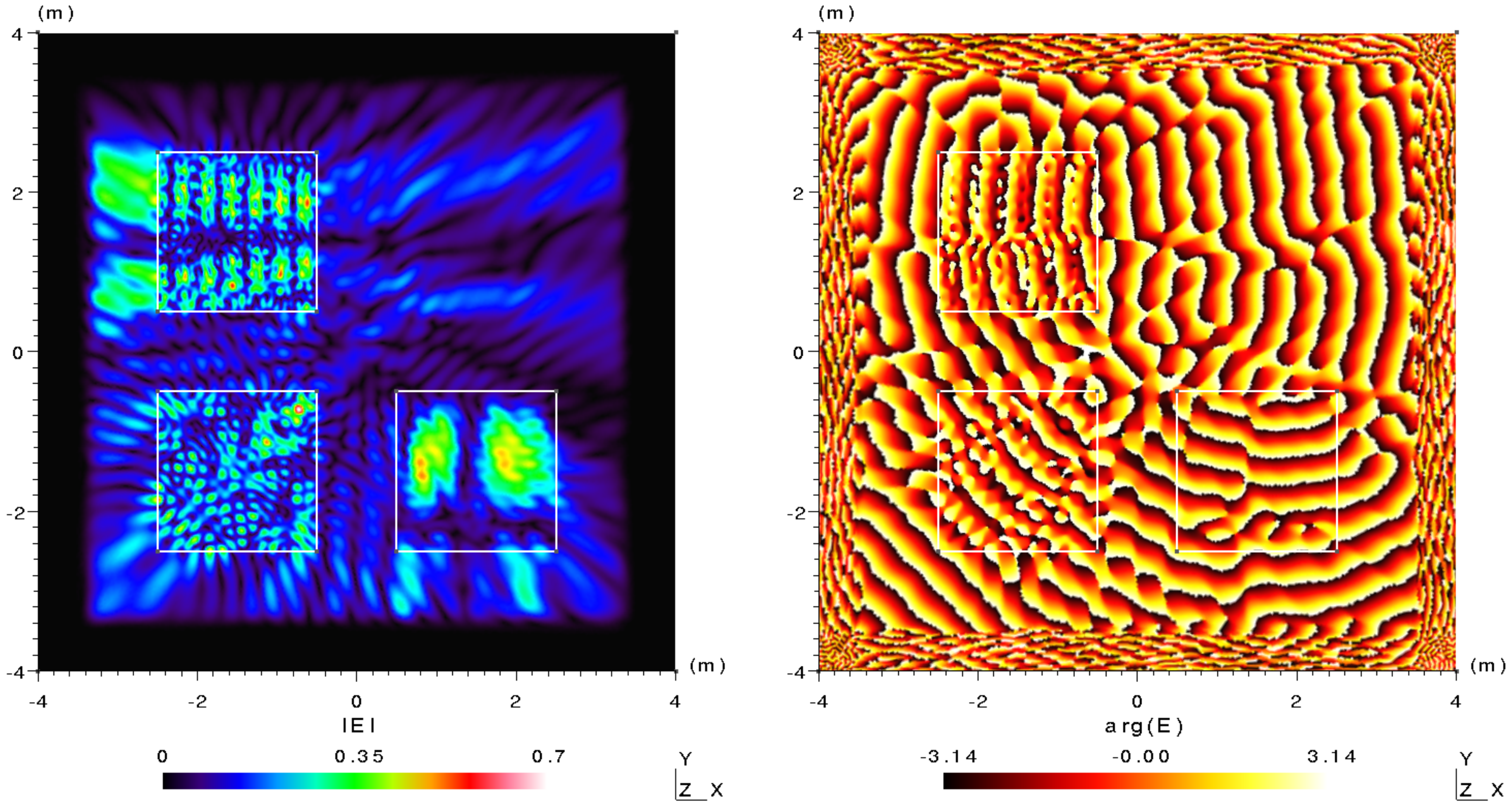
The interface problem is solved with the ***direct method***

$$L^2\text{-error} = \frac{||\mathbb{E}_1 - \mathbb{E}_2||^2}{||\mathbb{E}_1||^2}$$

where $\mathbb{E}_1$ is a solution of FEM

where $\mathbb{E}_2$ is a solution of FETI

L2-error of FETI-classical	L2-error of FETI-full
9.3415E-003	2.4155E-012



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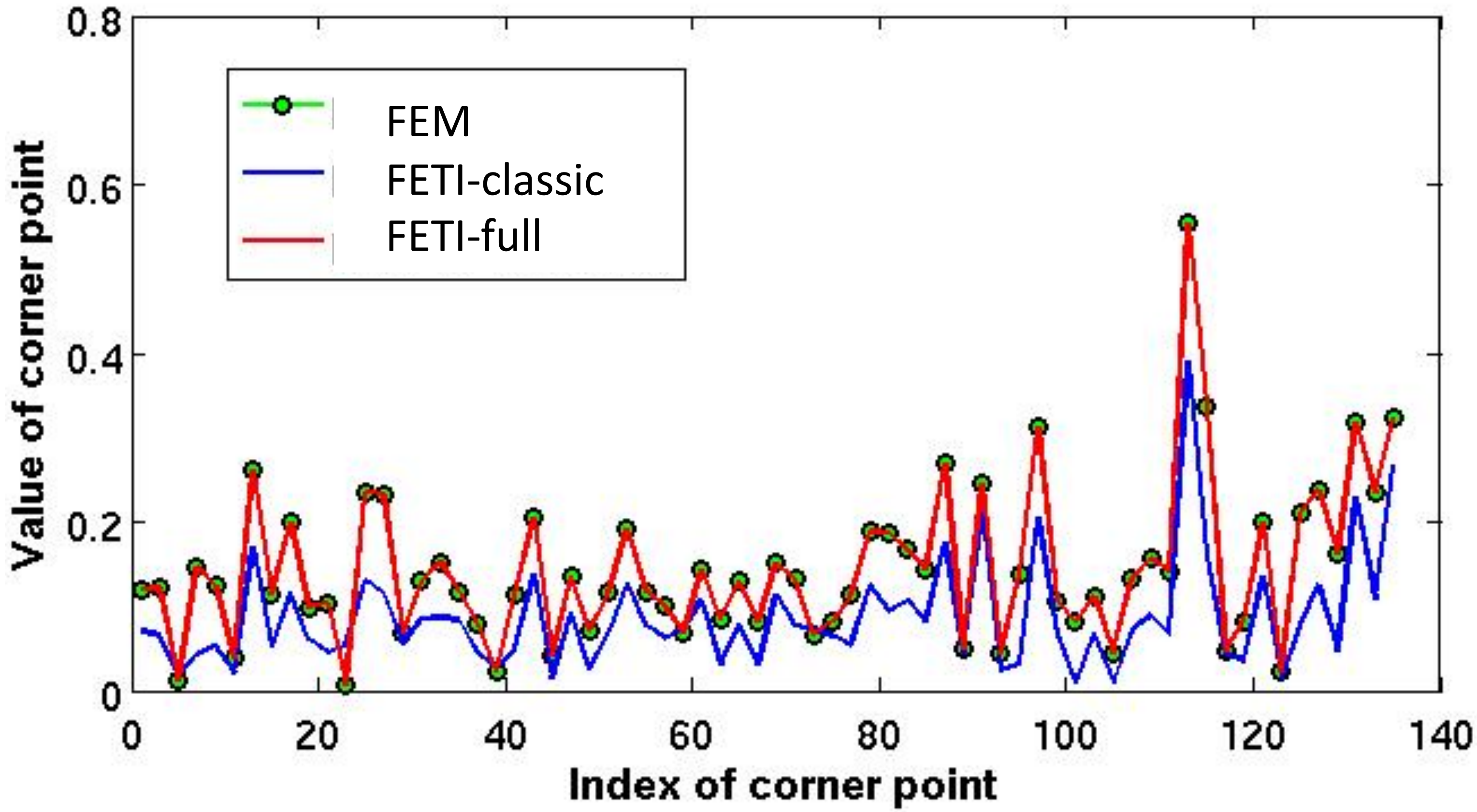
L2-error of FETI-classical	L2-error of FETI-full
9.3415**E-003**	2.4155**E-012**


FEM
FETI-classic
FETI-full
Value of corner point
Index of corner point
0
0.2
0.4
0.6
0.8
0
20
40
60
80
100
120
140

Inside air

$\varepsilon_0 = 8.85 \cdot 10^{-12} \mathrm{F} \cdot \mathrm{m}^{-1}$

$\mu_0 = 4\pi \cdot 10^{-7} \mathrm{H} \cdot \mathrm{m}^{-1}$

$f = 1$ GHz

The wavelength $\lambda \approx 0.3$ m

Domain of $\approx 2\lambda \times 2\lambda \times 2\lambda$

Excitation

Dipole located at $(0.05, 0.05, 0)$ oriented as $(1.0, 1.0, 0)$

Scatterers

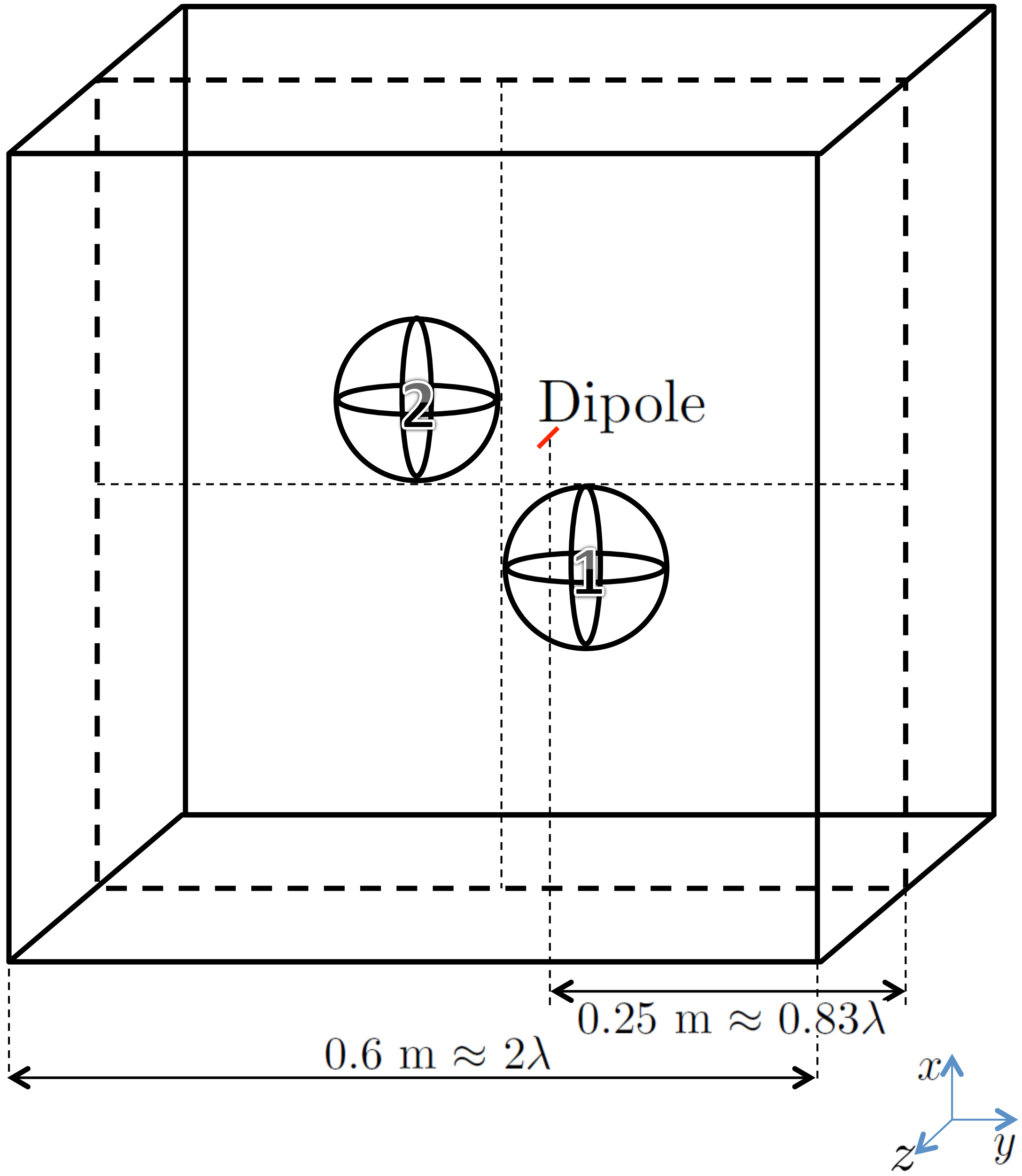
2 spheres

$\varepsilon_r^1 = 2.85$

$\varepsilon_r^2 = 5.00$

$\mu_1 = \mu_2 = 1.0$

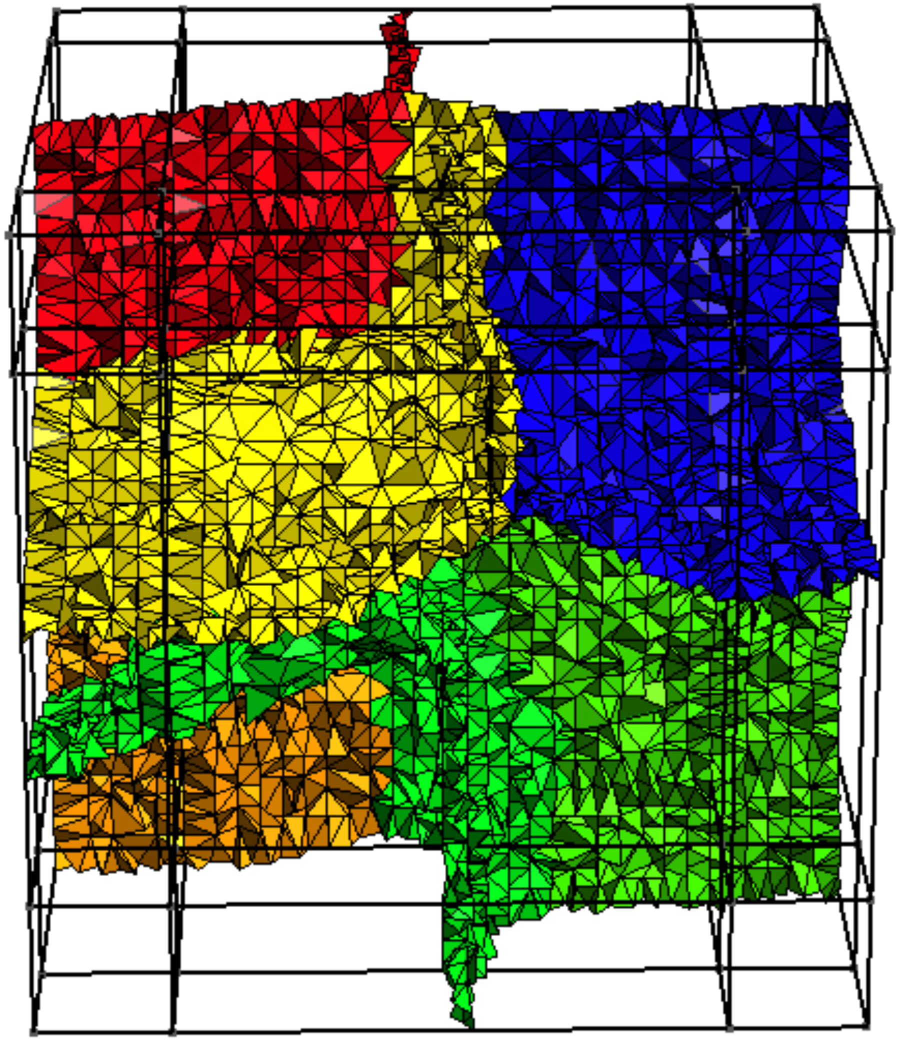
$R_1 = R_2 = 0.04\,\mathrm{m} \approx 0.13\lambda$



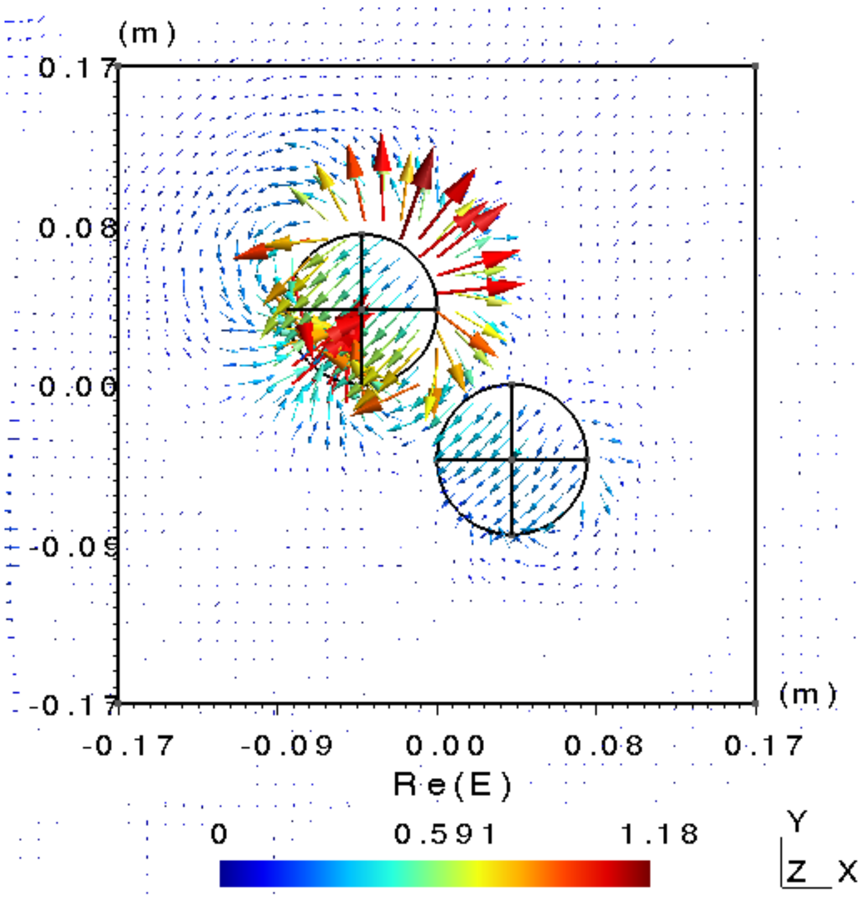
Total number of unknowns ($\mathbf{E}$): 302,561

10 subdomains
Size of interface problem (λ_r): 37,499 $\approx$ 12%
Total number of corner-edges ($\mathbf{E}_c$): 495

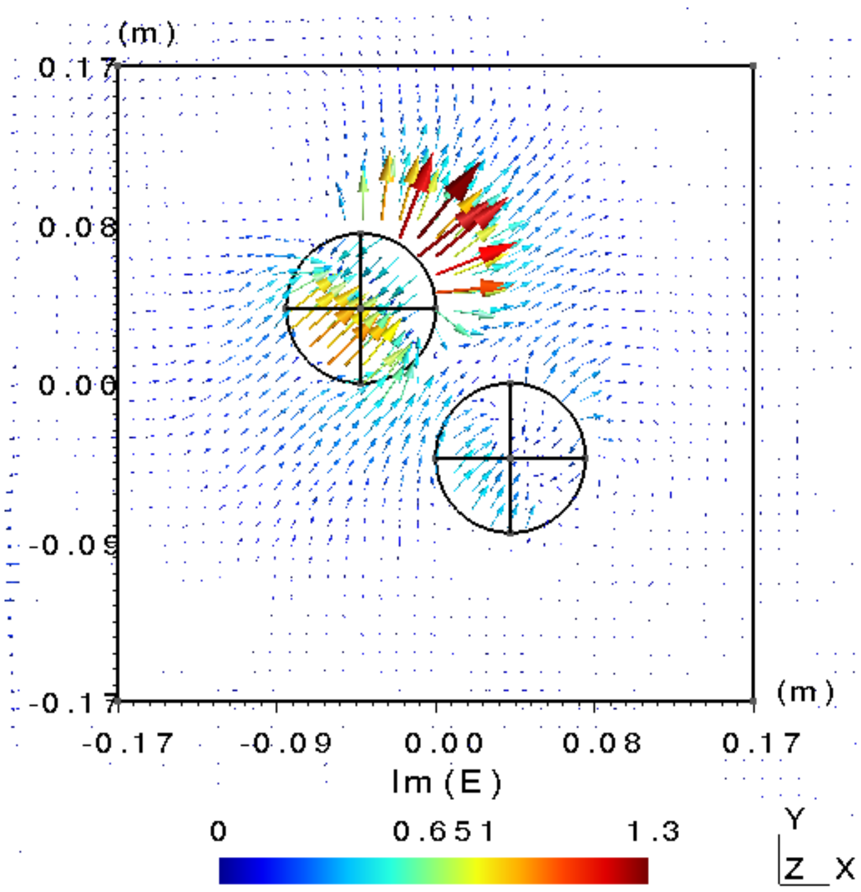
Total number of equations for λ_c: 1418 $\approx$ 0.5%



$Re(E^{sc})$



$Im(E^{sc})$



The interface problem is solved with the ***direct method***

L^2-error $= \frac{||\mathbb{E}_1 - \mathbb{E}_2||^2}{||\mathbb{E}_1||^2}$
where $\mathbb{E}_1$ is a solution of FEM
where $\mathbb{E}_2$ is a solution of FETI

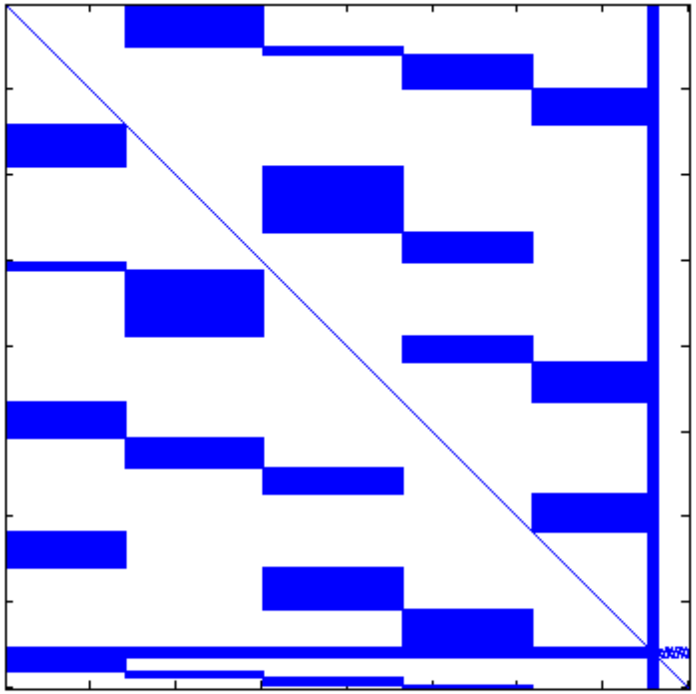
L^2-error $\mathbf{E}^{\text{tot}}$	L^2-error $\mathbf{E}^{\text{sc}}$
2.0187E-10	9.9886E-011

The Interface Problem (IP) Matrix

$$\begin{bmatrix} \mathrm{F}_{\lambda_r\lambda_r} & -\mathrm{F}_{\lambda_r\mathbf{E}_c} & 0 \\ -\mathrm{F}_{\mathbf{E}_c\lambda_r} & \mathrm{F}_{\mathbf{E}_c\mathbf{E}_c} & \mathrm{F}_{\mathbf{E}_c\lambda_c} \\ -\mathrm{F}_{\lambda_c\lambda_r} & -\mathrm{F}_{\lambda_c\mathbf{E}_c} & \mathrm{F}_{\lambda_c\lambda_c} \end{bmatrix} \begin{bmatrix} \lambda_r \\ \mathbf{E}_c \\ \lambda_c \end{bmatrix} = \begin{bmatrix} -\mathrm{d}_{\lambda_r} \\ \mathrm{d}_{\mathbf{E}_c} \\ -\mathrm{d}_{\lambda_c} \end{bmatrix}$$

Bottlenecks
Inverting and storing $\left(K_{rr}^{i}\right)^{-1}$ matrices
Storing the Interface Problem (IP) matrix

An iterative method?



GMRES

iterative method

The Interface Problem (IP) Matrix

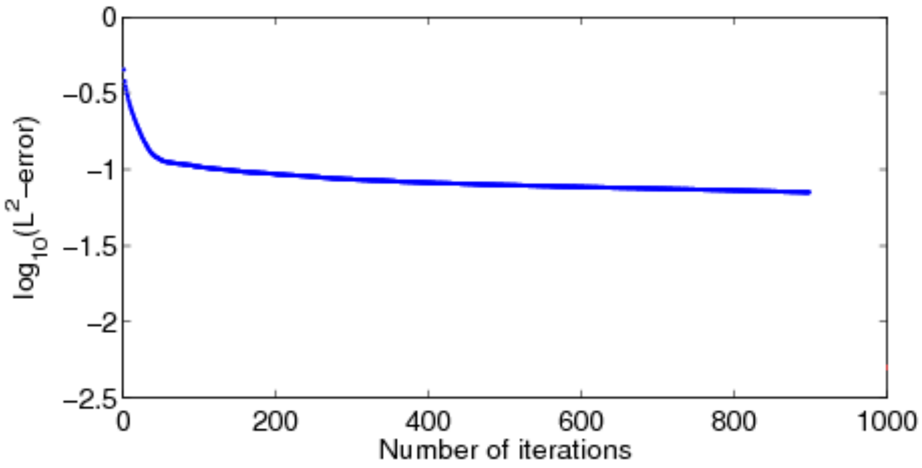
$$\begin{bmatrix} \mathrm{F}_{\lambda_r\lambda_r} & -\mathrm{F}_{\lambda_r\mathbf{E}_c} & 0 \\ -\mathrm{F}_{\mathbf{E}_c\lambda_r} & \mathrm{F}_{\mathbf{E}_c\mathbf{E}_c} & \mathrm{F}_{\mathbf{E}_c\lambda_c} \\ -\mathrm{F}_{\lambda_c\lambda_r} & -\mathrm{F}_{\lambda_c\mathbf{E}_c} & \mathrm{F}_{\lambda_c\lambda_c} \end{bmatrix} \begin{bmatrix} \lambda_r \\ \mathbf{E}_c \\ \lambda_c \end{bmatrix} = \begin{bmatrix} -\mathrm{d}_{\lambda_r} \\ \mathrm{d}_{\mathbf{E}_c} \\ -\mathrm{d}_{\lambda_c} \end{bmatrix}$$

GMRES

iterative method

The Interface Problem (IP) Matrix

$$\begin{bmatrix} F_{\lambda_r\lambda_r} & -F_{\lambda_r\mathbf{E}_c} & 0 \\ -F_{\mathbf{E}_c\lambda_r} & F_{\mathbf{E}_c\mathbf{E}_c} & F_{\mathbf{E}_c\lambda_c} \\ -F_{\lambda_c\lambda_r} & -F_{\lambda_c\mathbf{E}_c} & F_{\lambda_c\lambda_c} \end{bmatrix} \begin{bmatrix} \lambda_r \\ \mathbf{E}_c \\ \lambda_c \end{bmatrix} = \begin{bmatrix} -d_{\lambda_r} \\ d_{\mathbf{E}_c} \\ -d_{\lambda_c} \end{bmatrix}$$



GMRES

iterative method

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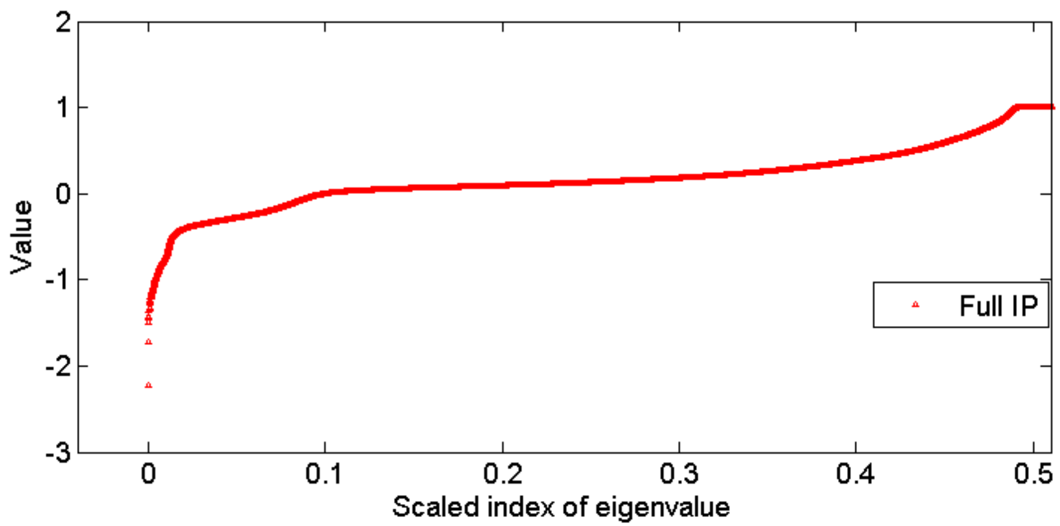
$$\begin{bmatrix} F_{\lambda_r\lambda_r} & -F_{\lambda_r\mathbf{E}_c} & 0 \\ -F_{\mathbf{E}_c\lambda_r} & F_{\mathbf{E}_c\mathbf{E}_c} & F_{\mathbf{E}_c\lambda_c} \\ -F_{\lambda_c\lambda_r} & -F_{\lambda_c\mathbf{E}_c} & F_{\lambda_c\lambda_c} \end{bmatrix} \begin{bmatrix} \lambda_r \\ \mathbf{E}_c \\ \lambda_c \end{bmatrix} = \begin{bmatrix} -d_{\lambda_r} \\ d_{\mathbf{E}_c} \\ -d_{\lambda_c} \end{bmatrix}$$

Convergence depends on $(A + A^T)/2$ [1,2]



References		
[1]	Saad, Schultz	1986
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The Interface Problem (IP) Matrix

$$\begin{bmatrix} F_{\lambda_r\lambda_r} & -F_{\lambda_r\mathbf{E}_c} & 0 \\ -F_{\mathbf{E}_c\lambda_r} & F_{\mathbf{E}_c\mathbf{E}_c} & F_{\mathbf{E}_c\lambda_c} \\ -F_{\lambda_c\lambda_r} & -F_{\lambda_c\mathbf{E}_c} & F_{\lambda_c\lambda_c} \end{bmatrix} \begin{bmatrix} \lambda_r \\ \mathbf{E}_c \\ \lambda_c \end{bmatrix} = \begin{bmatrix} -d_{\lambda_r} \\ d_{\mathbf{E}_c} \\ -d_{\lambda_c} \end{bmatrix}$$

The Reduced IP Matrix [3,4,5]

$$\left(F_{\lambda_r\lambda_r} + F_{\lambda_r\mathbf{E}_c}\left[\hat{F}^{-1}_{\mathbf{E}_c\mathbf{E}_c}\right]\hat{F}_{\mathbf{E}_c\lambda_r}\right)\lambda_r = -\hat{d}_{\lambda_r}$$

Convergence depends on $(A + A^T)/2$ [1,2]

References		
[1]	Saad, Schultz	1986
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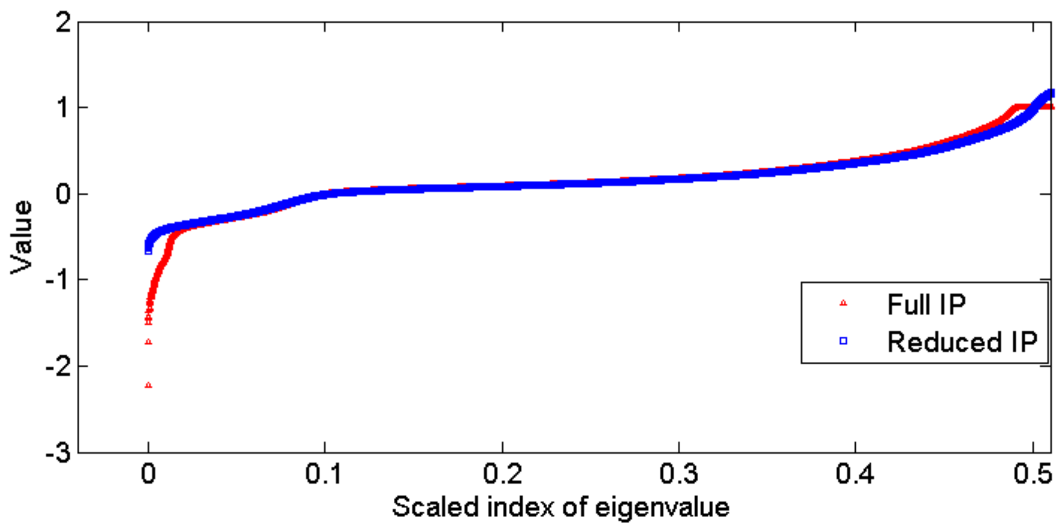

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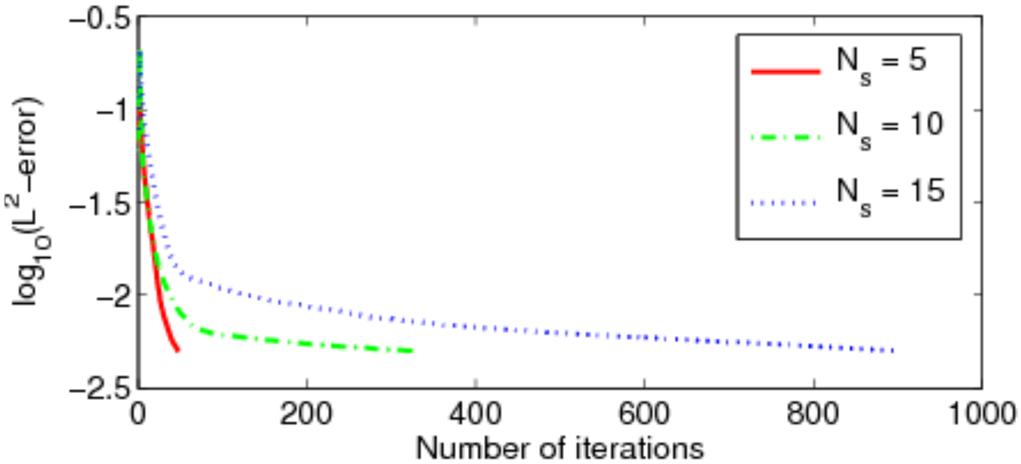

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The Domain Decomposition into $N_s = 50$ subdomains

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- ❑ The $L^2 - error\ ||E^{FEM} - E^{FETI}|| = 9.18\text{E-}002$

The Domain Decomposition into $N_s = 50$ subdomains

- The $L^2 - error\ \|E^{FEM} - E^{FETI}\| = 9.18\text{E-}002$
- But sometimes

Ω^i	27	28	29	31	41
L^2-error	0.5085	0.2435	0.3509	0.5687	0.2995

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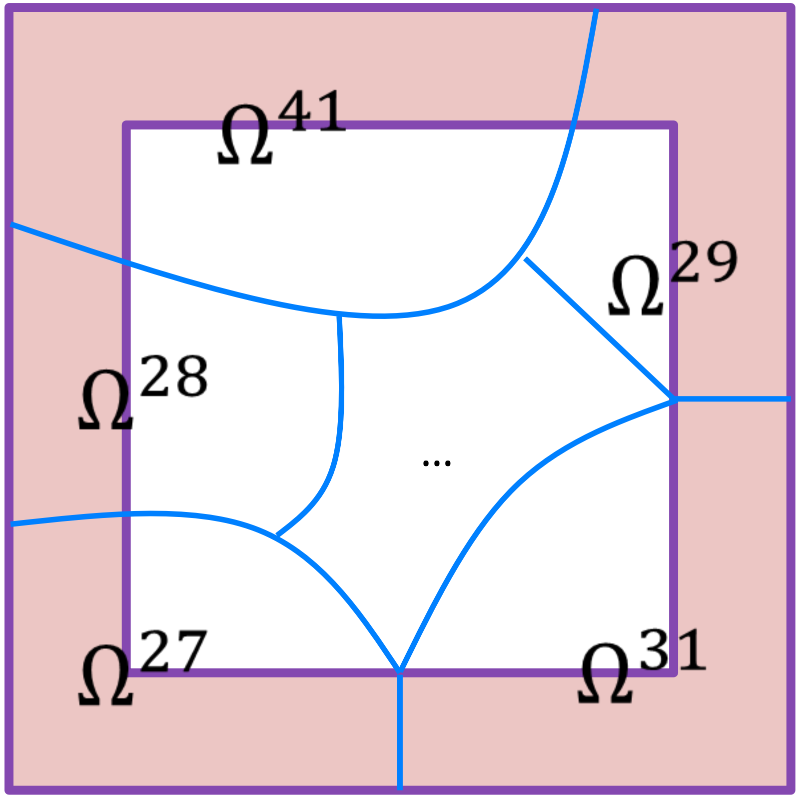

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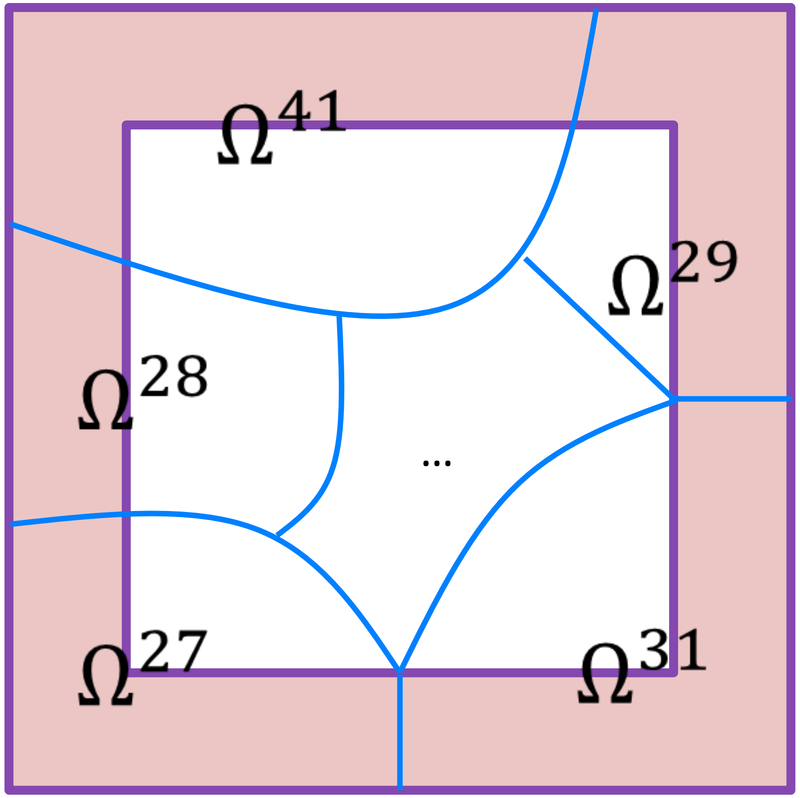
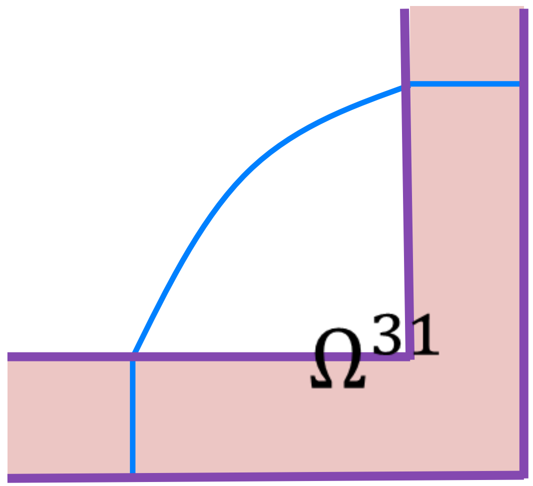


Conclusion

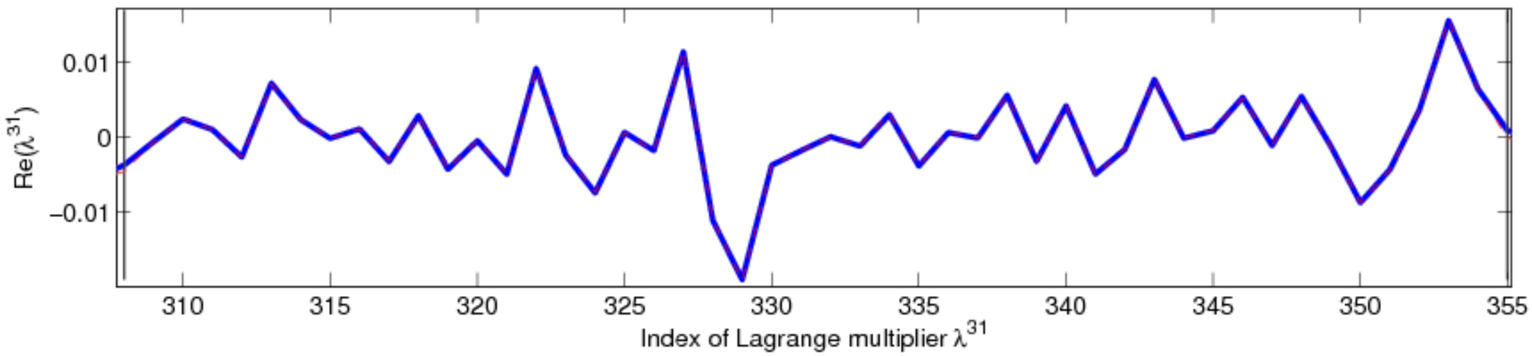
There is something really strange going on with the PML media

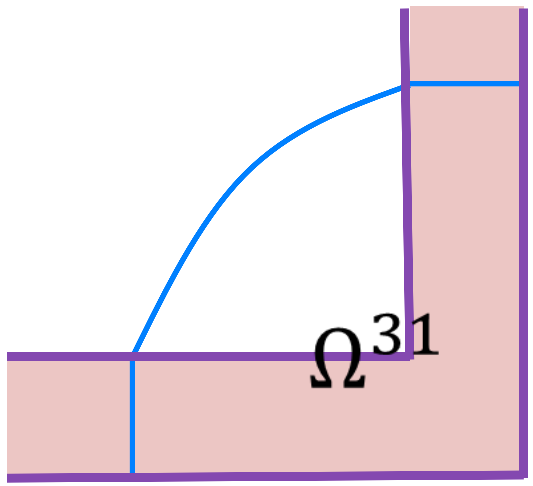
Convergence of Lagrange multipliers in subdomain Ω^{31}



Convergence of Lagrange multipliers in subdomain Ω^{31}

- Out of PML, after 10 iterations (— Exact, – – FETI)





Convergence of Lagrange multipliers in subdomain Ω^{31}

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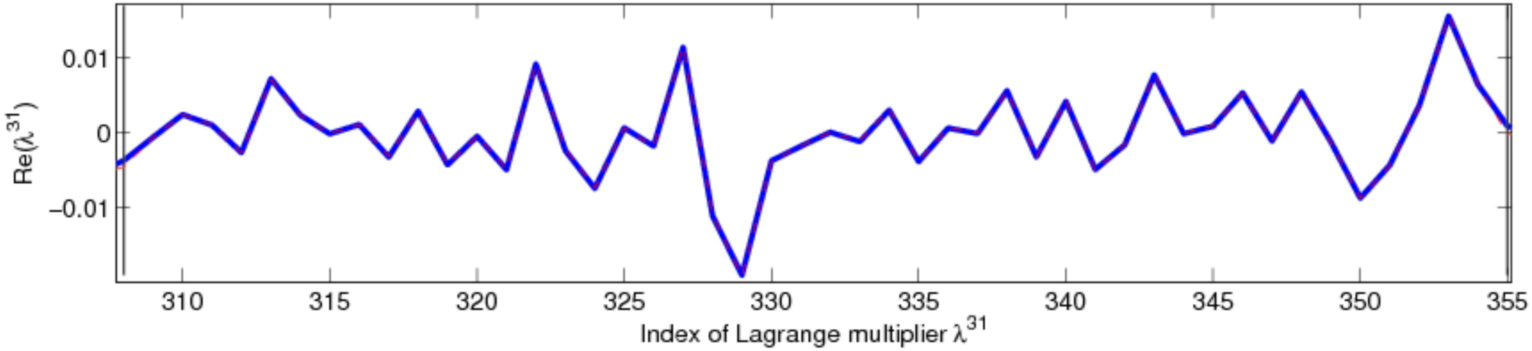
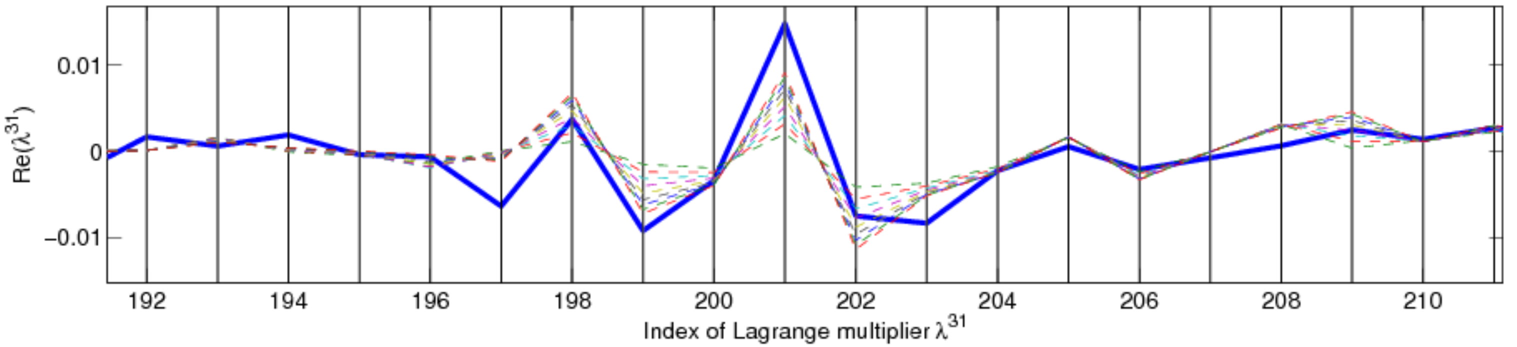
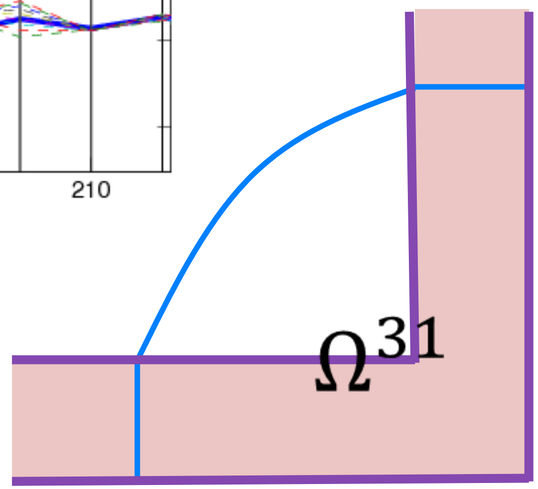


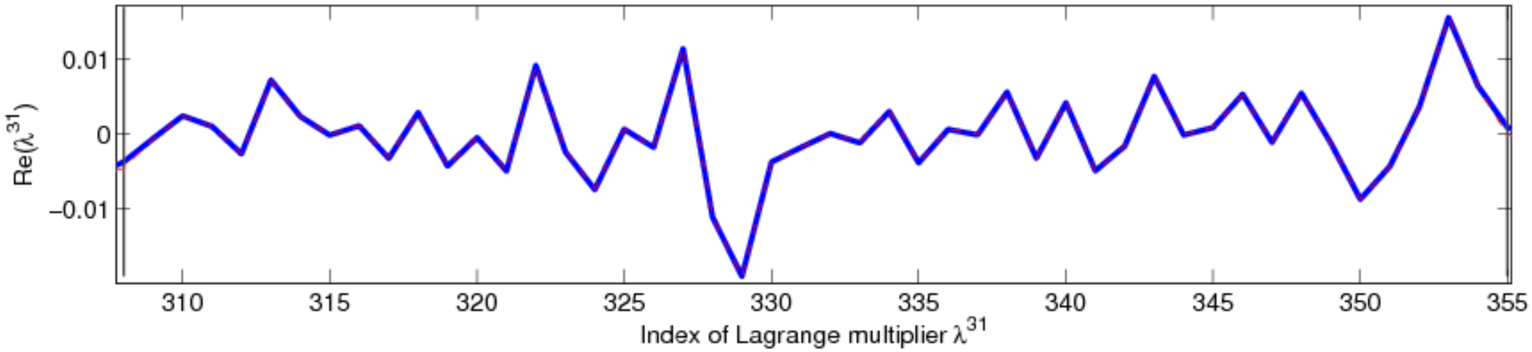
- In PML, after 100 iterations (— Exact, – – FETI)



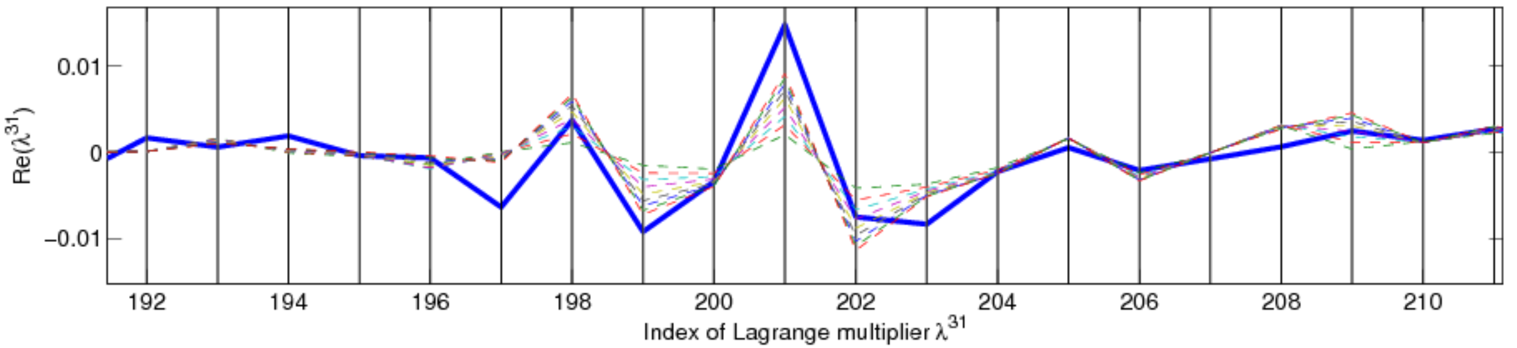


Convergence of Lagrange multipliers in subdomain Ω^{31}

- Out of PML, after 10 iterations (— Exact, – – FETI)

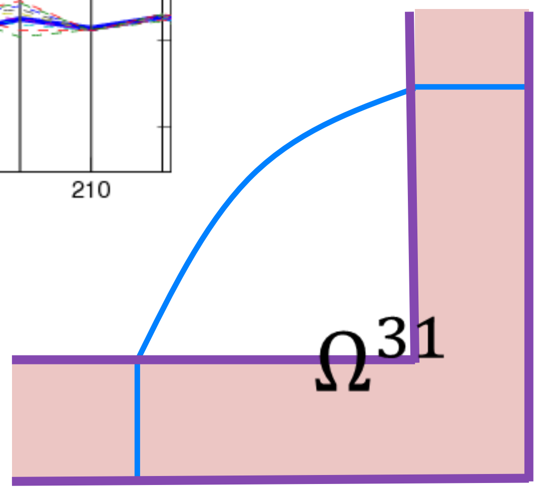


- In PML, after 100 iterations (— Exact, – – FETI)



Conclusion so far

- Bad influence of PML
- The error does not spread



What we can play with

- [ ] Number of Lagrange multipliers?

What we can play with

- ❑ Number of Lagrange multipliers?
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Robin-type Boundary Conditions

$$\vec{n} \times \left(\frac{1}{\mu_r} \nabla \times \mathcal{E}^i \right) + \alpha^i \vec{n} \times \vec{n} \times \mathcal{E}^i = \Lambda^i \text{ on } \Gamma^i$$

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Classical approach [1]

$$\alpha^i = j\, k_0$$

References

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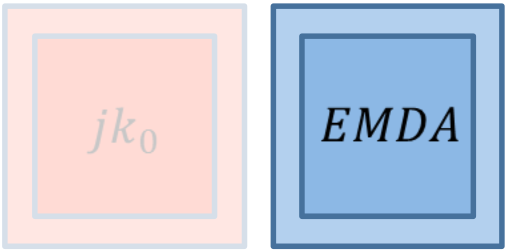

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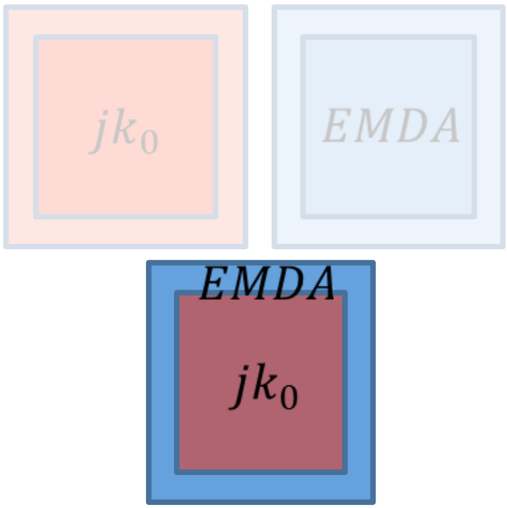

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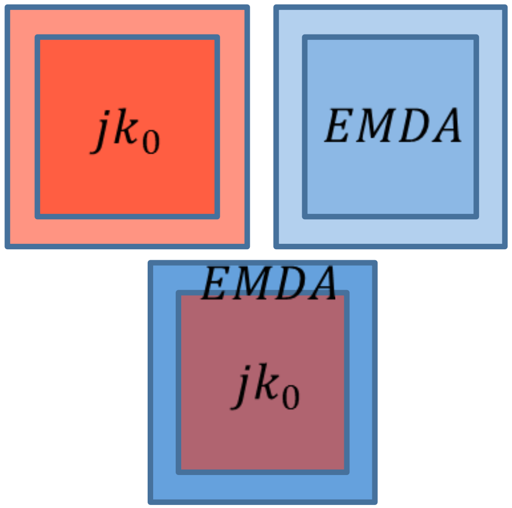

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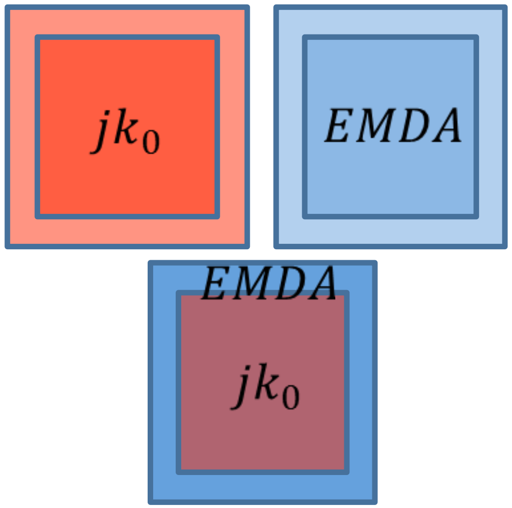

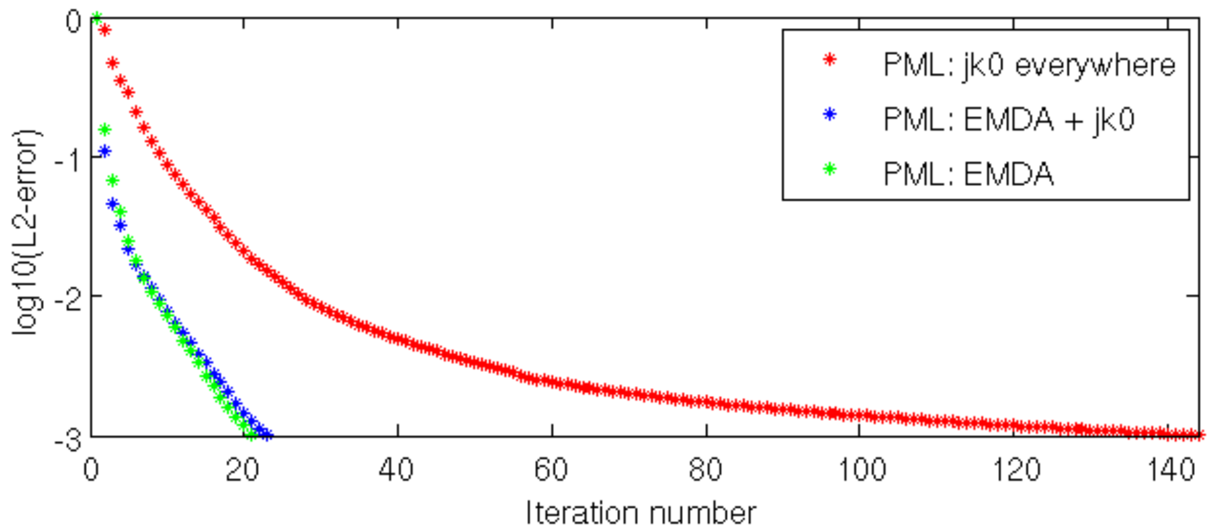

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jk_0 $EMDA$

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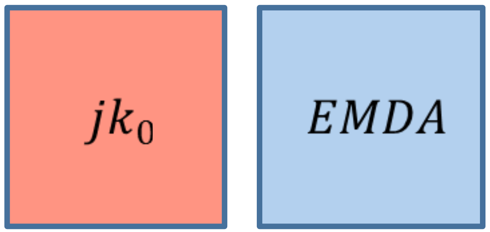

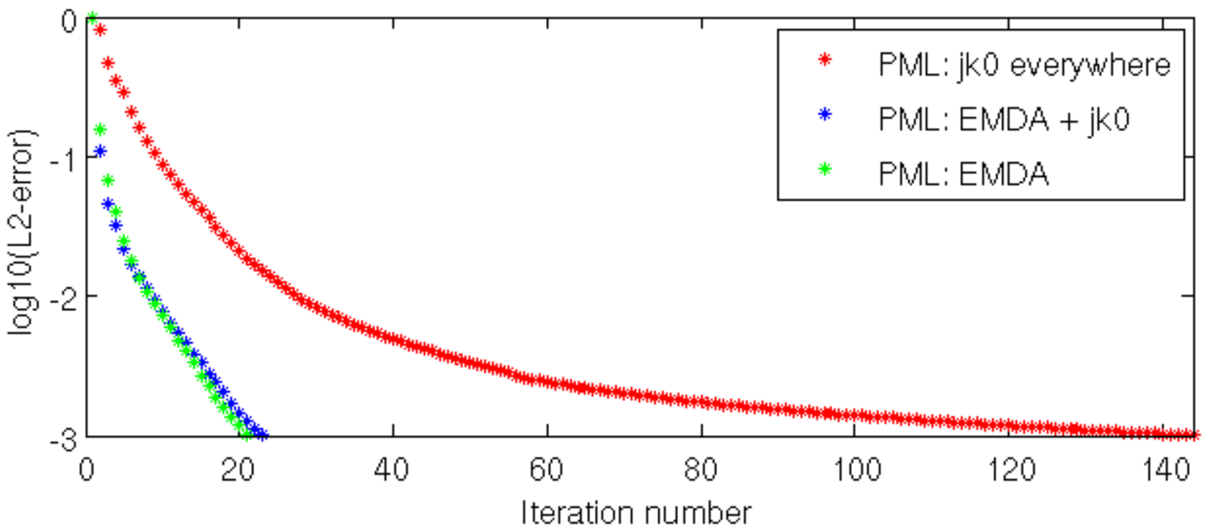

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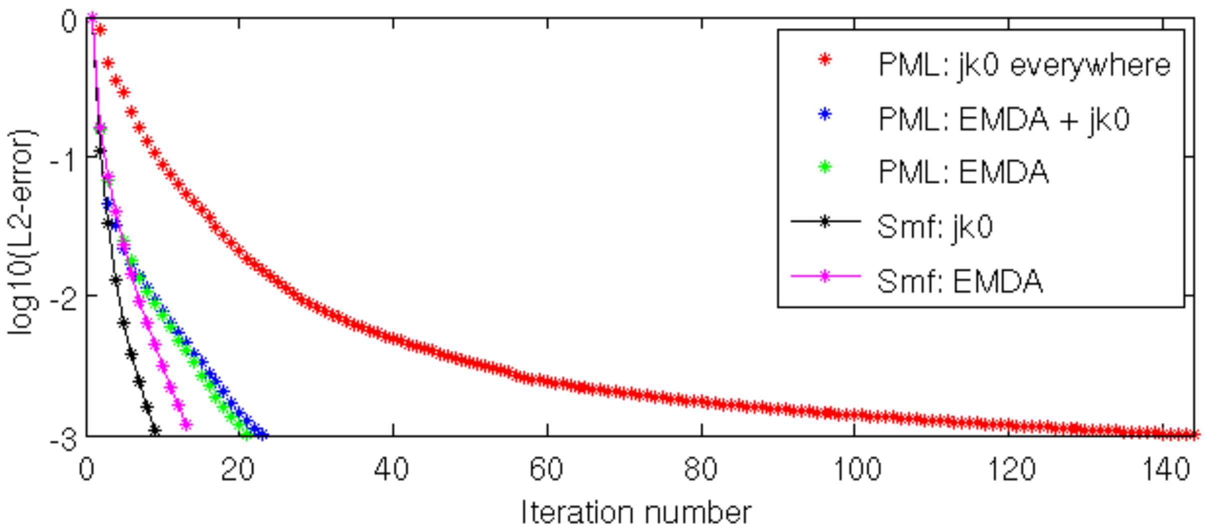

References		
[1]	Després	1991
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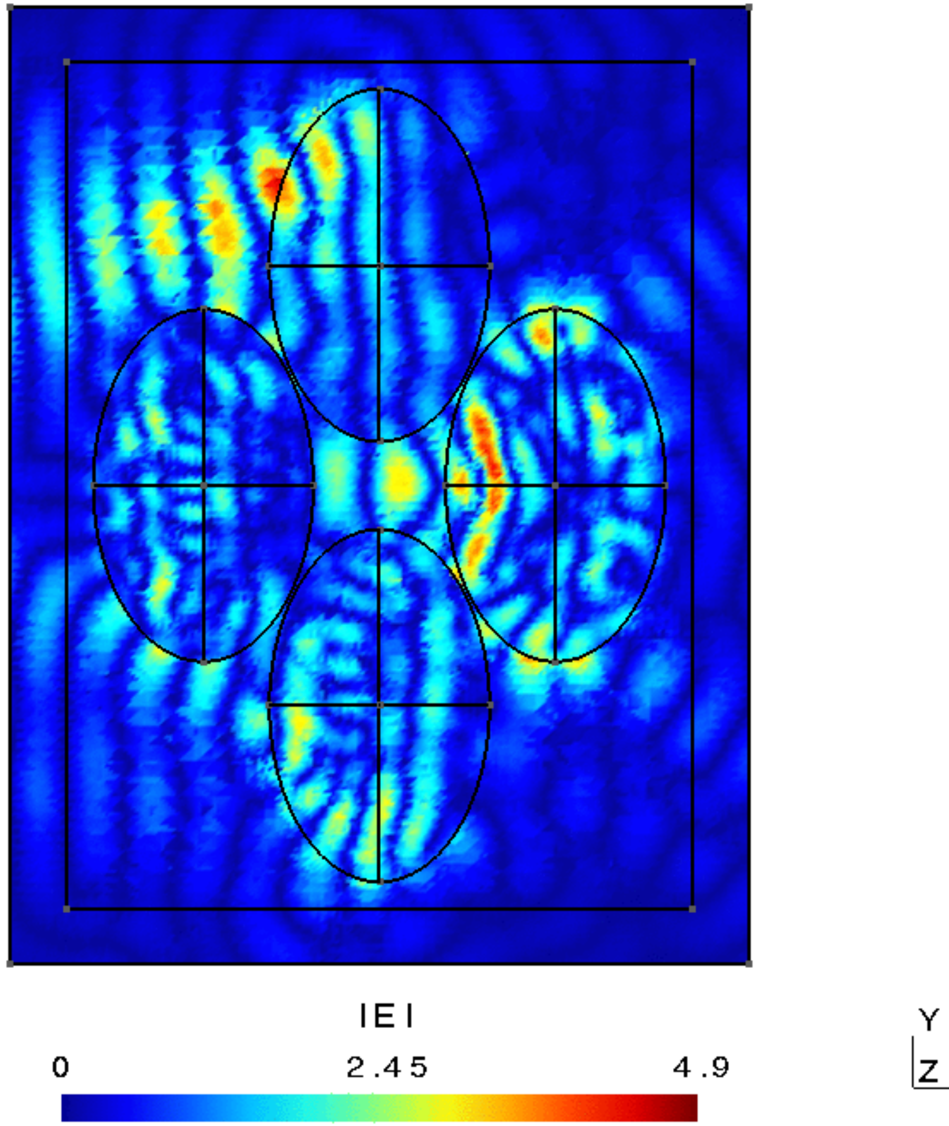
Recent conclusion

- ❑ Modified FETI-DPEM2-full method

Memory	
FEM	FETI-full
$\approx 1\,200\,000$	$\approx 3\,500\,000$

Time	
FEM	FETI-full
$2\,568\ sec$	$838\ sec$

Domain of 252 λ^3



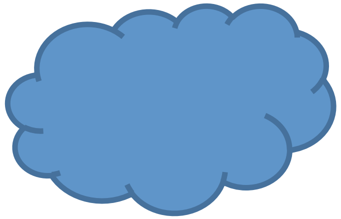
References

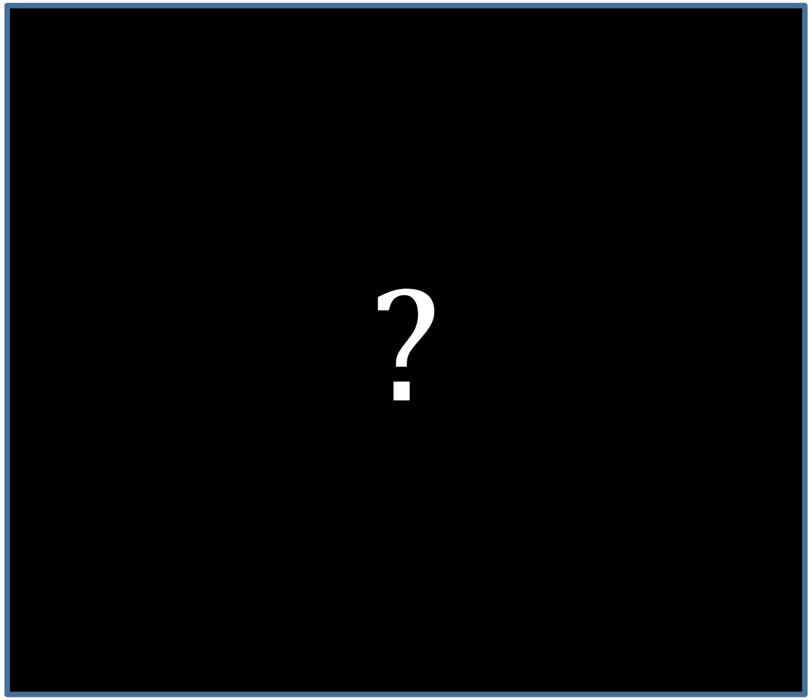
[1] Voznyuk et al (submitted) 2014

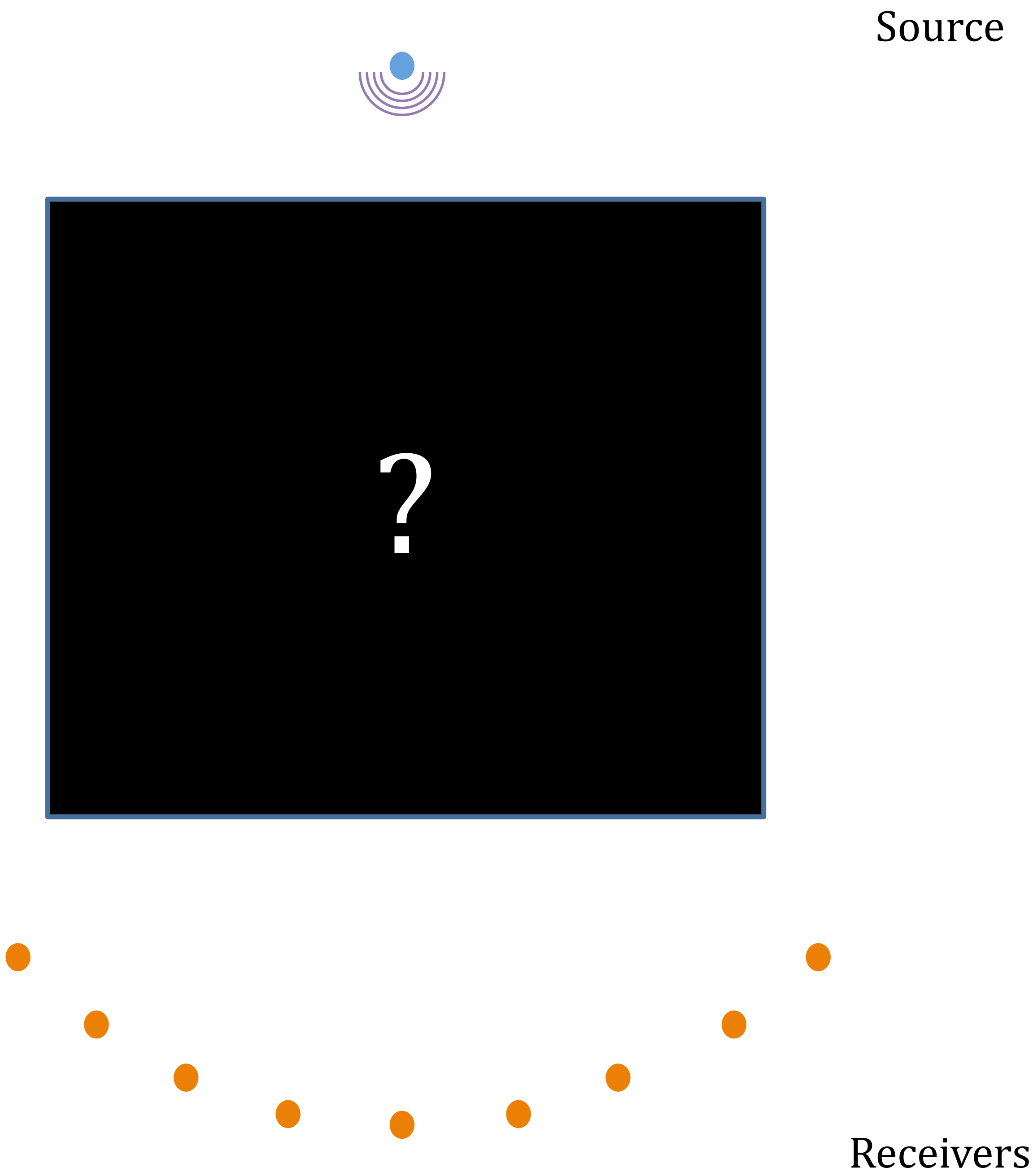
Outline

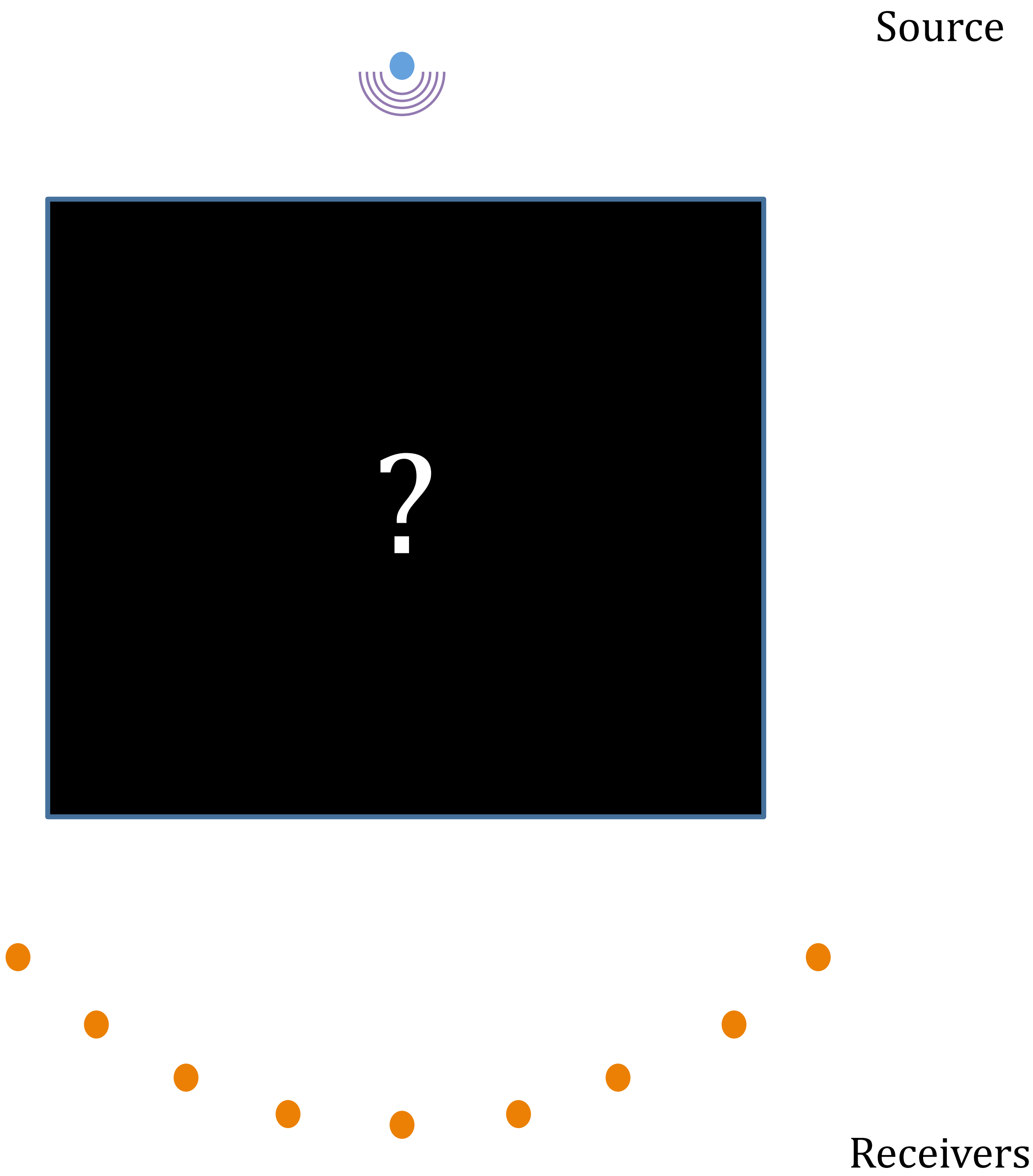
Just an object

Just an object






Source
?
Receivers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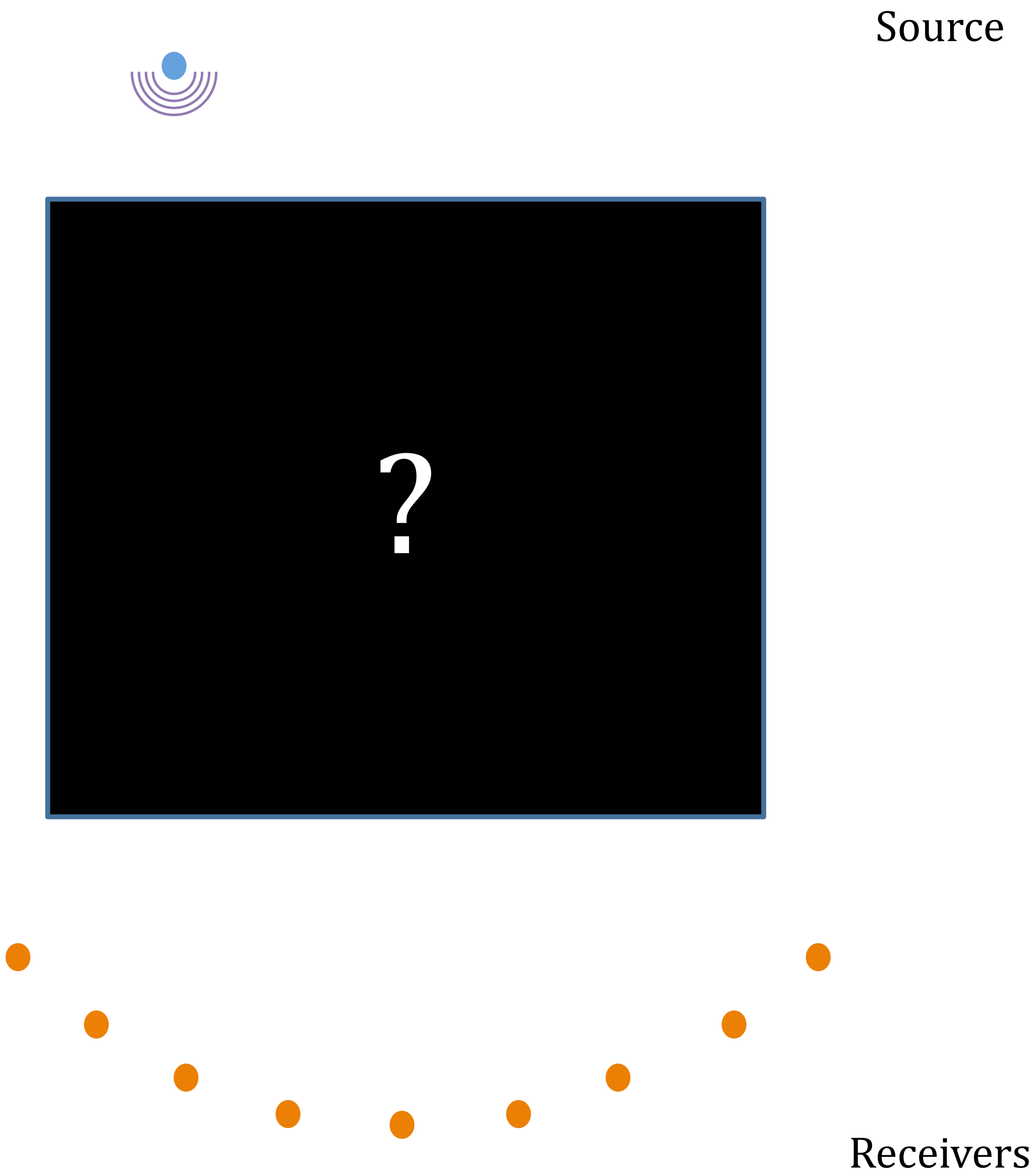
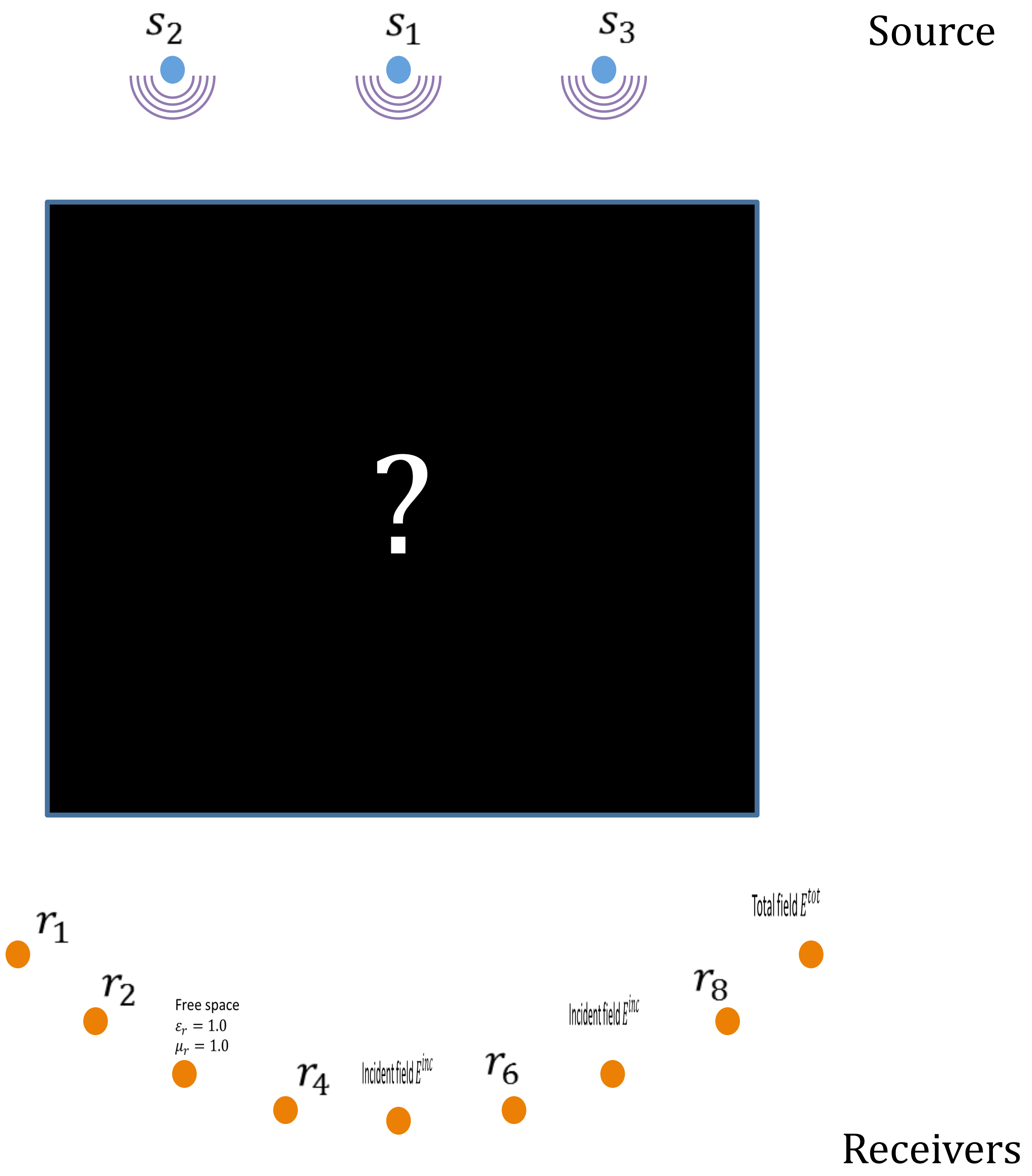

Source
?
Receivers

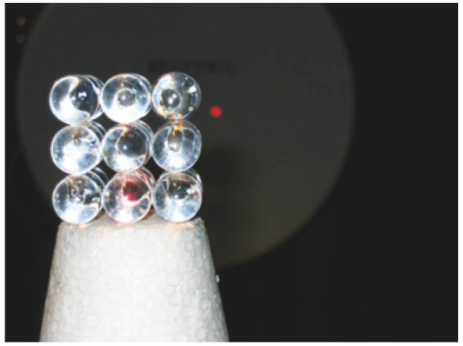
$E^{mes}: N_{src} \times N_{rec}$

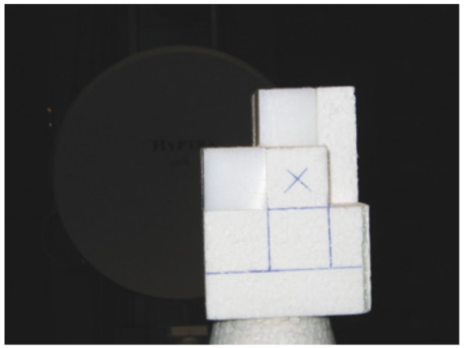


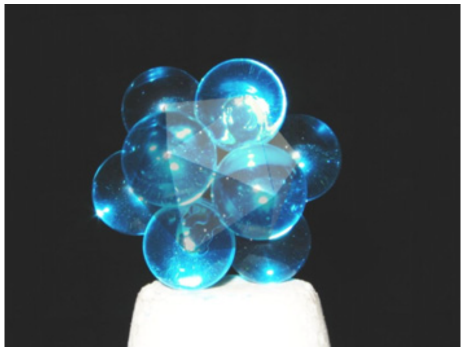
$$E^{mes}: N_{src} \times N_{rec}$$

3D Fresnel database [1]

- ❑ Set of homogeneous targets
- ❑ 162 transmitting dipoles
- ❑ 32 receiver positions
- ❑ Distance: $r = 1.796\ m$







Anechoic chamber

References

[1]	J.-M. Geffrin and P. Sabouroux	2009

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- Find ε_r such as the cost functional

$$J(\varepsilon_r, E^{far}) = \frac{1}{2}\sum_{s=1}^{N_{src}}\sum_{r=1}^{N_{rec}}\left\|E_{s,r}^{mes} - E_{s,r}^{far}(\varepsilon_r)\right\|_W^2$$

has to be minimized.

- W – covariance matrix, related to

$$E^{mes} = E^{far}(\varepsilon_r) + \underline{noise}$$

- $E_{s,r}^{far}(\varepsilon_r)$ – simulated far-field

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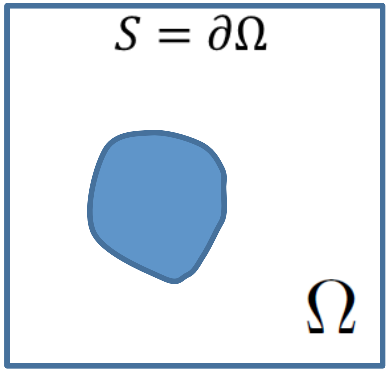

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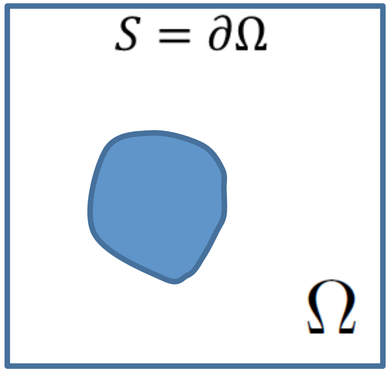

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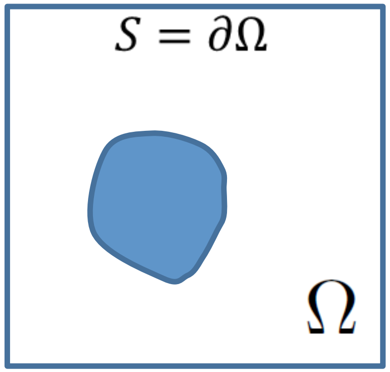
$$\mathbf{E}(r) = \oiint_S \{-jw\mu \left[\hat{n}' \times \mathbf{H}(r')\right] G_0(r,r') + \left[\hat{n}' \cdot \mathbf{E}(r')\right] \nabla' G_0(r,r') + + \left[\hat{n}' \times \mathbf{E}(r')\right] \times \nabla' G_0(r,r')\}\, dS'$$



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Characteristics of problem

- nonlinear
- ill posed
- underdetermined

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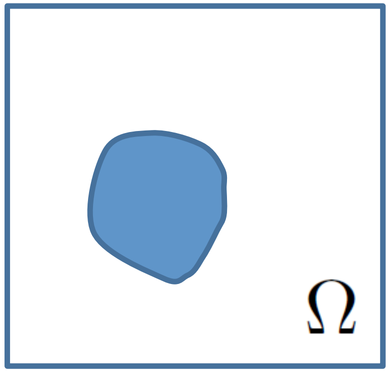

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underdetermined
There is not a unique solution

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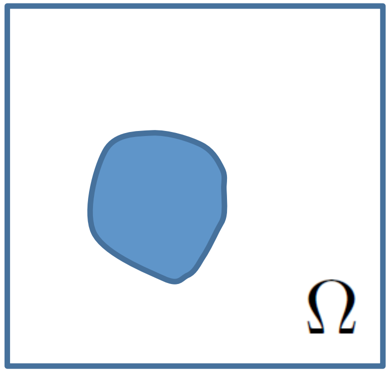

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Constraints

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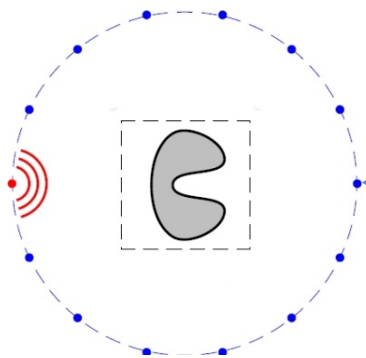
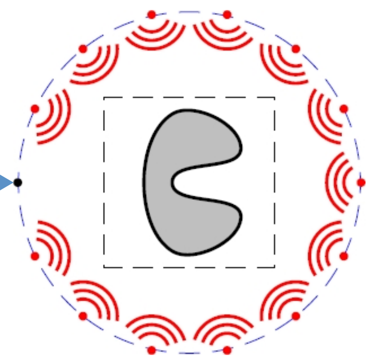
Total field ***Adjoint field***





$E^{mes} : N_{src} \times N_{rec}$

- Find ε_r such as the cost functional

$$J(\varepsilon_r, E^{far}) = \frac{1}{2}\sum_{s=1}^{N_{src}}\sum_{r=1}^{N_{rec}} \left\| E^{mes}_{s,r} - E^{far}_{s,r}(\varepsilon_r) \right\|^2_W$$

has to be minimized.

- An iterative quasi-Newton method with constraints

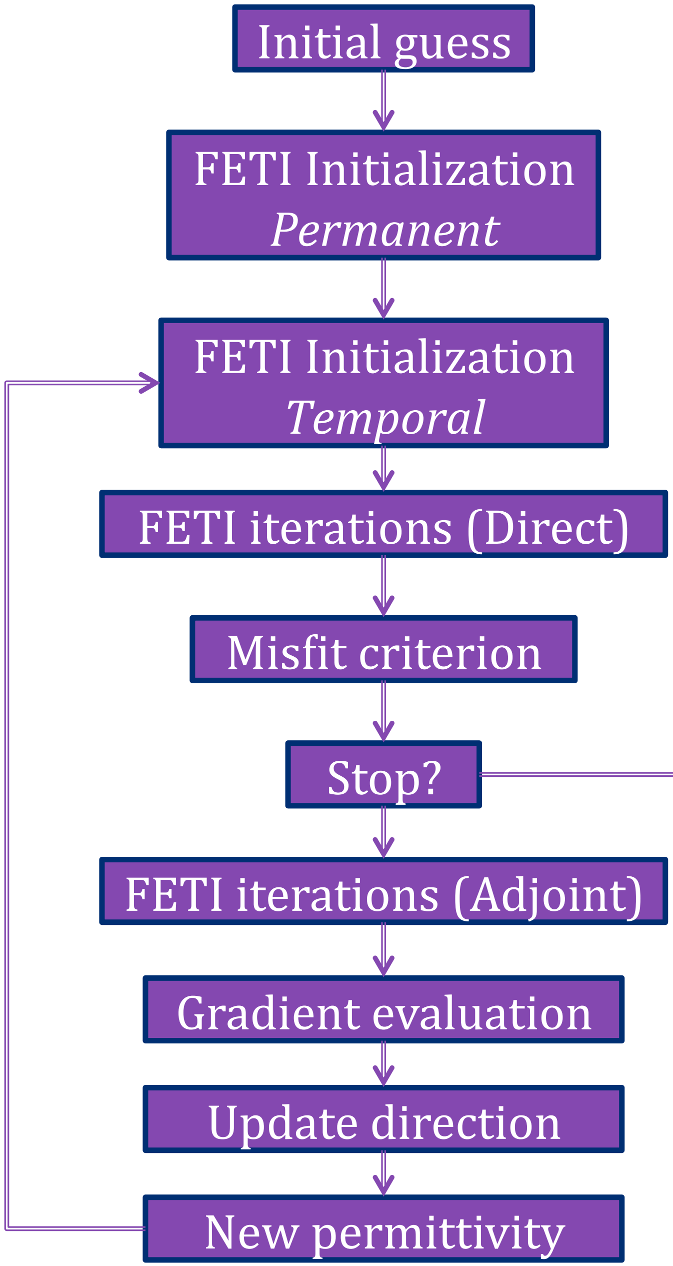
Constraints

$$\mathcal{L}(\varepsilon_r, E, P) = \mathcal{J}(\varepsilon_r, E) + \mathcal{R}e\left(P \middle| \triangle E + k^2 \varepsilon_r E - S\right)$$

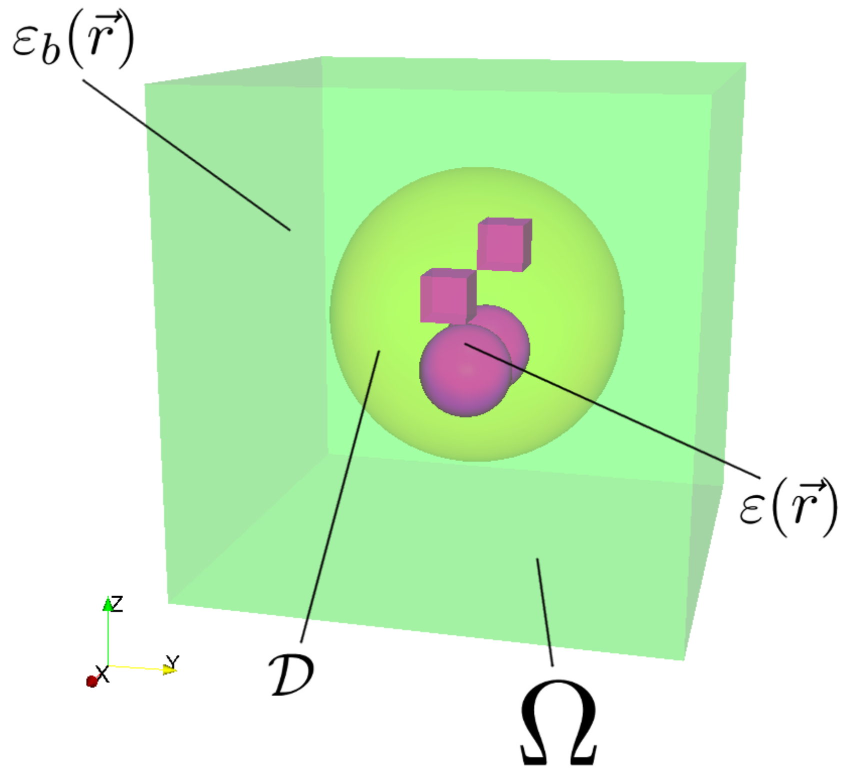
Total field

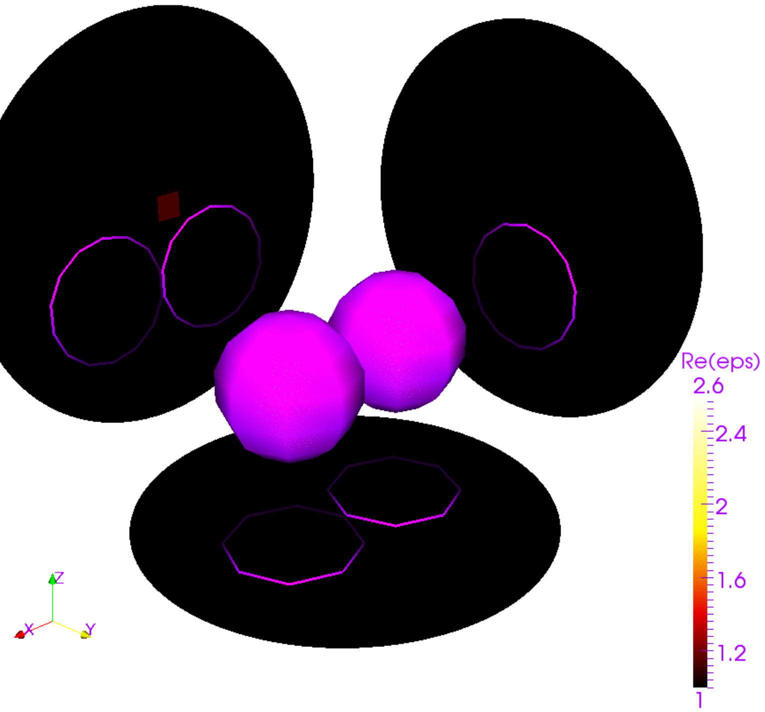
Adjoint field

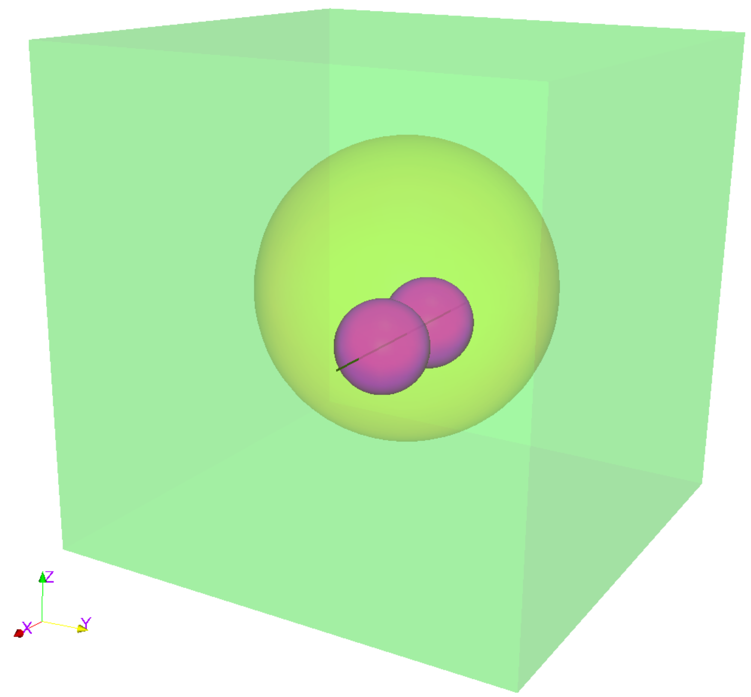
$$\nabla_{\varepsilon_r} \mathcal{J}(\varepsilon_r) = -\sum_{s=1}^{N_{src}} \overline{E_s \; P_s}$$



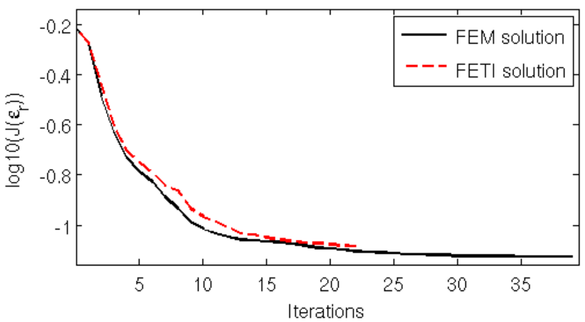
End of process
Max iteration
No evolution
Small misfit
Small gradient



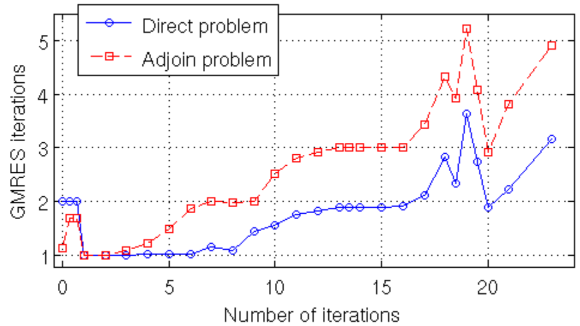




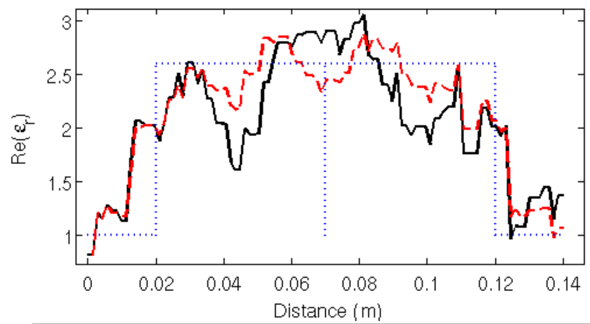
Cost Function convergence



FETI iterations



Profile cut comparison



Recent conclusion

- Successful combination of the Inversion algorithm and FETI method [1]

References

[1] Voznyuk et al (in preparation) 2014

Conclusions

- ❑ Development of the 3D FETI-DPEM2-***full*** method
- ❑ Implementation to the Large-Scale
  - ❖ Direct problems
  - ❖ Inverse quantitative problems

Perspective

- ❑ Play with the transmission conditions
- ❑ Parallelization
- ❑ Introduce *a-priori* information in inversion

Merci beaucoup

- The FETI-DPEM2-full method has been applied to the inversion algorithm
- In order to accelerate the inversion process we have studied some implementation issues of the proposed method such as the initialization of the FETI solution and the stopping criterion for GMRES
- The numerical code has been verified on some classical inversion problems of the Fresnel database

FUTURE:

- Improving the transmitting condition between subdomains
- Parallelization
- Level-set approach

Inside air

$f = 12$ GHz

Domain of $\approx 9\lambda \times 7\lambda \times 4\lambda$

The wavelength $\lambda \approx 0.025$ m

Excitation

Source moving $\theta_s = 0^\circ \div 180^\circ$

Receivers are $\theta_r = 0^\circ \div 360^\circ$

Polarization E_ϕ

Antenna step 2°

Scatterers

4 ellipsoids

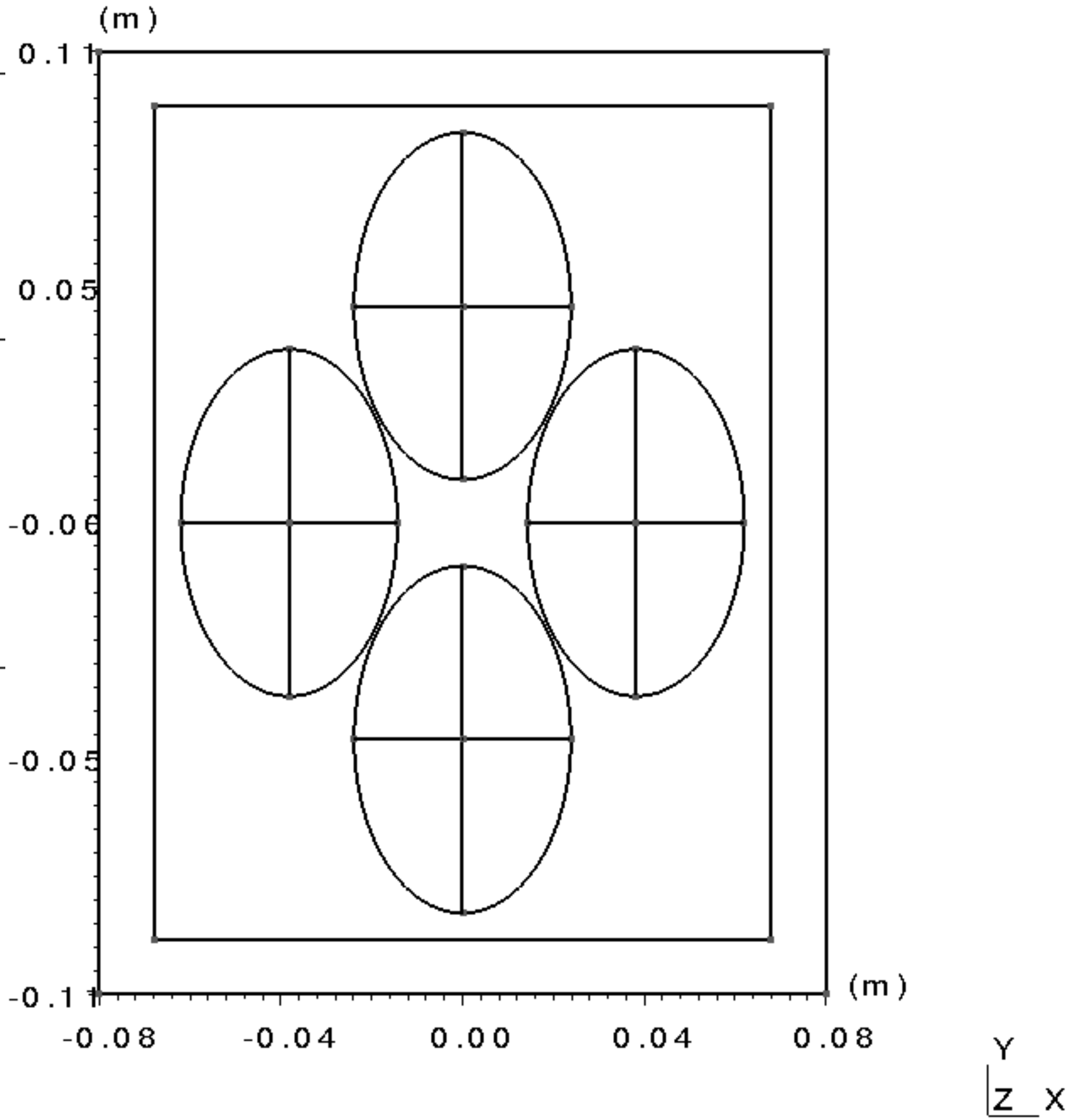
$\varepsilon_r^1 = 2.0$

$\varepsilon_r^2 = 3.0$

$\varepsilon_r^3 = 4.0$

$\varepsilon_r^4 = 5.0$

Taille $\approx 2\lambda \times 3.2\lambda \times 2\lambda$

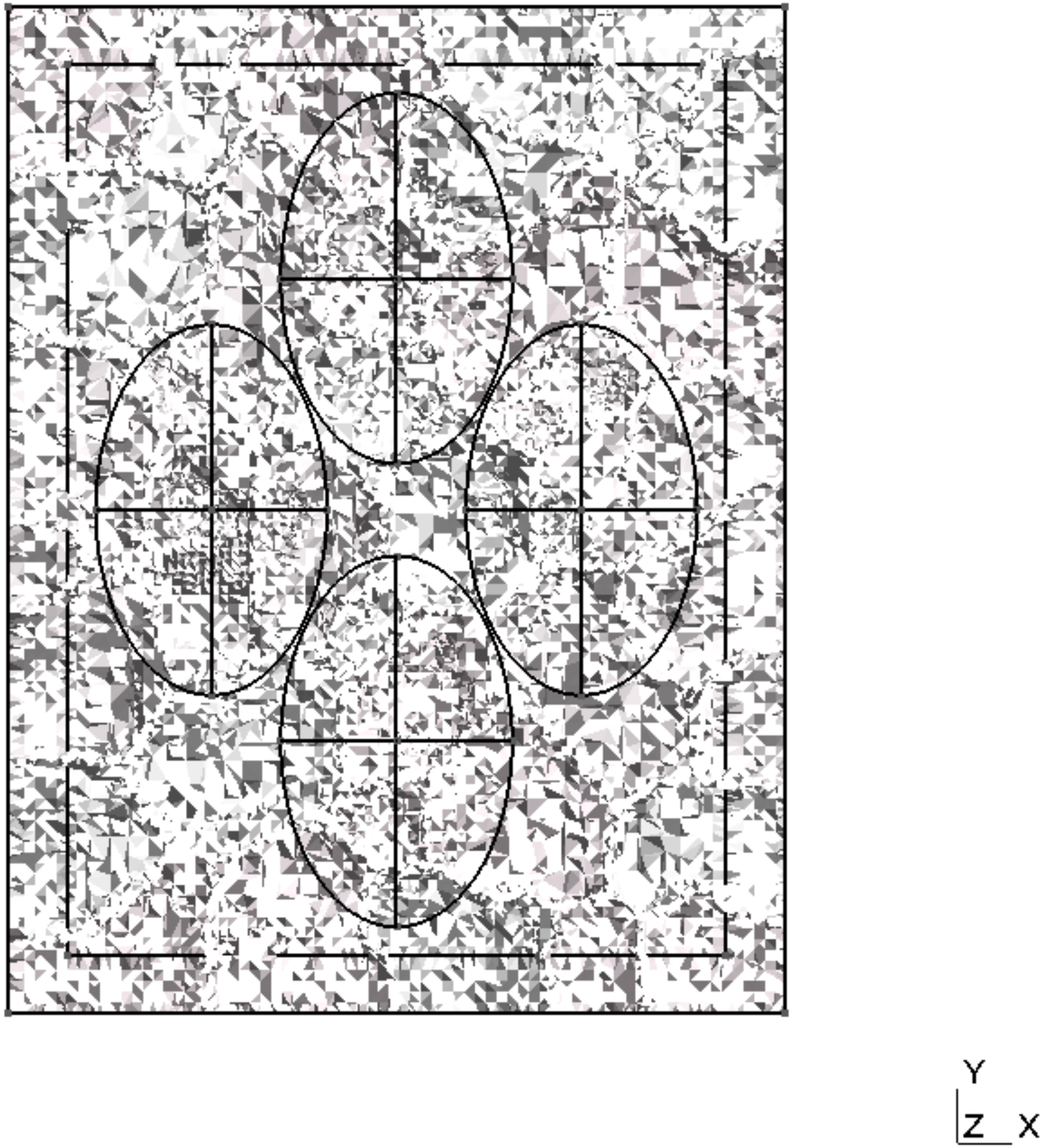


k = 13 points per wavelength

The limit of FEM: $\approx$ 1 200 000 unknowns

Total number of unknowns: **3 145 899**

Number of tetrahedras: 2 789 530
Number of points: 445 785



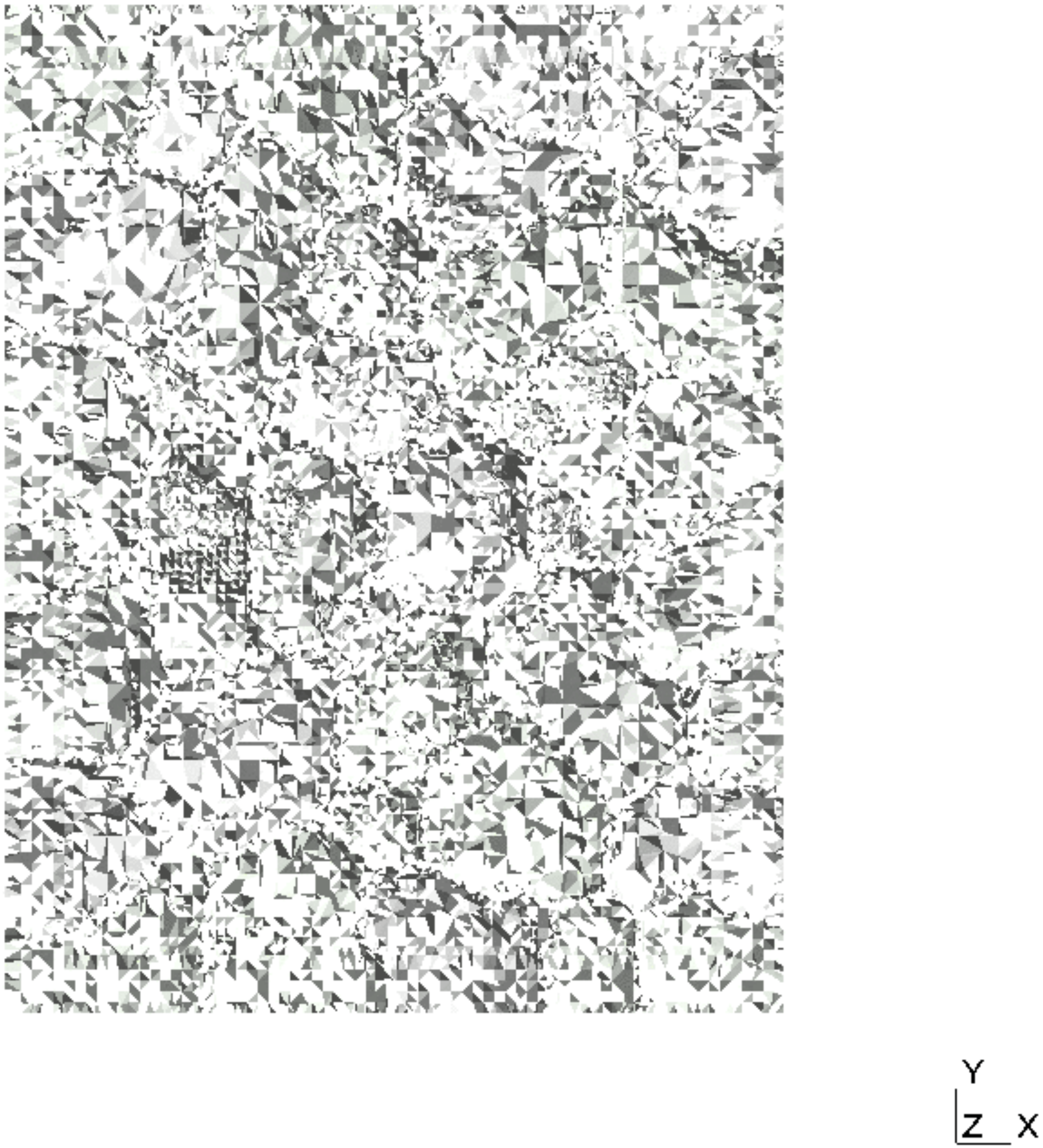
177 subdomains

Approximate size of one subdomain: 18 000

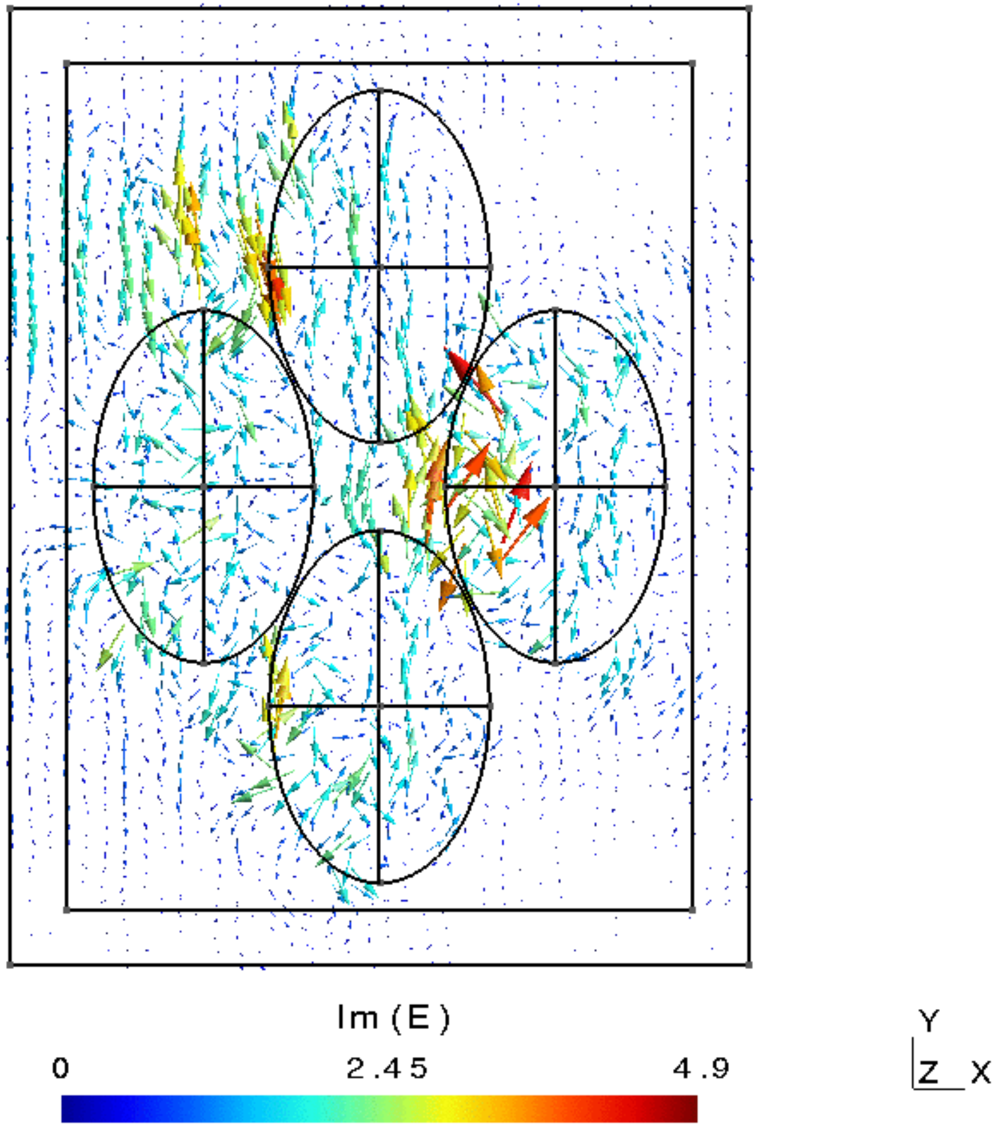
Size of interface problem (number of λ_r): 684 084

Total number of corner-edges: 18 622

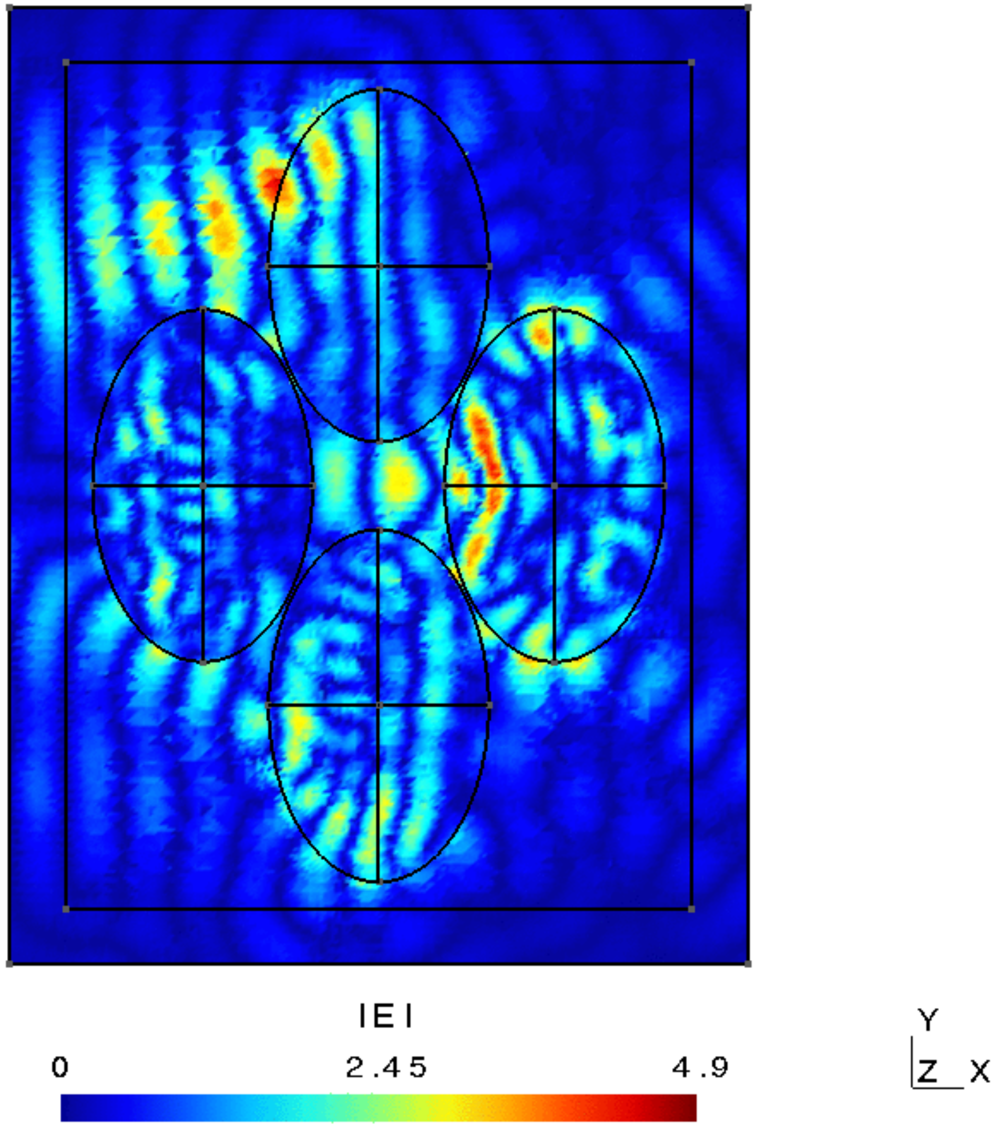
Total number of λ_c: 52 818



$Im(E^{sc})$



$Abs(E^{sc})$



Test case

Frequency $f = 7GHz$
Number of unknowns: 1 205 329

Memory

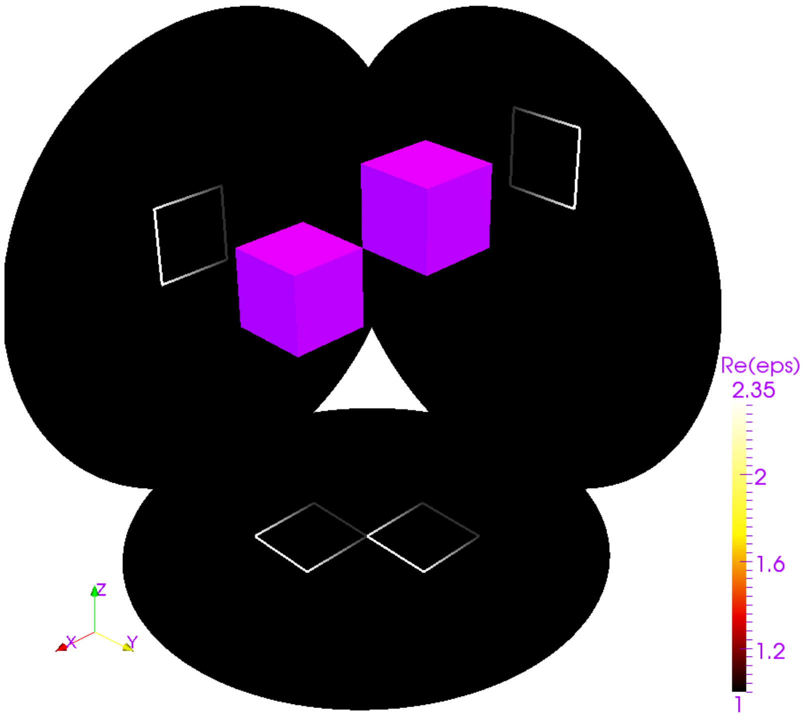
FEM	FETI
$\approx 1\ 200\ 000$	$\approx 3\ 500\ 000$

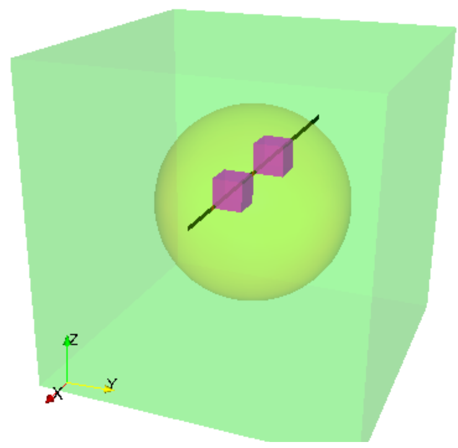
Time

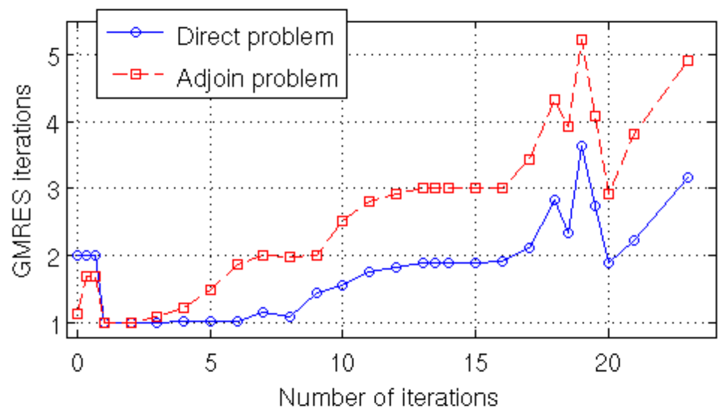
FEM LU dec $LUx = b$	FETI $(LU)^i$ $F_{E_cE_c}$ cst $F_{E_cE_c}$ dec 15 iters = 150 $F * \lambda$
2 568 sec	838 sec

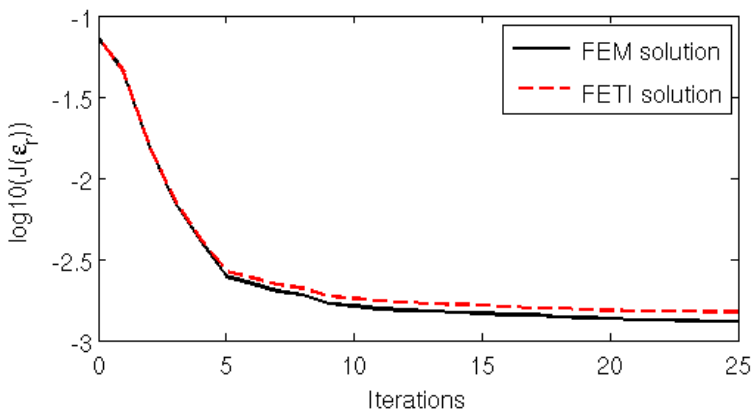
- In the framework of the tests based on the modeling the super-ellipsoids we studied the efficiency of the FETI-DPEM2-full method

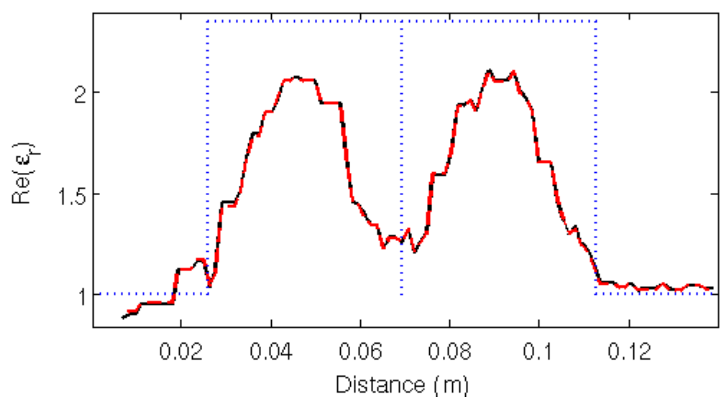
- Pros: Memory requirement, time of computation, natural parallelization
- Cons: Multi-source calculation


Re(eps)
2.35
2
1.6
1.2
1
Z
X
Y

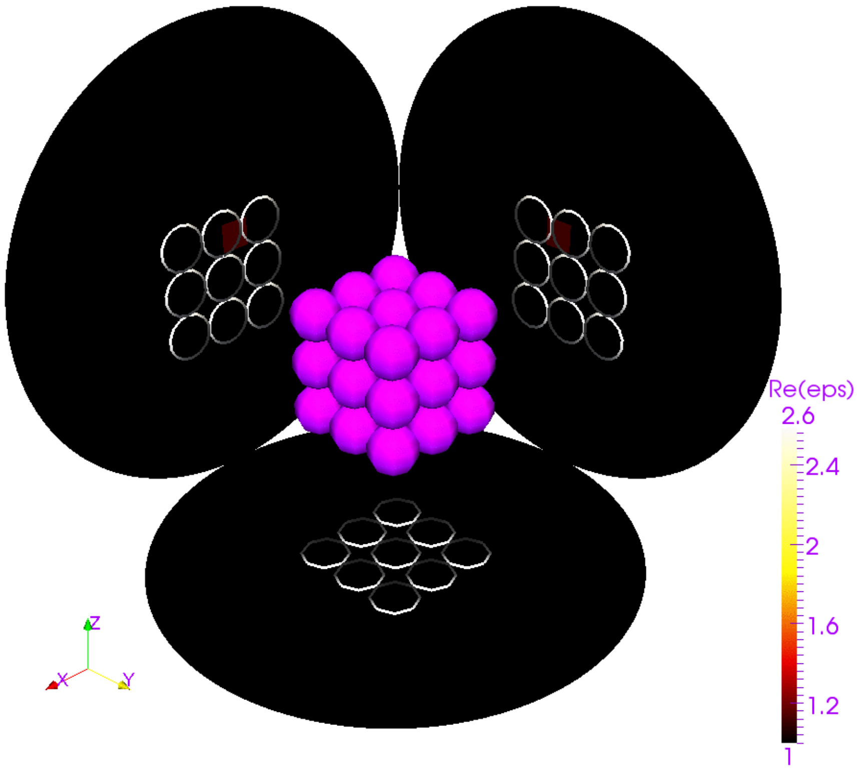

Z
X
Y

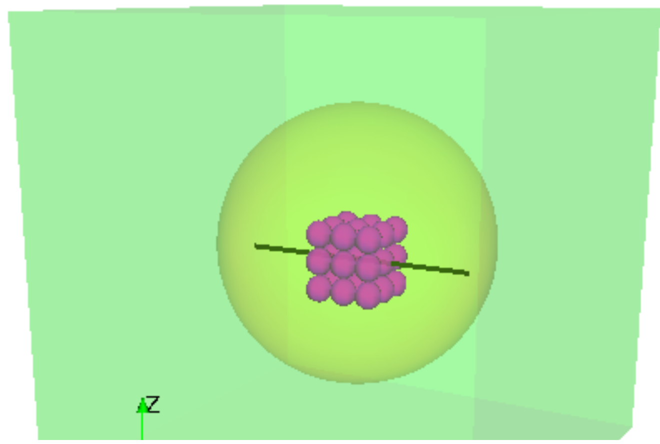

Direct problem
Adjoin problem
GMRES iterations
Number of iterations

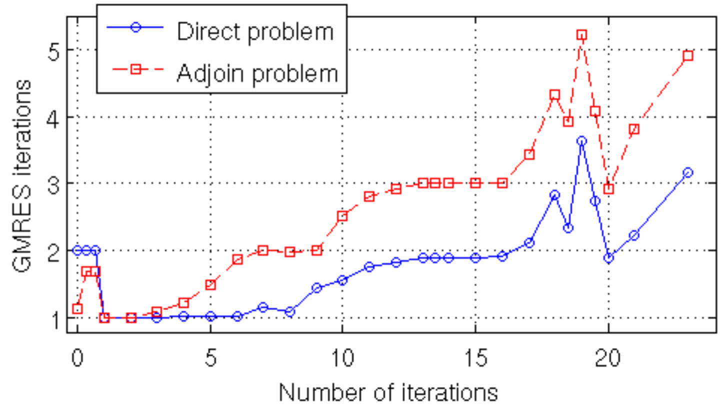

FEM solution
FETI solution
log10(J(ε_r))
Iterations

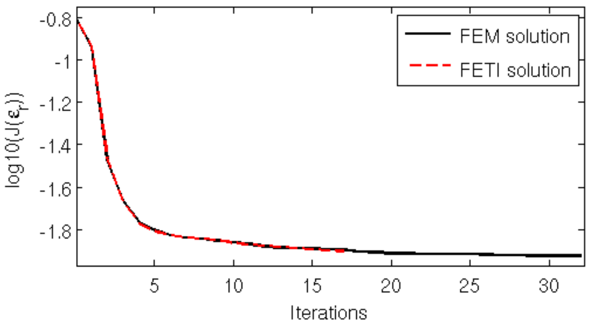

Re(ε_r)
Distance (m)

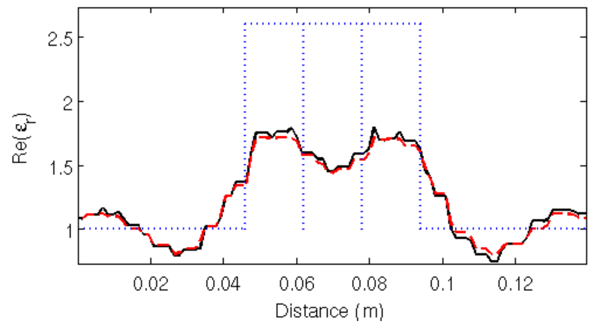
FETI. 2 points

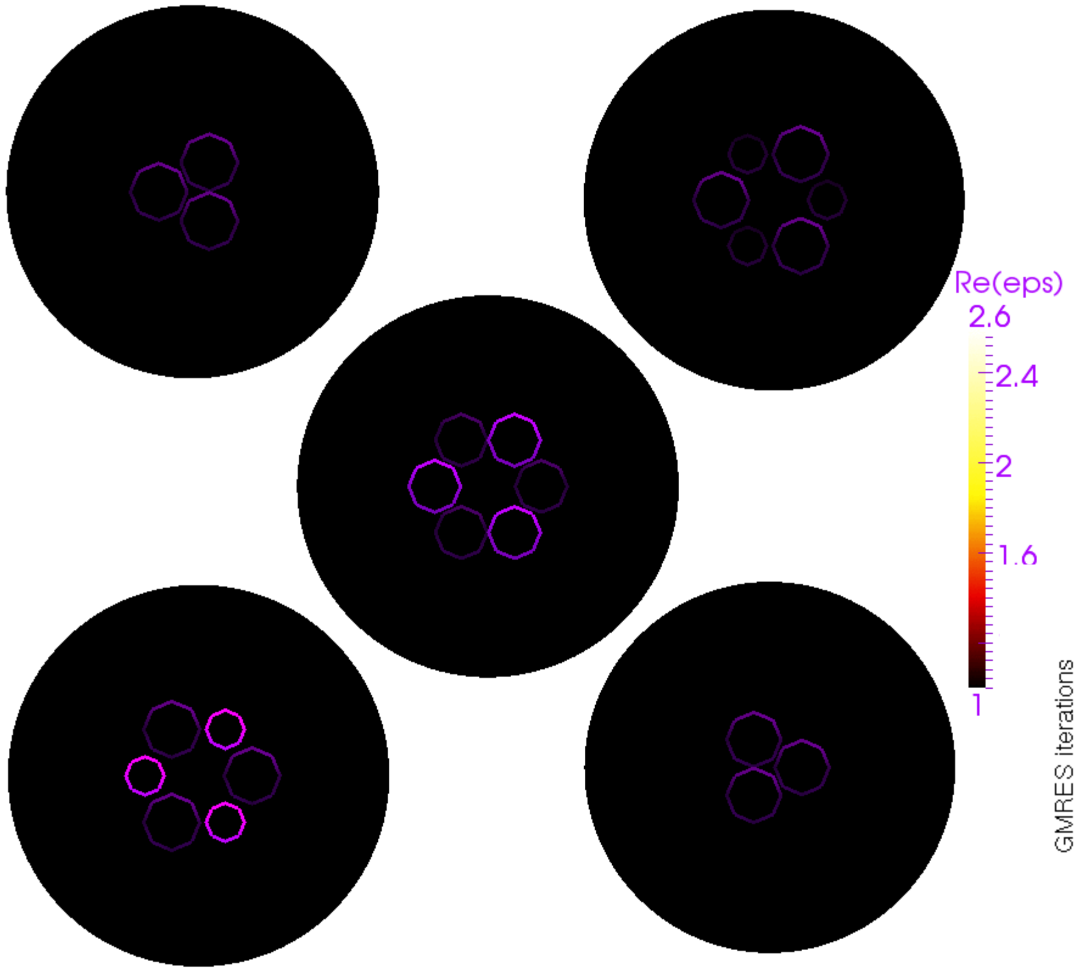


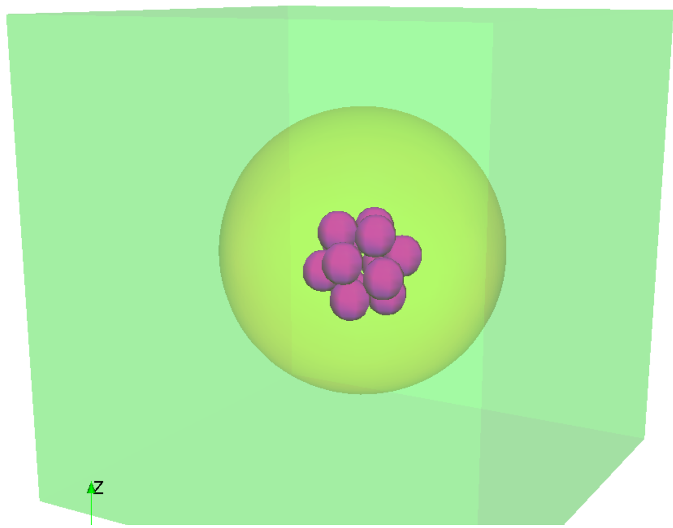


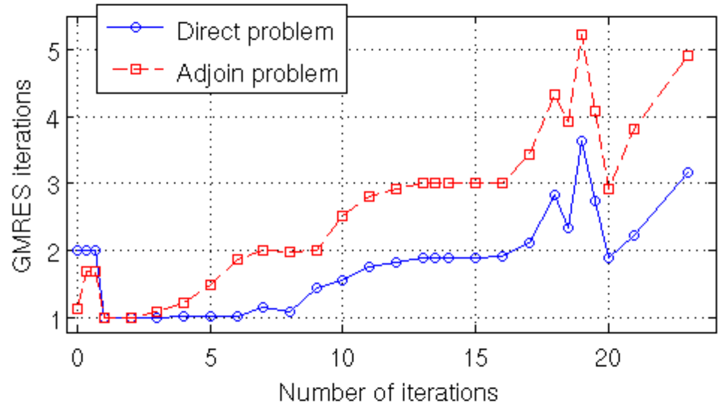


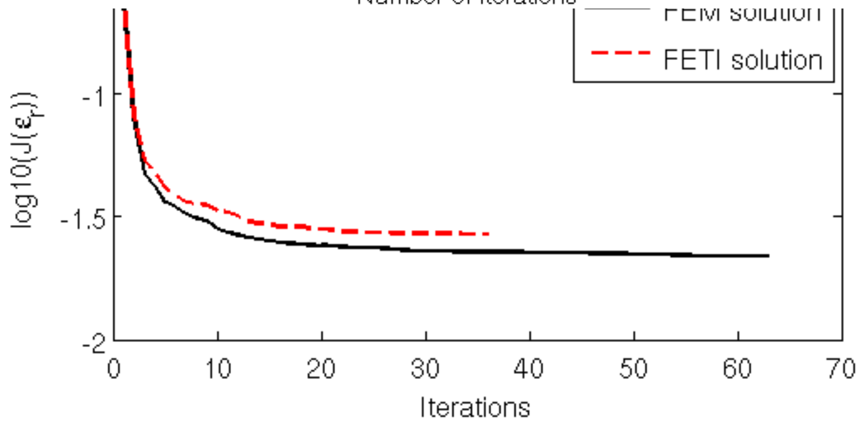





Re(eps)
2.6
2.4
2
1.6
1

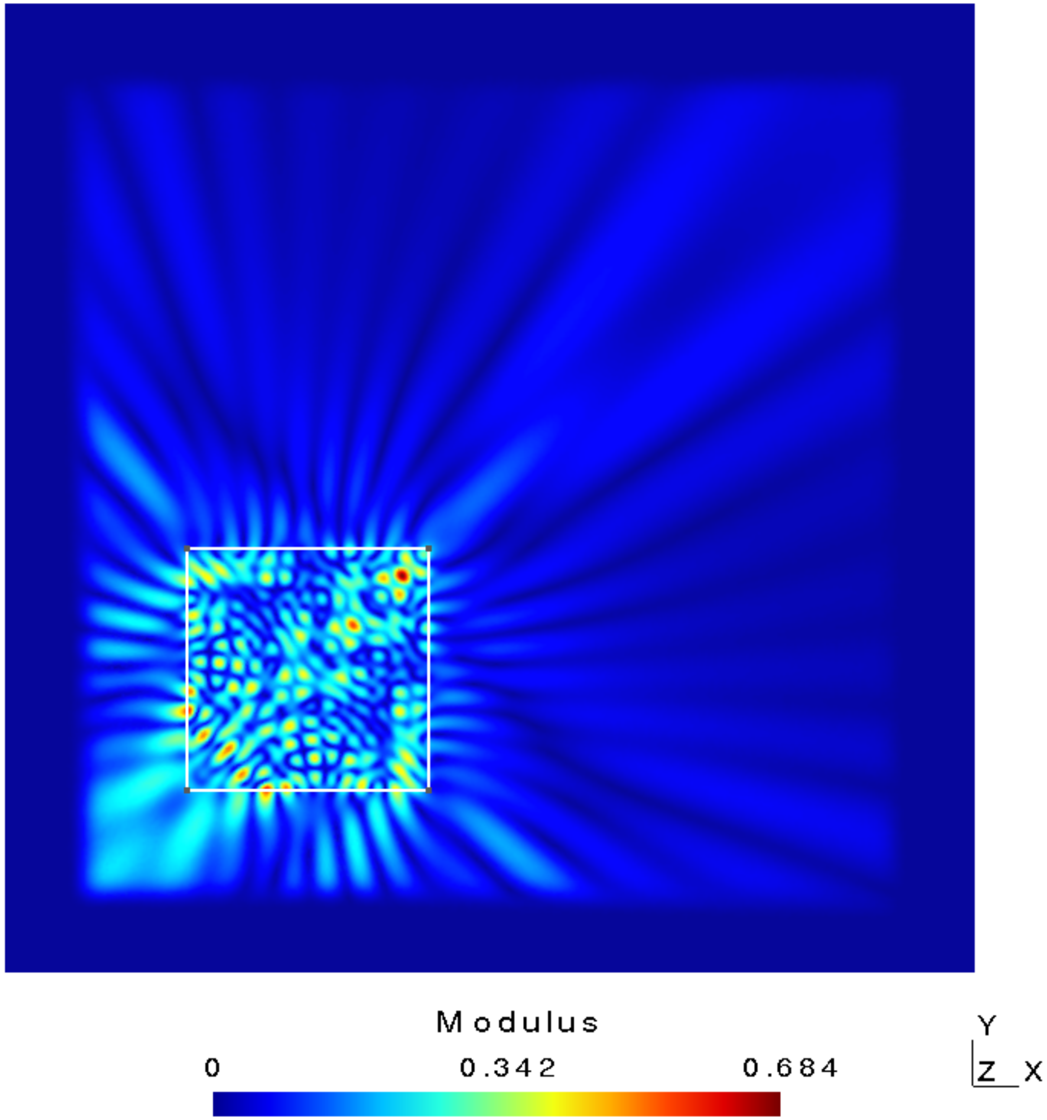

z

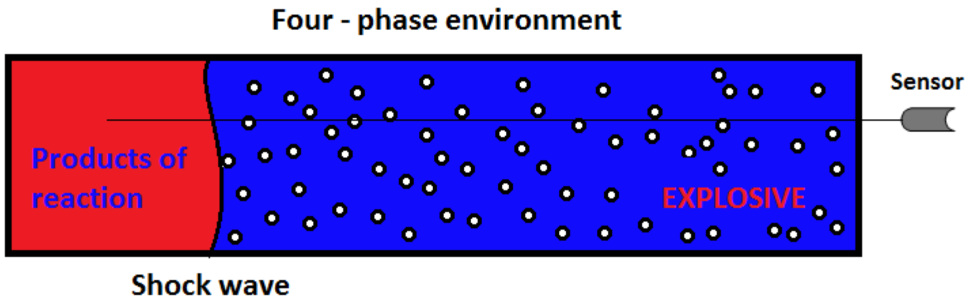

Direct problem
Adjoin problem
GMRES iterations
Number of iterations
0
5
10
15
20
1
2
3
4
5


FEM solution
FETI solution
log10(J(ε_r))
Iterations
-1
-1.5
-2
0
10
20
30
40
50
60
70

Incident field E^{inc}

Total field E^{tot}





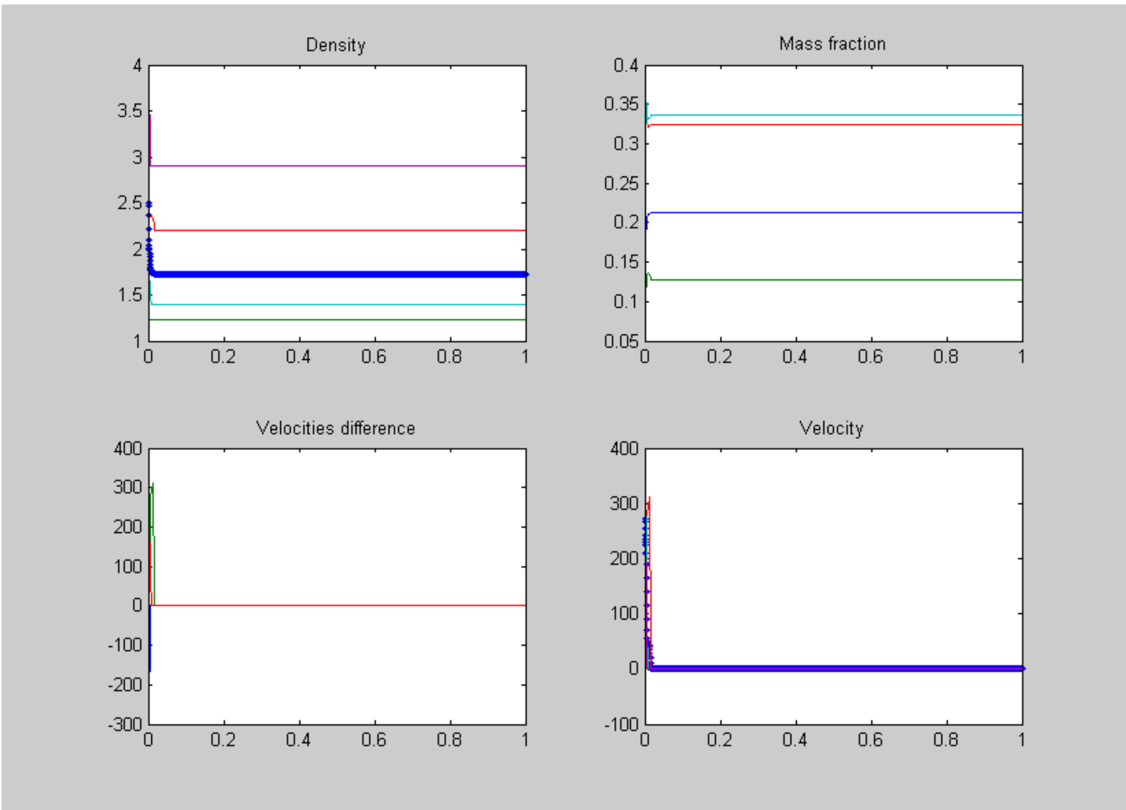
Four – phase environment:

Volume fraction:

- Air 10 - 50%
- Glass 1 - 10%
- Emulsion > 50%
- Products of reaction

Mass fraction:

- Air 1 - 5%
- Glass 10 - 50%
- Emulsion > 50%
- Products of reaction



Fifteen equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \left(\rho u^2 + \rho^2 E_\rho + \rho w E_w \right)}{\partial x} = 0$$

$$\frac{\partial \rho \alpha_i}{\partial t} + \frac{\partial \rho \alpha_i u}{\partial x} = 0, \; i = 1 \ldots 4$$

$$\frac{\partial \rho c_i}{\partial t} + \frac{\partial \left(\rho u c_i + \rho E_{w^i} \right)}{\partial x} = 0, \; i = 1 \ldots 4$$

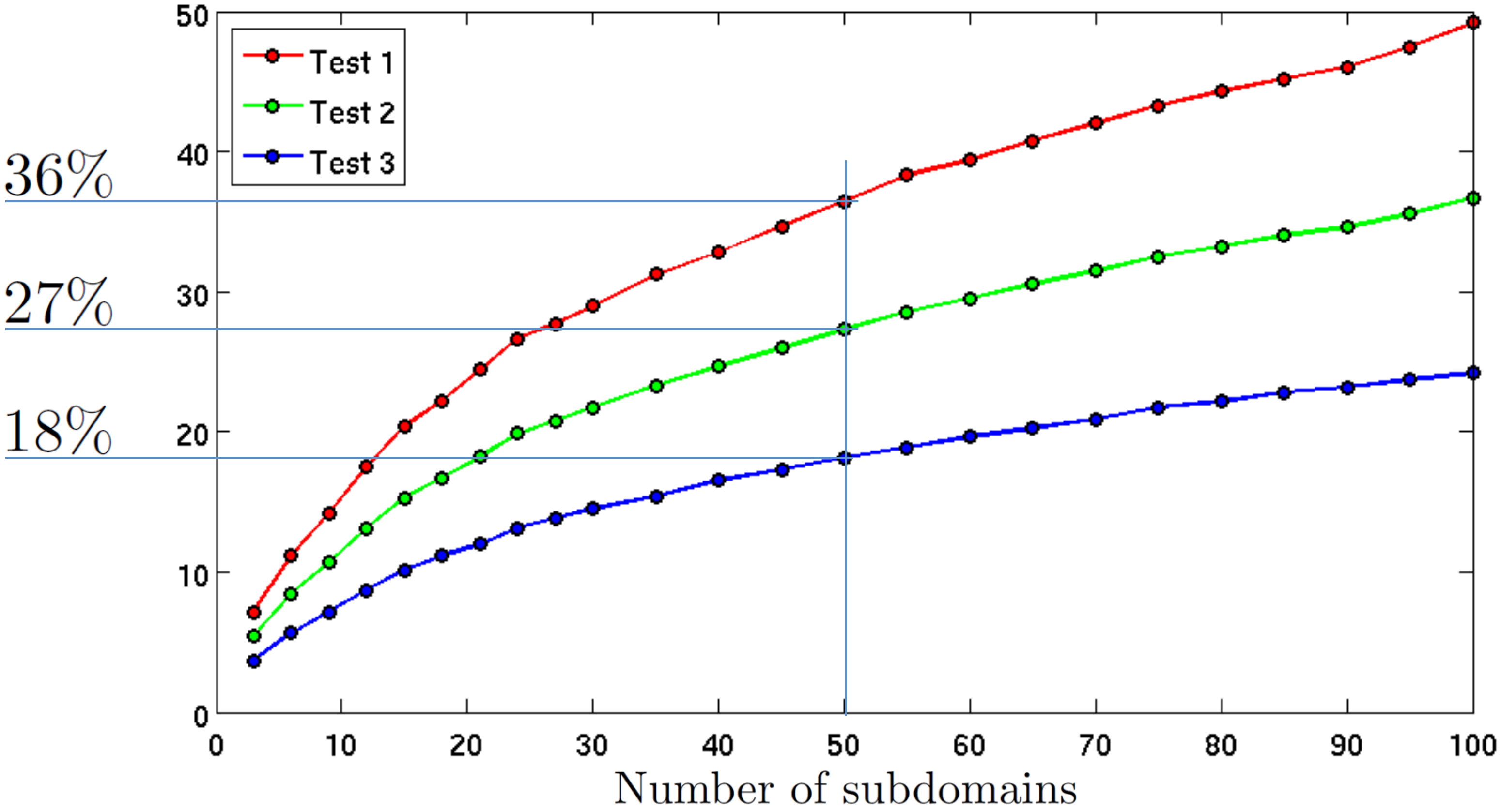
$$\frac{\partial w_j}{\partial t} + \frac{\partial \left(u w_j + E_{c_j} \right)}{\partial x} = 0, \; j = 1 \ldots 3$$

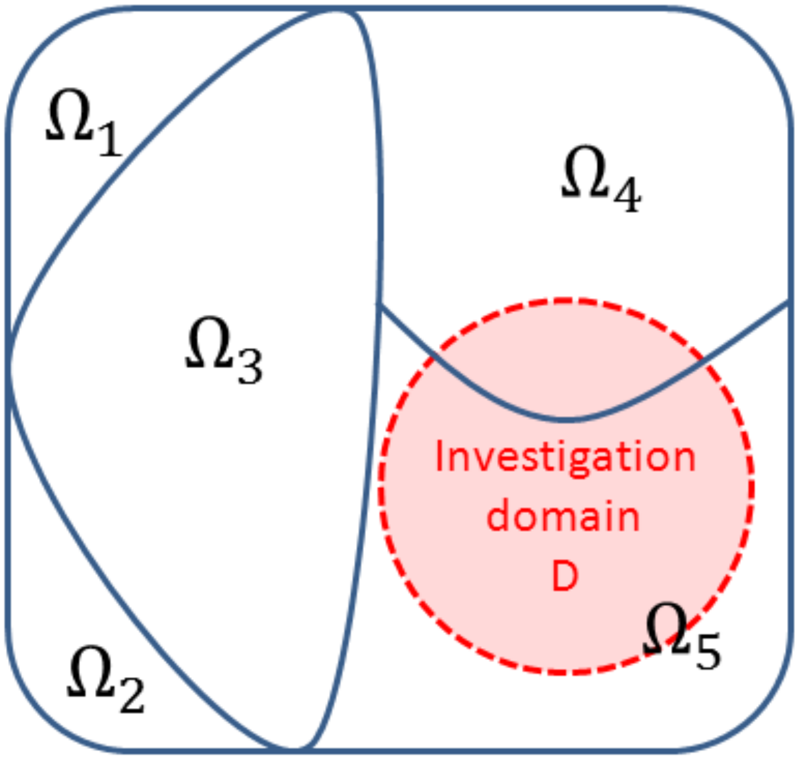
$$\frac{\partial \rho S}{\partial t} + \frac{\partial \rho S u}{\partial x} = 0$$

$$\frac{\partial \rho \left(E + \frac{u^2}{2} \right)}{\partial t} + \frac{\partial \left(\rho u \left(E + \frac{u^2}{2} + \rho E_\rho + w_l E_{w_l} \right) + \rho E_{c_l} E_{w_l} \right)}{\partial x} = 0$$

Numerical results of 3D task

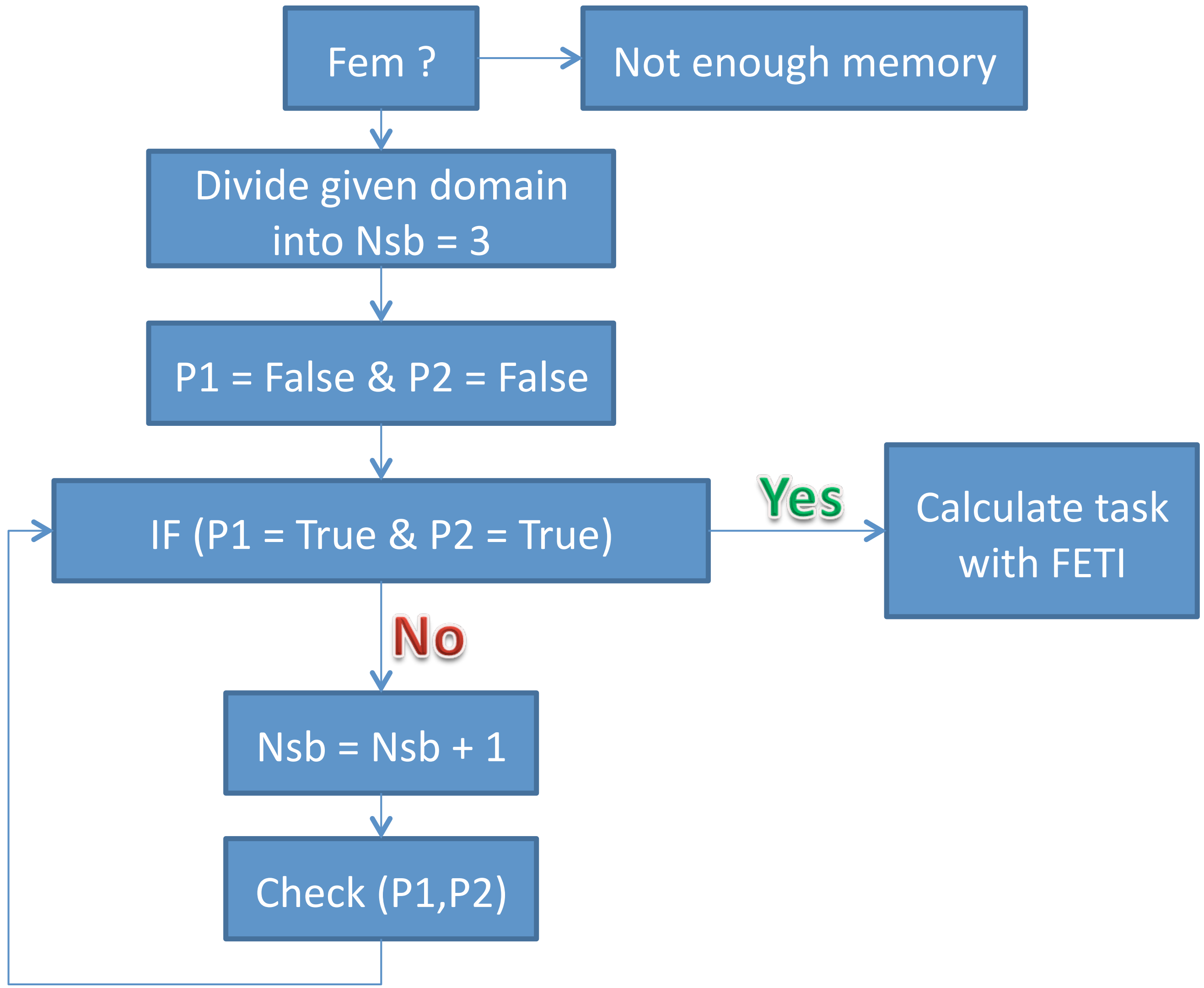
Test	Number of unknowns
1	219,283
2	1,013,587
3	1,844,154

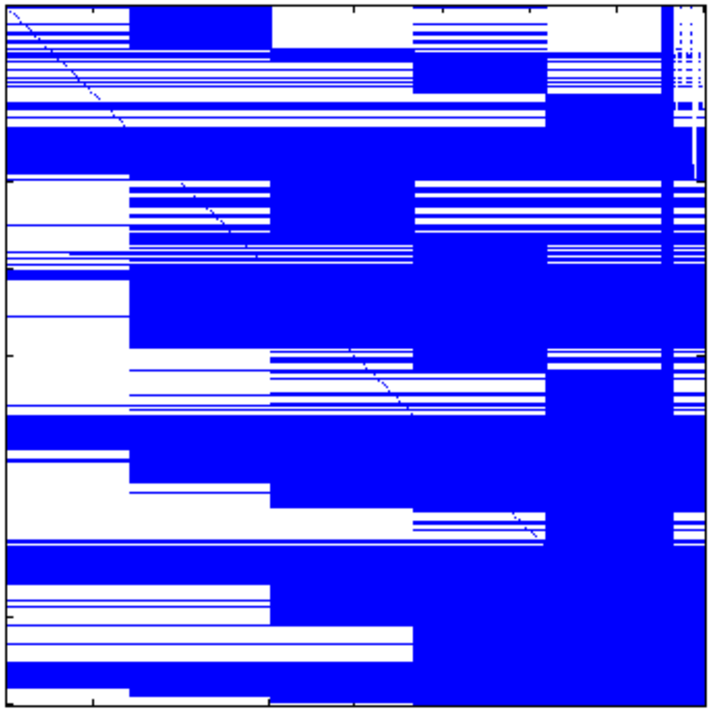



Ω_1
Ω_4
Ω_3
Investigation
domain
D
Ω_5
Ω_2

Scheme of FETI–idea…

P1 – if there is enough memory for IP
P2 – if there is enough memory for **1** inverse matrix

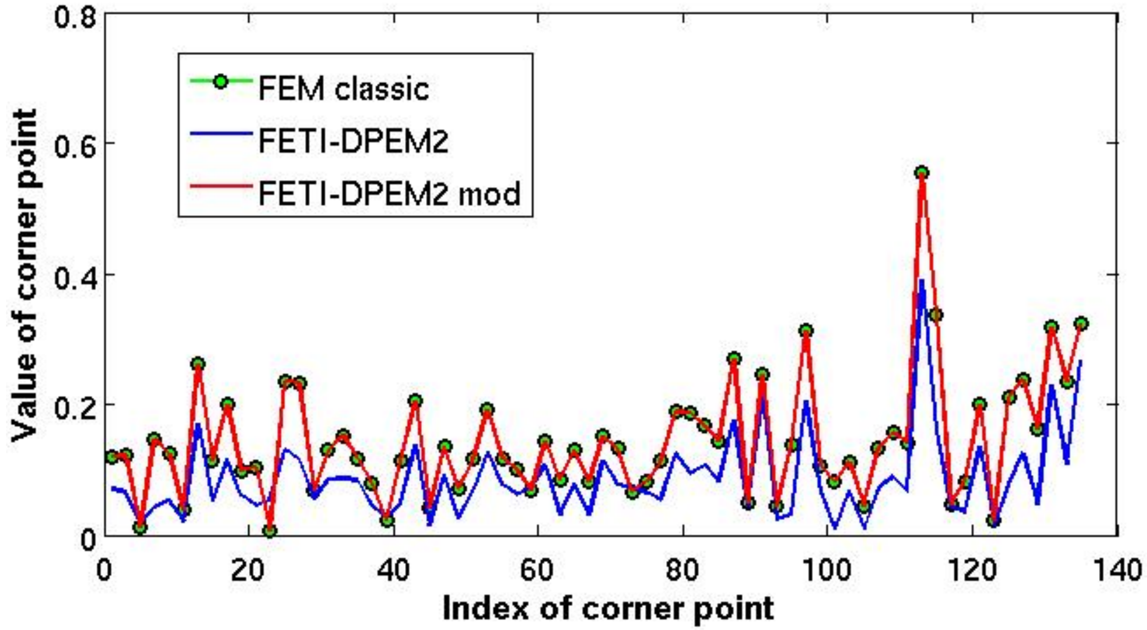




Numerical results. Physical statement of task.

Scheme of life …

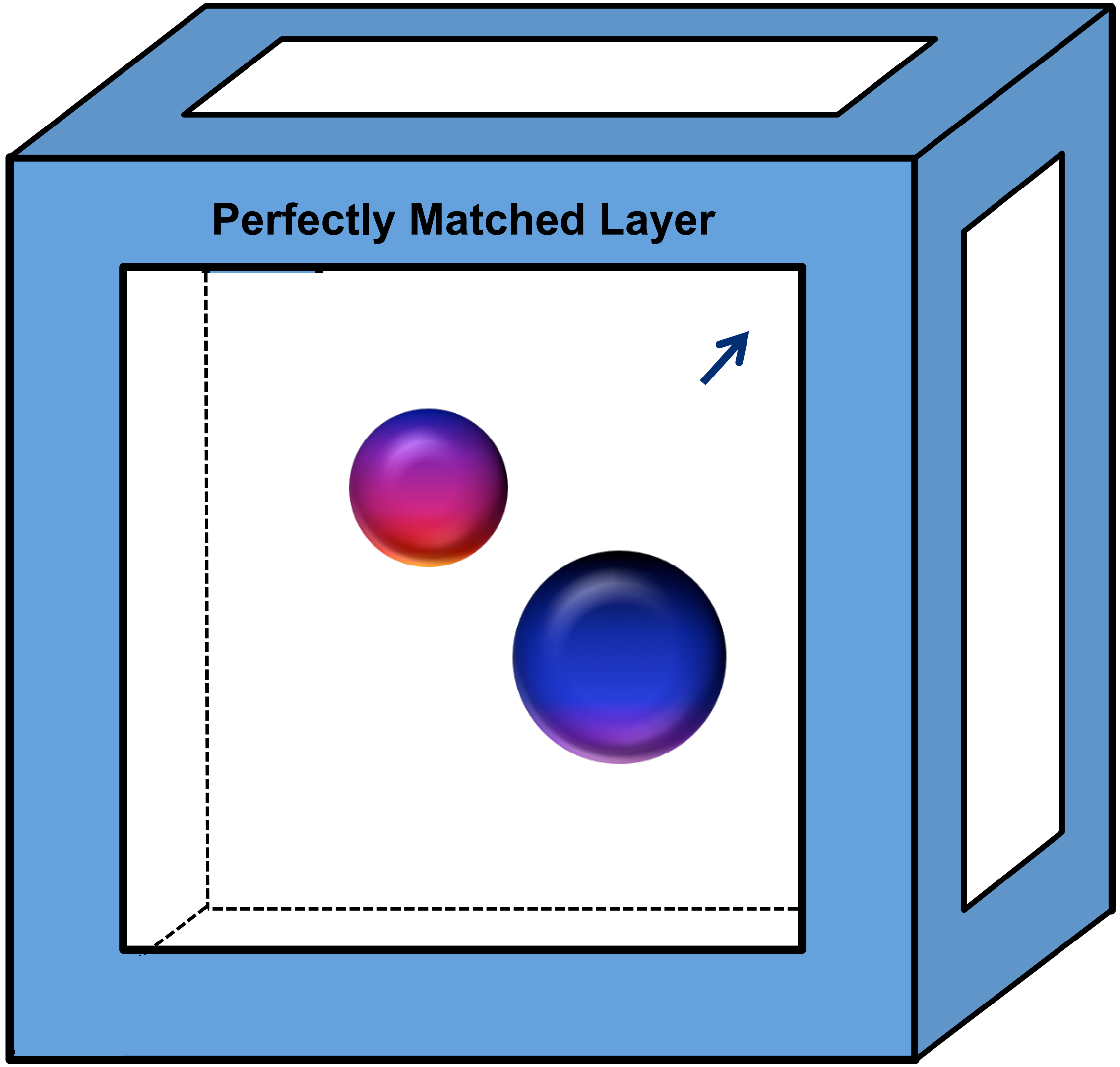
Numerical results. Physical statement of task.

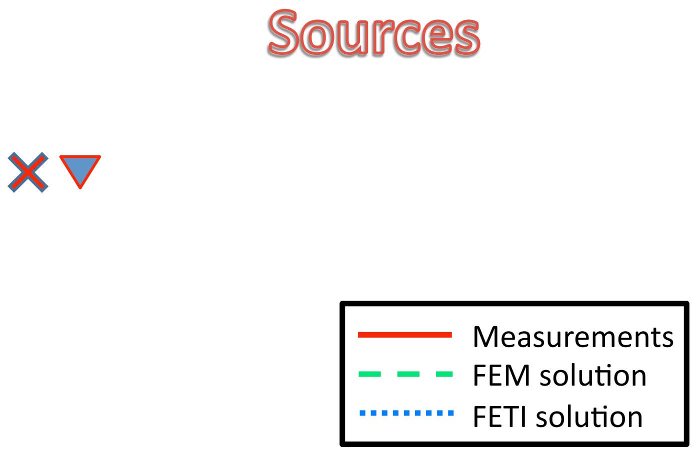


Rsphere = 0.075

Position_dipole = (0,0,0)

Direction = (1,1,0)




Sources
Measurements
FEM solution
FETI solution

❖ Helmholtz equation

$$-\mathrm{div}\left(\frac{1}{\mu_r(\vec{r})}\mathrm{grad}\, E(\vec{r})\right) - k_0^2\varepsilon_r(\vec{r})E(\vec{r}) = jk_0 Z_0 J(\vec{r}), \text{ in } \Omega$$

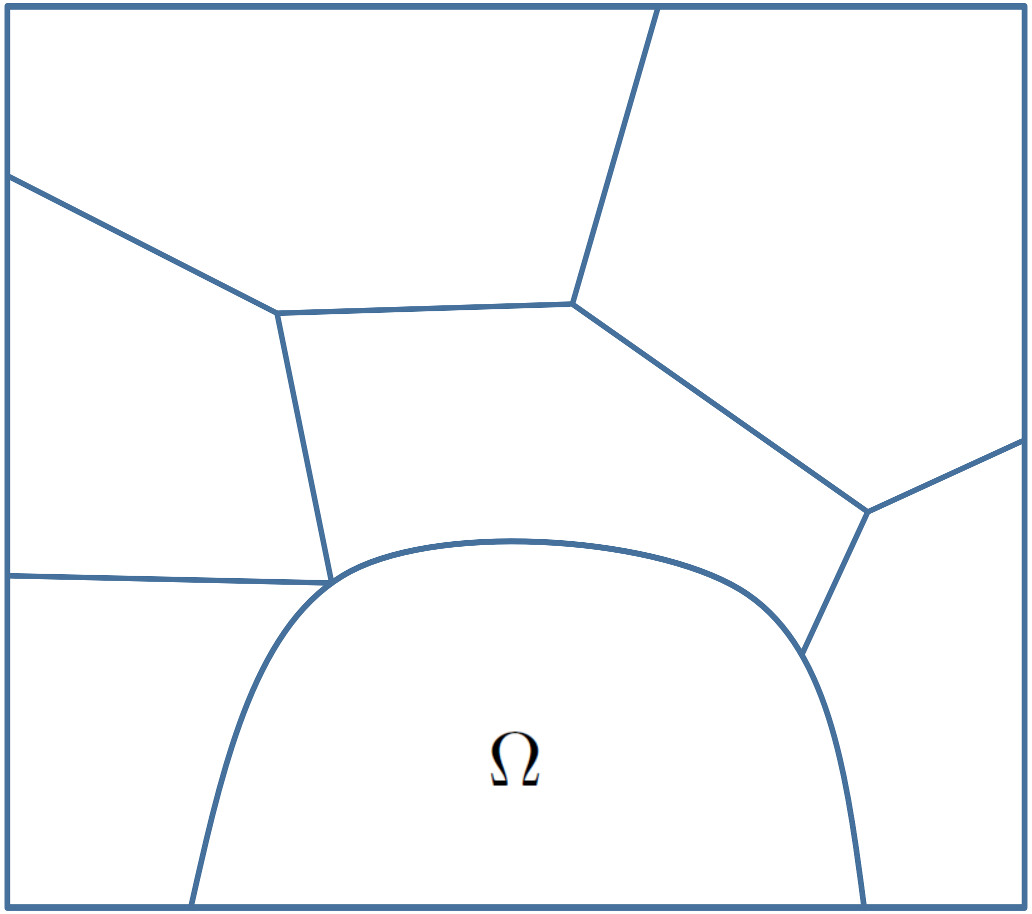
❖ Radiation boundary condition

$$\frac{1}{\mu_r(\vec{r})}\frac{\partial E(\vec{r})}{\partial n} - jk_0 E(\vec{r}) = 0, \quad \text{on } \Sigma$$

➢ Finite-elements discretization

$$\mathbb{K}\mathbf{E} = \mathbf{f}$$

$$\Sigma = \partial\Omega$$



- Helmholtz equation

$$-\mathrm{div}\left(\frac{1}{\mu_r(\vec{r})}\mathrm{grad}\, E(\vec{r})\right) - k_0^2\varepsilon_r(\vec{r})E(\vec{r}) = jk_0Z_0J(\vec{r}),\ \text{in } \Omega$$

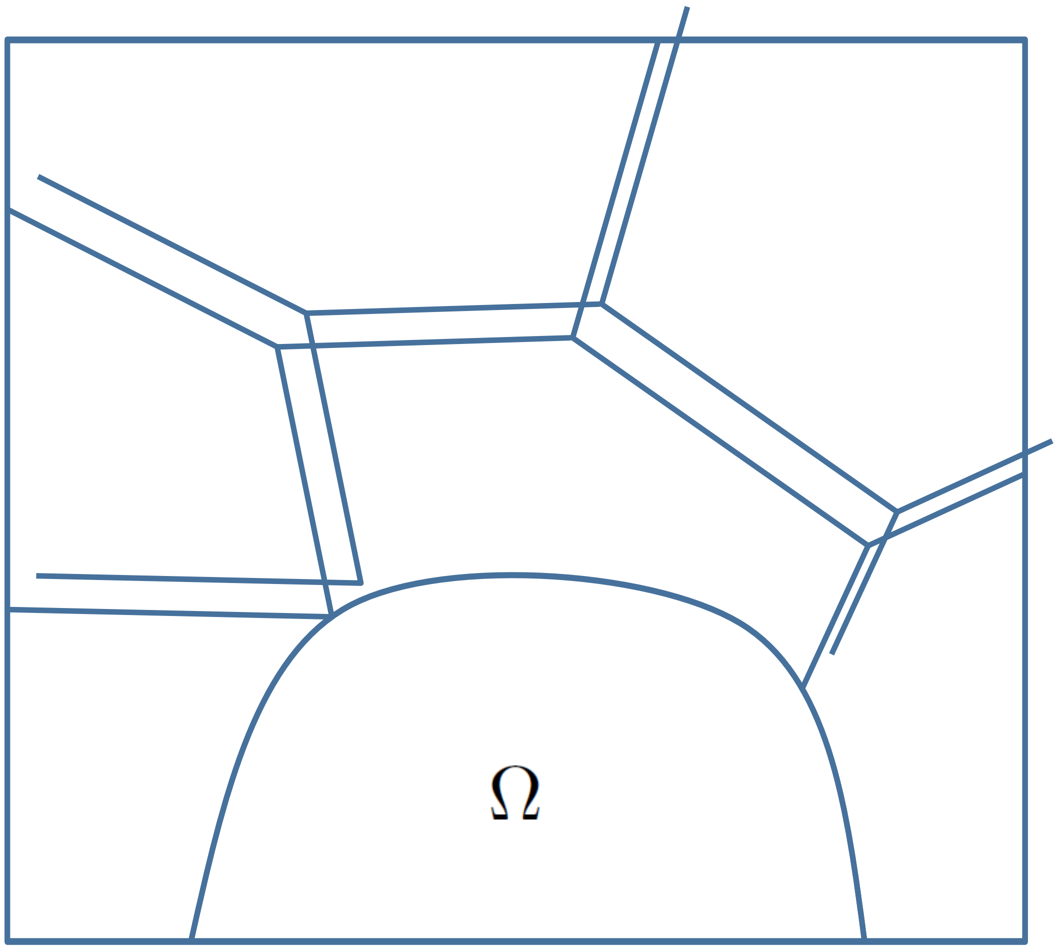
- Radiation boundary condition

$$\frac{1}{\mu_r(\vec{r})}\frac{\partial E(\vec{r})}{\partial n} - jk_0E(\vec{r}) = 0,$$
$$\text{on } \Sigma$$

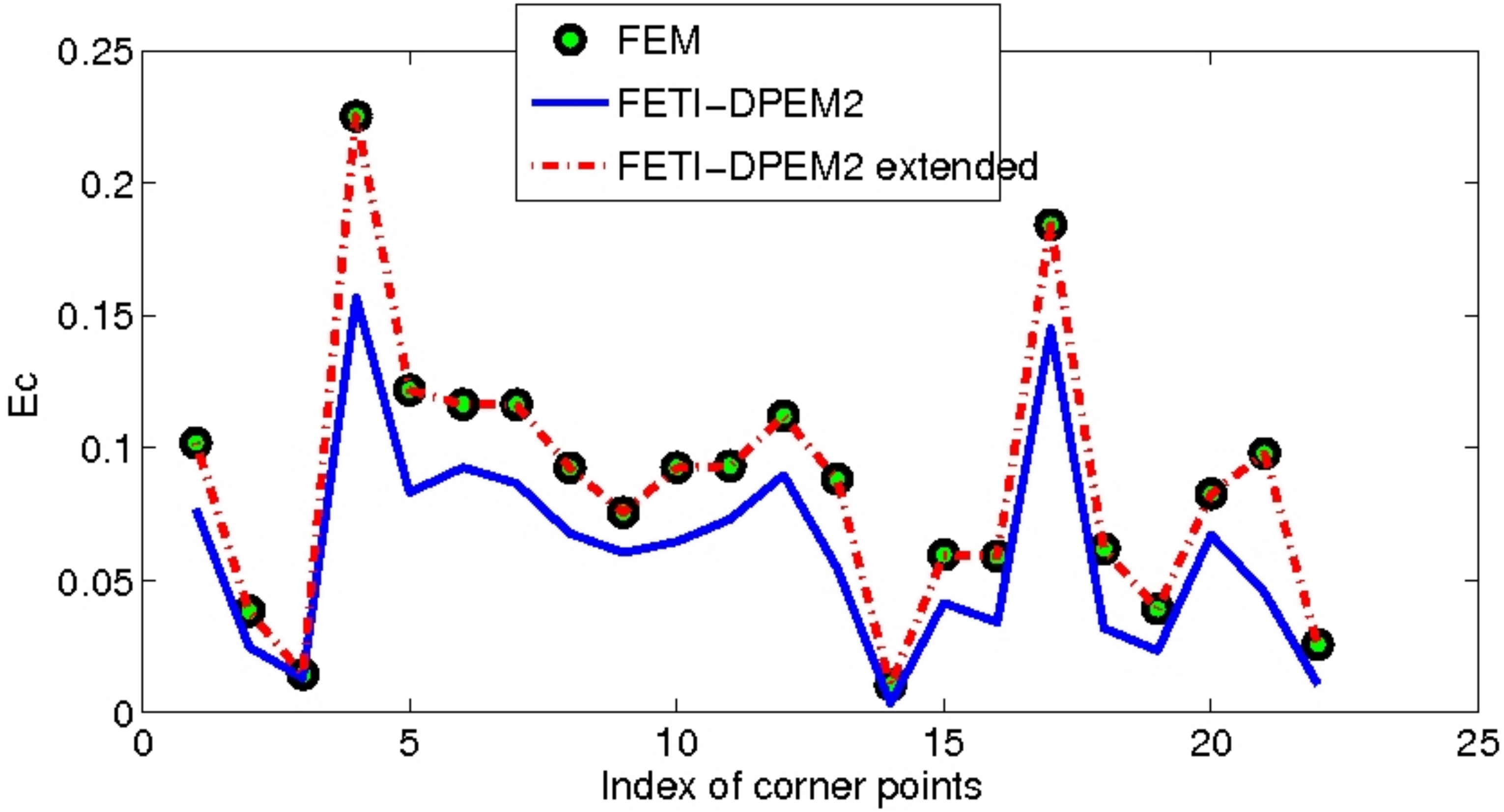
- Finite-elements discretization

$$\mathbb{K}\mathbf{E} = \mathbf{f}$$

$$\Sigma = \partial\Omega$$



Numerical results. Physical statement of task.



Scattering problem in 2D and 3D cases

2D Helmholtz equation:

$$\text{div}\left(\frac{1}{\mu_r^{\text{tot}}}\text{grad}\,\mathbf{E}^{\text{SC}}\right) + k_0^2\varepsilon_r^{\text{tot}}\mathbf{E}^{\text{SC}} = \mathbf{J}^{\text{SC}} \text{ in } \Omega$$

where:

$$\mathbf{J}^{\text{SC}} = -\text{div}\left(\left[\frac{1}{\mu_r^{\text{tot}}} - \frac{1}{\mu_r^{\text{inc}}}\right]\text{grad}\,\mathbf{E}^{\text{inc}}\right) - k_0^2\left[\varepsilon_r^{\text{tot}} - \varepsilon_r^{\text{inc}}\right]\mathbf{E}^{\text{inc}}$$

BC:

$$\frac{1}{\mu_r}\frac{\partial \mathbf{E}^{\text{SC}}}{\partial n} - jk_0\mathbf{E}^{\text{SC}} = 0 \text{ on } \Sigma$$

3D “rot-rot” equation:

$$\nabla \times \left(\frac{1}{\mu_r^{\text{tot}}}\nabla \times \mathbf{E}^{\text{SC}}\right) - k_0^2\varepsilon_r^{\text{tot}}\mathbf{E}^{\text{SC}} = \mathbf{J}^{\text{SC}}$$

where:

$$\mathbf{J}^{\text{SC}} = -\nabla \times \left(\left[\frac{1}{\mu_r^{\text{tot}}} - \frac{1}{\mu_r^{\text{inc}}}\right]\nabla \times \mathbf{E}^{\text{inc}}\right) + k_0^2\left[\varepsilon_r^{\text{tot}} - \varepsilon_r^{\text{inc}}\right]\mathbf{E}^{\text{inc}}$$

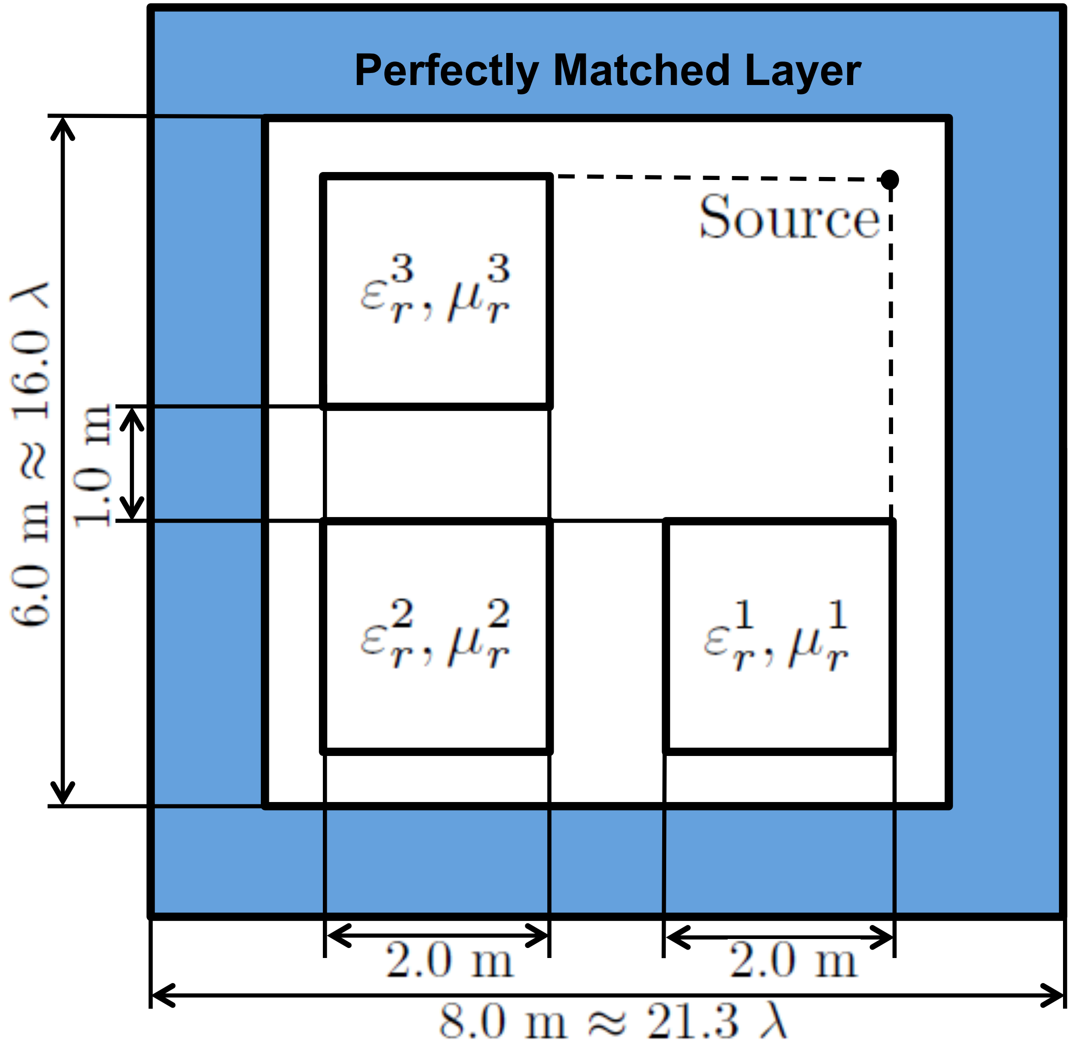
BC:

$$\vec{n} \times \left(\nabla \times \mathbf{E}^{\text{SC}}\right) + jk_0\vec{n} \times \vec{n} \times \mathbf{E}^{\text{SC}} = 0 \text{ on } \Sigma$$

Inside air
$\varepsilon_0 = 8.85 \cdot 10^{-12}\mathrm{F} \cdot \mathrm{m}^{-1}$
$\mu_0 = 4\pi \cdot 10^{-7}\mathrm{H} \cdot \mathrm{m}^{-1}$
1 source
$f = 800$ MHz

The wavelength $\lambda \approx 0.37m$
Domain of $\approx 21\lambda \times 21\lambda$

3 different areas
$\varepsilon_r^1, \varepsilon_r^2, \varepsilon_r^3$
and
$\mu_r^1 = \mu_r^2 = \mu_r^3 = 1.0$



Numerical results. Physical statement of 3D task.

Inside air

$\varepsilon_0 = 8.85 \cdot 10^{-12} \mathrm{F} \cdot \mathrm{m}^{-1}$

$\mu_0 = 4\pi \cdot 10^{-7} \mathrm{H} \cdot \mathrm{m}^{-1}$

Polarization of dipole $(1.0, 1.0, 0.0)$

$f = 1$ GHz

The wavelength $\lambda \approx 0.3$ m

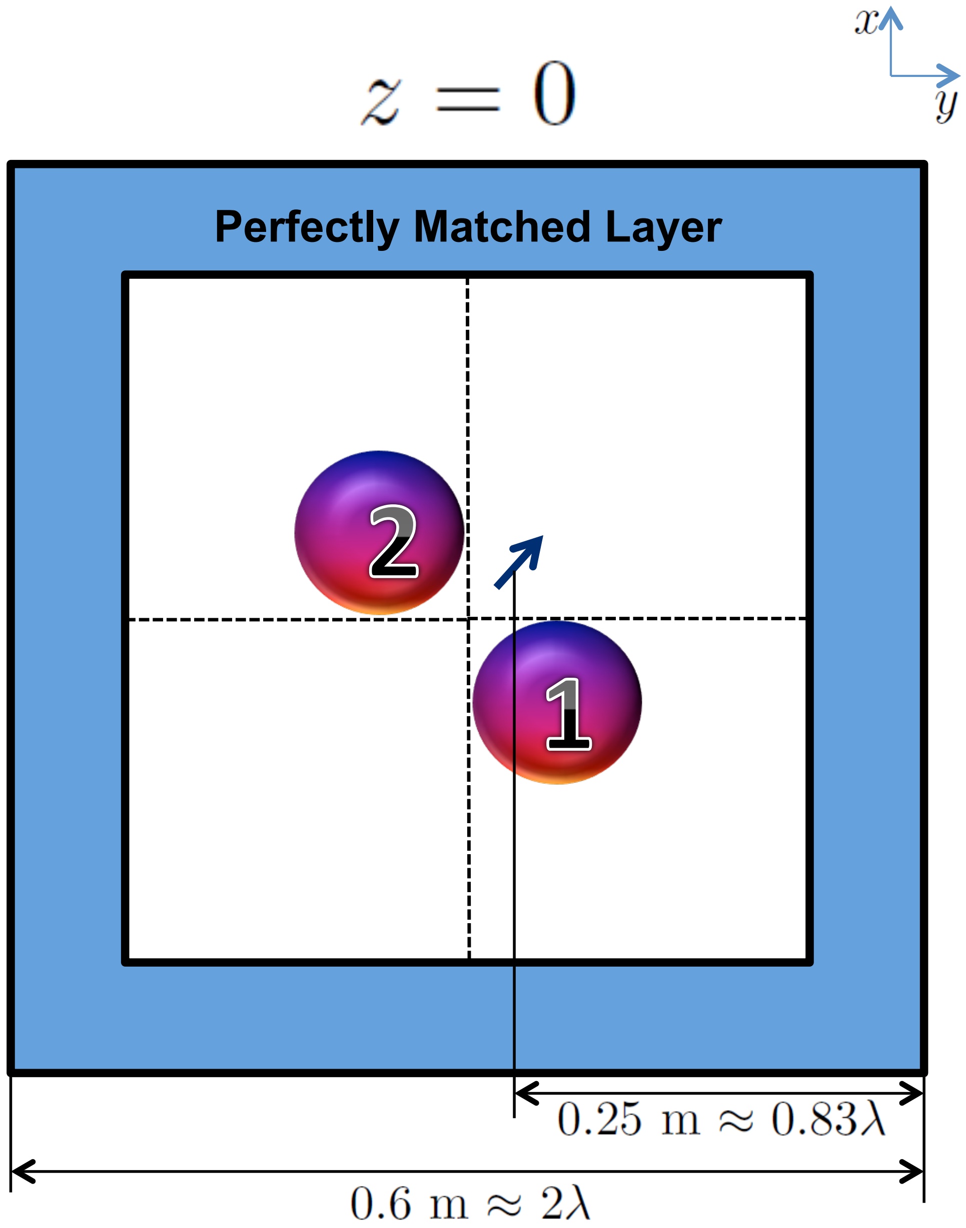
Domain of $\approx 2\lambda \times 2\lambda \times 2\lambda$

2 spheres

$\varepsilon_r^1 = 2.85$

$\varepsilon_r^2 = 5.00$

$R_1 = R_2 = 0.04\,\mathrm{m} \approx 0.13\lambda$



FETI-DPEM method

$$\begin{bmatrix} \mathbb{K}^i_{rr} & \mathbb{K}^i_{rc} \\ \mathbb{K}^i_{cr} & \mathbb{K}^i_{cc} \end{bmatrix} \begin{bmatrix} \mathbf{E}^i_r \\ \mathbf{E}^i_c \end{bmatrix} = \begin{bmatrix} \mathbf{f}^i_r \\ \mathbf{f}^i_c \end{bmatrix} - \begin{bmatrix} \lambda^i_r \\ \lambda^i_c \end{bmatrix}$$

FETI-DPEM2 classical:

$W_{rc} = W_{cr} = W_{cc} = 0$

$$\begin{bmatrix} \lambda_r \\ \lambda_c \end{bmatrix}^{i\to j} + \begin{bmatrix} \lambda_r \\ \lambda_c \end{bmatrix}^{j\to i} = - \begin{bmatrix} W_{rr} & W_{rc} \\ W_{cr} & W_{cc} \end{bmatrix}^{i\leftrightarrow j} \begin{bmatrix} E_r \\ E_c \end{bmatrix}^{j\to i}$$

M2 classical[2]:

$_{c\lambda_r} = \mathrm{F}_{\lambda_c E_c} = \mathrm{F}_{\lambda_c\lambda_c} = 0$

$$\begin{bmatrix} \mathrm{F}_{\lambda_r\lambda_r} & \mathrm{F}_{\lambda_r\mathbf{E}_c} & 0 \\ \mathrm{F}_{\mathbf{E}_c\lambda_r} & \mathrm{F}_{\mathbf{E}_c\mathbf{E}_c} & \mathrm{F}_{\mathbf{E}_c\lambda_c} \\ \mathrm{F}_{\lambda_c\lambda_r} & \mathrm{F}_{\lambda_c\mathbf{E}_c} & \mathrm{F}_{\lambda_c\lambda_c} \end{bmatrix} \begin{bmatrix} \lambda_r \\ \mathbf{E}_c \\ \lambda_c \end{bmatrix} = \begin{bmatrix} \mathrm{d}_{\lambda_r} \\ \mathrm{d}_{\mathbf{E}_c} \\ \mathrm{d}_{\lambda_c} \end{bmatrix}$$

[2] M.-F. Xue and J.-M. Jin *Nonconformal FETI-DP Methods for Large-Scale Electromagnetic Simulation*. IEEE, Transactions on Antennas and Propagation, Vol. 60, Sept. 2012

[1] I. Voznyuk, H. Tortel and A. Litman *Scattered field computation with an extended FETI-DPEM2 method*. Progress In Electromagnetics Research, 2013 (to appear)

FETI-DPEM2 classical:

FETI-DPEM2 modificated:

$$\begin{bmatrix} F_{\lambda_r\lambda_r} & F_{\lambda_r\mathbf{E}_c} \\ F_{\mathbf{E}_c\lambda_r} & F_{\mathbf{E}_c\mathbf{E}_c} \end{bmatrix} \begin{bmatrix} \lambda_r \\ \mathbf{E}_c \end{bmatrix} = \begin{bmatrix} d_{\lambda_r} \\ d_{\mathbf{E}_c} \end{bmatrix}$$ [1] [2]

[1] M.-F. Xue and J.-M. Jin *Nonconformal FETI-DP Methods for Large-Scale Electromagnetic Simulation*. IEEE, Transactions on Antennas and Propagation, Vol. 60, Sept. 2012

[2] I. Voznyuk, H. Tortel and A. Litman *Scattered field computation with an extended FETI-DPEM2 method*. Progress In Electromagnetics Research, 2013 (to appear)