Perfectly Matched Layers for Wave-Like Time-Dependent Problems

Design, Discretization and Optimization

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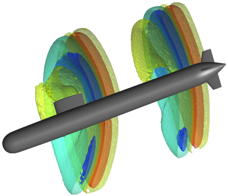
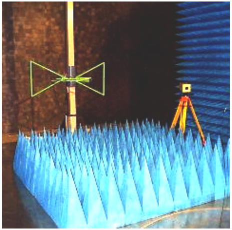
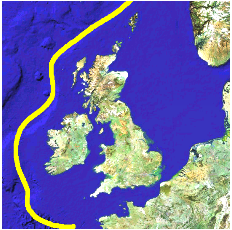
Wave-like phenomena occuring in large areas



(Aero)acoustics



Electromagnetic compatibility

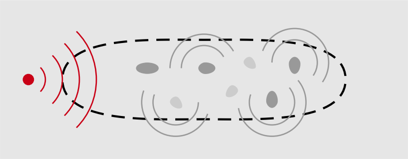


Regional oceanic modeling

Finite difference/finite volume/finite element numerical simulations require a limited computational domain

Need for a domain truncation strategy

Original problem



Modified problem

Artificial boundary conditions

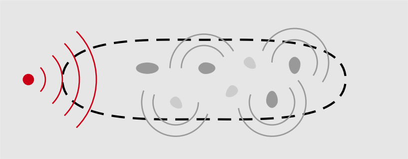
Radiation conditions

Sommerfeld, Silver-Muller,
Flather, Bayliss-Turkel, ...

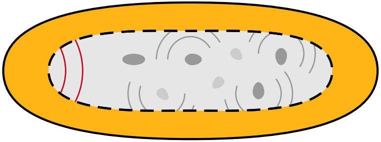
Hierarchical conditions

Exact conditions

Original problem



Modified problem



Artificial boundary conditions

Radiation conditions
Sommerfeld, Silver-Muller,
Flather, Bayliss-Turkel, ...
Hierarchical conditions
Exact conditions

Artificial layers

Absorbing/Sponge layers
Perfectly matched layers (PMLs)

Interesting properties of PMLs:
$\rightarrow$ dissipative medium
$\rightarrow$ perfect matching
$\rightarrow$ perfect absorption

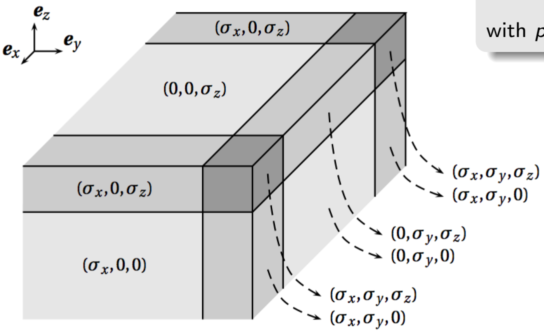
Original equations

$$\partial_t p + \rho c^2 \, (\partial_x u + \partial_y v + \partial_z w) = 0$$
$$\partial_t u + \rho^{-1} \, \partial_x p = 0$$
$$\partial_t v + \rho^{-1} \, \partial_y p = 0$$
$$\partial_t w + \rho^{-1} \, \partial_z p = 0$$

Bérenger's PML equations

$$\partial_t p_x + \rho c^2 \, \partial_x u = -\sigma_x p_x$$
$$\partial_t p_y + \rho c^2 \, \partial_y v = -\sigma_y p_y$$
$$\partial_t p_z + \rho c^2 \, \partial_z w = -\sigma_z p_z$$
$$\partial_t u + \rho^{-1} \, \partial_x p = -\sigma_x u$$
$$\partial_t v + \rho^{-1} \, \partial_y p = -\sigma_y v$$
$$\partial_t w + \rho^{-1} \, \partial_z p = -\sigma_z w$$

with $p = p_x + p_y + p_z$



$\sigma_x(x)$, $\sigma_y(y)$ and $\sigma_z(z)$
Absorption functions

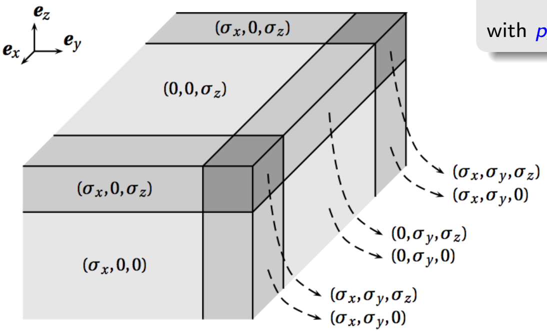
Original equations

$$\partial_t p + \rho c^2 \ (\partial_x u + \partial_y v + \partial_z w) = 0$$
$$\partial_t u + \rho^{-1} \, \partial_x p = 0$$
$$\partial_t v + \rho^{-1} \, \partial_y p = 0$$
$$\partial_t w + \rho^{-1} \, \partial_z p = 0$$

Bérenger's PML equations

$$\partial_t p + \rho c^2 \ (\partial_x u + \partial_y v + \partial_z w) = - \cdots$$
$$\partial_t p_x + \rho c^2 \, \partial_x w = -\sigma_x p_z$$
$$\partial_t p_y + \rho c^2 \, \partial_y v = -\sigma_y p_y$$
$$\partial_t u + \rho^{-1} \, \partial_x p = -\sigma_x u$$
$$\partial_t v + \rho^{-1} \, \partial_y p = -\sigma_y v$$
$$\partial_t w + \rho^{-1} \, \partial_z p = -\sigma_z w$$

with $p_x = p - p_y - p_z$



Original system
+ Source terms
+ 2 additionnal PDEs

$\sigma_x(x)$, $\sigma_y(y)$ and $\sigma_z(z)$
Absorption functions

Introduction. Some goals/requirements for novel PMLs

[adapted from Givoli, 2008]

1. Mathematical/numerical features
   - (strong) Well-posedness
   - (strong) Stability
   - Accuracy at both continuous and discrete levels
2. General geometry
   - Cuboidal truncated domains
   - Cylindrical or spherical truncated domains
   - Convex truncated domains
3. Complicated physics
   - Dispersive waves
   - Heterogeneous and anisotropic media
   - Multiple wave modes
   - Nonlinear dynamics
4. Scheme compatibility / Ease of implementation / Computational efficiency

In this talk:

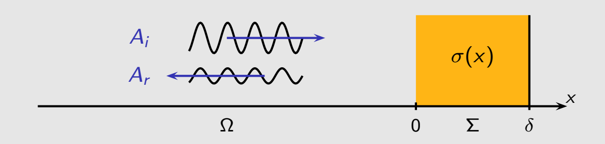
– Optimization (accuracy) at the discrete level
– PMLs for generally-shaped truncated domains

Optimization in discrete contexts

Two PMLs for generally-shaped domains

Numerical simulations

Consider a plane-wave solution with an incident (A_i) and a reflected wave (A_r)



- The effectiveness of the layer is quantified with the reflection coefficient

$$\boxed{r = \left|\frac{A_r}{A_i}\right|}$$

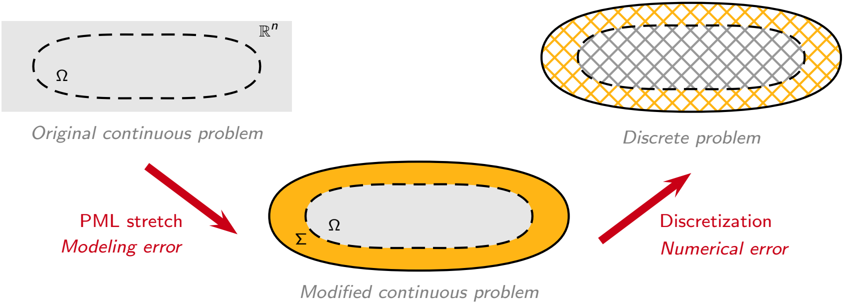
with $r = 0$ for a perfectly *absorbing* boundary
$r = 1$ for a perfectly *reflecting* boundary (wall)

- The reflection coefficients are

$$r_{interf} = 0 \qquad \textit{(infinite PML)}$$

$$r_{pml} = \exp\left\{-\frac{2}{c}\int_0^\delta \sigma(x')dx'\right\} \qquad \textit{(finite PML of thickness } \delta\textit{)}$$

The interface domain/layer is perfectly matched
A finite layer is perfectly absorbing if the integral of $\sigma(x)$ is infinite

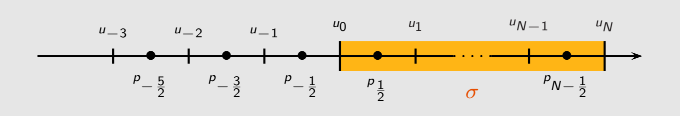


Goal: The solution of the discrete problem must be close to the solution of the original continuous problem

Two complementary viewpoints:

- Minimize the modeling error and the numerical error
- Minimize the reflection of discrete waves

Discretization of the problem with a *finite difference scheme*
Constant absorption function σ



- The discrete harmonic plane-wave solution is

$$\text{in } \Omega: \quad e^{i(kx_i - \omega t)} \quad \text{with } k = \pm\frac{2}{\Delta x}\arcsin\left(\frac{\omega \Delta x}{2c}\right)$$
$$\text{in } \Sigma: \quad e^{i(\beta x_i - \omega t)} \quad \text{with } \beta = \pm\frac{2}{\Delta x}\arcsin\left(\frac{\omega \Delta x}{2c} - i\frac{\sigma \Delta x}{2c}\right)$$

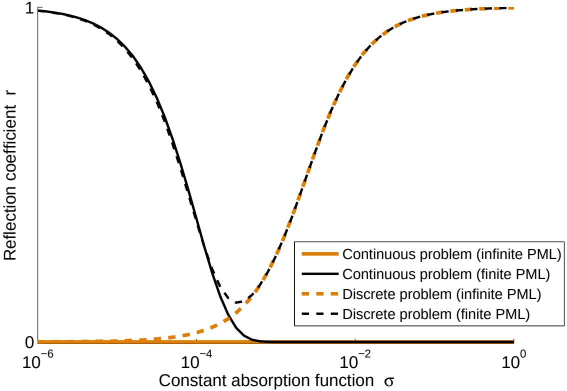
where h is the spatial step.

- The discrete reflection coefficients are

$$r^{\star}_{interf} = \left| \frac{e^{-\imath\beta\Delta x/2} - e^{-\imath k\Delta x/2}}{e^{-\imath\beta\Delta x/2} + e^{\imath k\Delta x/2}} \right| \qquad \textit{(infinite PML)}$$

$$r^{\star}_{pml} = \left| \frac{\imath\cos(\beta\delta + \beta\Delta x/2) - \sin(\beta\delta)\, e^{-\imath k\Delta x/2}}{\imath\cos(\beta\delta + \beta\Delta x/2) + \sin(\beta\delta)\, e^{\imath k\Delta x/2}} \right| \qquad \textit{(finite PML)}$$

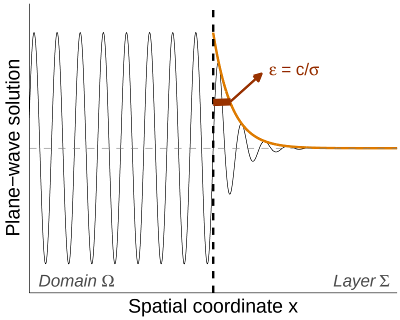
 Plane-wave analysis in 1D at a discrete level (constant σ)



Small σ: Same behavior in continuous and discrete context
Large σ: Different behavior – Discrete PML are reflective

Interpretation

Snapshot of an elementary harmonic solution

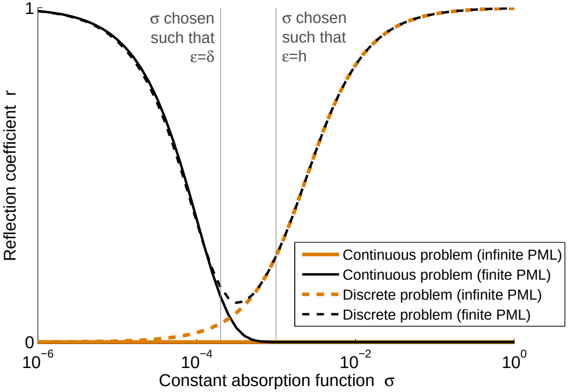


Too small σ: The outgoing waves are not enough damped in the PML ($\varepsilon > \delta$).
The continuous reflection coefficient is recovered *(modeling error)*

Too large σ: The characteristic scale of the exponential decay ε is too small ($\varepsilon < \Delta x$)
The discretization cannot represent the solution *(numerical error)*

$$\Delta x < \varepsilon < \delta$$

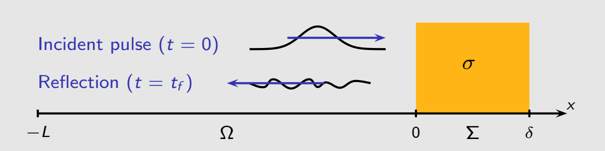
 Plane-wave analysis in 1D at a discrete level (constant σ)



$$\frac{c}{\delta} < \sigma_{opti} < \frac{c}{\Delta x}$$

Description of the 1D time-dependent benchmark

A Gaussian-shaped pulse moves to the right
At the end of the simulation ($t = t_f$), if the layer is perfectly reflecting, the pulse is back at its initial position



The effectiveness of the layer is quantified with the relative error ξ_r

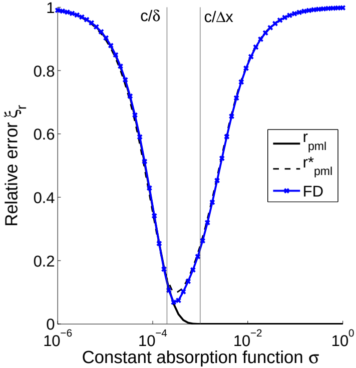
$$\xi_r = \sqrt{\frac{E_{\text{with pml}}}{E_{\text{with wall}}}}$$

$$\text{where } E = \int_{-L}^{0} \left(\frac{1}{2a} |p|^2 + \frac{1}{2b} |u|^2 \right) d\Omega$$

with
- $\xi_r = 0$ for a perfectly *absorbing* boundary
- $\xi_r = 1$ for a perfectly *reflecting* boundary (wall)

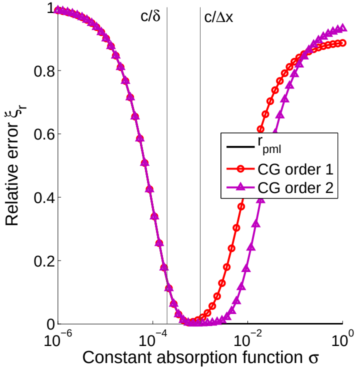
Numerical results

Finite difference scheme



Relative error ξ_r close to reflection coefficient r

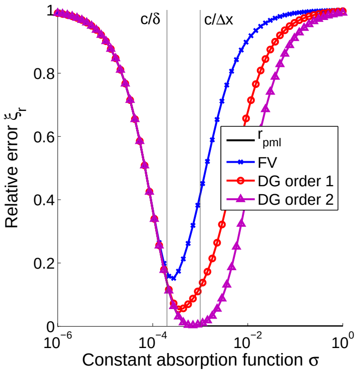
CG finite element scheme



Similar result:
$c/\delta < \sigma_{\text{opti}} < c/\Delta x$

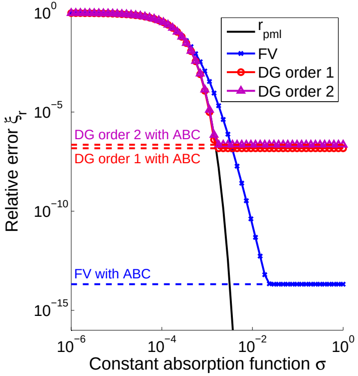
Numerical results

DG finite element scheme with centered fluxes



Similar result:
$c/\delta < \sigma_{\text{opti}} < c/\Delta x$

DG finite element scheme with upwind fluxes



For large σ:
works like an ABC
(the radiation BC)

~~Exponential decay of the solution~~

$\sigma(x)$ such that progressive decay of the solution

Polynomial functions

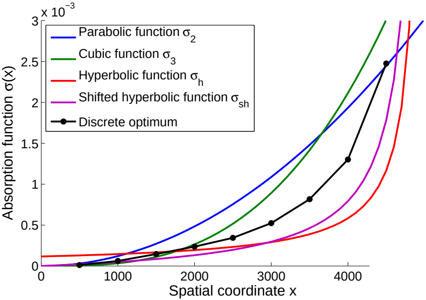
$$\sigma_2(x) = \alpha \left(\frac{x}{\delta}\right)^2$$

$$\sigma_3(x) = \alpha \left(\frac{x}{\delta}\right)^3$$

Hyperbolic functions

$$\sigma_h(x) = \frac{\alpha}{\delta - x}$$

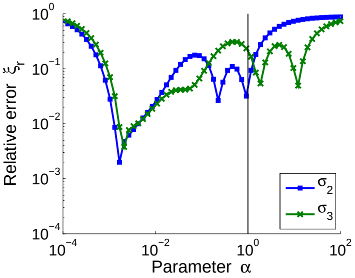
$$\sigma_{sh}(x) = \frac{\alpha}{\delta - x} - \frac{\alpha}{\delta}$$



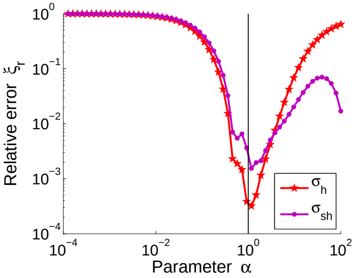
Bermúdez et al., 2007
Modave et al., 2010

Numerical results

With polynomial functions



With hyperbolic functions



(figures for CG scheme – similar figures for other schemes)

Comparison

- Optimum value of α

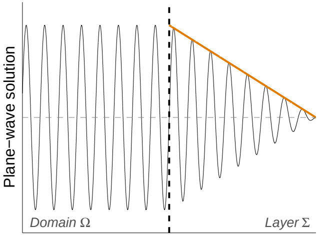
  σ_2 and σ_3: ?

  σ_h and σ_{sh}: approx. propagation velocity c
- Best absorption function

  FD scheme: optimized σ_2 and σ_3 better than σ_{sh},
  optimized σ_h the worst

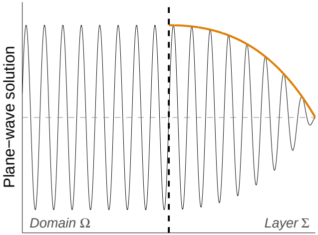
  CG and DG schemes: optimized σ_h equivalent or slightly better than others

σ_h *with* $\alpha = c$



Linear decay
Discontinuous derivative at the interface
OK with finite element schemes
NO with finite difference schemes

σ_{sh} *with* $\alpha = c$



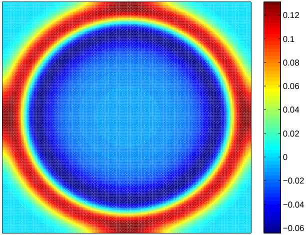
Exponential-linear decay
Continuous derivative at the interface
OK with all schemes

For oblique plane-wave solution (2D/3D), the shape of decay changes!

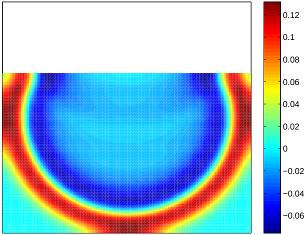
[Modave et al., ODyn, 2010]

Description of the 2D time-dependent benchmark

Reference numerical solution



Solution with the PML



The effectiveness of the layer is now quantified with the relative error

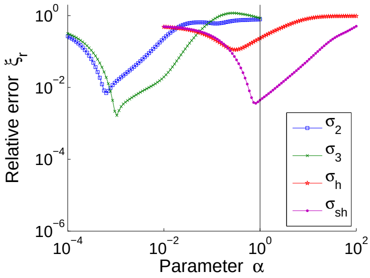
$$\xi_r = \sqrt{\frac{E_{\text{error with pml}}(t_f)}{E_{\text{error with wall}}(t_f)}}$$

$$\text{where } E_{\text{error}} = \int_{\Omega} \left(|p - p_{ref}|^2 + |\mathbf{u} - \mathbf{u}_{ref}|^2 \right) d\Omega$$

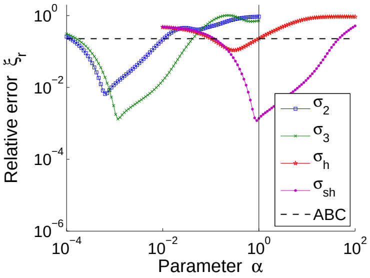
with
- $\xi_r = 0$ for a perfectly *absorbing* boundary
- $\xi_r = 1$ for a perfectly *reflecting* boundary (wall)

Numerical results

with DG and centered fluxes



with DG and upwind fluxes



With all the considered schemes:

- Optimized σ_2, σ_3 and σ_{sh} are nearly equivalent, far better than σ_h
- The optimum α remains close to c for σ_{sh}
- The absorption function σ_{sh} with $\alpha = c$ is the most convenient choice

[Modave et al., IJNME, 201X]

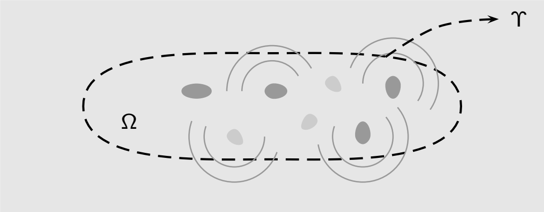
Optimization in discrete contexts

Two PMLs for generally-shaped domains

Numerical simulations

Design. Definition of the problem

Original domain $\mathbb{R}^d$



Scalar wave problem

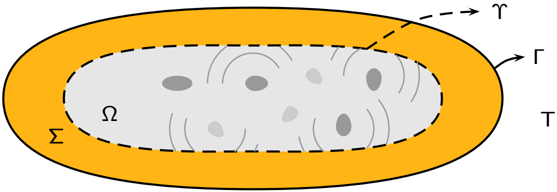
Find $p(t, \mathbf{x})$ and $\mathbf{u}(t, \mathbf{x})$ such that:

$$\begin{cases} \dfrac{\partial p}{\partial t} + \rho c^2 \nabla \cdot \mathbf{u} = 0, & \forall\, \mathbf{x} \in \mathbb{R}^d, t > 0 \\ \dfrac{\partial \mathbf{u}}{\partial t} + \dfrac{1}{\rho} \nabla p = 0, & \forall\, \mathbf{x} \in \mathbb{R}^d, t > 0 \end{cases}$$

with initial conditions on p and $\mathbf{u}$ at $t = 0$

p and $\mathbf{u}$ have initially a compact support in Ω

Convex truncated domain Ω with regular boundary surrounded with a PML Σ



The PML thickness $\delta = \text{dist}(\Upsilon, \Gamma)$ is constant

(Conformal PML)

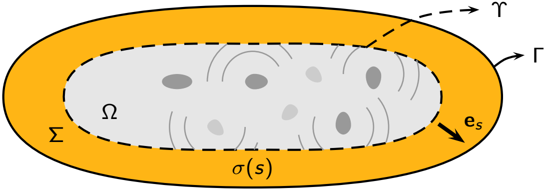
Scalar wave problem with PML

Find $p(t, \mathbf{x})$ and $\mathbf{u}(t, \mathbf{x})$ such that:

$$\begin{cases} \dfrac{\partial p}{\partial t} + \rho c^2 \nabla \cdot \mathbf{u} = 0, & \forall \mathbf{x} \in \Omega, t > 0 \\ \dfrac{\partial \mathbf{u}}{\partial t} + \dfrac{1}{\rho} \nabla p = 0, & \forall \mathbf{x} \in \Omega, t > 0 \end{cases}$$

with initial conditions on p and $\mathbf{u}$ at $t = 0$

Equations in Σ? Conditions at Υ and Γ?



Consider the time-harmonic equations

$$\begin{cases} -\imath\omega p + \rho c^2 \nabla \cdot \mathbf{u} = 0 \\ -\imath\omega \mathbf{u} + \dfrac{1}{\rho}\nabla p = 0 \end{cases}$$

Use the complex substitution

Lassas & Somersalo, 1999
Teixeira & Chew, 2000
Mouysset, 2006
Laurens, 2010

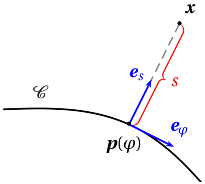
$$\mathbf{x} \quad \to \quad \tilde{\mathbf{x}}(\mathbf{x}) = \mathbf{x} - \frac{\mathbf{e}_s}{\imath\omega} \int_0^s \sigma(s')\,\mathrm{d}s'$$

where
- $\tilde{\mathbf{x}} \in \mathcal{U} \subset \mathbb{C}^d$
- $\mathbf{e}_s$ is the stretching direction
- s is the curvilinear coordinate in the direction $\mathbf{e}_s$
- $\sigma(s)$ is the (positive) absorption function

As a consequence, the elementary time-harmonic plane-wave solution changes:

$$e^{\imath(\mathbf{k}\cdot\mathbf{x}-\omega t)} \quad \to \quad \underbrace{e^{\imath(\mathbf{k}\cdot\mathbf{x}-\omega t)}}_{\textit{Propagation}} \; \underbrace{e^{-\frac{\mathbf{k}\cdot\mathbf{e}_s}{\omega}\int_0^s \sigma(s')\,\mathrm{d}s'}}_{\textit{Damping}}$$

Local orthogonal coordinates (s, φ, θ)
Local orthonormal frame $(\mathbf{e}_s, \mathbf{e}_\varphi, \mathbf{e}_\theta)$ *Darboux's frame*



The original system reads

$$\begin{cases} -\imath\omega\hat{\mathrm{p}} + \rho c^2 \left(\dfrac{\partial u_s}{\partial s} + \dfrac{1}{1+\kappa_\varphi s} \dfrac{\partial u_\varphi}{\partial \varphi} + \dfrac{1}{1+\kappa_\theta s} \dfrac{\partial u_\theta}{\partial \theta} \right) = 0 \\ -\imath\omega\hat{\mathbf{u}} + \dfrac{1}{\rho} \left(\mathbf{e}_s \dfrac{\partial p}{\partial s} + \dfrac{\mathbf{e}_\varphi}{1+\kappa_\varphi s} \dfrac{\partial p}{\partial \varphi} + \dfrac{\mathbf{e}_\theta}{1+\kappa_\theta s} \dfrac{\partial p}{\partial \theta} \right) = 0 \end{cases}$$

After the complex substitution and the parametrization

$$s \quad \to \quad \tilde{s}(s) = s - \frac{1}{\imath\omega} \int_0^s \sigma(s')\, \mathrm{d}s'$$

the system becomes

$$\begin{cases} -\imath\omega\hat{\mathrm{p}} + \rho c^2 \left(\dfrac{\imath\omega}{\imath\omega - \sigma} \dfrac{\partial u_s}{\partial s} + \dfrac{1}{1+\kappa_\varphi \tilde{s}} \dfrac{\partial u_\varphi}{\partial \varphi} + \dfrac{1}{1+\kappa_\theta \tilde{s}} \dfrac{\partial u_\theta}{\partial \theta} \right) = 0 \\ -\imath\omega\hat{\mathbf{u}} + \dfrac{1}{\rho} \left(\dfrac{\imath\omega\, \mathbf{e}_s}{\imath\omega - \sigma} \dfrac{\partial p}{\partial s} + \dfrac{\mathbf{e}_\varphi}{1+\kappa_\varphi \tilde{s}} \dfrac{\partial p}{\partial \varphi} + \dfrac{\mathbf{e}_\theta}{1+\kappa_\theta \tilde{s}} \dfrac{\partial p}{\partial \theta} \right) = 0 \end{cases}$$

where κ_φ and κ_θ are the main curvatures of the interface Υ at its point closest to $\mathbf{x}$

The time-harmonic PML equations can be written

$$\begin{cases} -\imath\omega p + \rho c^2 \tilde{\nabla} \cdot \mathbf{u} = 0 \\ -\imath\omega \mathbf{u} + \dfrac{1}{\rho}\tilde{\nabla} p = 0 \end{cases}$$

with the differential operator $\tilde{\nabla}$

$$\tilde{\nabla} = \nabla - \frac{\sigma}{\sigma - i\omega}[\mathbf{e}_s(\mathbf{e}_s \cdot \nabla)] - \frac{\bar{\kappa}_\varphi \bar{\sigma}}{\bar{\kappa}_\varphi \bar{\sigma} - i\omega}[\mathbf{e}_\varphi(\mathbf{e}_\varphi \cdot \nabla)] - \frac{\bar{\kappa}_\theta \bar{\sigma}}{\bar{\kappa}_\theta \bar{\sigma} - i\omega}[\mathbf{e}_\theta(\mathbf{e}_\theta \cdot \nabla)]$$

where $\bar{\kappa}_\varphi = (\kappa_\varphi^{-1} + s)^{-1}$, $\bar{\kappa}_\theta = (\kappa_\theta^{-1} + s)^{-1}$ and $\bar{\sigma} = \int_0^s \sigma(s')\mathrm{d}x'_s$

PML's equations (PML "with PDEs")

In the layer Σ, the fields p and $\mathbf{u}$ and two additional fields p_s and p_φ are governed by

$$\begin{cases} \dfrac{\partial p}{\partial t} + \rho c^2 \nabla \cdot \mathbf{u} = -\sigma p_s - \bar{\kappa}_\varphi \bar{\sigma} p_\varphi - \bar{\kappa}_\theta \bar{\sigma}(p - p_s - p_\varphi) \\ \dfrac{\partial \mathbf{u}}{\partial t} + \dfrac{1}{\rho}\nabla p = -\sigma \mathbf{e}_s(\mathbf{e}_s \cdot \mathbf{u}) - \bar{\kappa}_\varphi \bar{\sigma} \mathbf{e}_\varphi(\mathbf{e}_\varphi \cdot \mathbf{u}) - \bar{\kappa}_\theta \bar{\sigma} \mathbf{e}_\theta(\mathbf{e}_\theta \cdot \mathbf{u}) \\ \dfrac{\partial p_s}{\partial t} + \rho c^2 [\mathbf{e}_s(\mathbf{e}_s \cdot \nabla)] \cdot \mathbf{u} = -\sigma p_s \\ \dfrac{\partial p_\varphi}{\partial t} + \rho c^2 [\mathbf{e}_\varphi(\mathbf{e}_\varphi \cdot \nabla)] \cdot \mathbf{u} = -\bar{\kappa}_\varphi \bar{\sigma} p_\varphi \end{cases}$$

Cartesian case → Bérenger's PML system

[Modave et al., WAVES, 2012]

Well-posedness *[Kreiss and Lorenz, 1989]*

The Cauchy problem is *weakly/strongly well-posed* if there exists $K > 0$ and $\alpha \in \mathbb{R}$ such that the solution $U(t)$ satisfies an estimate on the type

$$\|U(.,t)\|_{L^2} \leq Ke^{\alpha t} \|U(.,0)\|_{H^s} \quad \text{with } s > 0 \qquad (\textit{weak})$$

$$\|U(.,t)\|_{L^2} \leq Ke^{\alpha t} \|U(.,0)\|_{L^2} \qquad (\textit{strong})$$

Stability *[Bécache and Joly, 2002]*

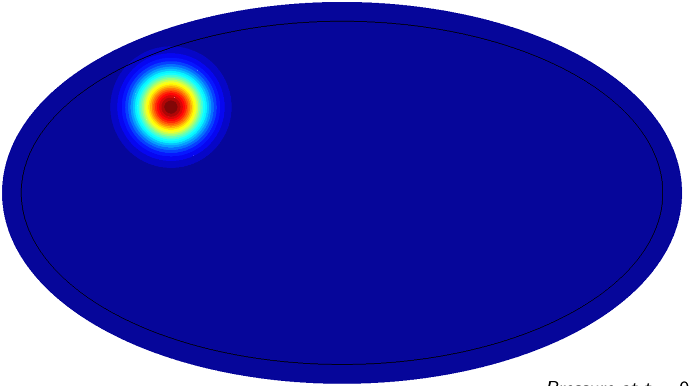
A system that is a 0^{th}-order perturbation of a 1^{st}-order hyperbolic system is *weakly/strongly stable* if there exists $K > 0$ and $A \in \mathbb{R}$ such that the solution $U(t)$ satisfies an estimate on the type

$$\|U(.,t)\|_{L^2} \leq K(1 + At)^s \|U(.,0)\|_{H^s} \quad \text{with } s > 0 \qquad (\textit{weak})$$

$$\|U(.,t)\|_{L^2} \leq K \|U(.,0)\|_{L^2} \qquad (\textit{strong})$$

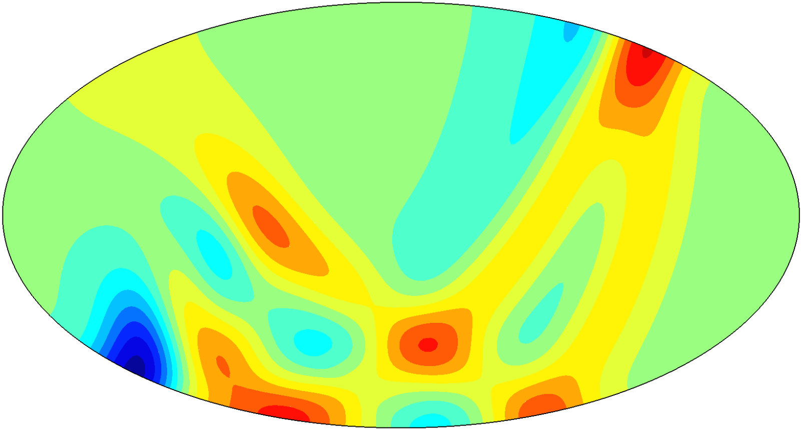
Description of the 2D time-dependent benchmark

Initial pulse — Elliptical truncated domain
DG finite element scheme with piecewise linear basis functions ($\mathfrak{p} = 1$)
About 230/310.000 discrete unknowns ($\sim 17 - 25\%$ in the PML for $\delta = 5\ell$)



Pressure at $t = 0$
Colorbar $[0, 1]$

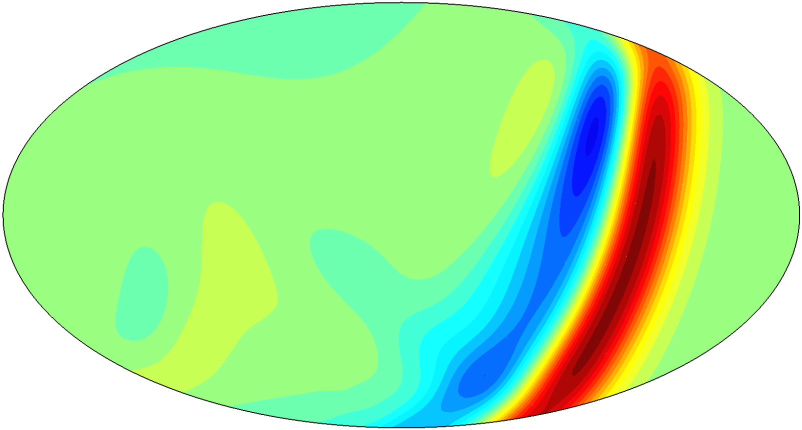
Wall BC case



Pressure at $t = t_{fin}$

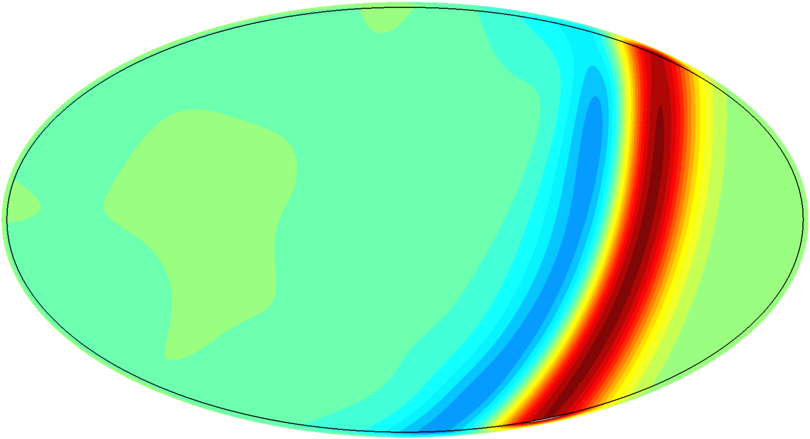
Colorbar $[-0.35, 0.35]$

Absorbing BC case



Pressure at $t = t_{fin}$
Colorbar $[-0.1, 0.1]$

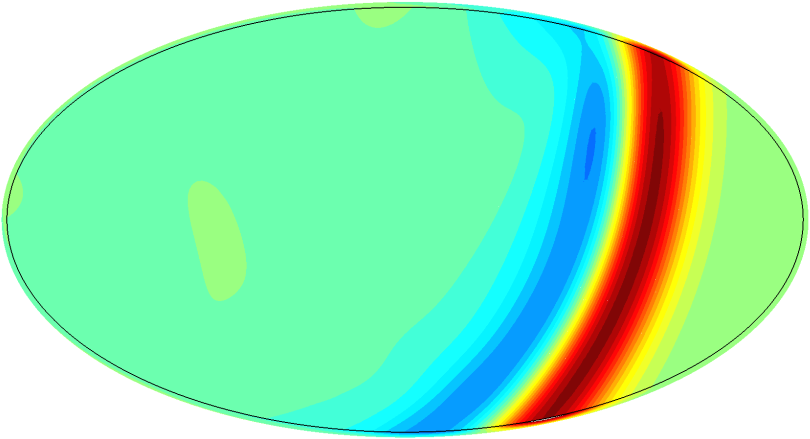
PML with PDEs and $\delta = 1h$



Pressure at $t = t_{fin}$

Colorbar $[-0.1, 0.1]$

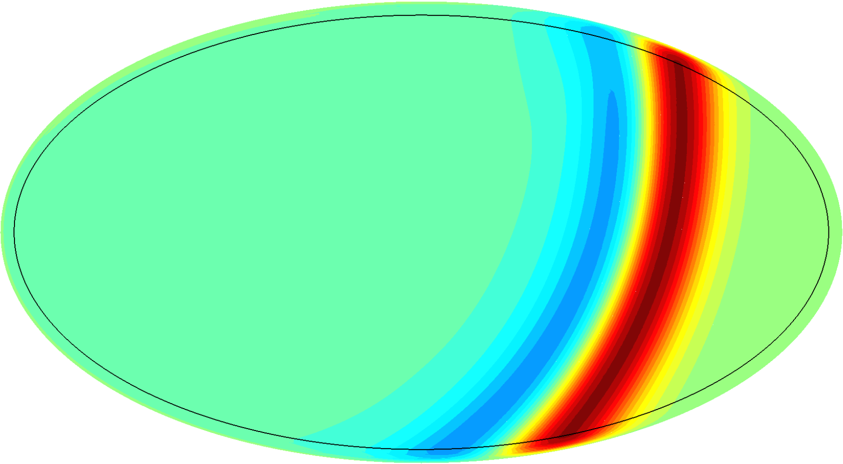
PML with ODEs and $\delta = 1h$



Pressure at $t = t_{fin}$

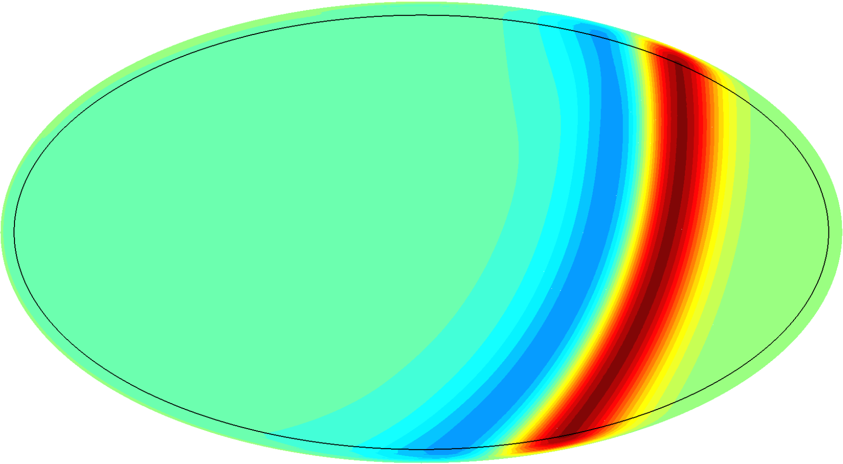
Colorbar $[-0.1, 0.1]$

PML with PDEs and $\delta = 3h$



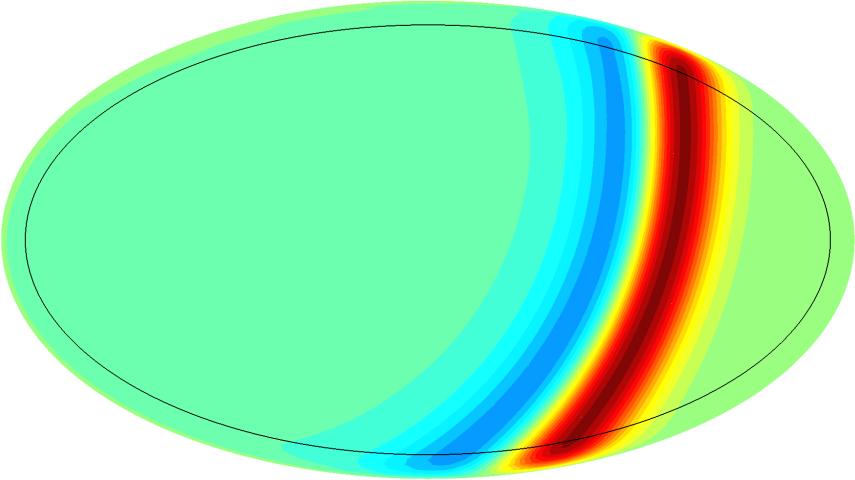
Pressure at $t = t_{fin}$
Colorbar $[-0.1, 0.1]$

PML with ODEs and $\delta = 3h$



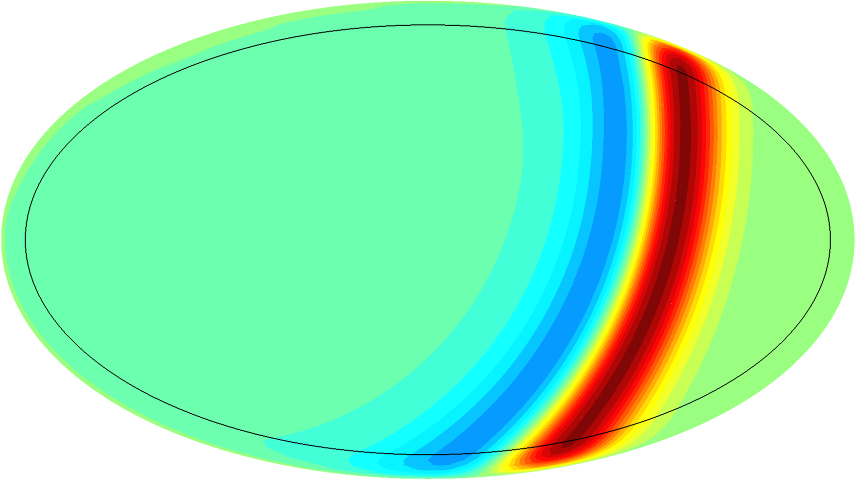
Pressure at $t = t_{fin}$
Colorbar $[-0.1, 0.1]$

PML with PDEs and $\delta = 5h$



Pressure at $t = t_{fin}$
Colorbar $[-0.1, 0.1]$

PML with ODEs and $\delta = 5h$

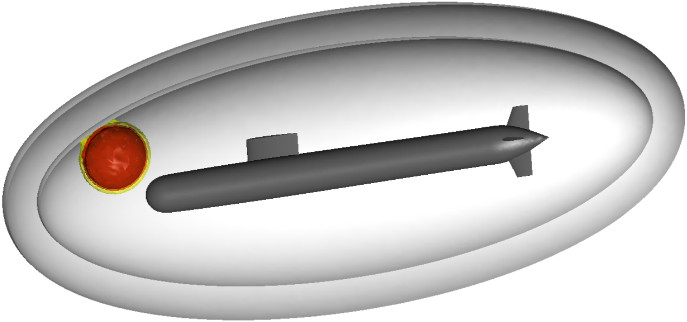


Pressure at $t = t_{fin}$

Colorbar $[-0.1, 0.1]$

Description of the 3D time-dependent benchmark

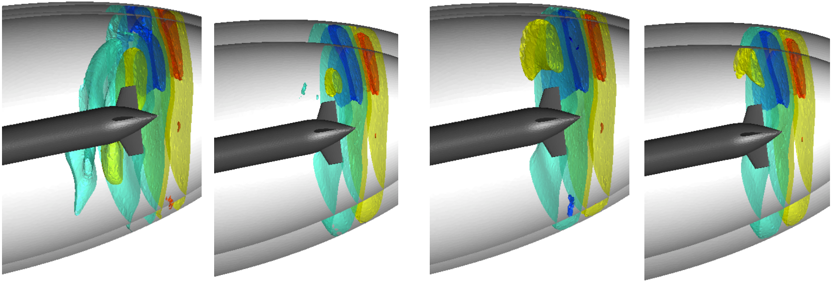
Initial pulse in the front of a submarine — Ellipsoidal truncated domain
DG finite element scheme with piecewise linear basis functions ($\mathfrak{p} = 1$)
About 17.8 millions discrete unknowns ($\sim$ 50% in the PML for $\delta = 5\ell$)



Iso-surfaces of the pressure at $t = 0$.

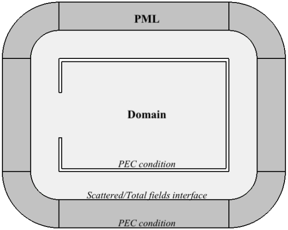
PML with PDEs terminated with a 'wall' — Thin layer | Thick layer

PML with PDEs terminated with an ABC — Thin layer | Thick layer



An ABC termination improves a poorly efficient PML

[see also *Petropoulos, 1998*]

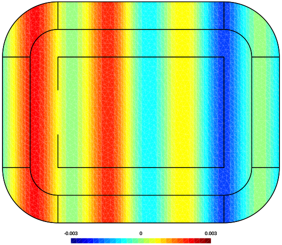


Shielding effectiveness of a cavity

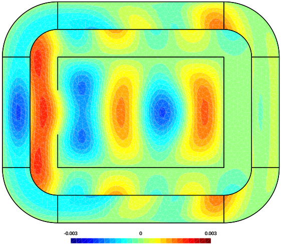
$$\mathrm{SE}_{\mathrm{dB}} = 20 \log_{10} \left| \frac{E^{\mathrm{inc}}}{E^{\mathrm{trans}}} \right|$$

where E^{inc} and E^{inc} are the wave amplitudes at the center of the cavity.

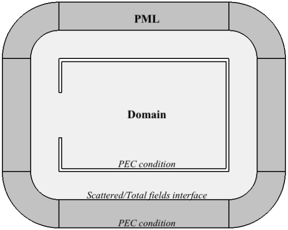
Time-domain simulation with the modulated transverse-magnetic incident field.



E^{incident}



E^{total} (in the domain)
$E^{\mathrm{scattered}}$ (in the layer)



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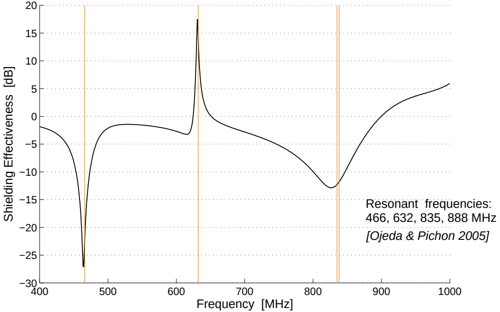
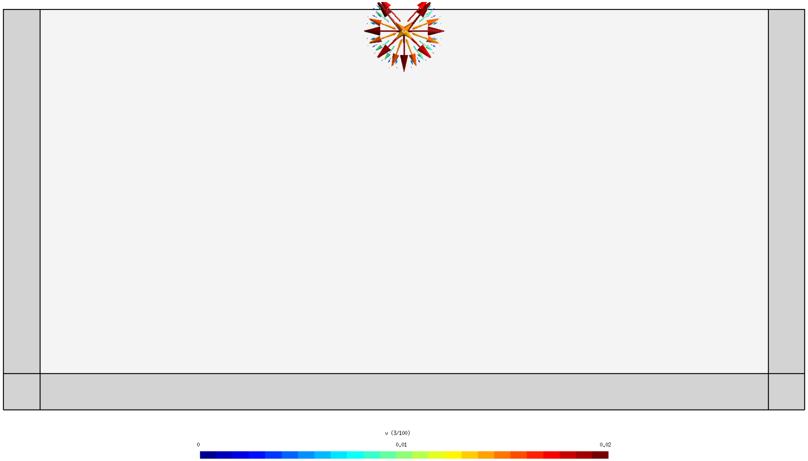
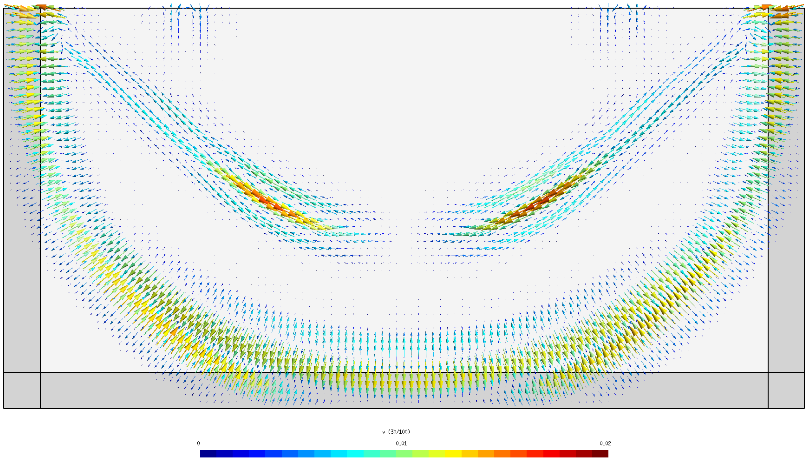


PML for elastic waves



Velocity at the beginning of the simulation

PML for elastic waves



Velocity ... later ...

Optimization in discrete contexts

Two PMLs for generally-shaped domains

Numerical simulations

Conclusion.

PML $\rightarrow$ An efficient way for domain truncation

Choice of the parameters in discrete context

- Effectiveness of the shifted hyperbolic function σ_{sh}
  - $\rightarrow$ Does not require a costly optimization procedure
  - $\rightarrow$ Interpretation of the optimum value of the free parameter
- Improving the effectiveness by increasing the layer thickness δ

Design of PML time-dependent formulations

- Valid for convex truncated domains with regular boundary
- Easy to implement in existing codes
- Good framework for further developments

Perfectly Matched Layers for Wave-Like Time-Dependent Problems

Design, Discretization and Optimization

Axel Modave

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University of Louvain (Belgium)

June 3rd, 2014

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