Invisibility cloaks and transformation optics

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Outline

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In collaboration with...

- Prof. Frédéric Zolla, Université de Provence,
- Dr. Sébastien Guenneau, former Ph. D. student and CR1 CNRS,
- Dr. Yacoub Ould Agha, former Ph. D. student,
- Dr. Alexandru I. Cabuz, former Postdoc.

Open geometries in static and quasistatic problems

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- Analytical map of the interior of a circle (disk) on the exterior of a circle (infinite domain), connecting the boundary to the boundary of the region of interest.
- Limited to 2D Laplace operator (harmonic problems).

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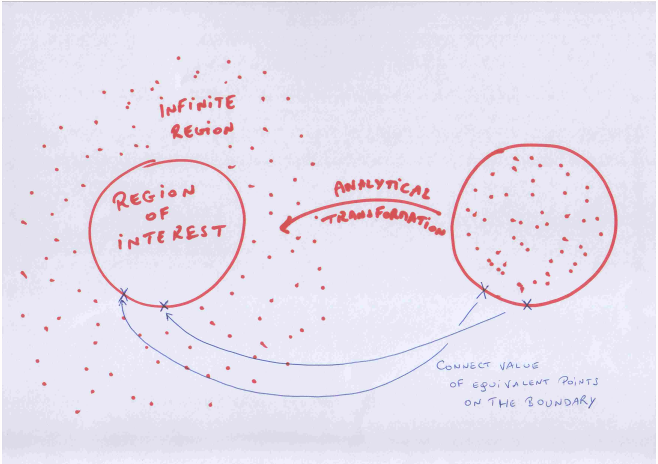

Solve a 2D harmonic problem on the disk.

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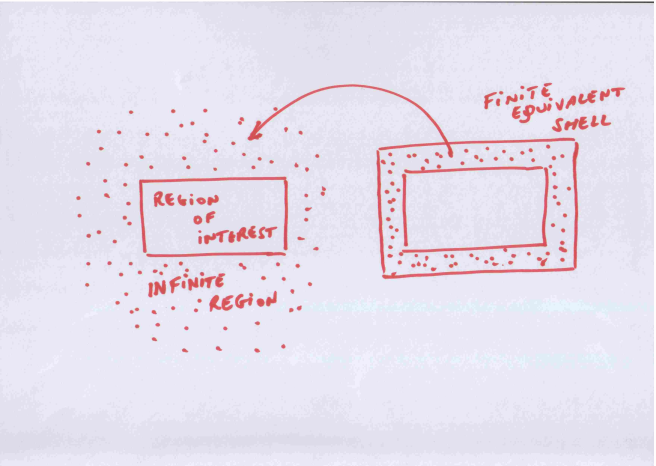
- Adapted to 3D problems and to various shapes, natural description of the geometry...
- Requires a modification of the operator.

Open geometries in static and quasistatic problems



How to modify simply the code to introduce the new coefficients ?

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Differential geometry of p-forms

Differential geometry of p-forms

- ***p*-forms** are rank p totally covariant tensors.
- **Exterior derivative** d = differential or gradient of functions, curl and divergence of vector fields...
- $dd = 0$ for **curl grad** $= 0$, div **curl** $= 0$ ...
- **Exterior product** $\wedge$ = multilinear antisymmetric map.
- **Geometric integration** $\int_\Sigma \alpha$ = line integrals of 1-forms, surface (flux) integrals of 2-forms, volume integrals of 3-forms...
- Stokes theorem $\int_\Sigma d\alpha = \int_{\partial\Sigma} \alpha$
- All are **metric free** operations !

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Electrodynamics

- **Maxwell's equations**

$$\begin{cases} \textbf{curl}\, H = J + \partial_t D \\ \textbf{curl}\, E = -\partial_t B \\ \mathrm{div}\, D = \rho \\ \mathrm{div}\, B = 0 \end{cases}$$

- **Poynting identity**

$$\mathrm{div}(E \times H) = J \cdot E + E \cdot \partial_t D + H \cdot \partial_t B$$

Electrodynamics

- **Maxwell's equations**

$$\begin{cases} dH = J + \partial_t D \\ dE = -\partial_t B \\ dD = \rho \\ dB = 0 \end{cases}$$

- **Poynting identity**

$$d(E \wedge H) = J \wedge E + E \wedge \partial_t D + H \wedge \partial_t B$$

- <u>These relations are metric free !</u>

Example of the Faraday equation

- In a general coordinate system $\{u, v, w\}$:
- The electric field is the 1-form $\mathbf{E} = E_u du + E_v dv + E_w dw$.
- The magnetic flux density is the 2-form $\mathbf{B} = B_u dv \wedge dw + B_v dw \wedge du + B_w du \wedge dv$.
- And $dE = -\partial_t B$ means
$$(\partial_u E_v - \partial_v E_u + \partial_t B_w) du \wedge dv +$$
$$(\partial_v E_w - \partial_w E_v + \partial_t B_u) dv \wedge dw +$$
$$(\partial_w E_u - \partial_u E_w + \partial_t B_v) dw \wedge du = 0$$

Metric

- **Distance, angle...**
- Hodge star operator $*$ maps p-forms on $(3 - p)$-forms.
- Example of Euclidean metric in Cartesian coordinates :

$$\left\{ \begin{array}{l} *dx = dy \wedge dz \\ *dy = dz \wedge dx \\ *dz = dx \wedge dy \end{array} \right.$$

- This simplicity hides metric aspects in Cartesian coordinates but the relations are more complicated with a general coordinate system...

Metric

- **... and electromagnetic constitutive laws !**
- For example in free space :

$$D = \varepsilon_0 * E$$

$$B = \mu_0 * H$$

- The Hodge star operator is necessary to transform fields (1-forms) into flux densities (2-forms) !

Coordinate transformation

- Considering a map from the coordinate system $\{u, v, w\}$ to the coordinate system $\{x, y, z\}$ given by the functions $x(u, v, w)$, $y(u, v, w)$, and $z(u, v, w)$,
- All the information is in the transformation of the differentials and is therefore given by the *chain rule* :

$$\begin{cases} dx = \frac{\partial x}{\partial u} du + \frac{\partial x}{\partial v} dv + \frac{\partial x}{\partial w} dw \\ dy = \frac{\partial y}{\partial u} du + \frac{\partial y}{\partial v} dv + \frac{\partial y}{\partial w} dw \\ dz = \frac{\partial z}{\partial u} du + \frac{\partial z}{\partial v} dv + \frac{\partial z}{\partial w} dw \end{cases}$$

Coordinate transformation

All the information is in the Jacobian matrix **J** :

$$\begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} = \mathbf{J} \begin{pmatrix} du \\ dv \\ dw \end{pmatrix}$$

with **J** defined as

$$\mathbf{J}(u,v,w) = \frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{pmatrix}$$

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Coordinate transformation

For a 1-form **E**, the transformation from $\{x, y, z\}$ to $\{u, v, w\}$ coordinates is performed as follows :

$$\mathbf{E} = E_x dx + E_y dy + E_z dz = (E_x \; E_y \; E_z) \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix}$$

$$= (E_x \; E_y \; E_z)\mathbf{J} \begin{pmatrix} du \\ dv \\ dw \end{pmatrix}$$

We have also :

$$\mathbf{E} = E_u du + E_v dv + E_w dw = (E_u \; E_v \; E_w) \begin{pmatrix} du \\ dv \\ dw \end{pmatrix}$$

and the following relation is obtained :

$$(E_x \; E_y \; E_z) \; \mathbf{J} = (E_u \; E_v \; E_w)$$

Pullback

Direct map :

New coordinates $\longrightarrow$ Cartesian coordinates

u, v, w $\quad$ $x(u,v,w), y(u,v,w), z(u,v,w)$

=Modelling space $\quad$ ="Physical space"

Pullback :

Forms in new coordinates $\longleftarrow$ Forms in Cartesian coordinates

$E_u du + E_v dv + E_w dw$ $\quad$ $E_x dx + E_y dy + E_z dz$

etc.

Scalar product of 1-forms

$$\mathbf{E} \wedge *\mathbf{E}' = (E_x\, E_y\, E_z)(E'_x\, E'_y\, E'_z)^T dx \wedge dy \wedge dz$$
(thanks to the simplicity of Cartesian coordinates)
$$= (E_u\, E_v\, E_w)\mathbf{J}^{-1}[(E'_u\, E'_v\, E'_w)\mathbf{J}^{-1}]^T dx \wedge dy \wedge dz$$
(using the previous relations between the coordinate systems)
$$= (E_u\, E_v\, E_w)\mathbf{J}^{-1}\mathbf{J}^{-T}(E'_u\, E'_v\, E'_w)^T \det(\mathbf{J}) du \wedge dv \wedge dw\,.$$
(transforming the volume form)

A **transformation matrix (related to metric tensor !) T** is defined by :

$$\mathbf{T}^{-1} = \mathbf{J}^{-1}\mathbf{J}^{-T}\det(\mathbf{J})$$

Scalar product of 1-forms

- For two 1-forms $\mathbf{E}$ and $\mathbf{E}'$, the scalar product is defined as follows :

$$\int_{\mathbb{R}^3} \mathbf{E} \wedge *\mathbf{E}'$$

- Practically, it may be computed as :

$$\int_{\mathbb{R}^3} \mathbf{E} \cdot \mathbf{T}^{-1}\mathbf{E}' dV$$

  where $\cdot$ denotes the “dot product in Cartesian coordinates” and dV the Lebesgue measure...

- Everything behaves as if $\mathbf{T}$ were an **anisotropic inhomogeneous tensor material property** (inverse permittivity) !

For 2-forms *e.g.* :

- $dx \wedge dy = [\frac{\partial x}{\partial u} du + \frac{\partial x}{\partial v} dv + \frac{\partial x}{\partial w} dw] \wedge [\frac{\partial y}{\partial u} du + \frac{\partial y}{\partial v} dv + \frac{\partial y}{\partial w} dw] = (\frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}) du \wedge dv + (\frac{\partial x}{\partial v}\frac{\partial y}{\partial w} - \frac{\partial x}{\partial w}\frac{\partial y}{\partial v}) dv \wedge dw + (\frac{\partial x}{\partial w}\frac{\partial y}{\partial u} - \frac{\partial x}{\partial u}\frac{\partial y}{\partial w}) dw \wedge du.$
- The cofactors of **J** are now involved in the transformation. These are the elements of $\mathbf{J}^{-T}\det(\mathbf{J})$.

$$\begin{pmatrix} dx \wedge dy \\ dy \wedge dz \\ dz \wedge dx \end{pmatrix} = \mathbf{J}^{-T}\det(\mathbf{J}) \begin{pmatrix} du \wedge dv \\ dv \wedge dw \\ dw \wedge dv \end{pmatrix}$$

Note : $\mathbf{J}^{-T}$ *means inverse of the transpose of* **J**.

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Scalar product of 2-forms

Given a 2-form :

$$\begin{aligned}\mathbf{D} &= D_x dy \wedge dz + D_y dz \wedge dx + D_z dx \wedge dy \\ &= D_u dv \wedge dw + D_v dw \wedge du + D_w du \wedge dv\end{aligned}$$

the following relation is obtained :

$$(D_x\ D_y\ D_z)\ \mathbf{J}^{-T} \det(\mathbf{J}) = (D_u\ D_v\ D_w)$$

and the matrix involved in the scalar product is here **T** (still equivalent to an inverse permittivity !) :

$$\int_{\mathbb{R}^3} \mathbf{D} \cdot \mathbf{T} \mathbf{D}' dV$$

Coordinate transformations...

...can be encapsulated in material properties.

Weak formulations (considering here possibly anisotropic materials) involve terms (3-forms) like

- $\mathbf{E} \cdot \underline{\underline{\varepsilon}}\mathbf{E}'$, $\mathbf{H} \cdot \underline{\underline{\mu}}\mathbf{H}'$ (scalar product of 1-forms type)
- $\mathbf{curl}\,\mathbf{E} \cdot \underline{\underline{\mu}}^{-1}\,\mathbf{curl}\,\mathbf{E}'$, $\mathbf{curl}\,\mathbf{H} \cdot \underline{\underline{\varepsilon}}^{-1}\,\mathbf{curl}\,\mathbf{H}'$ (scalar product of 2-forms type).

Introducing :
1-form transformation : $(E_x\ E_y\ E_z)\ \mathbf{J} = (E_u\ E_v\ E_w)$
2-form transformation : $(D_x\ D_y\ D_z)\ \mathbf{J}^{-T}\det(\mathbf{J}) = (D_u\ D_v\ D_w)$
and a Jacobian $\det(\mathbf{J})$ factor for the measure transformation...

All the terms in the weak formulations can be equivalently computed by introducing

equivalent material properties $\underline{\underline{\varepsilon}}_{eq}, \underline{\underline{\mu}}_{eq}$:

$$\underline{\underline{\varepsilon}}_{eq} = \mathbf{J}^{-1}\underline{\underline{\varepsilon}}\mathbf{J}^{-T}\det(\mathbf{J})$$
$$\underline{\underline{\mu}}_{eq} = \mathbf{J}^{-1}\underline{\underline{\mu}}\mathbf{J}^{-T}\det(\mathbf{J})$$

Consequences

- Map the transformed domain Ω' (modelling space) on the original domain Ω (physical space usually in Cartesian coordinates) and pullback the covariant objects (physical equations) to the transformed domain (new model equations).

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- The **material properties**, involving all the **metric** aspects, are written with the Hodge operator and contain all the coordinate dependent information.

Any change of coordinates can be translated into equivalent materials given by :

$$\underline{\underline{\varepsilon'}} = \varepsilon \mathbf{T}^{-1}, \quad \text{and} \quad \underline{\underline{\mu'}} = \mu \mathbf{T}^{-1}.$$

- with the matrix $\mathbf{T} = \mathbf{J}^T \mathbf{J} / det(\mathbf{J})$.

Equivalent materials mean that

you can work in the new coordinate system just as if you were still in Cartesian coordinates but for the ϵ and μ that have been turned to new equivalent material properties

$$\underline{\underline{\varepsilon'}} = \varepsilon \mathbf{T}^{-1} \text{ and } \underline{\underline{\mu'}} = \mu \mathbf{T}^{-1}$$

with the matrix $\mathbf{T} = \mathbf{J}^T \mathbf{J} / det(\mathbf{J})$!

- In the more general case where the initial $\underline{\underline{\varepsilon}}$ and $\underline{\underline{\mu}}$ are tensors corresponding to anisotropic properties, the equivalent properties become

$$\underline{\underline{\varepsilon'}} = \mathbf{J}^{-1}\underline{\underline{\varepsilon}}\mathbf{J}^{-T}det(\mathbf{J}) \,, \quad \text{and} \quad \underline{\underline{\mu'}} = \mathbf{J}^{-1}\underline{\underline{\mu}}\mathbf{J}^{-T}det(\mathbf{J}) \,.$$

where $\mathbf{J}^{-T}$ denotes the transpose of the inverse of $\mathbf{J}$.

Coordinate transformation and equivalent materials

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  where $\mathbf{J}^{-T}$ denotes the transpose of the inverse of $\mathbf{J}$.
- The two successive changes of coordinates are given by the Jacobian matrices $\mathbf{J}_{xX}$ and $\mathbf{J}_{Xu}$ so that

$$\mathbf{J}_{xu} = \mathbf{J}_{xX}\mathbf{J}_{Xu} \;.$$

  This rule naturally applies for an arbitrary number of coordinate systems.

All fields change...

- 1-form transformation : $(E_x\ E_y\ E_z)\ \mathbf{J} = (E_u\ E_v\ E_w)$ for the electric field **E** and also for the magnetic field **H**, the vector potential **A**...
- 2-form transformation : $(D_x\ D_y\ D_z)\ \mathbf{J}^{-T}\ \det(\mathbf{J}) = (D_u\ D_v\ D_w)$ for the electric displacement **D** and also for the magnetic induction **B**, the current density **J**, the Poynting vector **S**...
- **What does it REALLY means "equivalent" then ?**

Global integral quantities are conserved !

- Line integrals of 1-forms are conserved (because curves and 1-forms experience dual transformations) such as electromotive force, magnetomotive force, magnetic flux (evaluated with **A**)...
- Surface flux integrals of 2-forms are conserved (because surfaces and 2-forms experience dual transformations) such as electric flux, magnetic flux (evaluated with **B**), total current, power flow...
- Volume integrals of 3-forms are conserved (because volumes and 3-forms experience dual transformations) such as total charge, powers...
- **Measurements are preserved !**

$\mathbf{T}^{-1}$ for an open domain (exterior of a disk) mapped on a circular annulus.

Consider the radial transformation
$r = f(r') = (R_1 - R_2)r'/(r' - R_2)$
so that $r' = R_1 \Rightarrow r = R_1$ and $r' = R_2 \Rightarrow r \to \infty$.
Define $c_{11}(r') = \frac{df(r')}{dr'}$ and

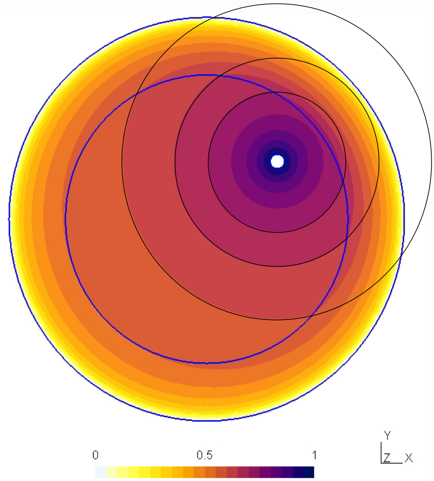
$$\mathbf{T}^{-1} = \mathbf{R}(\theta')\mathbf{diag}\left(\frac{f(r')}{c_{11}(r')r'}, \frac{c_{11}(r')r'}{f(r')}, \frac{c_{11}(r')f(r')}{r'}\right)\mathbf{R}(\theta')^T$$

where r' and θ' are the well known functions
$r'(x', y') = \sqrt{x'^2 + y'^2}$ and $\theta'(x', y') = 2\,\mathrm{arctan}\left(\frac{y'}{x'+\sqrt{x'^2+y'^2}}\right)$ and
$\mathbf{R}(\theta')$ is a rotation matrix.

x', y' are the Cartesian coordinates in the annulus configuration.

Equivalent material for unbounded electrostatic problem

Electrostatic potential : circular cylinder $V = 1,\ V(r \to \infty) = 0$
$\Rightarrow$ circular equipotential lines !



Open propagation problems : Perfectly Matched Layers (PML)

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- Nowadays, may be introduced as a complex mapping

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- Roughly turns $e^{-i\gamma\rho}$ into $e^{-i\gamma\tilde{\rho}}$ exponentially decreasing because $\tilde{\rho}$ is complex (with the good sign of the imaginary part of $\tilde{\rho}$) and it damps outgoing waves and other leaky modes...

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- Roughly turns $e^{-i\gamma\rho}$ into $e^{-i\gamma\tilde{\rho}}$ exponentially decreasing because $\tilde{\rho}$ is complex (with the good sign of the imaginary part of $\tilde{\rho}$) and it damps outgoing waves and other leaky modes...
- The global coordinate transformation is taken to be **identity ($s_\rho = 1$) inside a region of interest ($\rho < R^*$)** and **the complex change of coordinates (s_ρ complex valued) outside the region of interest ($\rho > R^*$)** so that the transformed problem provides directly the required fields in the region of interest (This will be more clear on an example !)

PML $\mathbf{T}_{PML}$ matrix

The inverse matrix $\mathbf{T}_{PML}^{-1}$ corresponding to the complex stretch is given by :

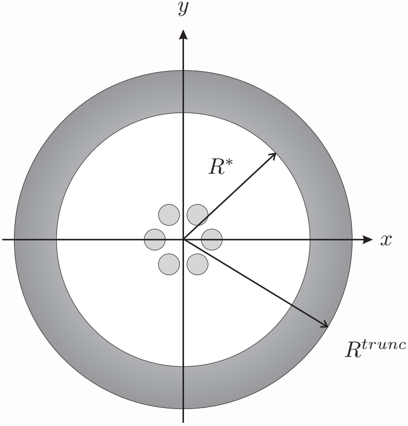
$$\begin{pmatrix} \frac{\rho s_\rho \sin(\theta)^2}{\tilde{\rho}} + \frac{\tilde{\rho}\cos(\theta)^2}{\rho s_\rho} & \sin(\theta)\cos(\theta)\left(\frac{\tilde{\rho}}{\rho s_\rho} - \frac{\rho s_\rho}{\tilde{\rho}}\right) & 0 \\ \sin(\theta)\cos(\theta)\left(\frac{\tilde{\rho}}{\rho s_\rho} - \frac{\rho s_\rho}{\tilde{\rho}}\right) & \frac{\rho s_\rho \cos(\theta)^2}{\tilde{\rho}} + \frac{\tilde{\rho}\sin(\theta)^2}{\rho s_\rho} & 0 \\ 0 & 0 & \frac{\tilde{\rho} s_\rho}{\rho} \end{pmatrix}.$$

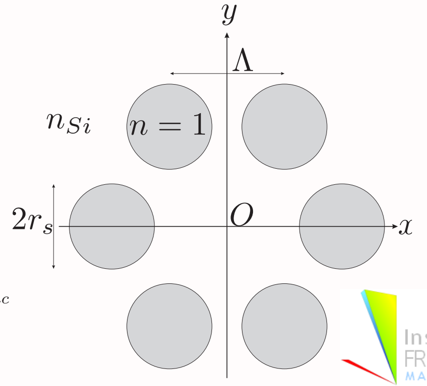
All the quantities involved in the previous expression can be given as explicit functions of x and y "pseudo Cartesian modelling coordinates" : $\theta = 2\arctan\left(\frac{y}{x+\sqrt{x^2+y^2}}\right)$,
$\rho = \sqrt{x^2+y^2}$, $s_\rho(\rho) = s_\rho(\sqrt{x^2+y^2})$, and $\tilde{\rho} = \int_0^{\sqrt{x^2+y^2}} s_\rho(\rho')d\rho'$.

A strong test for PML : leaky modes in Microstructured Optical Fibres

Cross section of a six air hole in silica MOF structure

The hole structure is with $\Lambda = 6.75\,\mu m$, $r_s = 2.5\,\mu m$, the surrounding annulus used to set up the PML has $R^* = 30\mu m$, $R^{trunc} = 40\mu m$





Comparison with the Multipole Method

$\lambda_0 = 1.55\,\mu m$ is considered for which the index of silica is about $\sqrt{\varepsilon_{\mathrm{Si}}} = n_{\mathrm{Si}} = 1.444024$.
The corresponding complex effective index $n_{\mathrm{eff}} = \beta/k_0$ is

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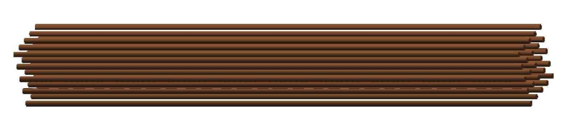
COMSOL Multiphysics®, about $16,800$ second order triangular elements, 150 seconds on a Pentium M 1.86 GHz, 1Go RAM laptop computer.

What happens if a Microstructured Optical Fibre is twisted ?

- From translational invariance...



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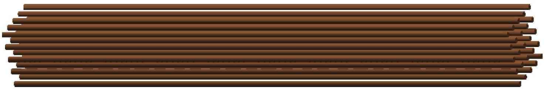
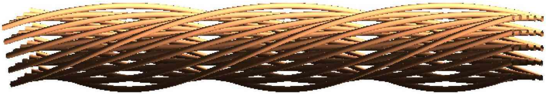


- ... to twisted structures :



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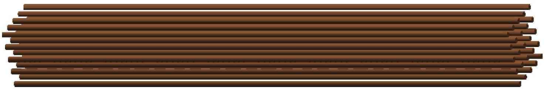
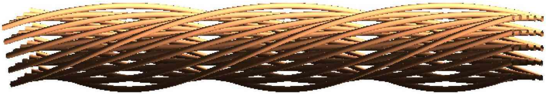


- ... to twisted structures :



- We are going to look for a simple and efficient model i.e. still 2D and rigorous !

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Helicoidal coordinates

- Helicoidal coordinates :

$$\begin{cases} x_1 = \xi_1 \cos(\alpha\xi_3) + \xi_2 \sin(\alpha\xi_3) \ , \\ x_2 = -\xi_1 \sin(\alpha\xi_3) + \xi_2 \cos(\alpha\xi_3) \ , \\ x_3 = \xi_3 \ , \end{cases} \quad (1)$$

where α is a parameter which characterizes the torsion of the structure.

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where α is a parameter which characterizes the torsion of the structure.

A twisted problem

is a problem whose cross section is independent from ξ_3.

Jacobian matrix of the Cartesian/helicoidal coordinate transformation

- This coordinate system is characterized by the Jacobian matrix of the transformation :

$$\mathbf{J}(\xi_1, \xi_2, \xi_3) = \frac{\partial(x_1, x_2, x_3)}{\partial(\xi_1, \xi_2, \xi_3)}$$

$$= \begin{pmatrix} \cos(\alpha\xi_3) & \sin(\alpha\xi_3) & \alpha\xi_2\cos(\alpha\xi_3) - \alpha\xi_1\sin(\alpha\xi_3) \\ -\sin(\alpha\xi_3) & \cos(\alpha\xi_3) & -\alpha\xi_1\cos(\alpha\xi_3) - \alpha\xi_2\sin(\alpha\xi_3) \\ 0 & 0 & 1 \end{pmatrix},$$

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- which does depend on the three variables ξ_1, ξ_2 and ξ_3.

$\mathbf{T}_h$ matrix for equivalent material in helicoidal coordinates

- The transformation matrix $\mathbf{T}_h$ for helicoidal coordinates is :

$$\mathbf{T}_h(\xi_1, \xi_2) = \frac{\mathbf{J}^T \mathbf{J}}{det(\mathbf{J})} = \begin{pmatrix} 1 & 0 & \alpha\xi_2 \\ 0 & 1 & -\alpha\xi_1 \\ \alpha\xi_2 & -\alpha\xi_1 & 1 + \alpha^2(\xi_1^2 + \xi_2^2) \end{pmatrix} ,$$

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- which only depends on the first two variables ξ_1 and ξ_2...
- ... allowing a **two-dimensional formulation** of the twisted waveguide problem !

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Compose the transformation into helicoidal coordinates with a complex stretch to obtain the **twisted PML** inverse matrix $\mathbf{T}_{hPML}^{-1}$ given by :

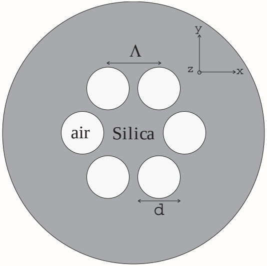
$$\begin{pmatrix} \frac{\tilde{\rho}\cos^2(\varphi)}{\rho s_\rho} + \frac{\rho(1+\alpha^2\tilde{\rho}^2)s_\rho \sin^2(\varphi)}{\tilde{\rho}} & \frac{\sin(2\varphi)(\tilde{\rho}^2-\rho^2(1+\alpha^2\tilde{\rho}^2)s_\rho^2)}{2\rho\tilde{\rho}s_\rho} & -\alpha\tilde{\rho}s_\rho\sin(\varphi) \\ \frac{\sin(2\varphi)(\tilde{\rho}^2-\rho^2(1+\alpha^2\tilde{\rho}^2)s_\rho^2)}{2\rho\tilde{\rho}s_\rho} & \frac{\tilde{\rho}\sin^2(\varphi)}{\rho s_\rho} + \frac{\rho(1+\alpha^2\tilde{\rho}^2)s_\rho \cos^2(\varphi)}{\tilde{\rho}} & \alpha\cos(\varphi)\tilde{\rho}s_\rho \\ -\alpha\tilde{\rho}s_\rho\sin(\varphi) & \alpha\cos(\varphi)\tilde{\rho}s_\rho & \frac{\tilde{\rho}s_\rho}{\rho} \end{pmatrix}$$

All the quantities involved in the previous expression can be given as explicit functions of the two “helicoidal pseudo Cartesian modelling coordinates ” ξ_1, ξ_2 :

$\varphi = 2\arctan\left(\frac{\xi_2}{\xi_1+\sqrt{\xi_1^2+\xi_2^2}}\right)$, $\rho = \sqrt{\xi_1^2+\xi_2^2}$, $s_\rho(\rho) = s_\rho(\sqrt{\xi_1^2+\xi_2^2})$,

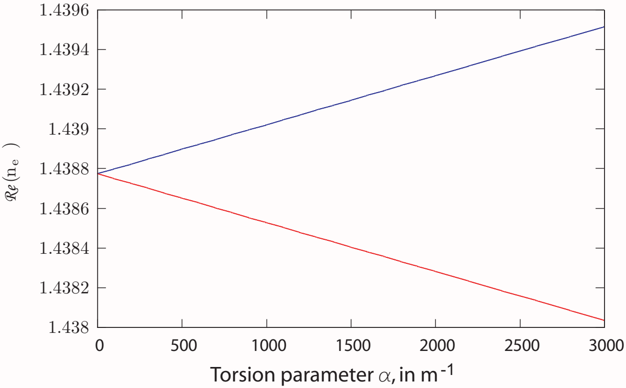
and $\tilde{\rho} = \int_0^{\sqrt{\xi_1^2+\xi_2^2}} s_\rho(\rho')d\rho'$.

Evolution of the Real Part of Effective Index

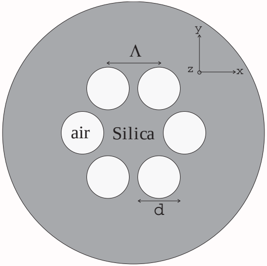


Parameters

- $\Lambda = 6.75\mu m$
- $\mathrm{d} = 5\mu m$
- $n_{Si.} = 1.444024$
- $\lambda = 1.55\mu m$



Evolution of the Imaginary Part of Effective Index



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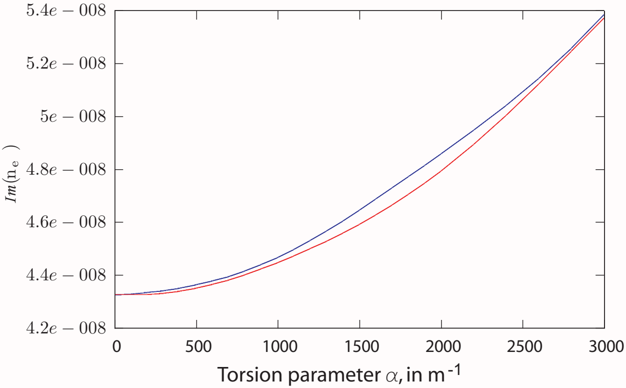
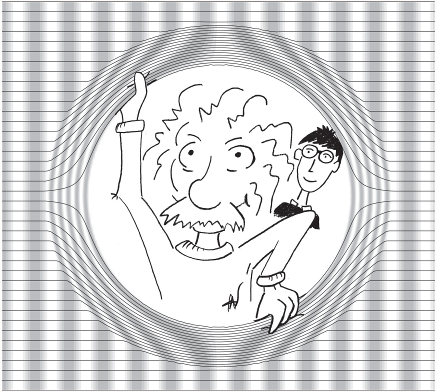


From invisibility cloak...



... to polyjuice potion !

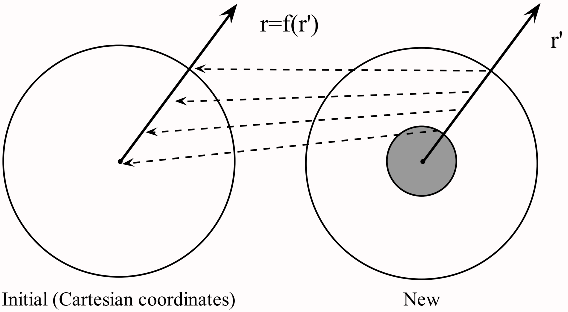
Pendry's map for cylindrical **invisibility cloaks**

Map an annulus on a circle and let the **EQUIVALENT MATERIAL** to become a **NEW PHYSICAL MATERIAL** !
Consider a geometric transformation which maps the field within the disk $r \leq R_2$ onto the annulus $R_1 \leq r \leq R_2$:

$$\begin{cases} r = f(r') = (r' - R_1)R_2/(R_2 - R_1) \text{ for } R_1 \leq r' \leq R_2, \\ \theta = \theta', \ z = z'. \end{cases}$$

where r', θ' and z' are "radially contracted cylindrical coordinates". Moreover, this transformation maps the field for $r \geq R_2$ onto itself by the identity transformation.

Pendry's map for cylindrical **invisibility cloaks**



The material properties of the invisibility cloak are given by :

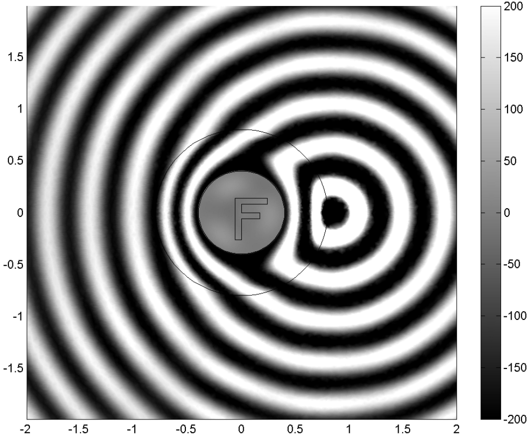
$$\mathbf{T}^{-1} = \mathbf{R}(\theta')\mathbf{diag}\left(\frac{r'-R_1}{r'}, \frac{r'}{r'-R_1}, c_{11}^2\frac{r'-R_1}{r'}\right)\mathbf{R}(\theta')^T$$

where $c_{11} = R_2/(R_2 - R_1)$, r' and θ' are the well known functions $r'(x', y') = \sqrt{x'^2 + y'^2}$ and $\theta'(x', y') = 2\,\mathrm{arctan}\left(\frac{y'}{x'+\sqrt{x'^2+y'^2}}\right)$, and $\mathbf{R}(\theta')$ is a rotation matrix.

x', y' are the Cartesian coordinates in the annulus configuration.

Cloaking

- ... and we have got the recipe for the cylindrical circular cloak !



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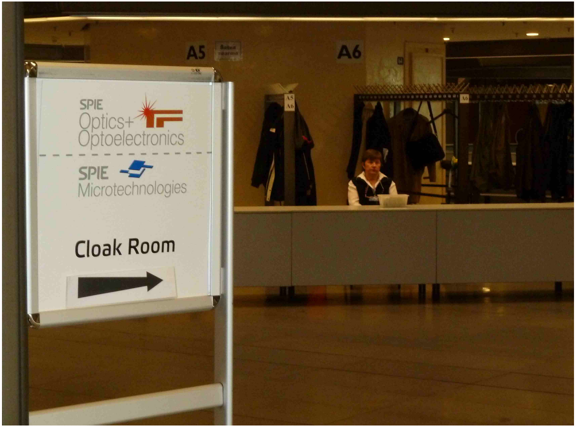
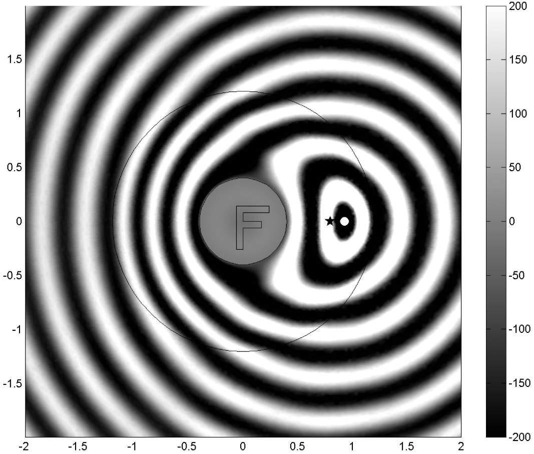
Cloaking

- In Photonics conferences, courtesy requires that you leave your cloak in an appropriate place !



Mirage effect

- The light source can also be inside the cloak !
  In this case, the object in the cavity is still invisible and the light seems to be emitted from a shifted position (mirage effect).



Cloak of arbitrary shape

Make the radii depend on θ : $R_1(\theta)$, $R_2(\theta)$.
The geometric transformation which maps the field within the full domain $r \leq R_2(\theta)$ onto the hollow domain $R_1(\theta) \leq r \leq R_2(\theta)$:

$$\begin{cases} r'(r,\theta) = R_1(\theta) + r(R_2(\theta) - R_1(\theta))/R_2(\theta) \; , \; 0 \leq r \leq R_2(\theta) \\ \theta' = \theta \; , \; 0 < \theta \leq 2\pi \\ z' = z \; , \; z \in \mathbb{R} \; , \end{cases}$$

and the transformation maps the field for $r \geq R_2(\theta)$ onto itself by the identity transformation. This leads to

$$\mathbf{J}_{rr'} = \frac{\partial(r(r',\theta'),\theta,z)}{\partial(r',\theta',z')} = \begin{pmatrix} c_{11}(r',\theta') & c_{12}(r',\theta') & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Cloak of arbitrary shape

$$\mathbf{J}_{rr'}(r',\theta') = \frac{\partial(r(r',\theta'),\theta,z)}{\partial(r',\theta',z')} = \begin{pmatrix} c_{11}(r',\theta') & c_{12}(r',\theta') & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $c_{11}(r',\theta') = R_2(\theta')/(R_2(\theta') - R_1(\theta'))$ for $0 \leq r' \leq R_2(\theta')$
and $c_{11} = 1$ for $r' > R_2(\theta')$

and $c_{12}(r',\theta') = \frac{(r'-R_2(\theta'))R_2(\theta')\frac{dR_1(\theta')}{d\theta'} - (r'-R_1(\theta'))R_1(\theta')\frac{dR_2(\theta')}{d\theta'}}{(R_2(\theta')-R_1(\theta'))^2}$

for $0 \leq r' \leq R_2(\theta')$

and $c_{12} = 0$ for $r' > R_2(\theta')$.

Cloak of arbitrary shape

Finally, the properties of the cloak are given by :

$$\mathbf{T}^{-1} = \mathbf{R}(\theta') \begin{pmatrix} \frac{c_{12}^2+f_r^2}{c_{12} f_r r'} & -\frac{c_{12}}{f_r} & 0 \\ -\frac{c_{12}}{f_r} & \frac{c_{11} r'}{f_r} & 0 \\ 0 & 0 & \frac{c_{11} f_r}{r'} \end{pmatrix} \mathbf{R}(\theta')^T$$

with

$$f_r = \frac{(r' - R_1)R_2}{(R_2 - R_1)}.$$

The central matrix depends on θ' !

Elliptical cloak as a particular case

- Parametric representation of the ellipse

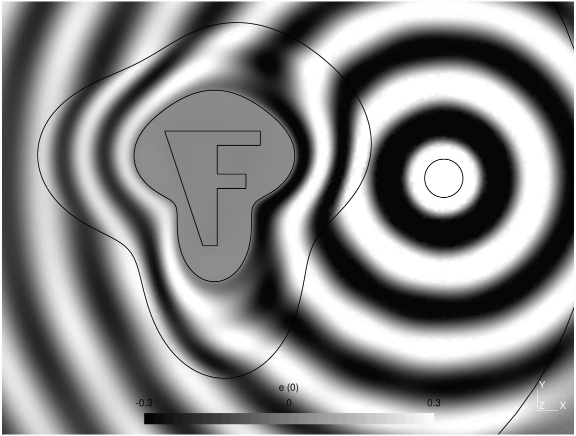
$$R(\theta) = \frac{ab}{\sqrt{a^2 \cos(\theta)^2 + b^2 \sin(\theta)^2}}$$

Cloak of arbitrary shape

Use Fourier series $R(\theta) = a_0 + \sum_{i=1}^{n}(a_i \cos(i\theta) + b_i \sin(i\theta))$ to obtain general shapes :
R_1 is with $a_0 = 1, b_1 = 0.1, a_2 = -0.15, b_3 = 0.2, a_4 = 0.1,$
R_2 is with $a_0 = 2, a_2 = -0.1, a_3 = -0.15, b_3 = 0.3, a_4 = 0.2,$
all the other coefficients $= 0$.

Cloak of arbitrary shape

- Zoom on the cloak and the source...



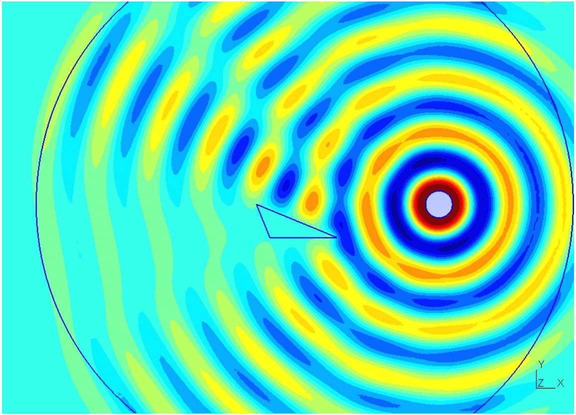
The geometric transformation is performed pointwise on the disk :

$$\underline{\underline{\varepsilon}}'(\mathbf{x}') = \mathbf{J}^{-1}(\mathbf{x}')\underline{\underline{\varepsilon}}(\mathbf{x}(\mathbf{x}'))\mathbf{J}^{-T}(\mathbf{x}')\det(\mathbf{J}(\mathbf{x}')),$$
$$\underline{\underline{\mu}}'(\mathbf{x}') = \mathbf{J}^{-1}(\mathbf{x}')\underline{\underline{\mu}}(\mathbf{x}(\mathbf{x}'))\mathbf{J}^{-T}(\mathbf{x}')\det(\mathbf{J}(\mathbf{x}')).$$

These formulae are valid for any initial content of the central disk !

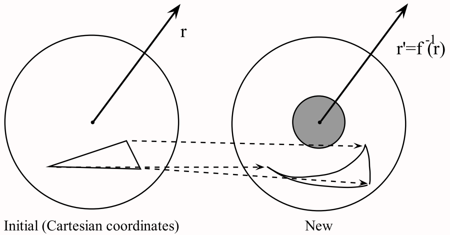
Masking effect

Scattering of cylindrical waves (real part of E_z) by a conducting object of triangular cross section :



Masking effect

Transforming a non homogeneous region :



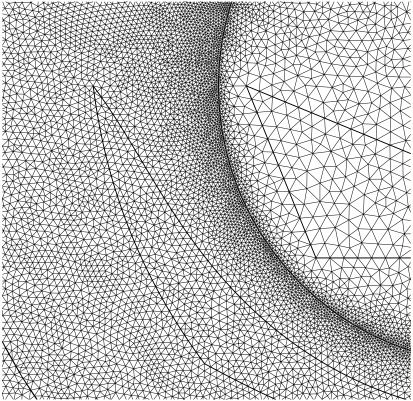
- The cross sections of the boundaries of homogeneous regions (jumps of material properties) are curves (here, the triangle) and are therefore **contravariant**.
- They are **pushed forward** along the $r' = f^{-1}(r)$ inverse map (that is fortunately simple) :

$$\mathbf{x}'(t) = f^{-1}(\mathbf{x}(t)) = (\frac{R_2 - R_1}{R_2} + \frac{R_1}{\|\mathbf{x}(t)\|})\mathbf{x}(t),$$

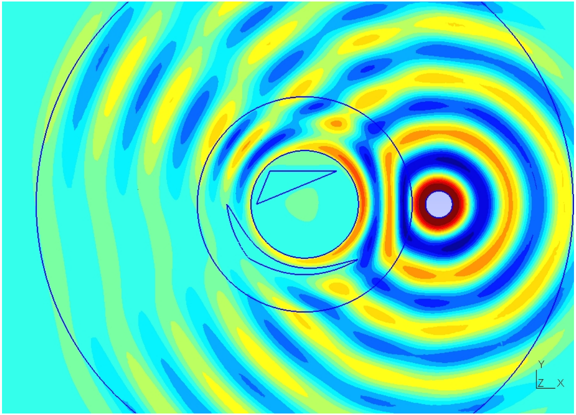
- The anamorphosis of the triangle is described by three splines interpolating each 40 points.

Masking effect

- Geometry description and meshing are performed with GetDP and Gmsh.
- The inner boundary of the cloak must be very finely meshed because of the singularity of the material properties.



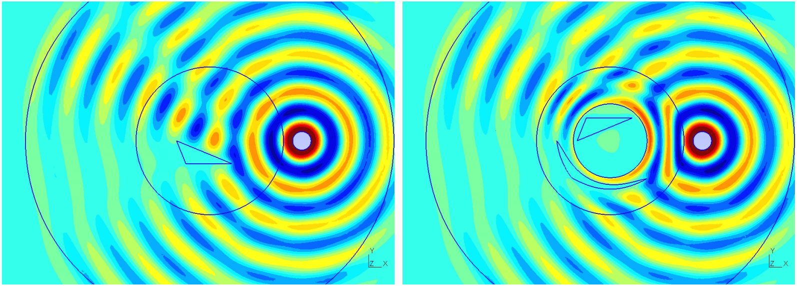
Masking effect



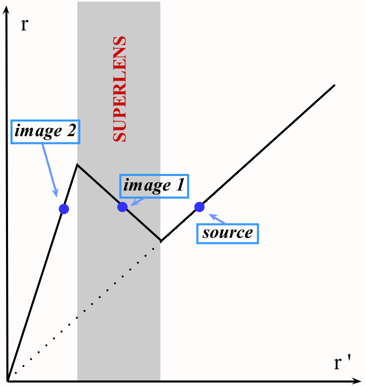
Outside the cloak, the scattered field is left unchanged with respect to the initial situation !

Masking effect

Compare the field outside the disks :



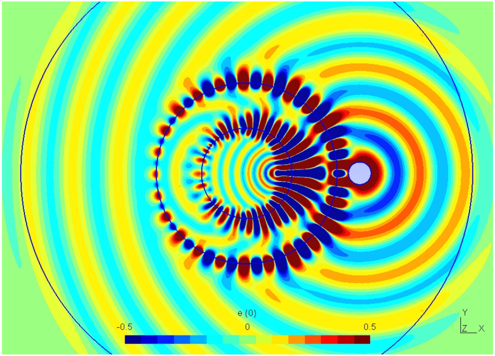
A multivalued map



The negative slope corresponds to Negative Refraction Index materials and should provide superlenses.

J B Pendry, Phys. Rev. Lett. 85, 3966 (2000)
Ulf Leonhardt and Thomas G Philbin, New J. Phys. 8 247 (2006)
Min Yan, Wei Yan, and Min Qiu, Phys. Rev. B 78, 125113 (2008)

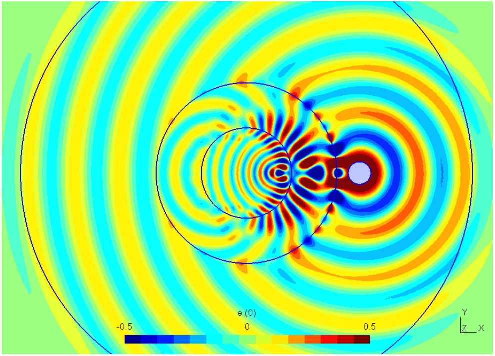
Superscatterer



Without losses : anomalous resonances* are dazzling !

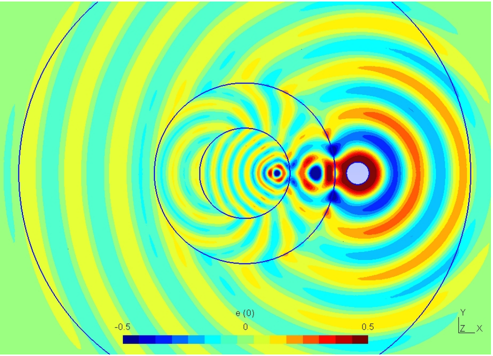
* N. A. Nicorovici, G. W. Milton, R. C. McPhedran, and L. C. Botten, Optics Express, Vol. 15, Issue 10, pp. 6314-6323 (2007)

Superscatterer



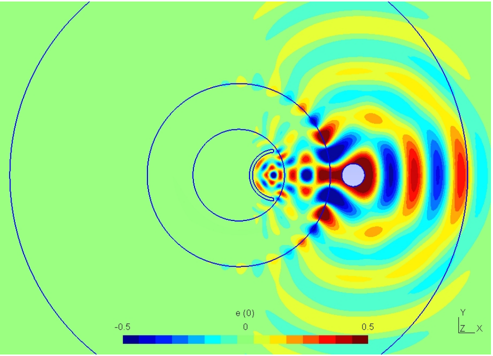
With $1/1000$ losses in the superlens : the two images of the sources appear !

Superscatterer



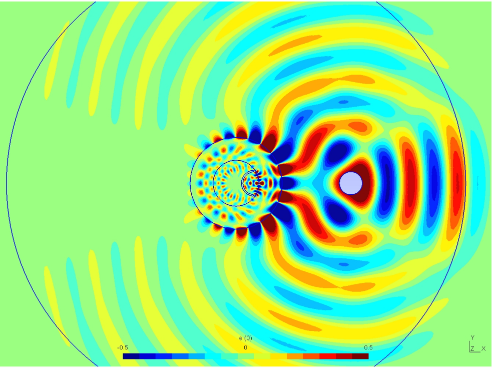
With $1/100$ losses in the superlens : the disturbance is still reasonable and the three copies of the source appear clearly !

Superscatterer

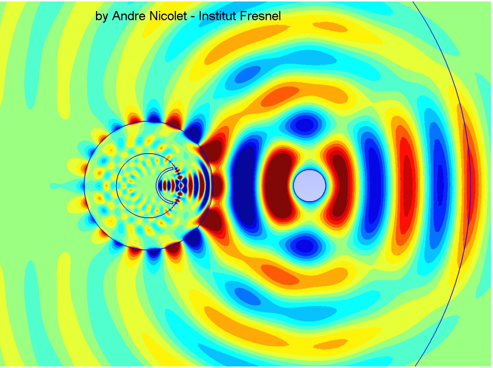


Remote control : a small scatterer close to the interior image of the source acts as a large one (magnification factor = 4) on the source ($1/100$ losses in the superlens) !

Superscatterer

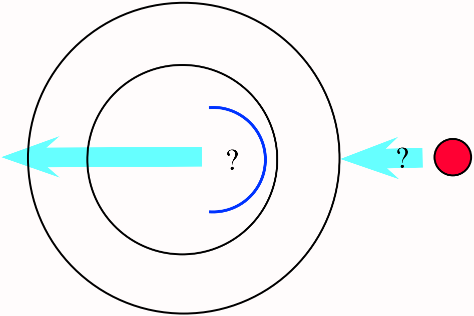


Remote control : a small scatterer close to the interior image of the source acts as a large one (magnification factor = 8) on the source ($1/100$ losses in the superlens) !



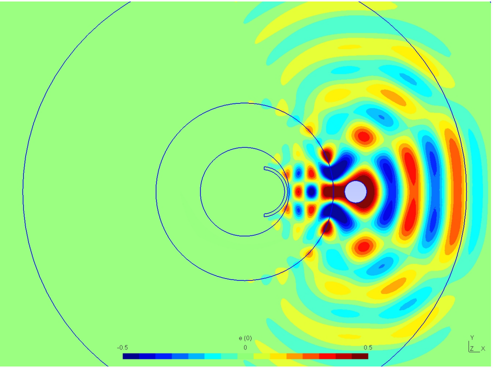
Remote control : a small scatterer close to the interior image of the source acts as a large one (magnification factor = 8) on the source ($1/100$ losses in the superlens) ! (zoomed)

Superscatterer



Can you force the light to go to the left with a device located on the left of the source ?

Superscatterer



No ! The scatterer is a PERTURBATION of the folded geometry and its presence prevents the correct formation of the image source !

- Periodic structures

Metamaterials in electromagnetism

- Periodic structures
  - 1D : gratings, Bragg mirrors...

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- Plasmonic - metallic nanostructures for optics ($Re(\varepsilon) < 0$, $Im(\varepsilon)$ very small, size $< \lambda$),
  Split Ring Resonators for microwaves

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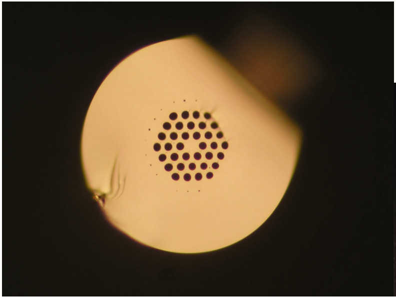
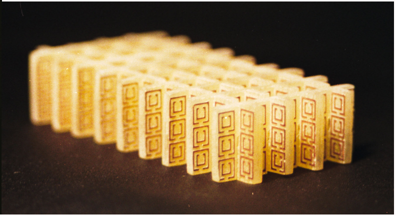
- **Wire media.**

Metamaterials in electromagnetism





Microstructured optical fibre made of chalcogenide glass manufactured at the Université de Rennes (ext. diam. = 139 μm).

Split Ring Resonator device with printed copper "double C" for microwave experiments.

Metamaterials in electromagnetism

Purpose : obtain new artificial properties :

- Negative Refraction Index for Perfect Lenses

Metamaterials in electromagnetism

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- Photonic Band Gap and Epsilon Near Zero for guiding light in defects
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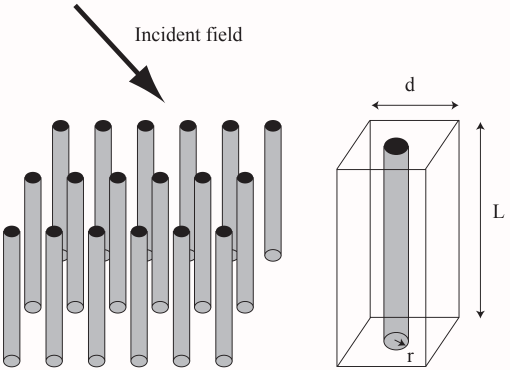
Homogenization :

Find equivalent material properties (ε, μ ...) in order to represent the metamaterial as an homogeneous medium !

Wire Media

Set of parallel metallic or dielectric wires of FINITE length periodically disposed.

This media is completely transparent for waves without a component of the electric field transverse to the wires !



Wire Media : Bouchitté - D. Felbacq approach

In

[BF] *Homogenization of a Wire Photonic Crystal : the Case os Small Volume Fraction, G. Bouchitté and D. Felbacq, SIAM J. APPL. MATH., Vol. 66, No. 6, pp. 2061-2084, 2006.*

the medium is not represented by equivalent ε and μ but by non local homogeneous properties : equivalent sources given by a PDE coupled to the field and involving homogeneous coefficients !

Wire Media : asymptotic model

Define : $k_0 := 2\pi/\lambda$ (wave number), $\eta := d/\lambda$ (relative size of the cell), $f := \pi r^2/d^2$ (filling factor), and $Z_0 := \sqrt{\frac{\mu_0}{\varepsilon_0}}$ (free space impedance)

λ, L are finite

and

$d, r, \sigma^{-1} \to 0$ with

$$\kappa = \frac{\sigma f Z_o}{k}$$

$$\gamma = -[\frac{1}{2}\log(\frac{f}{\pi})\eta^2]^{-1}$$

remaining FINITE (critical behaviour).

Wire Media : equivalent system [BF]

$\mathcal{E}$ electric field, $\mathcal{H}$ magnetic field, **y** unit vector , j equivalent sources (**y** component = direction of the parallel wires)

$$\begin{cases} \textbf{curl}\,\mathcal{E} = i\omega\mu_0\mathcal{H} \\ \textbf{curl}\,\mathcal{H} = -i\omega\varepsilon_0\left(\mathcal{E} + ij\mathbf{y}\right) \\ \dfrac{\partial^2 j}{\partial y^2} + K^2 j = 2i\pi\gamma\mathcal{E}\cdot\mathbf{y} \end{cases}$$

with $K^2 := k_0^2 + \frac{2i\pi\gamma}{\kappa}$.
+ Boundary Conditions (cf. [BF])

Note : γ *is geometrical and therefore real while* κ *depends on the medium and is possibly complex and even purely imaginary in the case of high permittivity dielectrics* $(\sigma + \omega\varepsilon'' - i\omega\varepsilon')$.

Wire Media : direct validation (A. Cabuz)

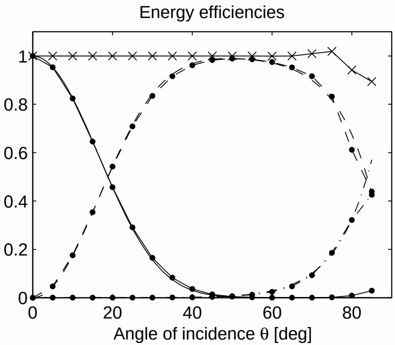


FIG.: Transmission (solid), reflection (dot-dashed) and absorption (dashed) efficiency curves comparing the finite element solution (dot markers) and the effective medium (no markers) as a function of angle of incidence. The wire conductivity is that of Toray T300® carbon fibers $\sigma = 5.89 \cdot 10^4 (\Omega m)^{-1}$. The structure has period $d_0 = 0.01m$, and dimensionless parameters $L/d_0 = 80$, $\lambda/d_0 = 20$, $r/d_0 = 3.5 \cdot 10^{-4}$, and $\delta/r = 15$. Energy conservation of the finite element model ($\times$ markers) is respected to within better than one percent for most angles of incidence. The departure around 80° is explained by the poor performance of the PML absorbing layers when close to grazing incidence.

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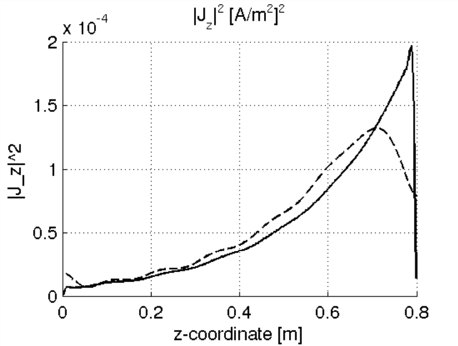

FIG.: Squared magnitude of the current density for the effective medium model (dashed) and the finite element solution (solid) as a function of position within the slab (which is positioned in $z \in (0, L)$). The structure is the same as in previous Fig., illuminated at an angle of incidence $\theta = 40^\circ$ from the top. Note that the surface areas under the two curves are the same because they are proportional to the Joule dissipation rates, which are seen to be equal from previous Fig. at the given angle of incidence.

Theoretical tool : two-scale convergence.

- Theorem : Given a frequency, a real number h, and an arbitrary real symmetric tensor M, by the homogenization of a periodic structure made of parallelepipeds containing inclusions that are an homogenized wire media (iterated homogenization), it is possible to build a media such that the effective permittivity has a real part equal to M and an imaginary part bounded by h (and the effective permeability is equal to 1).
- It is possible to obtain strong artificial magnetism with a periodic structure of dielectric inclusions.

Conclusion

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- **Thank you for your attention !**
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