Directed acyclic graphs with the unique dipath property

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Abstract

Let $\mathscr{P}$ be a family of dipaths of a directed graph G. The load of an arc is the number of dipaths containing this arc. Let $\pi(G, \mathscr{P})$ be the maximum of the load of all the arcs and let $w(G, \mathscr{P})$ be the minimum number of wavelengths (colors) needed to color the family of dipaths $\mathscr{P}$ in such a way that two dipaths with the same wavelength are arc-disjoint. Let G be a DAG (Directed Acyclic Graph). An internal cycle is an oriented cycle such that all the vertices have at least one predecessor and one successor in G (said otherwise every cycle contain neither a source nor a sink of G).

There exist DAGs such that the ratio between $w(G, \mathscr{P})$ and $\pi(G, \mathscr{P})$ cannot be bounded. We prove that, if that if G is a DAG, then for any family of dipaths $\mathscr{P}$, $w(G, \mathscr{P}) = \pi(G, \mathscr{P})$ if and only if G is without internal cycle.

We also consider an apparently new class of DAGs, which is of interest in itself, those for which there is at most one dipath from a vertex to another. We call these digraphs UPP-DAGs. For these UPP-DAGs we show that the load is equal to the maximum size of a clique of the conflict graph. We prove that the ratio between $w(G, \mathscr{P})$ and $\pi(G, \mathscr{P})$ cannot be bounded (a result conjecured in an other article). For that we introduce "good labelings" of the conflict graph associated to G and $\mathscr{P}$, namely labelings of the edges such that for any ordered pair of vertices (x, y) there do not exist two paths from x to y with increasing labels.

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