An unimportant (?) yet addictive (??) question

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The Zykov graphs compose a family of triangle-free graphs with unbounded chromatic number. It is obtained in the following way.

Let Z_1 be the single-vertex graph. For each $i \geq 2$, the graph Z_i is obtained by first taking one copy H_j of every graph Z_j with $j < i$. Then, for each sequence $s := v_1, v_2, \ldots, v_{i-1}$ of vertices such that $v_j \in V(H_j)$ for each index $j < i$, we create a new vertex x_s joined to (and only to) the vertices of the sequence s.

Thus, Z_2 is an edge while Z_3 is a 5-cycle. It can be checked that Z_i is triangle-free and has chromatic number i.

The question is to determine the fractional chromatic number a_i of Z_i. By analogy with what happens for the Mycielski graphs [2], Tony Jacobs conjectured [1] that

$$\forall i \geq 1, \quad a_{i+1} = a_i + a_i^{-1}\,.$$

He checked the correctness of his conjecture for $i \leq 5$.

If I am not mistaken, it can be shown that the proposed value is indeed an upper bound on a_{i+1}. To prove the conjecture, it would thus suffice to find a fractional clique in Z_{i+1} of weight $a_i + a_i^{-1}$. However, I doubt about the validity of the conjecture, and the recurrence relation might be more cryptic...

References

[1] Tony Jacobs: Fractional colorings and the Mycielski graphs. *Master Thesis*, Portland State University, (2006).

[2] M. Larsen, J. Propp, and D. Ullman: The fractional chromatic number of Mycielskis graphs. J. Graph Theory, 19(3):411–416, 1995.