Approximation for weighted coloring

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We consider weighted coloring for specific graphs, and we ask for good approximation algorithms, namely either $(1+\varepsilon)$ approximations or constant ones. In the weighted version, a vertex with weigth w must receive w different colors. We denote $\chi(G, w)$ the chromatic number of G with weigth function w.

- For grid like (hexagonal, triangular lattice) and planar graphs, we ask if there exists a constant approximation. One can prove that there exists algorithm returning a proper coloring with $(1+1/k)\chi(G, w)$ colors. This fact is mainly derived from the property that the maximum weigth independent set of planar graphs can be computed with precision $1+1/k$ in polynomial time (for any fixed k). As byproduct of the proof one also prove that $\chi(G, w)$ is up $1+\varepsilon$ egal to the maximum chromatic number of bounded subgraphs of G (which mean that this value is almost local); moreover the ratio $\chi(G, W)/\chi'(G, w)$ converge to 1 when both number goes to infinity. Note also that the size of the maximum clique is not giving a good approximation and that no constant approximation can be achieved by considering only bounded subgraphs (long odd cycles).

  We wonder if there exists a polynomial algorithm computing a coloring with $\chi(G, w)+b$ for some fixed arbitrary constant, or one computing $\chi(G, w) + \sqrt{\chi(G, w)}$ (any $o(x)$ function could be used). Intuitively coloring weighted planar graphs seems *easy* when one get a few extra colors.

- In the case of the following conflicts graphs we ask for $1+\varepsilon)$ approximation algorithms :

  - Arbitrary set of dipaths of a directed tree (two dipaths are connected if they share an arc)
  - Circular arc coloring : Arbitrary sets of path on a ring , two paths are adjacent if they share a link – ie intersection pattern of intervals of a circle.

  In both case rather good approximation do exist. In the case of circular interval graphs tucker gave the first approximations, but he was mainly interested i deriving relation between χ and the maximum clique. He also proved hardness relating the problem to finding disjoints paths (intergral multicommoidy) flow in an auxilliary graph which he reduced to expressing a permutation as a product of permutations lying in some subgroups of S_n.

  In both cases, the best existing approximation algorithm (Kumar, Ferreira & al) both rely on rounding a fractionnal coloring which can be related to solving Tucker flow problem (in the case of tree of variation of it) fractionnaly. Those algorithms perform much better than more classical algorithm that would not use linear programming. We conjecture that the ratio $\chi(G, w)/\chi'(G, w)$ converge to 1 when both number goes to infinity, this would lead to $(1+\varepsilon)$ approximation algorithms since in those graphs the fractionnal chromatic number can be computed in polynomial time.

Note that we do not exlcude hardness of approximation within some ratio better than 1.00000000.....1 but it seems unlikely. The hardness result seems difficult to turn into a non approximation one.