Cooperative colouring

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November 2008

Let $\mathcal{G} = (G_1, \dots, G_k)$ be a collection of graphs on the same vertex set V. A *cooperative colouring* of $\mathcal{G}$ is a family of subsets I_i, $1 \leq i \leq k$ of V such that I_i is stable in G_i and $\bigcup i = 1^k I_i = V$. According to a result of Haxell on independent system of representatives, if $k \geq 2\Delta(\mathcal{G})$ with $\Delta(\mathcal{G}) = \max \Delta(G_i)$ then there is a cooperative colouring of $\mathcal{G}$. Aharoni, Haxell and Holzman conjectured that if $k \geq \Delta(\mathcal{G}) + 2$ then there is a cooperative colouring of $\mathcal{G}$. This would be tight as Holzman showed a collection G of k graphs of maximum degree at most $k-1$ without cooperative colouring. This conjecture is trivaillay true if $\Delta(\mathcal{G}) + 1$ of the G_i are the same. It is not difficult to see that it also holds if $\Delta(\mathcal{G})$ of them are the same. The next step would be to show that it holds if $\Delta(\mathcal{G}) - 1$ of the G_i are the same. In particular, if these $\Delta(\mathcal{G}) - 1$ graphs are union of cliques one need to show the following. If G_1, G_2 and G_3 are three graphs of degree at most k on the same vertex set V and $(V_1, \dots, V_m)$ is a partition of V such that $|V_i| = k+1$ for every $1 \leq i \leq m$ then there exists I_1, I_2 and I_3 three independent sets of G_1, G_2 and G_3 respectively such that $|(I_1 \cup I_2 \cup I_3) \cap V_i| \geq 2$ for every $1 \leq i \leq m$.