Résumé de thèse

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Mascotte - Boréon

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Plan de l'exposé (et de la thèse..)

- Partie I: Groupage de trafic
- Partie II: Sous-graphes avec contraintes sur le degré
- Partie III: Autres problèmes (plus ou moins reliés)
- Conclusions

Traffic Grooming

General idea

- WDM networks (Wavelength Division Multiplexing)
  - 1 wavelength = up to 40 Gb/s
  - 1 fiber = hundreds of wavelengths = Tb/s

- Idea: **Traffic grooming** consists in grouping low-speed traffic flows into higher speed streams

  $\longrightarrow$ we allocate the same wavelength to several low-speed requests (TDM, Time Division Multiplexing)

- Objectives:
  - Efficient usage of bandwidth
  - Minimize network cost

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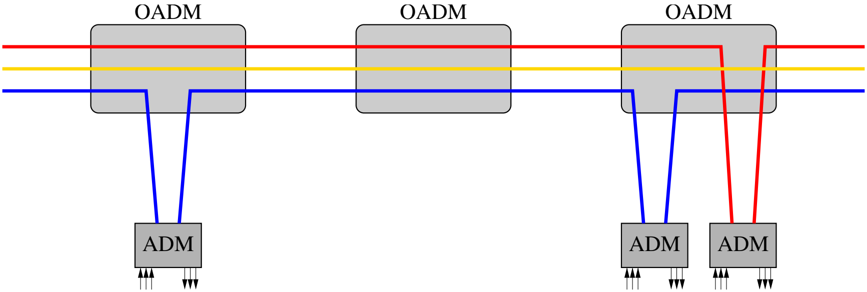
ADM and OADM

- **OADM** (Optical Add/Drop Multiplexer)= adds-extracts a wavelength from a fiber
- **ADM** (Add/Drop Multiplexer)= add-extracts OC/STM (low speed signal) from a wavelength



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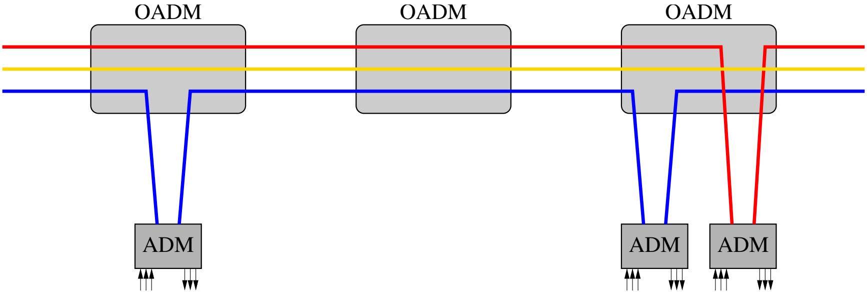

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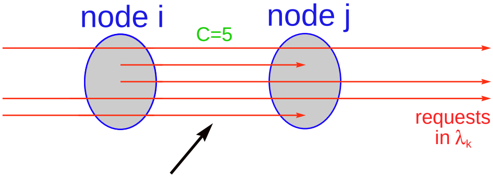
Definitions

- **Request** (i, j): pair of nodes (i, j) that want to exchange (low-speed) traffic
- **Grooming factor C:**



For each wavelenght and each arc between 2 nodes, there can be only C requests routed through this arc

- **Load** of an arc on a wavelength: number of requests using this arc on this wavelength ($\leq C$)

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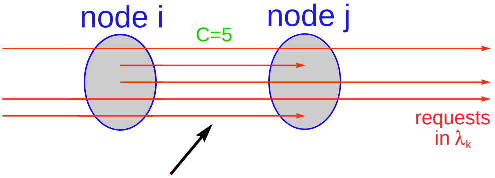

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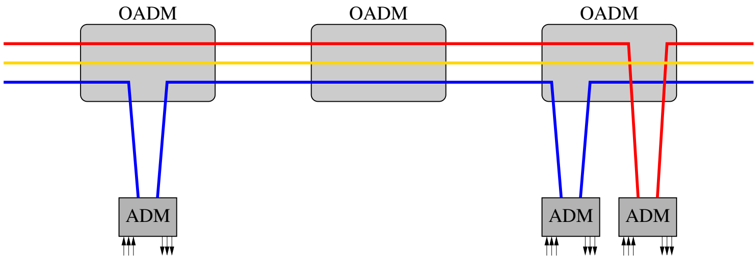

- <u>**Idea:**</u> Use **ADMs only at the endpoints of a *request* (lightpaths)** to save as many ADMs as possible

Model

- Model:

Topology	$\rightarrow$	graph G
Set of requests	$\rightarrow$	graph R
Grooming factor	$\rightarrow$	integer C
Requests on a **wavelength**	$\rightarrow$	edges of a **subgraph** of R
ADM on a wavelength	$\rightarrow$	vertex of a subgraph of R

- An important case: $G = \overrightarrow{C}_n$ (**unidirectional ring**)
- Typically, one considers **symmetric requests**

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Statement of the problem

Traffic Grooming in Unidirectional Rings

Input A cycle C_n on n nodes (network);
An *undirected* graph R on n nodes (request set);
A grooming factor C.

Output A partition of $E(R)$ into subgraphs
$R_1, \ldots, R_W$ with $|E(R_i)| \leq C$, i=1,...,W.

Objective Minimize $\sum_{\omega=1}^{W} |V(R_\omega)|$.

Example: $n = 4$, $R = K_4$, and $C = 3$

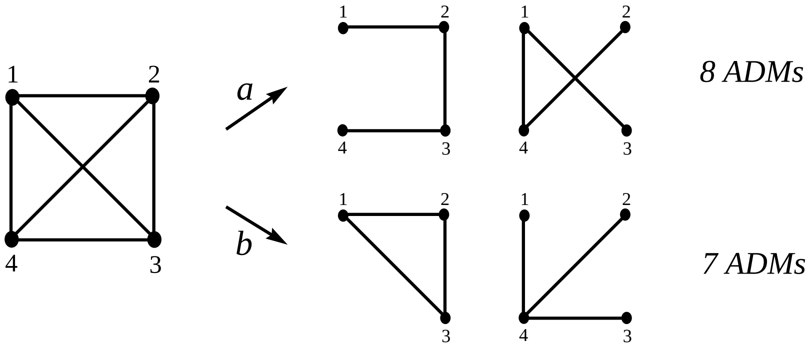

Figure: Two valid partitions of K_4 in a unidirectional ring for $C = 3$.

Traffic grooming in my Ph.D

- Hardness and approximation in rings and paths
  With *Omid Amini* and *Stéphane Pérennes*.
- Bidirectional rings
  With *Jean-Claude Bermond* and *Xavier Muñoz*.
- 2-period traffic grooming in unidirectional rings
  With *Jean-Claude Bermond*, *Charles J. Colbourn*, *Lucia Gionfriddo* and *Gaetano Quattrocchi*.
- Bounded degree request graph in unidirectional rings
  With *Xavier Muñoz* i *Zhentao Li*.
- Stars and trees...?
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Hardness and approximation in rings and paths

With *Omid Amini* and *Stéphane Pérennes*

- The problem of finding the maximum number of edge-disjoint triangles in a tripartite graph is APX-hard.

- TRAFFIC GROOMING is APX-hard in rings and paths.

- Using a known algorithm for the k-DENSE SUBGRAPH problem, we provide an $\mathcal{O}(n^{1/3} \log^2 n)$-algorithm for any $C \geq 1$. This is the first approximation algorithm whose approximation guarantee and running times are independent of the grooming factor.

  (n is the number of nodes of the network)

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Bidirectional rings

With *Jean-Claude Bermond* and *Xavier Muñoz*

- We consider the all-to-all case.
- Statement of the problem and general lower/upper bounds.
- Optimal solutions for some infinite families of values of n, C.

2-period traffic grooming in unidirectional rings

With *Jean-Claude Bermond*, *Charles J. Colbourn*, *Lucia Gionfriddo* and *Gaetano Quattrocchi*

- There is a subset of nodes that need more bandwidth $\Rightarrow$ two grooming factors C, C', with $1 \leq C' < C$.
- The problem consists in finding a partition of the edges of K_n that *embeds* another partition with different grooming factor.
- We solve the cases $C = 4$ and $C' \in \{1, 2, 3\}$.

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Bounded degree request graph in unidirectional rings

With *Xavier Muñoz* i *Zhentao Li*

- We introduce a new model that allows the network to support dynamic traffic, as far as the maximum degree of the request graph is at most a constant Δ.
- The problem consists in finding the least integer $M(C, \Delta)$ such that the edges of any graph with maximum degree at most Δ can be partitioned into subgraphs with at most C edges and each vertex appears in at most $M(C, \Delta)$ subgraphs.
- We establish the value of $M(C, \Delta)$ for many more cases, leaving open only the case where $\Delta \geq 5$ is odd, $\Delta \pmod{2C}$ is between 3 and $C - 1$, $C \geq 4$, and the request graph does not contain a perfect matching.

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Degree-constrained subgraph problems

Broad family of problems

- A *typical* **DEGREE-CONSTRAINED SUBGRAPH PROBLEM**:

  Input:

  - a (*weighted* or *unweighted*) graph G, and
  - an integer d.

  Output:

  - a (*connected*) subgraph H of G,
  - satisfying some degree constraints ($\Delta(H) \leq d$ or $\delta(H) \geq d$),
  - and optimizing some parameter ($|V(H)|$ or $|E(H)|$).

- Several problems in this broad family are classical widely studied NP-hard problems.

- They have a number of applications in interconnection networks, routing algorithms, chemistry, ...

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First problem

- **MINIMUM SUBGRAPH OF MINIMUM DEGREE** $\geq d$ **(MSMD$_d$)**:

  Input: an undirected graph $G = (V, E)$ and an integer $d \geq 3$.

  Output: a subset $S \subseteq V$ with $\delta(G[S]) \geq d$, s.t. $|S|$ is minimum.

- For $d = 2$ it is the GIRTH problem (find the length of a shortest cycle), which is in P.

- Motivation: relation with DENSE k-SUBGRAPH problem and TRAFFIC GROOMING problem in optical networks.

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Second problem

- **MAXIMUM d-DEGREE-BOUNDED CONNECTED SUBGRAPH (MDBCS$_d$):**

  Input:
  - an undirected graph $G = (V, E)$,
  - an integer $d \geq 2$, and
  - a weight function $\omega : E \to \mathbb{R}^+$.

  Output:
  a subset of edges $E' \subseteq E$ of **maximum weight**, s.t. $G' = (V, E')$
  - is **connected**, and
  - has **maximum degree** $\leq d$.

- It is one of the classical **NP**-hard problems of *[Garey and Johnson, Computers and Intractability, 1979]*.
- If the output subgraph is not required to be connected, the problem is in **P** for any d (using matching techniques).
- For *fixed* $d = 2$ it is the well known LONGEST PATH problem.

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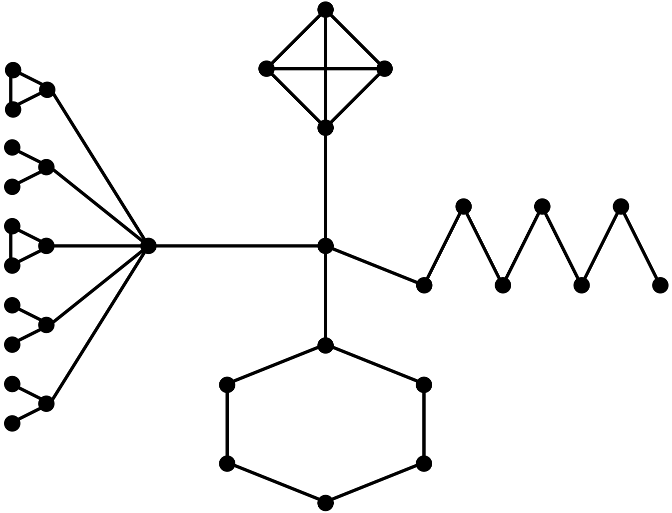
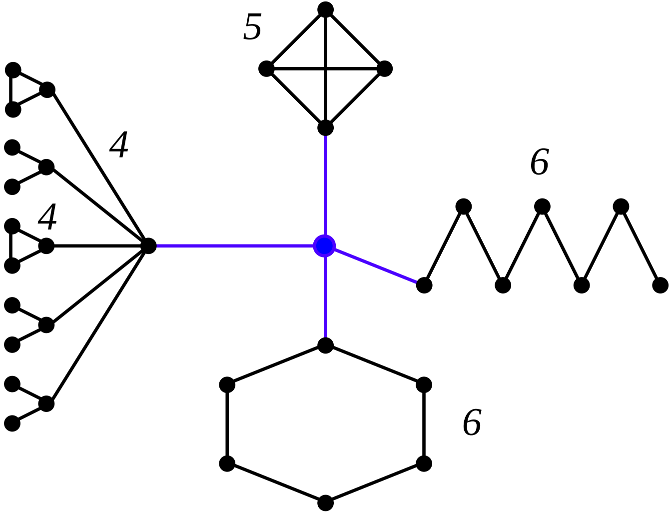
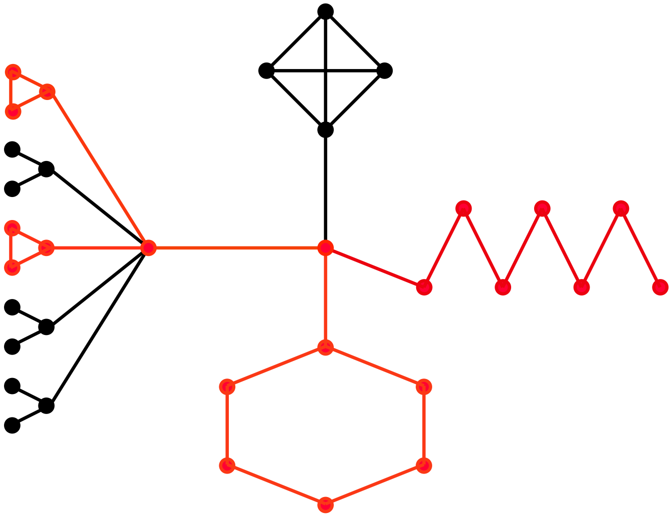
Example with $d = 3$, $\omega(e) = 1$ for all $e \in E(G)$



Example with $d = 3$ (II)



Example with $d = 3$ (III)



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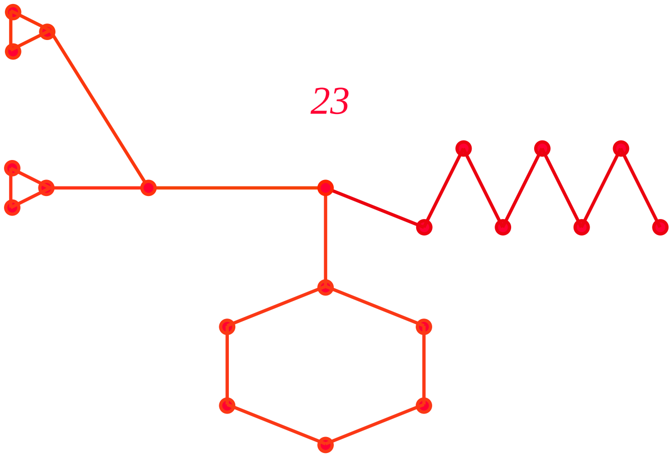

Degree-constrained subgraph problems in my Ph.D

- Parameterized complexity of MSMD_d

  With *Omid Amini* and *Saket Saurabh*.

- Hardness and approximation of degree-constrained subgraph problems

  With *Omid Amini*, *David Peleg*, *Stéphane Pérennes* and *Saket Saurabh*.

- Subexponential parameterized algorithms for bounded-degree connected subgraph problems on planar graphs

  With *Dimitrios M. Thilikos*.

- Non-crossing partitions and dynamic programming in graphs on surfaces

  With *Dimitrios M. Thilikos* and *Juanjo Rué*.

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- Non-crossing partitions and dynamic programming in graphs on surfaces

  With *Dimitrios M. Thilikos* and *Juanjo Rué*.

Other problems

Some other problems (...in my Ph.D?) 1/2

- **Packet routing problems in plane grids**
  - Permutation routing in triangular grids
    With *Janez Zerovnik*.
  - (ℓ, k)-routing in plane grids
    With *Omid Amini*, *Florian Huc* and *Janez Zerovnik*.
- **Tolerance graphs**
  - New intersection model and improved coloring and maximum clique algorithms
    With *George Mertzios* and *Shmuel Zaks*.
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- **Tunnels in GMPLS networks**
  - The case of the path

    With *Jean-Claude Bermond*, *David Coudert*, *Joanna Moulierac*, *Stéphane Pérennes*, *Hervé Rivano* and *Fernando Solano*.
  - Hardness and approximation in general networks

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- **Self-duality of branchwidth in graphs of bounded genus**

  With *Dimitrios M. Thilikos*.

- **Edge-simple circuits through 10 ordered vertices in square grids**

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- **Weighted coloring in P_4-sparse graphs**

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