Routing and Scheduling Problem in Wireless Networks

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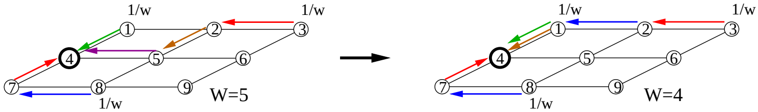
The Round Weighting Problem [1]

Definition:

- Joint *d-distance weighted edge coloring* and *routing* problem.
- **Input:**
  - Network graph G (sources and destinations),
  - Routers bandwidth demands (units/W),
  - Interference distance *d*.
- **Output:** a routing (edge weights with flow conservation) that requires the minimum quantity of colors W (time units).

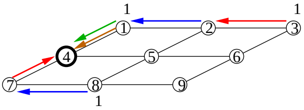


[1] Klasing,Perennes,Morales

The Round Weighting Problem

Objective: Minimize the weight of the **rounds** covering the **routing**.

Round: a collection of links that can be simultaneously activated in the network, for example $d = 1$ (matching in G) or $d = 2$ (induced matching in G).



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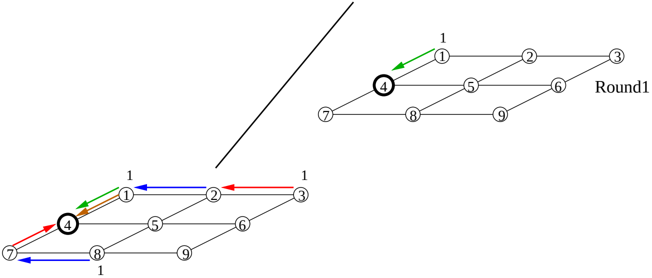

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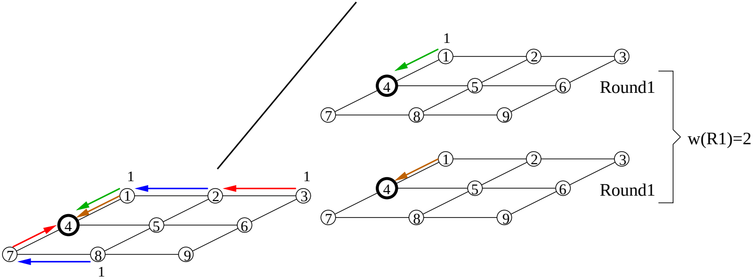

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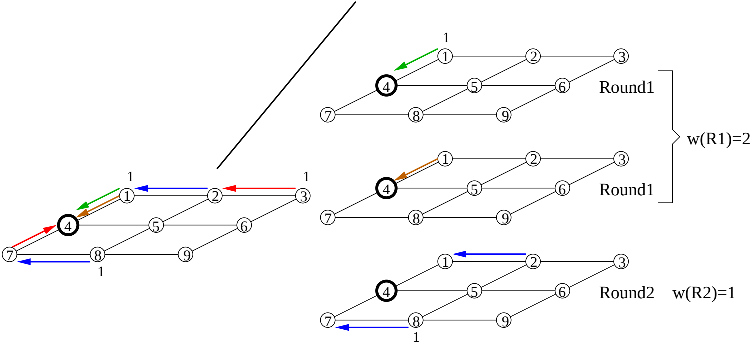

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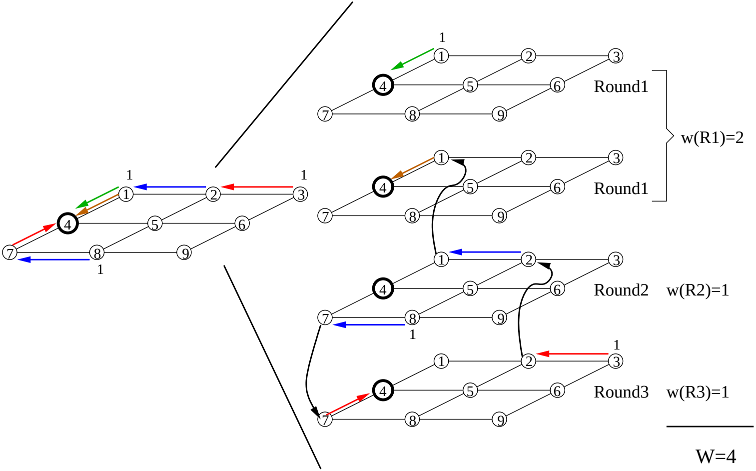

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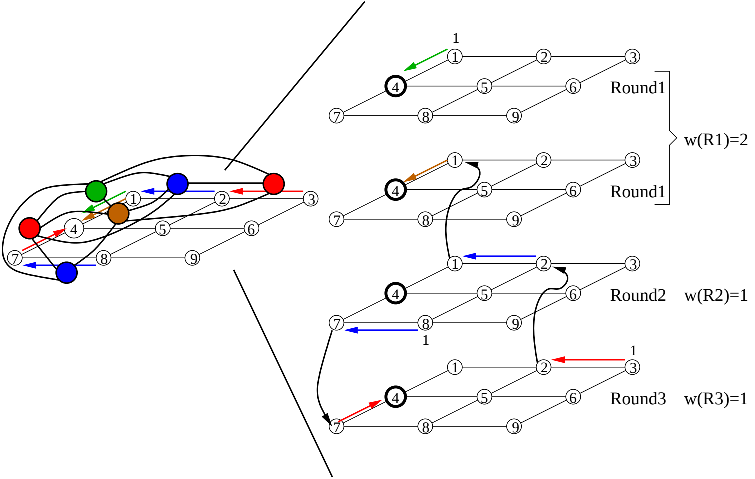


The Round Weighting Problem

Network Characteristics

- Continuous traffic;
- Time division multiple access (TDMA).

Specificity of our case

- Binary interference model (any two links either interfere with each other, or they can be active simultaneously, e.g. Rounds = induced matchings);
- Convergent traffic;
- Application: Wireless Mesh Network;
- Fractional Flow.

Resolution Methods

Column Generation (CG)

CG is used to avoid dealing with the whole exponential set of rounds.

RWP Decomposition [2]:

- *Master problem:* Routing problem (polynomial);
- *Sub-problem:* Maximum weighted round problem (reduced to the max. weighted independent set).

[2] based on Zhang et al [ZWZL05]

Column Generation (CG)

Mathematical Formulation [3]

Master problem: Routing problem

$$\sum_{i \in V_{/}(v,i) \in E} x^v_{v,i} = d_v, \forall v \in V_r \qquad (1)$$

$$\sum_{j \in V_g} \sum_{i \in V_r/(i,j) \in E} x^v_{i,j} = d_v, \forall v \in V_r \qquad (2)$$

$$\sum_{i \in V_r/(i,j) \in E} x^v_{i,j} - \sum_{k \in V/(j,k) \in E} x^v_{j,k} = 0, \forall j, v \in V_r \qquad (3)$$

$$\sum_{r \in \mathcal{R}} a^r_{i,j}.w_r - \sum_{v \in V_r} x^v_{i,j} \geqslant 0, \forall i, j \in E \qquad (4)$$

Sub-problem: Maximum weighted round problem

$$\max \sum_{(i,j) \in E} p_{(i,j)} u_{(i,j)} \qquad (5)$$

$$u_{(i,j)} + u_{(k,l)} \leqslant 1 + F^{(k,l)}_{(i,j)}, \forall (i,j) \in E, \forall (k,l) \in E \qquad (6)$$

[3] Implementation: AMPL (script), CPLEX (solver)

Column Generation and Multi-objective Analysis [4]

Minimize the communication time (RWP objective)

- maximizing equally the bandwidth of the routers,

$$obj_1 = \min \sum_{r \in \mathcal{R}} w_r.$$

Minimize the maximum load (Load balancing)

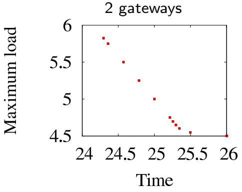
- increasing the security in case of failure,

$$obj_2 = \min \max_{v \in V_r} (l_v).$$

[4] C. Gomes, G. Huiban [GH07]

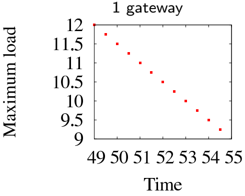
Column Generation and Multi-objective Analysis

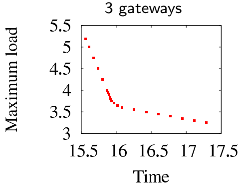
Results: Pareto set obtained by the ϵ-restricted technique.

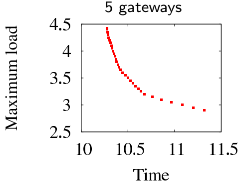


Mesh network with 39 nodes

Test networks: 39, 54, 65 nodes
1,2,3 and 5 gateways







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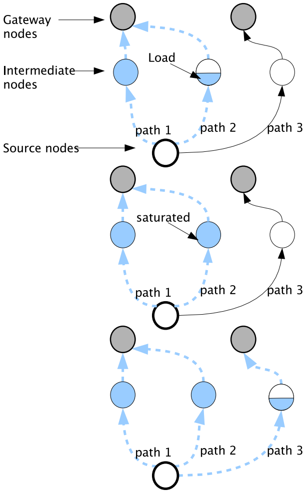

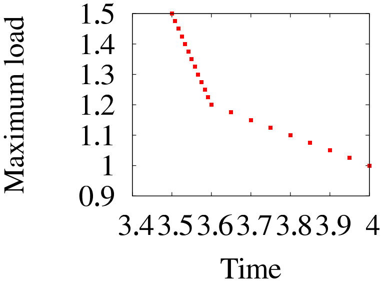

Column Generation and Multi-objective Analysis

Results:

- Each disruption is due to a saturation of a new node;
- The saturated nodes (bottleneck) are usually located around the gateway(s);
- The relation between the maximum load and the transmission time is convex and piecewise linear (Pareto set).

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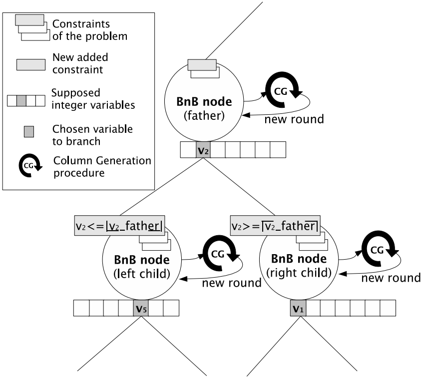
Branch and Price Algorithm [6]

BnP combines Branch-and-bound (BnB) with Column Generation [5]



- Motivation: Mono-routing (integer flow b(v)=1) to avoid dealing with the packet-reordering problem.
- Approach: Depth-First.

[5] Implementation: AMPL, Concert Tecnology (Java), CPLEX (solver)
[6] C. Gomes, H. Rivano, S. Perennes [GPR08]

Branch and Price Algorithm

Results (d=2, b(v)=1):

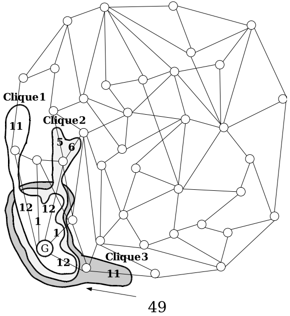
Network	Gateways	Nodes	Edges	W_{frac}	W_{int}
A	1	11	34	16	16
A	2	11	34	9.5	10
B	1	12	18	15	15
C	1	15	22	17.666	18
C	3	15	22	7.71428	8
D	1	16	49	18.5	19
D	3	16	49	6.6666	7
E	1	25	45	54	54
E	3	25	45	14.5	15
F	1	28	41	38	38
G	1	39	172	49	49

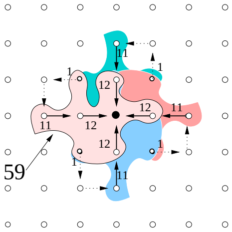
- **Interger round-up property** seems to hold for the *RWP* in our tests results, $W_{int} = \lceil W_{frac} \rceil$;
- **The bottleneck** remains at the gateway in our tests.

Bounds

Lower Bound: MinMax weighted clique [7]

The clique is given by the best routing in a localized region (probable bottleneck region).



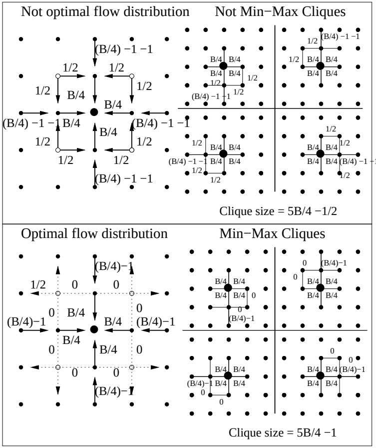


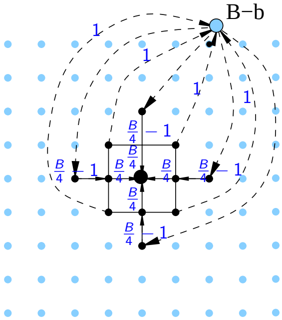
- A clique is a set of interfering calls (edges).
- It is known that $\omega(G) \leqslant \chi_f(G)$ for any graph G (W=$\chi_f(G_r)$);

[7] C. Gomes, H. Rivano, S. Perennes [GPR08]

Lower Bound: MinMax weighted clique

The clique is given by the best routing in a limited region (the probable bottleneck region).





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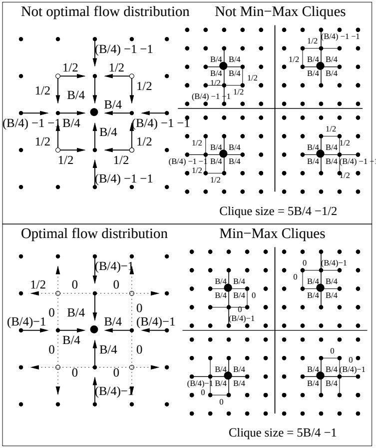
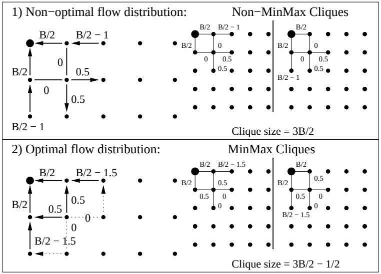
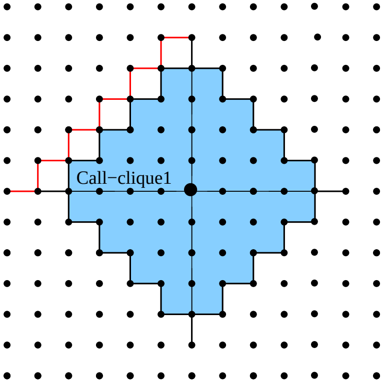




Critical Region (CR): definition for general graphs [8]

CR is a definition of the bottleneck region.



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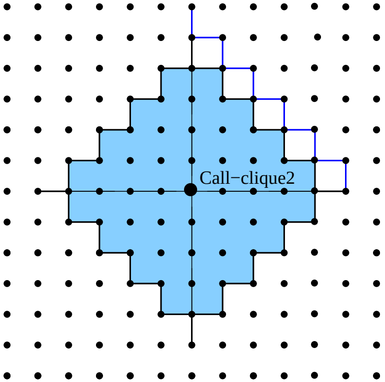

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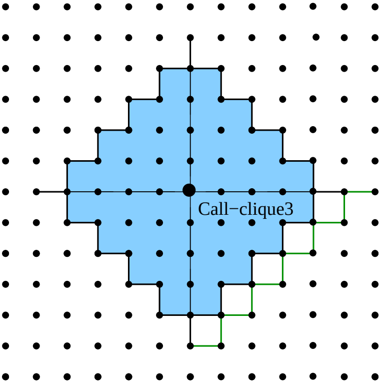

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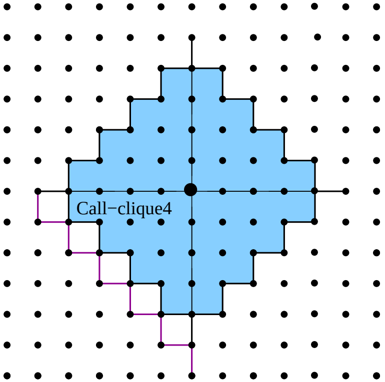

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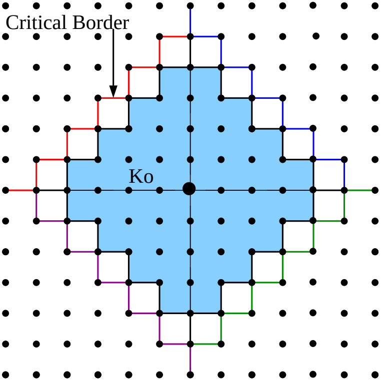

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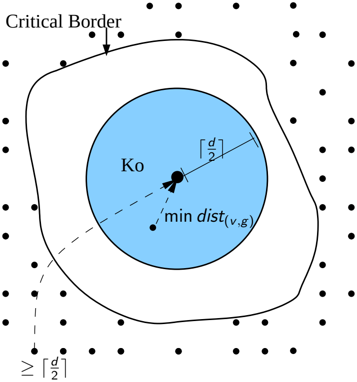

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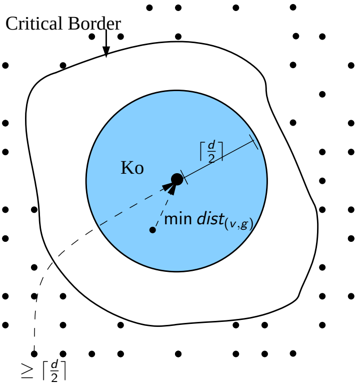

Results:

- Call-clique is a LB;
- Bounds for individual nodes;
- $W_{local}(CR) \leqslant W$ (Call-clique$< \chi_f(CR_r)$).

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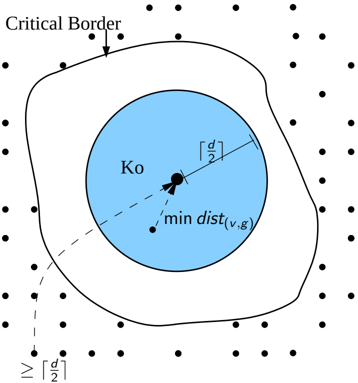

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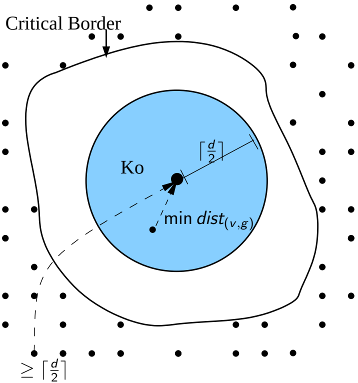

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Lower Bound formulas for grid graphs [9]

Considering:

- Gateway placed at the center;
- Uniform traffic;
- Any interference distance d.

General formula:

$W = (B - |K_0'|)p(v \notin K_0') + p(v \in K_0')$.

- **Grid with odd** d**:**
  $(N^2 - 1 - (\sum_{i=1}^{\lceil \frac{d}{2} \rceil - 1} 4i + 4))\lceil \frac{d}{2} \rceil + \sum_{i=1}^{\lceil \frac{d}{2} \rceil - 1} 4i^2 + 4\lceil \frac{d}{2} \rceil$.
- **Grid with even** d**:**
  $(N^2 - 1 - \sum_{i=1}^{\lceil \frac{d}{2} \rceil} 4i)(\frac{4\lceil \frac{d}{2} \rceil + 1}{4}) + \sum_{i=1}^{\lceil \frac{d}{2} \rceil} 4i^2$.

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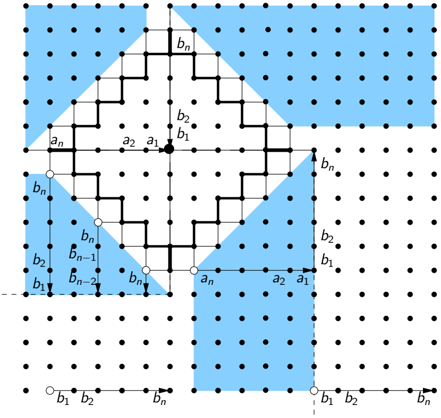
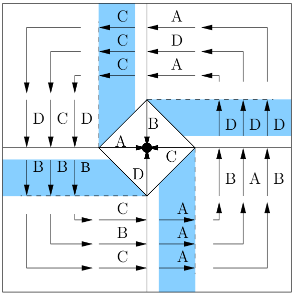
Lower Bound is tight for grid graphs

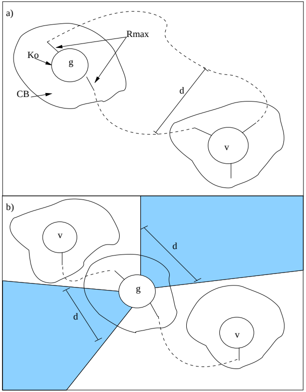
Protocol Description (Upper Bound)





Lower Bound is tight

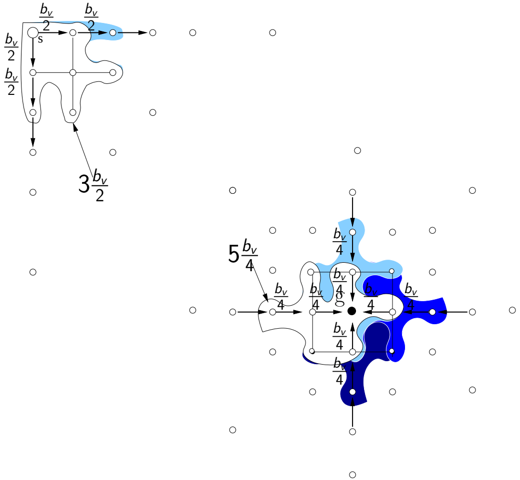
Network Characteristics:



- **Disjoint paths** guarantee the paths are completely covered by the colors of the critical region.
- At least one edge of the critical region is contained in each Round.

Lower Bound: MinMax weighted clique

Sparse demands and other bottleneck positions:



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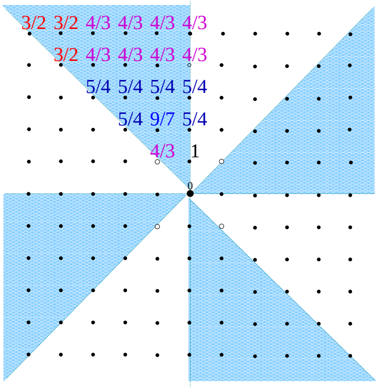
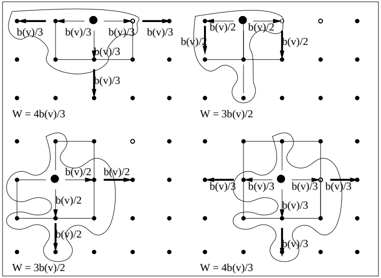
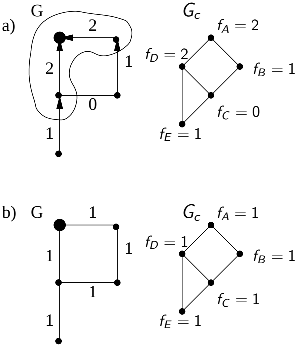


Figure: Cost to send one unit of flow from each position individually to the gateway (d=2).



Lower Bound: MinMax weighted clique

Dominant Edges:



$d_A = 1$
$d_B = 0$
$d_C = 0$
$d_D = 1$
$d_E = 0$
$\sum_{I \in \mathcal{I}} w_I = 4$

$d_A = \frac{1}{2}$
$d_B = \frac{1}{2}$
$d_C = \frac{1}{2}$
$d_D = \frac{1}{2}$
$d_E = \frac{1}{2}$
$\sum_{I \in \mathcal{I}} w_I = \frac{5}{2}$

Fractional coloring dual problem

$$\max \sum_{(i,j) \in E(G')} d_{(i,j)} f_{(i,j)}$$

$$\sum_{(i,j) \in R} d_{(i,j)} \leqslant 1, \forall R \in \mathcal{R}$$

$$d_{(i,j)} \geqslant 0, \forall (i,j) \in E(G')$$

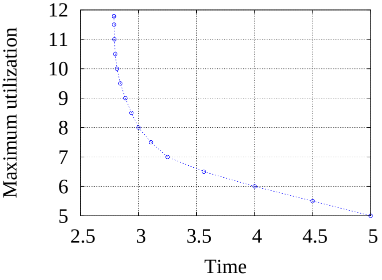
- Traffic (routing) that causes **highly loaded edges** (dominant edges) in a localized region (e.g. cumulative traffic).

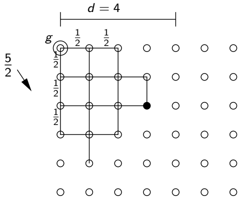
Lower Bound: MinMax weighted clique

Sparse demands and other bottleneck positions

Multi-objective Analysis:

MinMax Utilization x Min Time (RWP objective)





- Dual weights spread to other areas of the network ($W - W_{local}(CR)$ increases);
- Without dominant edges ⇒ strong trade-off time x utilization;
- With dominant edges ⇒ weak trade-off time x utilization (almost shortest path routing).

Lower Bound: MinMax weighted clique

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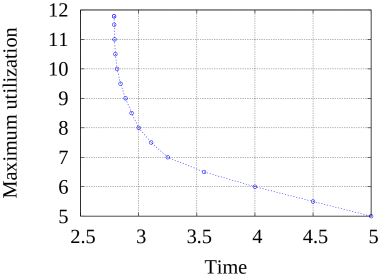

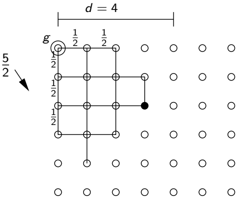

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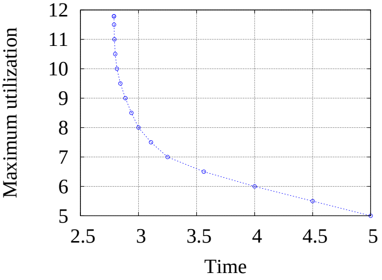

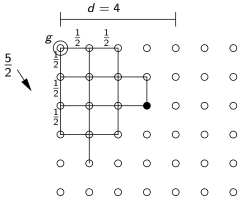

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Another interference model (considering SINR)

Optimal and Fair Transmission Rate Allocation Problem [10]

The problem is how to define the cumulative utility functions $\mathcal{U}_r(\alpha_r)$ for each relay node in a way to represent the utility functions $U_t(\rho_t)$ of its relayed terminals.

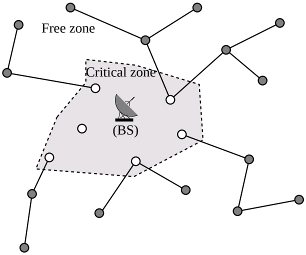


Figure: Multi-hop cellular network.

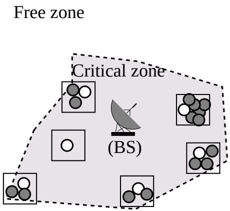


Figure: Multi-hop cellular network reduced in single-hop.

Objective: Max the sum of utility functions over all the nodes.

[10] Current work (C. Gomes, J. Galtier)

Optimal and Fair Transmission Rate Allocation Problem

Optimal rate allocation model [11]

$$\max \sum_{r \in \mathcal{R}} \mathcal{U}_r(\alpha_r) \tag{7}$$

subject to

$$\begin{cases} \alpha_r \gamma \leqslant \frac{p_{r,b} g_{r,b}}{N_o + g_{r,b} \sum_{s \neq r} p_{s,b}} = SINR_r, \forall r \in \mathcal{R} \\ \sum_{r \in \mathcal{R}} p_{r,b} \leqslant KN_o. \end{cases} \tag{8}$$

[11] Implementation: AMPL, Interior Point OPTimizer (IPOPT) - COIN-OR.

Optimal and Fair Transmission Rate Allocation Problem

Technical assumption and hypothesis:

- The mobiles' utility functions $U_t(.)$ are assumed to be strictly increasing concave functions and satisfy the condition $U_t^{''}(x) \leqslant \frac{-1}{x^2}$;
- All terminal nodes in $\mathcal{T}_r$ pass by an unique relay node $r \in \mathcal{R}$;
- We only consider interferences between the relay nodes in $\mathcal{R}$.

Optimal and Fair Transmission Rate Allocation Problem

Optimal rate allocation model

Problem (P_r)

$$\mathcal{U}_r(\alpha_r) = \max \sum_{t \in \mathcal{T}_r} U_t(\rho_t) \qquad \text{subject to} \qquad \alpha_r = \sum_{t \in \mathcal{T}_r} \rho_t, \forall r \in \mathcal{R}.$$

Problem (P'_r): local version of P_r

$$\max \sum_{t \in \mathcal{T}_r} U_t(\beta_t \alpha_r) \qquad \text{subject to} \qquad \begin{cases} \beta_t >= 0, \forall t \in \mathcal{T}_r \\ \sum_{t \in \mathcal{T}_r} \beta_t = 1. \end{cases}$$

Optimal and Fair Transmission Rate Allocation Problem

Problem (P'_r): local version of P_r (fixed feasible α_r)

$$\max \sum_{t\in\mathcal{T}_r} U_t(\beta_t\alpha_r) \qquad \text{subject to} \qquad \begin{cases} \beta_t >= 0, \forall t \in \mathcal{T}_r \\ \sum_{t\in\mathcal{T}_r} \beta_t = 1. \end{cases}$$

The Lagrangian of P'_r:

$$L(\beta) = \sum_{t\in\mathcal{T}_r} U_t(\beta_t\alpha_r) - \sum_{t\in\mathcal{T}_r} \lambda_t\,(-\beta_t) - \mu\left(\sum_{t\in\mathcal{T}_r} \beta_t - 1\right) - \nu\left(\sum_{t\in\mathcal{T}_r} -\beta_t + 1\right)$$

As $\beta^* = (\frac{\rho'_t}{\alpha_r}, ..., \frac{\rho'_{|\mathcal{T}_r|}}{\alpha_r})$ ($\rho^* = (\rho'_1, ..., \rho'_{|\mathcal{T}_r|})$) is necessarily a vector of optimal solutions for the Lagrangian. So it verifies KKT's optimality conditions and we obtain: $U'_t(\rho'_t) = C, \forall t \in \mathcal{T}_r$ where C is a constant.

Optimal and Fair Transmission Rate Allocation Problem

Cumulative utility function $\mathcal{U}_r(\alpha_r)$:

$$h_r(C) = \beta_1^{'}\alpha_r + ... + \beta_{|\mathcal{T}_r|}^{'}\alpha_r = \alpha_r$$

$$C = h_r^{-1}(\alpha_r)$$

$$h_t(C) = h_t \circ h_r^{-1}(\alpha_r)$$

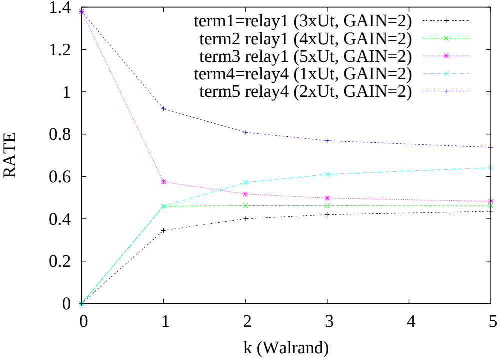
$$\rho_t^{'} = h_t \circ h_r^{-1}(\alpha_r)$$

$$U_t(\rho_t^{'}) = U_t \circ h_t \circ h_r^{-1}(\alpha_r)$$

$$\sum_{t \in \mathcal{T}_r} U_t(\rho_t^{'}) = \sum_{t \in \mathcal{T}_r} U_t \circ h_t \circ h_r^{-1}(\alpha_r)$$

Optimal and Fair Transmission Rate Allocation Problem

Fairness



Utility function : $U_t(x) = \frac{x^{1-\kappa}}{1-\kappa}$ [Walrand00].

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