Cheat Sheet

Terminology

Context
```
x : T
S : {set T}
xS : x \in S
```
The bar
```
===============
```
Goal
```
forall y, y == x -> y \in S
```
$\underbrace{\texttt{forall y, y == x ->}}_{\textit{Assumptions}}\ \underbrace{\texttt{y \textbackslash in S}}_{\textit{Conclusion}}$

Top is the first assumption, `y` here

Stack alternative name for the list of *Assumptions*

Popping from the stack

Note: in the following example we assume *cmd* does nothing, exactly like `move`, to focus on the effect of the intro pattern.

cmd`=> x px`

Run *cmd*, then pop Top, put it in the context naming it `x` then pop the new Top and names it `px` in the context

```
=========
 forall x,
  P x -> Q x -> G
```
→
```
x : T
px : P x
=========
Q x -> G
```

cmd`=> [|x xs] //`

Run *cmd*, then reason by cases on Top. In the first breanch do nothing, in the second one pop two assumptions naming then `x` nd `xs`. Then get rid of trivial goals. Note that, since only the first branch is trivial, one can write `=> [// | x xs]` too. $\boxed{\text{caveat}}$: Immediately after `case` and `elim` it does not perform any case analysis, but can still introduce different names in different branches

```
=========
 forall s : seq nat,
  0 < size s -> P s
```
→
```
x : nat
xs : seq nat
=========
0 < size (x :: xs) -> P (x ::
     xs)
```

cmd`=> /andP[pa pb]`

Run *cmd*, then apply the view `andP` to Top, then destruct the conjuction and introduce in the context the two parts naming the `pa` and `pb`

```
=========
 A && B -> C -> D
```
→
```
pa : A
pb : B
=========
C -> D
```

cmd`=> /= {px}`

Run *cmd* then simplify the goal then discard `px` from then context

```
 x : nat
 px : P x
=========
 true && Q x -> R x
```
→
```
x : nat
=========
Q x -> R x
```

cmd`=> [y -> {x}]`

Run *cmd* then destruct the existential, then introduce y, then rewrite with Top left to right and discard the equation, then clear x

```
 x : nat
=========
 (exists2 y, x = y & Q y)
   -> P x
```
→
```
y : nat
=========
Q y -> P y
```

cmd`=> /(_ x) h`

Introduce `h` specialized to `x`

```
P : nat -> Prop
x : nat
=====================
(forall n, P n) -> G
```
→
```
P : nat -> Prop
x : nat
h : P x
================
G
```

Pushing to the stack

Note: in the following *cmd* is not `apply` or `exact`. Moreover we display the goal just before *cmd* is run.

cmd`: (x) y`

Push `y` then push `x` on the stack. `y` is also cleared

```
x, y : nat
px : P x
=========
Q x y
```
→
```
x : nat
px : P x
=========
forall x0 y, Q x0 y
```

cmd`: {-2}x (erefl x)`

Push the type of `(erefl x)`, then push `x` on the stack binding all but the second occurrence

```
x : nat
=========
P x
```
→
```
x : nat
=========
forall x0,
 x0 = x -> P x0
```

cmd`: _.+1 {px}`

Clear `px` and generalize the goal with respect to the first match of the pattern `_.+1`

```
x : nat
px : P x
=========
x < x.+1
```
→
```
x : nat
=========
forall x0, x < x0
```

Proof commands

`rewrite Eab (Exc b).`

Rewrite with `Eab` left to right, then with `Exc` by instantiating the first argument with `b`

```
Eab : a = b
Exc : forall x, x = c
=========
P a
```
→
```
Eab : a = b
Exc : forall x, x = c
=========
P c
```

`rewrite -Eab {}Eac.`

Rewrite with `Eab` right to left then with `Eac` left to right, finally clear `Eac`

```
Eab : a = b
Eac : a = c
=========
P b
```
→
```
Eab : a = b
=========
P c
```

`rewrite /(_ && _).`

Unfold the definition of `&&`

```
a : bool
=========
a && a = a
```
→
```
a : bool
=========
if a then a else false = a
```

`rewrite /= -[a]/(0+a) -/c.`

Simplify the goal, then change `a` into `0+a`, finally fold back the local definition `c`

```
a, b : nat
c := b + 3 : nat
=========
true && (a == b + 3)
```
→
```
a, b : nat
c := b + 3 : nat
=========
0 + a == c
```

`apply: H.`

Apply `H` to the current goal

```
H : A -> B
=========
B
```
→
```
=========
A
```

`case: ab.`

Eliminate the conjunction or disjunction

```
ab : A /\ B
=========
G
```
→
```
=========
A -> B -> G
```

`case: (leqP a b).`

Reason by cases using the `leqP` spec lemma

```
a, b : nat
=========
a <= b && b > a
```
→
```
a, b : nat
=========
true && false
```
```
a, b : nat
=========
false && true
```

`elim: s.`

Perform an induction on `s`

```
s : seq nat
=========
P s
```
→
```
=========
P [::]
```
```
=========
forall n s,
 P s -> P (n :: s)
```

`elim/last_ind: s`

Start an induction on `s` using the induction principle `last_ind`

```
s : seq nat
=========
P s
```
→
```
=========
P [::]
```
```
=========
forall s x,
 P s ->
  P (rcons s x)
```

`have pa : P a.`

Open a new goal for `P a`. Once resolved introduce a new entry in the context for it named `pa`

```
a : T
=========
G
```
→
```
a : T
=========
P a
```
```
a : T
pa : P a
=========
G
```

`by [].`

Prove the goal by trivial means, or fail

```
=========
0 <= n
```
→

`exact: H.`

Apply `H` to the current goal and then assert all remaining goals, if any, are trivial. Equivalent to `by apply: H.`

```
H : B
=========
B
```
→

Reflect and views

`reflect P b`
States that `P` is logically equivalent to `b`

`apply: (iffP V)`
Proves a reflection goal, applying the view lemma `V` to the propositional part of `reflect`. E.g. `apply: (iffP idP)`

```
P : Prop               P : Prop         P : Prop
b : bool               b : bool         b : bool
=============     →    =========        =========
reflect P b            b -> P           P -> b
```

`apply/V1/V2`
Prove boolean equalities by considering them as logical double implications. The term `V1` (resp. `V2`) is the view lemma applied to the left (resp. right) hand side. E.g. `apply/idP/negP`

```
b1 : bool            b1 : bool         b1 : bool
b2 : bool      →     b2 : bool         b2 : bool
=========            =========         =========
b1 = ~~ b2           b1 -> ~ b2        ~ b2 -> b1
```

`rewrite: (eqP Eab)`
rewrite with the boolean equality `Eab`

```
Eab : a == b           Eab : a == b
=============     →    =========
P a                    P b
```

Idioms

`case: b => [h1| h2 h3]`
Push `b`, reason by cases, then pop `h1` in the first branch and `h2` and `h3` in the second

`have /andP[x /eqP->] : P a && b == c`
Open a subgoal for `P a && b == c`. When proved apply to it the `andP` view, destruct the conjunction, introduce `x`, apply the view `eqP` to turn `b == c` into `b = c`, then rewrite with it and discard the equation

`elim: n.+1 {-2}n (ltnSn n)=> {n}// n`
General induction over `n`, note that the first goal has a false assumption `forall n, n < 0 -> ...` and is thus solved by `//`

```
                 n : nat
n : nat          =========
=========   →    (forall m, m < n -> P m) ->
P n               forall m, m < n.+1 -> P m
```

`rewrite lem1 ?lem2 //`
Use the equation with premises `lem1`, then get rid of the side conditions with `lem2`

Searching

`Search _ addn (_ * _) "C" in ssrnat`
Search for all theorems with no constraints on the main conclusion (conclusion head symbol is the wildcard `_`), that talk about the `addn` constant, matching anywhere the pattern `(_ * _)` and having a name containing the string `"C"` in the module `ssrnat`

Misc notations

```
"f1 \o f2" := (comp f1 f2)
"x \in A" := (in_mem x (mem A))
"x \notin A" := (~~ (x \in A))
"[ /\ P1 , P2 & P3 ]" := (and3 P1 P2 P3)
"[ \/ P1 , P2 | P3 ]" := (or3 P1 P2 P3)
"[ && b1 , b2 , .. , bn & c ]" :=
  (b1 && (b2 && .. (bn && c) .. ))
"[ || b1 , b2 , .. , bn | c ]" :=
  (b1 || (b2 || .. (bn || c) .. ))
"#| A |" := (card (mem A))
"n .-tuple" := (tuple_of n)
"'I_ n" := (ordinal n)
"f1 =1 f2" := (eqfun f1 f2)
"b1 (+) b2" := (addb b1 b2)
```

Notations for natural numbers: `nat`

```
"n .+1" := (succn n)
"n .-1" := (predn n)
"m + n" := (addn m n)
"m - n" := (subn m n)
"m <= n" := (leq m n)
"m < n" := (m.+1 <= n)
"m <= n <= p" := ((m <= n) && (n <= p))
"m * n" := (muln m n)
"n .*2" := (double n)
"m ^ n" := (expn m n)
"n `!" := (factorial n)
"m %/ d" := (divn m d)
"m %% d" := (modn m d)
"m == n %[mod d ]" := (m %% d == n %% d)
"m %| d" := (dvdn m d)
"pi .-nat" := (pnat pi)
```

Notations for lists: `seq T`

```
"x :: s" := (cons _ x s)
"[ :: ]" := nil
"[ :: x1 ]" := (x1 :: [::])
"[ :: x & s ]" := (x :: s)
"[ :: x1 , x2 , .. , xn & s ]" :=
  (x1 :: x2 :: .. (xn :: s) ..)
"[ :: x1 ; x2 ; .. ; xn ]" :=
  (x1 :: x2 :: .. [:: xn] ..)
"s1 ++ s2" := (cat s1 s2)
```

Notations for iterated operations

```
"\big [ op / idx ]_ i F" :=
"\big [ op / idx ]_ ( i | P ) F" :=
"\big [ op / idx ]_ ( i <- r | P ) F" :=
"\big [ op / idx ]_ ( m <= i < n | P ) F" :=
"\big [ op / idx ]_ ( i < n | P ) F" :=
"\big [ op / idx ]_ ( i \in A | P ) F" :=
"\sum_ i F" :=
"\prod_ i F" :=
"\max_ i F" :=
"\bigcap_ i F" :=
"\bigcup_ i F" :=
```

$\boxed{\text{caveat}}$: in the general form, the iterated operation `op` is displayed in prefix form (not in infix form) $\boxed{\text{caveat}}$: the string `"big"` occurs in every lemma concerning iterated operations

Rewrite patterns

`rewrite [pat]lem [in pat2]lem2 [X in pat3]lem3`
Rewrite the subterms selected by the pattern `pat` with `lem`. Then in the subterms selected by the pattern `pat2` match the pattern inferred from the left hand side of `lem2` and rewrite the terms selected by it. Last, in the sub terms selected by `pat3` rewrite with `lem3` the sub terms identified by `X` exactly

`rewrite {3}[in X in pat1]lem1`
Like in `rewrite [X in pat1]lem1` but use the pattern inferred from `lem1` to identify the sub terms of `X` to be rewritten. Of these terms, rewrite only the third one. Example: `rewrite {3}[in X in f _ X]E.`

```
E : a = c
=========                                    →
a + f a (a + a) = f a (a + a) + a

E : a = c
=========
a + f a (a + a) = f a (c + a) + a
```

`rewrite [e in X in pat1]lem1`
Like before, but override the pattern inferred from `lem1` with `e`

`rewrite [e as X in pat1]lem1`
Like `rewrite [X in pat1]lem1` but match `pat1[X := e]` insted of just `pat1`

`rewrite /def1 -[pat]/term /=`
Unfold all occurrences of `def1`. Then match the goal against `pat` and change all its occurrences into `term` (pure computation). Last simplify the goal

`rewrite 3?lem2 // {hyp} => x px`
Rewrite from 0 to 3 times with lem2, then try to solve with `by []` all the goals. Finally clear `hyp` and introduce `x` and `px`

Pattern matching detailed rules

pattern a term, possibly containing `_`

key The head symbol of a pattern

The sub terms selected by a pattern:

1. the goal is traversed outside in, left to right, looking for verbatim occurrences of the key
2. the sub terms whose key matches verbatim are higher order matched (i.e. up to definition unfolding and recursive function computation) against the pattern
3. if the matching fails, the next sub term whose key matches is tried
4. if the matching succeeds, the sub term is considered to be the (only) instance of the pattern
5. the sub terms selected by the pattern are then all the copies of the instance of the pattern
6. these copies are searched looking again at the key, and higher order comparing the arguments pairwise

Note: occurrence numbers can be combined with patterns. They refer to the list of of sub terms selected by the (last) pattern (i.e. they are processed at the very end).

`set n := {2 4}(_ + b)`
Put in the context a local definition named `n` for the second and fourth occurrences of the sub terms selected by the pattern `(_ + b)`

```
=========
a + c + (a + b) + (a + b) =                  →
 a + (a + b) + (0 + a + b) + c

 n := a + b
=========
 a + c + (a + b) + n =
  a + (a + b) + n + c
```