# Further analysis of the 2-2 wire-driven parallel crane

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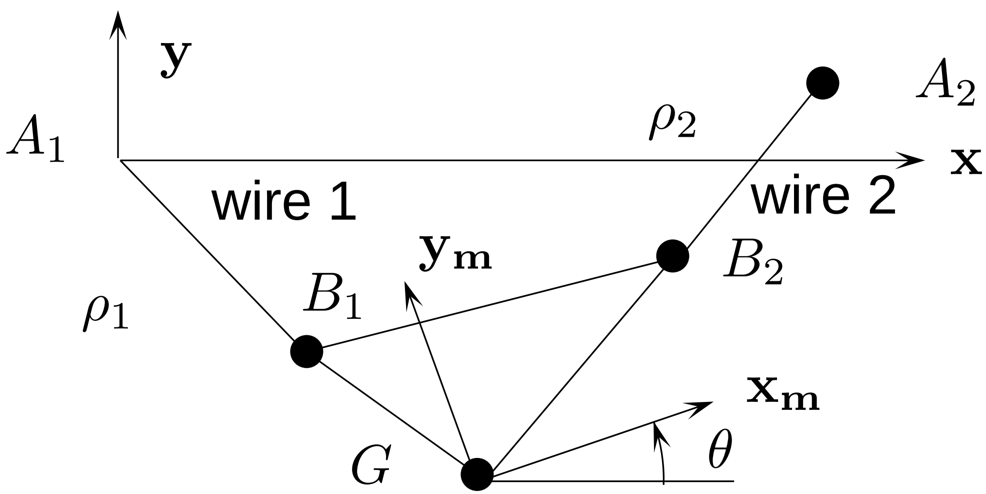
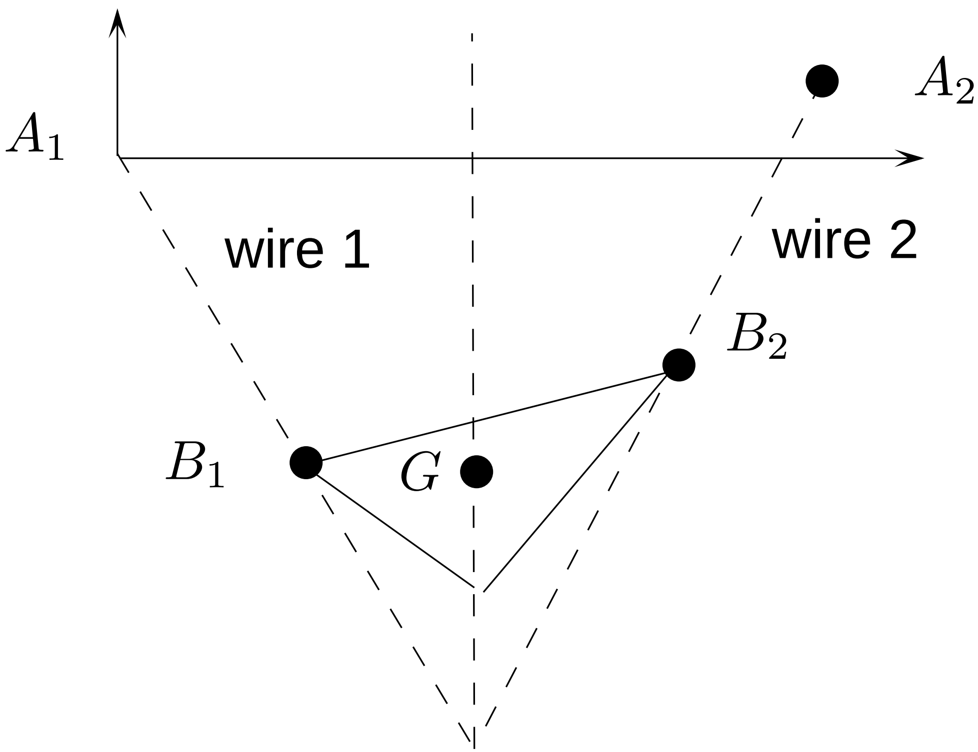
# The 2-2 WDPR

- robot with 2 coilable wires
- output point of the winches: $A_1, A_2$
- attachment point of the wire on the platform: $B_1, B_2$
- $G(x_g, y_g)$: center of mass of the platform
- wire length: $\rho_i$, wire tension $\tau_i$



# Mechanical equilibrium

- necessary condition: lines $A_1B_1, A_2B_2$ intersect the vertical line going through $G$ at the same point
- not sufficient: $\tau > 0$



# Mechanical equilibrium

let $X_i, Y_i$ be the components of vector $\mathbf{GB_i}$

condition for mechanical equilibrium:

$$(Y_2 - Y_1)x_g^2 + (X_1 - X_2)x_g y_g + X_1(y_{a_2}X_2 - x_{a_2}Y_2)$$
$$(x_{a_2}(Y1 - Y2) + y_{a_2}X_2 + Y_2X_1 - Y_1X_2)x_g - x_{a_2}X_1y_g = 0$$

assume $\theta$ is fixed $\Rightarrow X_i, Y_i$ are fixed

- $G$ moves on an hyperbola
- whose principal axes makes an angle $\phi$ with the $\mathbf{x}$ axis with $\tan(2\phi) = (X_1 - X_2)/(Y_2 - Y_1)$

# Mechanical equilibrium

Analysis of the hyperbola allows one to determine if a given orientation $\theta$ is reachable

**Example**:

- coefficient of $y_g$ cancels if $x_g = x_{a_2} X_1 / (X_1 - X_2)$
- if this value lie in the range $[0, x_{a_2}]$ then
  - the workspace is separated into two components
  - it is not possible to maintain the given orientation over the workspace of the crane.

# Inverse kinematics

the 2-2 is a 2 dof robot but the platform has 3 dof

The number of solution of the IK will vary according to the choice of the controlled dof

- $x_g, \theta$ fixed $\Rightarrow$ one solution for $y_g$
- $y_g, \theta$ fixed $\Rightarrow$ up to two solutions for $x_g$
- $x_g, y_g$ fixed $\Rightarrow$ up to four solutions for $\theta$

# Forward kinematics

Known results

- solutions can be obtained by solving a 12th order univariate polynomial
- BUT if out of the plane motion are possible there may be up to 24 solutions

# Forward kinematics

But getting numerically the solutions by solving the polynomial may not be a good idea because of numerical round-off errors

One may transform this solving in a eigenvalue problem

- numerically more robust
- but still not guaranteed

# Forward kinematics

We consider using interval analysis which provides guaranteed results

FK may be formulated as solving a system of $n$ equations in $n$ unknowns with $n =$ 3, 4, 5, 6, 8

# Forward kinematics

We have conducted a numerical evaluation of the 5 formulations

Most efficient

- 6 unknowns ($x_g, y_g, x_1, y_1, x_2, y_2$) $\longrightarrow$ computation time: 0.49s
- 4 unknowns ($x_g, y_g, \sin\theta, \cos\theta$) $\longrightarrow$ computation time: 0.08s

# Forward kinematics

Over a set of 400 randomly selected set of wire lengths the FK has

- 2 solutions in 34% of the cases
- 3 solutions in 8.75%,
- 4 solutions in 45.5%
- 5 solutions in 0.75%,
- 6 solutions in 8.5%,
- 7 solutions in 0.5 %,
- 8 solutions in 2%

# Working in the joint space

# Working in the joint space

Classical static analysis imposes to know $\mathbf{X}$ for computing the wire tensions $\tau$

$$\Downarrow$$

imposes solving the FK

# Working in the joint space

Can we solve some statics problems using only $\rho_1, \rho_2$ measurements ?

- determining the wire tensions
- determining the region of the joint space such that $\tau_1, \tau_2 \leq \tau_{max}$

# Working in the joint space

From joint space to **statics**

- force equilibrium are linear in $x_g, y_g$
- remains 3 equations in $\theta, \tau_1, \tau_2$
- resultant in $T = \tan(\theta/2)$ leads to 2 polynomials in $\tau_1, \tau_2$ of degree 6
- resultant of these polynomials leads to a polynomial of degree 12

# Working in the joint space

Finding the region $\mathcal{W}$ of the joint space such that $\tau_1, \tau_2 \leq \tau_{max}$

Main idea: find the border of the region $\mathcal{W}$

# Working in the joint space

- set $\tau_1 = \tau_{max}$
- $F_x$ equilibrium linear in $\tau_2$
- $\rho_1^2 - \rho_2^2$ linear in $x_g$
- $F_y$ equilibrium linear in $y_g$
- remains 2 equations in $\theta, \rho_1, \rho_2$
- elimination of $\theta$ leads to 2 polynomials:
  - polynomial of degree 6 in $\rho_1$ and degree 2 in $\rho_2^2$
  - polynomial of degree 16 in $\rho_1$, 12 in $\rho_2$, total degree 16

The two later polynomials define partly the border of the region

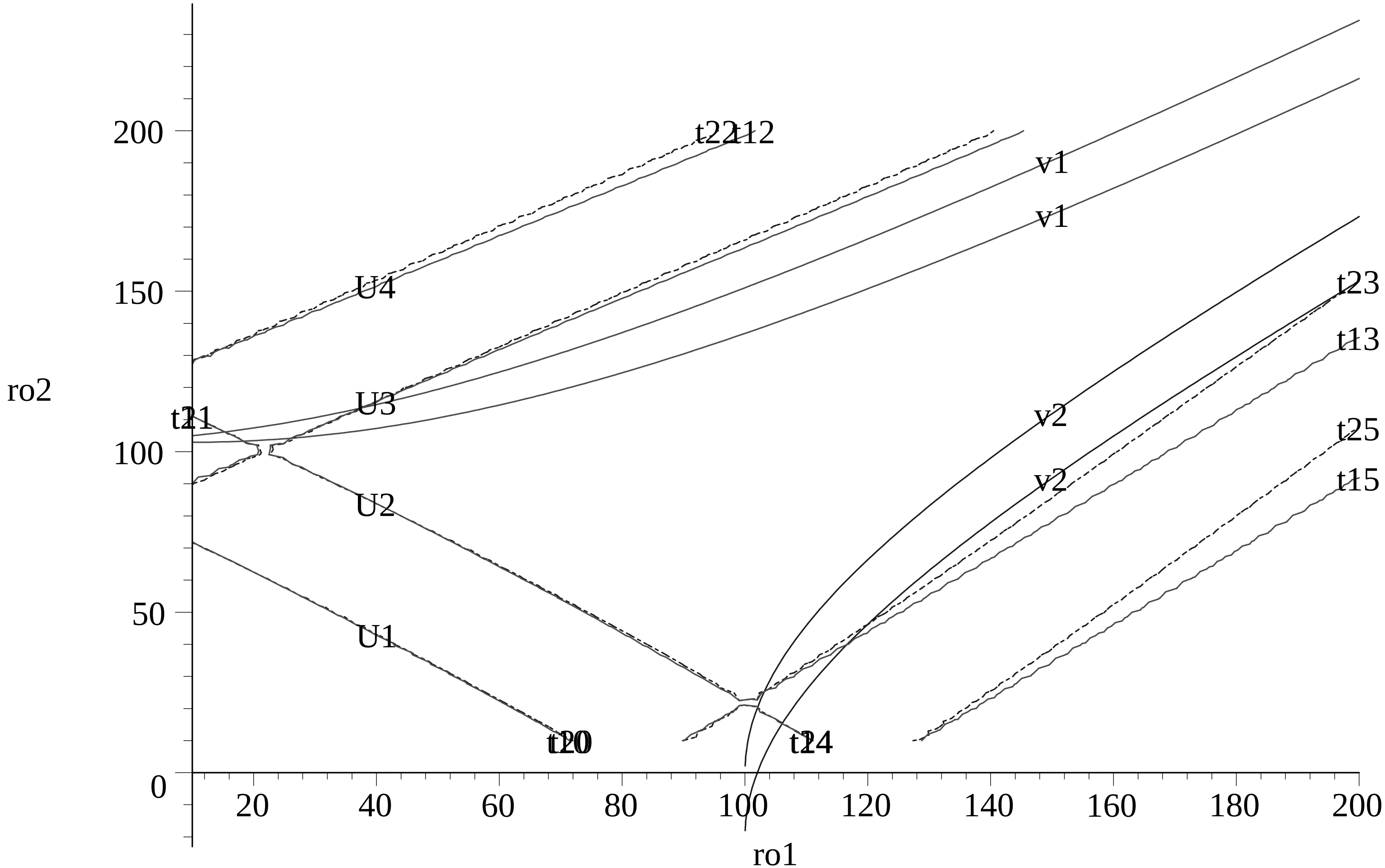
# Working in the joint space

Computing $\mathcal{W}$

- compute the polynomials when setting $\tau_1 = \tau_{max}$
- compute the polynomials when setting $\tau_2 = \tau_{max}$
- intersect all pairs of polynomials for finding extreme points $P_1, \ldots P_n$
- points $P_i$ on a polynomial curve split this curve in arcs whose extremities are such that $\tau_1 = \tau_{max}$ or $\tau_2 = \tau_{max}$
- takes the mid-point $M$ of each arc and compute the normals $N_1, N_2$ vector to the curve at $M$
- if $\partial\tau_{1,2}/\partial N_i < 0$ and $\partial\tau_1/\partial N_j > 0$ or $\partial\tau_2/\partial N_j > 0$, then arc is part of the border of $\mathcal{W}$
- join the border arcs to calculate the border of $\mathcal{W}$

# Working in the joint space

## Example



# Working in the joint space

## Singularity

- $||B_1B_2|| < ||A_1A_2||, \tau < \tau_{max} \Rightarrow$ classical singularity cannot occur

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- BUT there is **another singularity event**: when one wire becomes slack

Occurs in the joint space if:

$$\rho_2^2 \geq \left(\frac{y_{b_1}\, x_{b_2} - x_{b_1}\, y_{b_2}}{d_1} - x_{a_2}\right)^2 + \left(-\rho_1 \pm d_1 + \frac{x_{b_1}\, x_{b_2} + y_{b_1}\, y_{b_2}}{d_1} - y_{a_2}\right)^2$$

# Conclusion

- the 2-2 is the simplest WDPR
- still its kinematic analysis is very rich
- next objectives
  - pursue IK and FK analysis for spatial robots
  - getting similar joint space results for spatial robots