Direct kinematics of CDPR with extra cable orientation sensors: the 2 and 3 cables case with perfect measurement and sagging cables

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***Abstract*— Direct kinematics (DK) of cable-driven parallel robots (CDPR) based only on cable lengths measurements is a complex issue even with ideal cables and consequently even harder for more realistic cable models such as sagging cable. A natural way to simplify the DK solving is to add sensors. We consider here sensors that give a partial or complete measurement of the cable direction at the anchor points and/or measure the orientation of the platform of CDPR with 2 or 3 cables and we assume that the measurements are exact. We provide a solving procedure and maximal number of DK solutions for an extensive combination of sensors for CDPR with sagging cables. We show that at least two measurements are necessary for the planar 2 cables case while six are necessary for the spatial 3 cables case. For spatial CDPR with n cables we prove that at least $2n$ additional sensors will be required to get a closed-form solution of the DK.**

I. INTRODUCTION

We consider cable-driven parallel robot (CDPR) with 3 cables whose output points on the base will be denoted by A_i with anchor point B_i on the platform (figure 1). The

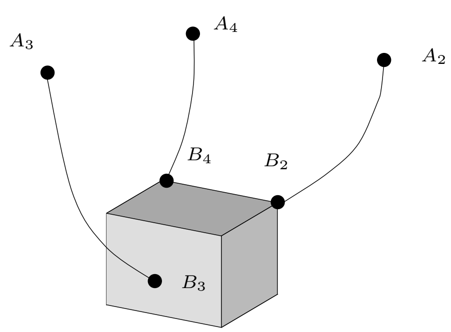

Fig. 1. A CDPR with sagging cables

winch system assume that the cable is not submitted to any deformation and is able to control and measure the length at rest L_0 of the cable. But the elasticity and own mass of the cable induce a sagging effect on the cable that modify its shape and length. We will assume here that the sagging of the cable is such that the whole cable lies in a vertical plane $\mathcal{P}$ that includes A, B (see figure 5). A sagging cable model, called the Irvine model [1], may be defined in $\mathcal{P}$. In this plane the coordinates of A, B are assumed to be respectively (0,0), $(x_b \geq 0, y_b)$ while F_z, F_x denote the vertical and horizontal forces exerted on the cable at point B. With this notation the coordinates of B are related to the forces F_x, F_z by:

$$x_b = F_x\left(\frac{L_0}{EA_0} + \frac{sinh^{-1}(F_z) - sinh^{-1}((F_z - \frac{\mu g L_0}{F_x})}{\mu g}\right) \quad (1)$$

$$z_b = \frac{\sqrt{F_x^2 + F_z^2} - \sqrt{F_x^2 + (F_z - \mu g L_0)^2}}{\mu g} + \frac{F_z L_0}{EA_0} - \frac{\mu g L_0^2}{2EA_0} \quad (2)$$

where E is the Young modulus of the cable material, μ its linear density and A_0 the surface of the cable cross-section.

The direct kinematic (DK) problem amounts to find all the possible pose(s) of the platform being given the L_0. Although this may seen to be mostly a theoretical problem, DK solving is important also in practice for a better understanding of singularity and workspace and also for providing an initializing solution for the real time DK that is then less problematic [2], [3]. Although relatively well mastered for parallel robots with rigid legs, DK is still an open issue for CDPR. Even if we assume ideal cable (with no elasticity and no deformation of the cable due to its own mass) the DK problem leads to a larger number of equations than in the rigid leg case [4] and consequently to solving problems [5], [6], [7], [8], [9], [10], [3]. The DK problem with the full Irvine model has been addressed in [11] where it has been shown that the DK always amount to solve a square system of equations whatever the number of cables is. A solving algorithm has also been presented in [11] but is computer intensive. A more efficient solving scheme has then been presented in [12] but it is still quite intensive. A major problem with the Irvine model is that the non algebraic nature of equations (1,2) prohibits to use elimination methods that have been quite successful for parallel robots with rigid legs to reduce the DK to the solving an univariate polynomial. Another issue is that the DK has usually several solutions while the real CDPR is always in a given pose among all the possible solutions. The later use of the real-time DK then imposes to visually determine what is the current pose of the platform among all the DK solutions.

An intuitive approach to avoid or reduce this non-unicity problem and to speed up the solving time of the DK is to add sensors that provide additional information on the cable beside the cable lengths, as already proposed for classical parallel robot [13], [14], [15], [16]. A natural candidate will be to measure the cable tensions as they play an important role in the DK solving. Unfortunately force measurement are usually noisy and measuring these tensions on a moving platform submitted to various mechanical noises appears to be difficult [17], [18]. Although several attempts have been made of integrating force sensing in CDPR, none of

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them have presented clear result about the reliability of the measurement.

In this paper we are considering another measurement possibility which consists in getting complete or partial information on the cable direction at the anchor points A and B. The measurement at point A are (figure 2):

- the angle θ_V between the x axis and the vertical plane that includes the cable
- the angle θ_H between the horizontal direction and the cable tangent at A

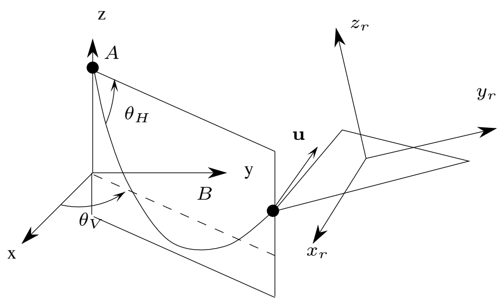

Fig. 2. Orientation sensors at A may provide the value of θ_V and/or θ_H

We introduce a mobile frame attached to the platform whose center is G, the center of mass of the platform, and vectors $\mathbf{x_r}, \mathbf{y_r}, \mathbf{z_r}$. Let $\mathbf{u}$ be the unit vector of the cable tangent at B and $\mathbf{u_p}$ its projection in the $\mathbf{x_r}, \mathbf{y_r}$ plane. The measured angles (figure 3) may be

- the angle α_V between $\mathbf{u_p}$ and $\mathbf{x_r}$
- the angle α_H between $\mathbf{u_p}$ and $\mathbf{u}$

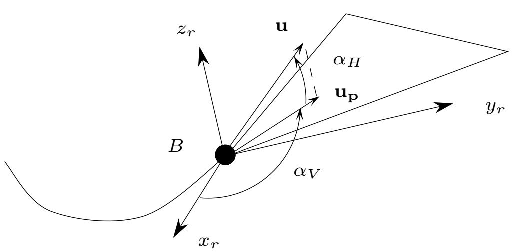

Fig. 3. Orientation sensors at B may provide the value of α_V and/or α_H

Realizing such measurement has already been considered: for example our CDPR MARIONET-Assist uses a simple rotating guide at A whose rotation is measured by a potentiometer in order to obtain the measurement of θ_V while our CDPR MARIONET-VR is instrumented with a more sophisticated cable guiding system which allows for the measurement of both θ_V and θ_H (figure 4). Our first trials with such a simple system have shown that the accuracy is poor as soon as the cable tension is low. We are therefore considering a non contact system: the idea is to have a range meter mounted on a rotary head in front of the A point (figure 5). The head rotates around the x axis until the range meter detects the cable at point M. The measurement of the distance d between A' and M and of the rotation angle β allows one to calculate the coordinates of M. The

Fig. 4. On the left the rotation guide of MARIONET-Assist which allows for the measurement of θ_V. On the right the system used on MARIONET-VR for the measurement of both θ_V and θ_H

coordinates of the points A and M allow us to determine the cable plane and therefore the θ_V angle, while the angle between AM and the x axis provides the θ_H angle. This non contact sensing method should provide a better accuracy than the mechanical guides of figure 4. To the best of our knowledge no similar system has been proposed for the platform. For measuring theses angles we may also consider a vision system as proposed in [19].

We may also consider having a 3D accelerometrer on the platform and assume that the CDPR motion is sufficiently slow to neglect the acceleration due to the motion. In that case the accelerometer will provide 2 orientation angles of the platform. Such a sensor will be called an IMU and will be counted as 1 sensor.

II. Fundamental equations

In this section a superscript ^{j} will denote an unknown of cable j. The components of $\mathbf{u}$ in the mobile frame are $\mathbf{u_m} = \cos(\alpha_V)\cos(\alpha_H), \cos(\alpha_V)\sin(\alpha_H), \sin(\alpha_H)$. If $\mathbf{R}$ is the rotation matrix between the reference frame and the mobile frame, $\mathbf{R}$ being expressed as functions of 3 angles ψ, θ, ϕ, then $\mathbf{u}$ in the reference frame is $\mathbf{R}\mathbf{u_m}$. The vector $\mathbf{u}$ should be perpendicular to the normal to $\mathcal{P}$. A possible way to express this constraint is to consider the rotation matrix $\mathbf{R_1}$ corresponding to a rotation around the vertical of angle θ_V so that the vector $\mathbf{R_1}\mathbf{R}\mathbf{u_m}$ has 0 as second component. This constraint may be written as

$$\mathbf{R_1}\mathbf{R}\mathbf{u_m}.(0,1,0)^T = 0 \tag{3}$$

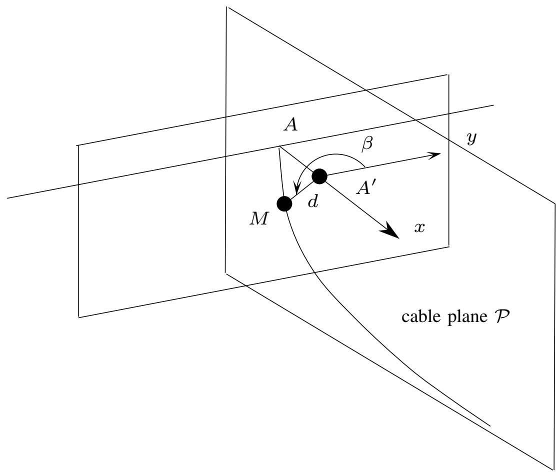

Fig. 5. A new sensor arrangement for measuring the θ_V, θ_H angles. A range meter is mounted on a rotary head located at A' rotating around the x axis. The range meter is rotated until it detects the cable and the system provides the distance d between A' and the cable and the rotation angle β

This constraint is a function of $\alpha_V, \alpha_H, \theta_V, \psi, \theta, \phi$. Let $\mathbf{u_v}$ be the 2D vectors whose components are the first and third component of $\mathbf{R_1 R u_m}$. In the plane frame the unit vector of the tangent to the cable at B is expressed as $(u_v(1), u_v(2))$. If we define $\tau_B = \sqrt{F_x^2 + F_z^2}$ the components of this vector are also $(F_x/\tau_B, F_z/\tau_B)$. This allows to define a new constraint

$$F_z/F_x = u_v(2)/u_v(1) \tag{4}$$

This constraint is a function of $F_x, F_z, \alpha_V, \alpha_H, \theta_V$ and of ψ, θ, ϕ. At A the unit tangent vector in the cable plane is $F_x/\tau_A, (F_Z - \mu g L_0)/\tau_A$ with $\tau_A = \sqrt{F_x^2 + (F_z - \mu g L_0)^2}$ so that we have

$$(F_Z - \mu g L_0)/F_x = tan(\theta_H) \tag{5}$$

This constraint only involve θ_H, F_x, F_z. We may also have to use the mechanical equilibrium of the platform, assuming that it is submitted only to gravity. The force equilibrium may be written as $\mathbf{R_1}^T(F_x, 0, F_z)^T = (0, 0, mg)$ where m is the platform mass, which leads to

$$\sum_{j=1}^{j=3} -\cos(\theta_V^j)F_x^j = 0 \quad \sum_{j=1}^{j=3} \sin(\theta_V^j)F_x^j = 0$$
$$\sum_{j=1}^{j=3} -F_z^j - mg = 0 \tag{6}$$

These 3 constraints are evidently only function of the θ_V, F_x, F_Z. The moment equations may be written as

$$\sum_{j=1}^{j=3} \mathbf{GB_j} \times (\mathbf{R_1}^T(F_x, 0, F_z)^T) = \mathbf{0} \tag{7}$$

which are functions of θ_V, F_x, F_Z and of ψ, θ, ϕ. We may also use the fact that if the coordinates x_b, z_b have been determined, then the components of vector $\mathbf{OB}$ in the reference may be obtained as

$$\mathbf{OB} = \mathbf{OA} + \mathbf{R_1^T}(x_b, 0, z_b)^T \tag{8}$$

provided that the 3 angles θ_V are known. Our objective is now to investigate various sensor placements and determine how the DK solutions may be obtained through only algebraic manipulation. In this paper we will assume that all sensor measurements are exact: we are aware that it is not realistic but this paper is a preliminary step for investigating which sensor placement may be used to determine theoretical solutions while uncertainties on the measurement will be studied next. It must be noted that the non algebraic Irvine equations (1,2) may be used only to determine first the x_b, z_b (provided that the F_z, F_z have been calculated) and then the coordinates of the B (provided that θ_V has been determined). Therefore our objective is not use these 6 equations in the DK solving unless the F_x, F_z, θ_V have been obtained.

Let's now analyze how many sensors are required. If we assume that only the θ_V, θ_H are measured we have as constraints the 3 equations (5) and the 3 force equilibrium (6) with a total of 12 unknowns. For getting a system with a finite number of solutions we must therefore have 6 measurements. If we add the moment equilibrium (7) we have nine constraints but we add 3 unknowns (ψ, θ, ϕ) for a total of 15 unknowns. Here again we will need 6 measurements to end up with a square system.

If we measure only the α_V, α_H we have as unknowns 6 α_V, α_H, the F_x, F_z and $\psi, \theta, \phi, \theta_V$ (because any constraints dealing with α_V, α_H use these 6 later variables) for a total of 18 unknowns. The constraints are the 6 force and moment equilibrium (6, 7), the 3 equations (3) and the 3 equations (4) for a total of 12 equations. Therefore we need at least 6 measurements to get a square system.

More generally assume that we are measuring $n_1\theta_V$, $n_2\theta_H$, $n_3\alpha_V$, $n_4\alpha_H$ (on the same B than the measured α_V), $n_5\alpha_H$ (on a B that has no α_V measurement) and n_6 measurement on the ψ, θ, ϕ. Assuming that n_3 or n_5 are greater than 0 we have as unknowns: 6 F_x, F_z, $3n_1$ θ_V, $3n_6$ ψ, θ, ϕ, $|n_4 - n_3|$ α_H or α_V (on the same B) and n_5 α_H for a total of $12 + |n_4 - n_3| + n_5 - n_1 - n_6$. In terms of equations we have the 6 equilibrium, $\text{Sup}(n_3, n_4)$ equations (3), $\text{Sup}(n_3, n_4)$ equations (4) and n_2 equations (5) for a total of $6 + 2\text{Sup}(n_3, n_4) + n_2 + n_5$. Hence to get a square system imposes to have $6 \leq n_2 + 2\text{Sup}(n_3, n_4) + n_1 + n_6 - |n_4 - n_3|$. This formula allows on to show that **at least 6 measurements are necessary** to solve the DK in analytic form.

III. Planar case, 2 cables

If we consider a planar robot with 2 cables we have only θ_H and α_H sensors and the equations of the previous section are simpler. If θ denote the orientation of the platform the mechanical equilibrium may written as

$$F_x^1 + F_x^2 = 0 \qquad F_z^1 + F_z^2 = mg \tag{9}$$

$$\sum_{i=1}^{i=2} -\mathbf{GB_i}(1)F_z^i + \mathbf{GB_i}(2)F_x^i = 0 \tag{10}$$

the later equation being a function of θ and of the F_x, F_z. As for the sensors we have

$$tan(\theta + \alpha_H) = \frac{F_z}{F_x} \quad (11)$$

$$tan(\theta_H) = (F_Z - \mu g L_0)/F_x \quad (12)$$

For solving the DK in symbolic form we cannot use the Irvine equations (1, 2) unless we have determined the F_x, F_z. Hence if we use the two first equations of (10), then we have 4 unknowns and 2 constraints, while if we use the 3 equations (10), then we have 5 unknowns. Hence **it is necessary to have at least 2 sensor measurements to solve the DK in analytic form**.

If we measure 2 θ_H, then the 2 first equations of (10) and the 2 equations of (12) constitutes a system of 4 linear equations in the F_x, F_z. Solving this system and reporting the result in the Irvine equations leads to the location of B_1, B_2. In turn this leads to 2 solutions for the DK, one with G above B_1B_2, which is unstable, and one with G below B_1B_2, which is stable.

If we measure 2 α_H we may obtain the F_x, F_z by solving the linear system (9),(11) which are linear in the F_x, F_z. These unknowns are obtained as function of θ and reporting their value in (10) leads to a function of $\sin\theta, \cos\theta$ which is transformed into a 6th order algebraic equation using the tangent half-angle substitution. Hence there will be at most 6 stable DK solutions.

If we measure one θ_H and one α_H we may obtain the F_x, F_z by solving the linear system (9),(11),(12) which are linear in the F_x, F_z. These unknowns are obtained as function of θ and reporting their value in (10) leads to a function of $\sin\theta, \cos\theta$ which is transformed into a 4th order algebraic equation using the tangent half-angle substitution. Hence there will be at most 4 stable DK solutions.

IV. Spatial case, 3 cables

A. DK with 6 sensors: 3-$\theta_V\theta_H$

In this section we assume that all A anchor points have both θ_V, θ_H sensors. If we consider the constraints (5, 6) we get a system of 6 equations in the 6 F_x, F_z which is linear in these unknowns. Solving for these unknowns allows one to calculate the coordinates x_b, z_b for each cable. As the 3 angles θ_V are known we may then use equation (8) to determine the coordinates of the three B in the reference frame and therefore the pose of the platform. In summary **this placement allows to determine a single DK solutions with the effort of solving a linear system**.

B. DK with 6 sensors: 2-$\theta_V\theta_H$, 1 θ_V and IMU

In this section we assume that the anchor points A_1, A_2 have both θ_V, θ_H sensors while A_3 has only a θ_V sensor. An accelerometer is located on the platform. The 2 first equations of (5), the three equations (6) and the equations of (7) have as unknowns ψ, θ, ϕ and the F_x, F_z while being linear in these later unknowns. We select the first five equations and the first equation of the moment equilibrium (7) to get a linear system of 6 equations in the F_x, F_z. After solving this system the second and third equations of (7) are only function of ψ, θ, ϕ. As the accelerometer provides 2 of these unknowns these 2 equations are just function of the sine and cosine of the remaining unknown angle. If we use the Euler's angles the second equation factors out in 2 terms that are linear in the cosine, sine of any of the angle ψ, θ, ϕ. The first equation is linear in $\sin(\psi), \cos(\psi)$ and therefore the 2 equations constitute a linear system in these 2 unknowns. Hence we will obtain at most 2 DK solutions by solving 2 linear systems.

C. DK with 5 sensors: 2-$\theta_V\theta_H$ and IMU

In this section we assume that the anchor points A_1, A_2 have both θ_V, θ_H sensors while an accelerometer is located on the platform. The 2 first equations of (5), the 6 equilibrium equations (6) and (7) have as unknowns ψ, θ, ϕ and the F_x, F_z while being linear in these later unknowns. We select the first five equations and the first equation of (7) to get a linear system of 6 equations in the F_x, F_z. After solving this system the second and third equations of (7) are only functions of $\psi, \theta, \phi, \theta_V^3$. As the accelerometer provides 2 of these unknowns these 2 equations are just functions of the sine and cosine of the remaining unknown angle. If we use the Euler's angles the second equation factors out in 2 terms. The first factor is linear in the cosine, sine of any of the angle ψ, θ, ϕ and does not involve θ_V^3. The first equation is linear in $\sin(\psi), \cos(\psi)$ and therefore the first factor and this equation constitute a linear system in these 2 unknowns. After solving this system the constraint $\sin^2\psi + \cos^2\psi - 1$ involves only the sine and cosine of θ_V^3. By using the half-angle tangent substitution this equation becomes a 4th order polynomial in $T = tan(\theta_V^3/2)$ and hence **we will obtain at most 4 DK solutions**.

The second factor is linear in $\sin(\theta_V^3)$ and after solving for this variable it appears that the first equation may be written as $\cos\theta_V^3 F(\psi, \theta, \phi)$. As $\cos\theta_V^3 = 0$ implies also $\sin\theta_V^3 = 0$ only the equation F has to be considered. This equation is of degree 4 in $T_1 = tan(\psi/2)$, $T_2 = tan(\theta/2)$ and $T_3 = tan(\phi/2)$. Whatever is the chosen variable we get for each root a **special case** where the mechanical equilibrium is not dependent upon the value of θ_V^3 while $F_x^1, F_x^2, F_z^1, F_z^2, F_z^3$ have constant values (meaning that B_1, B_2 have a fixed position) and F_x^3 is obtained as $a/\cos\theta_V^3$ where a is a constant. Using he Irvine equations (1,2) we get x_b^3, z_b^3 as a function of $\cos\theta_V^3$ and using equation (8) we obtain the coordinates of B_3 as a function of θ_V^3. At the same time B_3 must lie on a circle centered in a point located on the line going through B_1, B_2, the circle being perpendicular to this line. The center and the radius of this circle may easily be calculated from the known distances between (B_1, B_3), (B_1, B_2) and (B_2, B_3). As B_3 must belong to this circle it induces 2 constraint equations on θ_V^3 and unfortunately there is no way to determine the maximum number of solutions of this system as it is not algebraic. However we have found numerical examples with 2 solutions.

D. DK with 6 sensors: 3-$\alpha_V \alpha_H$

In this section we assume that the anchor points B have both α_V, α_H sensors. The 3 equations of the force equilibrium (6) and the equations (4) are linear in the F_x, F_z. Solving this system leads to 6 equations, the 3 moment equilibrium (7) and the 3 equations (3), whose unknowns are the 3 θ_V and ψ, θ, ϕ. Hence this system (which may be transformed into an algebraic form) has in general a finite number of solution but we have been unable to reduce this system to an univariate polynomial.

E. DK with 5 sensors: 3-θ_V, 1 θ_H and IMU

In this section we assume that the A anchor points have all θ_V sensors while A_1 has also a θ_H sensor and an accelerometer is located on the platform. We consider the system constituted of the 3 equations of the force equilibrium (6), the 2 first equations of the moment equilibrium (7) and the first equation of (5). This system has as unknowns ψ, θ, ϕ and the F_x, F_z and is linear in the 6 later variables. Solving this system and reporting the result in the last equation of the moment equilibrium (7) leads to an expression that factors out into 2 components, both of which are linear in the sine, cosine of any angle ψ, θ, ϕ considered independently. As two of these variables are provided by the accelerometer we end up with two systems that can be written as $U \cos\beta + V \sin\beta + W = 0$, β being any angle in the set ψ, θ, ϕ, which admits two solutions in β. Therefore we get up to 4 solutions in the unknown angle. For each of the solution we get the position of the B of each cable in the cable plane and equation (8) allows to calculate them in the reference frame: hence **there may up to 4 solutions of the DK that are obtained by solving two quadratic polynomials**.

F. DK with 5 sensors: 3-θ_H, 1 θ_V and IMU

In this section we assume that the A anchor points have all θ_H sensors while A_1 has also a θ_V sensor. An accelerometer is located on the platform. We consider the system constituted of the 3 equations of the force equilibrium (6) and the 3 equations (4). This system of 6 equations has as unknowns θ_V^2, θ_V^3, and the F_x, F_z and is linear in the 6 later variables. Solving this system and reporting the result in the 3 moment equations (7) lead to a system having $\psi, \theta, \phi, \theta_V^2, \theta_V^3$ as unknowns. Any of the 3 equations of the moment equilibrium is linear in $\sin\theta_V^2, \cos\theta_V^2$. We consider any two pair of these equations to solve in this variables and report the result in the remaining equation of (7) and in the constraint equation $\sin^2\theta_V^2 + \cos^2\theta_V^2 - 1 = 0$. This leads to 2 equations in the unknown $\theta_V^3, \psi, \theta, \phi$. Using the half-angle tangent substitution on θ_V^3 and calculating the resultant of the 2 equations in T_3 leads to a single equation in ψ, θ, ϕ. As the IMU provides two of these angles we have therefore an univariate equation and using the half-angle substitution on any of the angle leads to a polynomial of degree 32. Each of the root of this polynomial leads to a single value for θ_V^2, θ_V^3 and for the F_x, F_z. This allows one to calculate the x_b, z_b for each cable and using equation (8) a single position for the B. **Hence the DK may have up to 32 solutions**.

G. DK with 5 sensors: 2-$\alpha_V \alpha_H$ and IMU

In this section we assume that the B_1, B_2 anchor points have both α_V, α_H sensors and that an accelerometer is located on the platform. The unknowns are therefore the 6 F_x, F_z, the 3 θ_V that are used to calculate the x_b, z_b and the angles ψ, θ, ϕ, two of which will be provided by the IMU for a total of 10 unknowns. In terms of constraint we have the 6 equilibrium constraints (6,7), the two equations in (3) and in (4) involving the measured α_V, α_H for a total of 10 constraints. The 3 force equations (6), the two equations (4) and the first moment equation of (7) are linear in the F_x, F_z and are used to find these variables. We assume that θ is not measured by the IMU (but the process will the same whatever is the not measured angle). The 2 equations (3) are linear in the unknowns $\sin\theta, \cos\theta$ and are used to determine these variables with the additional constraint $\sin^2\theta + \cos^2\theta - 1 = 0$ which is now a function of θ_V^1, θ_V^2. The two remaining equations of the moment equilibrium (7) are now function of the 3 θ_V. Hence we have now 3 equations in the 3 θ_V that are converted into algebraic form using the tangent half-angle substitution. Successive resultant in T_1, T_3 leads to a single polynomial in T_2 which factors out in 2 polynomial of degree 4 and one polynomial of degree 16. Each root of these polynomials leads to a single value of the 3 θ_V and of θ which in turn leads to a single value for the F_x, F_z allowing to calculate first the x_b, z_b of all cables and using (8) a single location for all the B. **Hence the DK may have up to 24 solutions**.

V. USING A SIMPLER CABLE MODEL: THE PARABOLA CASE

It must be noted that Irvine has proposed as simplified cable model where the shape of the cable is a parabola. This model leads to a single solution for the inverse kinematic [20], while they are multiple solutions with the full model [21]. From the DK view point the unknowns are the 3 coordinates (x_i, y_i, z_i) of the B_i and the F_x, F_z for a total of 15 unknowns as the θ_V may be expressed as functions of the x_i, y_i. The constraint are the 6 equilibrium equations (6, 7) and the 3 equations (4) (which are identical to the equations (5)) which amount to:

$$F_z = \frac{\mu g L_0}{2} + F_x \frac{z_b}{x_b} \tag{13}$$

where x_b, z_b are the coordinates of B in the cable plane, that are derived from the (x_i, y_i, z_i) Three other equations are obtained by writing that the distance d_{ij} between the pair of points B_i, B_j is a known constant, leading to quadratic equations in the x_i, y_i, z_i. The final 3 constraints are that the L_0 may be expressed as non linear, non algebraic equations (involving the logarithm function) of the unknowns, that therefore cannot be used to have an algebraic system. Hence we end up with a square system of 15 equations, 12 of them being algebraic. A benefit of this approach is that we theoretically need only 3 measurements to get algebraic system that may provide solution(s) in closed-form. For that purpose we consider the force equilibrium equations (6) and the 3 equations (13) that constitutes a linear system in the F_x, F_z. We may solve this system in these variables but the presence of z_b/x_b in equation (13) leads to relatively large expression for the equations (7) and our trials have shown that even with 5 additional sensors the elimination process leads to an univariate polynomial of high degree.

VI. Extension for a CDPR with n cables

If we consider a spatial CDPR with n cables the unknowns are the $2n$ F_x, F_z, the n θ_V and the 6 parameters of the pose for a total of $3n+6$ unknowns. The constraints are the $2n$ Irvine equations (1, 2), the n constraints (3) and the 6 equations of the mechanical equilibrium (6, 7) for a total of $3n+6$ constraints. Hence if we want to get rid of the non algebraic equations the measurements should provide $2n$ data. The simplest case is to measure all θ_V, θ_H: in that case the same reasoning than in section IV-A allows one to show that the DK will have a single solution.

VII. Conclusion

This investigation on additional sensors for simplifying the direct kinematics of CDPR with sagging cable has shown that the best sensor arrangement is to use ground-based sensors that measure the θ_V, θ_H angles for all cables. This arrangement has the advantages of a simple implementation and of providing usually a single DK solution. We have also shown that for a CDPR with n cables at least $2n$ additional measurement are required for getting the DK solutions in closed-form and that the θ_V, θ_H arrangement on all cables will provide a single DK solution whatever n is.

Having sensors on the platform is still a possibility but leads to a more complex solving process that may be simplified if we have an estimate of the platform orientation with the drawback of leading to multiple DK solutions. Accelerometer may provide this information but their accuracy in practice is doubtful.

We have proceeded under the assumption that all measurements are exact, an assumption that is clearly not true in practice. Taking into account the effective accuracy of the orientation sensors and its influence on the DK solving is another issue that has to be investigated.

Another issue is that adding sensors beside the cable lengths provides sensor redundancy. To reduce the effect of uncertainties on the DK solution we may consider the DK as an optimization problem. However using the deterministic approach to provide an initial guess for the optimization may make sense.

At this stage it is uncertain if adding orientation sensors on the cable will provide a reliable solution for the DK problem and may lead to an improvement of the accuracy of the CDPR. Possibly direct estimation of the platform pose using vision, optical markers or telemeter may provide a better accuracy even for large CDPR.

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