An experimental investigation of extra measurements for solving the direct kinematics of cable-driven parallel robots

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***Abstract*— Solving the direct kinematics (DK) of cable-driven parallel robots (CDPR) based only on the cable length measurements is a demanding problem that is still not well mastered, especially for robots having sagging cables. A model-based approach may be used to solve this problem but the model parameters and measurements are uncertain, thereby leading to positioning inaccuracy. A possible way to improve the accuracy and speed up the solving is to add extra measurements. For that purpose a preliminary step is to determine what type of measurements are possible and then to estimate how accurate they are. For that purpose we have used a CDPR with 4 cables that has been instrumented with various types of extra measurements: cable tensions and orientations, platform orientation. Ground truth has been established and we have compared the data provided by the extra sensors with their real values. This work shows that cable tensions sensors and platform orientation sensors are not good candidates to be used for the DK while cable orientations may be obtained with a good accuracy both in static poses or during a quasi-static motion.**

I. INTRODUCTION

Cable-driven parallel robots (CDPR) which use coiling cables as actuators are currently being investigated especially for applications that require a large workspace. Beside the classical advantages inherent to a parallel structure (improved accuracy, excellent load/weight ratio) CDPR have the huge advantages of mechanical simplicity and excellent lifting capacity. For example our MARIONET-CRANE robot has a lifting capacity of 2.5 tons for a 75m $\times$ 35m $\times$ 25m workspace [1] while other robots with large workspace have been proposed [2],[3], [4],[5],[6].

In this paper we define a reference frame and a mobile frame that is attached to the platform. The vector $\mathbf{X}$ denotes the parameters of the pose of the platform which is constituted of the coordinates of a specific point C of the platform and the three Euler's angles ψ, θ, ϕ. Cable i exit from its winch at a fixed point A_i and is connected to the platform at point B_i, whose coordinate in the mobile is supposed to be known. For non deformable cable ρ_i will denote the cable length while τ_i is the cable tension. The vector τ sum up all cable tensions while ρ sum up all cable lengths.

Any CDPR model-based control requires to solve the kinematics of the robot. Inverse kinematics (finding the cable lengths to reach a given pose) is straightforward if there no cable deformation while it is more complex if the cables are sagging [7]. As for the direct kinematics (DK) (finding the pose(s) of the robot for given cable lengths) is always a complex problem [8], [9], [10], [11], [12] even with non deformable cables and even more complex for sagging cables [13], [14] as soon as one want to determine all solutions. Indeed for a CDPR with n cables we have geometrico-static equations that relates ρ to the cable tension τ and the pose $\mathbf{X}$ of the platform.

$$H(\rho, \tau, \mathbf{X}) = 0 \quad (1)$$

where H is dependent upon the cable model. At the same time the CDPR should satisfy a mechanical equilibrium conditionL

$$E(\rho, \mathbf{X}, \tau, \mathcal{F}) = 0 \quad (2)$$

where E depends upon the cable model. In general the DK solver has to consider both equations systems (1), (2) to obtain a square system of equations that usually admit a finite set of solutions.

Real-time DK solving has the purpose not to determine all solutions but only the current pose of the platform. It is usually based on a guess of the solution and is less problematic than the general DK problem [12], [15] as soon as a proper approach is used to ensure the convergence of the method toward the current pose [16]. However the error in the determination of the pose is heavily dependent upon the accuracy of the measurements and of the parameters of the model, some of which are changing with time.

To reduce this error a possibility is to add measurements to overconstrain the equations system, possibly leading to a single solution as shown for parallel robots with rigid legs [17], [18], [19]. In that case the first problem is to identify what physical quantities may be measured, the sensor configuration(s) and the corresponding DK solving algorithm, a topic that has been addressed in [20], [21] for various cable models, assuming perfect measurements. But this assumption is unrealistic, thereby leading to the following theoretical problems: being given a CDPR with extra sensors what should be the sensor accuracy so that, in general, a DK solver will provide a single solution that is more accurate than the classical DK? As solving these difficult problems is dependent upon the sensor types, configuration and errors it makes sense to determine first what will be the sensor accuracy for different types of sensing modes in order to focus the theoretical analysis on the most promising sensors. This is the purpose of this paper.

II. THE POSSIBLE EXTRA SENSORS

The purpose of this section is to determine what physical quantities may be measured on a CDPR in order to facilitate

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the DK solving and to improve the accuracy. Beside measuring directly the platform pose, that may be difficult for large scale robot, it is possible to consider:

- *cable tensions*: this is clearly an important item as cable tensions appear directly in the DK equations. There has been numerous attempts to measure cable tensions [22], [23], [24], [25] for purpose of control but, to the best of the author knowledge, none of them have studied the errors in the tension measurements, while it has been shown that cable tensions are extremely sensitive to the cable physical parameters such as elasticity [26]
- *cable orientations*: fully determining the cable orientation requires to measure two angles: α between the cable plane and a reference plane and γ between the cable and the horizontal axis. Some CDPR has been instrumented to measure α or both α, γ [1] through a mechanical system while vision has also been used [27], [28] although it is difficult to use outdoor
- *platform pose*: for large CDPR it is difficult to measure the location of a specific point of the platform with respect to a reference frame but it may be considered to measure, at least partly, the platform orientation [29].

III. The robot and the instrumented platform

For this experiment we have used our MARIONET-ASSIST robot with 4 cables (numbered as 1, 2, 3 and 6) which has been presented in [30]. This CDPR is a modular robot with up to 6 cables with dimensions roughly 6 $\times$ 4 $\times$ 3 meters which has been designed to perform transfer operation for elderly. This robot is assumed to be a $N-1$ robot, i.e to have all the 4 cables connected at the same point on the platform so that it has 3 translational d.o.f. In practice however the attachment points on the platform are never exactly in the same location so that the robot is an *under-constrained* CDPR. The cables have a diameter of 4 mm a linear density of 0.01 kg/m and have a Kevlar kernel with a polyester coating so that their elasticity can be neglected if the cable tension is lower than 200 N. The robot has been fully geometrically calibrated using accurate measurements of the distances between its A points [31].

To measure the sensor errors we have designed a planar instrumented platform (figure 1) allowing to perform all three types of measurements presented in the previous section. Each cable is connected at point B to a Delrin cylinder C_1 that can slide within another Delrin cylinder C_2 rigidly fixed on the platform. The end-point of C_1 is connected to an accurate strain gage force sensor CZL616C with a maximal load of 7.8 N, that has been calibrated beforehand. The purpose of this sensing system is to measure the cable tension while avoiding to have the strain gages mounted on a U joint (for aligning the stress with the gage axis) as the sensorsrotation may lead to inaccurate measurement.

A Phidget accelerometer is mounted at the bottom of the platform with the purpose of measuring two of the three rotational d.o.f. of the platform. Another accelerometer, with a weight of 10 grams, is mounted on each cable at a distance of about 10 cm from the B point, so that one of its side lies along the cable direction. The purpose of this sensor is to measure the γ angle of the cables with minimal disturbance on the direction of the cables. Two other sensing systems are used to measure the orientation of the cable. Two rotating heads are mounted on the platform with a Sharp distance sensor (range: 0-30cm) in a calibrated position on each head. The head allows for a sweeping motion of the distance sensor whose measurements allow to detect points on a cable. Being given the rotation angle of the head and the distance measurement we are able to determine the coordinates of a point P on the cable in the platform frame and as the coordinates of the B points in the same frame have been accurately determined the vector $\mathbf{BP}$ provides the full direction of the cable with minimal disturbance.

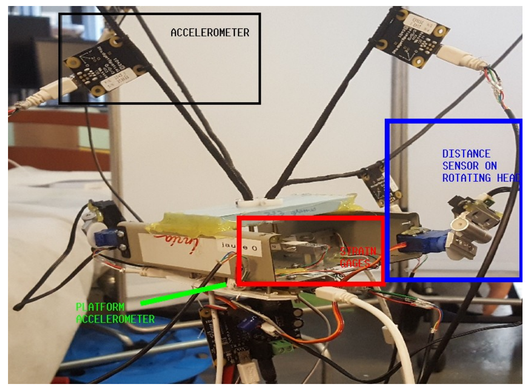

Fig. 1. The instrumented platform

IV. Static tests

In this test the lengths of the cables are fixed and the CDPR platform is still. The lengths of the cable have been measured with a laser distance sensor with an accuracy of 0.01mm. We first assume that the cables are not deformable and we use a variant of an interval analysis based DK solver [10] that is able to provide all solutions of the DK based on the equations (1, 2), that is a square system, taking into account that the B are exactly at the same position. This variant takes into account the remaining uncertainties on the location of the A, B and on the cable lengths to compute regions that include all solutions for the real robot, that are usually very small. In our case the solver provides usually 4 distinct solution regions but a manual inspection allows us to determine the current CDPR pose. The selected region allows one to determine a ground truth of the state of the CDPR that will then be compared to the measurements. About one hundred poses have been tested with coherent results. First we consider the cable tensions and the angle γ. Table I shows a typical example in which one of the cable is slack (cable 2) while the other cables have various tensions. The cable tension error are 25.7% (cable 1), 11.44% (cable 3) and 12.69% (cable 6), they decrease with the amplitude of the tension but are relatively large. We have then substituted the last 3 equations of (2) by 3 additional constraints involving the measurements in order to to determine if the obtained

cable	real τ	measured τ	real γ	measured γ
1	1.79	1.33	47.96	38.79
2	0	0.048	NA	14.47
3	3.49	3.09	62.31	61.25
6	1.95	1.71	51.5	60.81

TABLE I

COMPARISON BETWEEN THE REAL AND MEASURED CABLE TENSION (IN N) AND BETWEEN THE ANGLE γ BETWEEN THE CABLE AND THE HORIZONTAL AS MEASURED BY THE ACCELEROMETER (IN DEGREES)

square system has a single solution but it appears that in most cases this system has no solution. We have then determined the platform pose that minimizes the sum of the square of the equations. The differences between the components of $\mathbf{OC}$ and the ground truth are 0.67, 0.105 and 0.644 cm, which is reasonable, but we have a large error on the θ angle (17.96 degrees). In summary it appears that measuring the cable tensions is not a good option to improve the DK.

We investigate now the measurements provided by the accelerometer located on the platform. We are especially interested in the angle θ between the platform normal and the vertical. Our test have shown that in the static case the error on the angle, $\Delta\theta$, is about 1.5 degrees.

We consider now the use of the distance sensors on the rotating head. Figure 2 presents the scene viewed by the distance sensor and the inverse of the distance data during several sweeping motion, which last about 4 seconds. The first signal (around time 0) corresponds to the leftmost cable which is the farthest away from the sensor. The second peak at 1.8s is the second cable from the left (which is the closest to the sensor). The third peak at 2.2 s is the rightmost cable. As may be seen from the data the signal is quite repetitive. Being given the measured rotation angle β of the

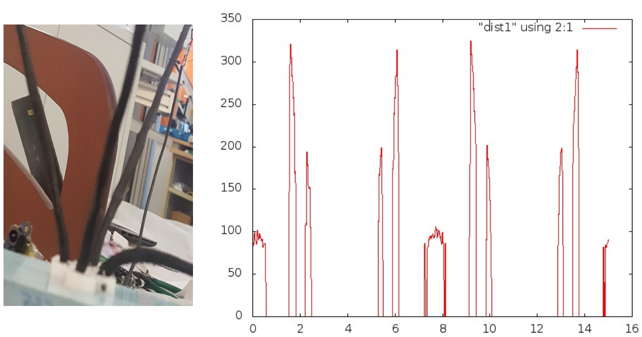

Fig. 2. The scene as viewed from the distance sensor and the inverse of the distance data during several sweeping motion

head and the distance d provided by the sensor it is possible to determine the location of a point M on the cable in the mobile frame. As the location of B in this frame is also known it is therefore possible to determine the unit vector $\mathbf{n_m}$ of the cable direction. Note that several points on a given cable are obtained during the sensor motion i.e. several pairs (d, β) may be used to determine $\mathbf{n_m}$. We have calculated the angular error between the measured cable direction and the real one for all measured (d, β) pairs and a typical plot of these errors is presented in figure 3. It may be seen that with

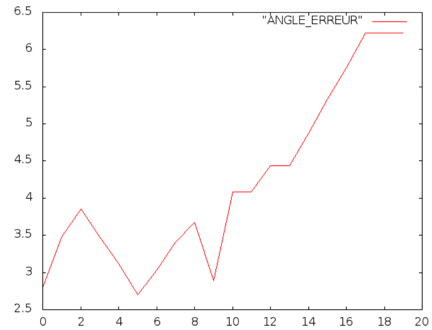

Fig. 3. Angular error in degrees between the real cable direction and the one measured by the distance sensor during a sweep motion. Here 20 points have been identified for cable 1 and the plot presents the angular error for each of them

a proper processing it will be possible to determine the cable direction with an error of less than 4 degrees. Note that it will be better to have this system at the A points. Indeed with three angular measurements and the cable lengths it is possible to determine the location of three B points on the platform, which is sufficient to fully calculate the pose of the platform. But for large CDPR the even small angular error will lead to large errors on the platform pose.

In summary for a fixed pose the most accurate measurements are obtained from the accelerometers both on the platform and on the cables. The sensing from the distance sensor is of relatively good quality while the measurement of the cable tensions is unreliable.

V. DYNAMIC TESTS

We are now interested in the measurements when the platform is moving at a slow speed so that there is no vibration problem. For that purpose we have fixed the length of cables 1, 3, 6 so that these cables initially are under tension while the initial length ρ_2^0 of cable 2 was such that this cable is slack. We then coil cable 2 for about 10 seconds and then uncoil it until its length became again ρ_2^0, the process being repeated (the accompanying video presents the experiments). Visually, when coiling cable 2 cables 1 and 6 remain under tension. The initially slack cable 2 become taught at some time while tension in cable 3 decreases until it is slack. This is confirmed by the cable tension as measured by the strain gages (figure 4). As we have seen previously the tension provided by the strain gages are not accurate enough to contribute to the DK but it appears here that they are repeatable and can be used to determine which cable(s) have the highest tension or are close to be slack.

But a finer analysis may be obtained by looking at the cable angles provided by the accelerometers. Figure 5 presents the angles between the platform plane and the cable during a repetitive motion. At the initial configuration (time 0 on figure 4 and time 22 on figure 5) the platform is supported

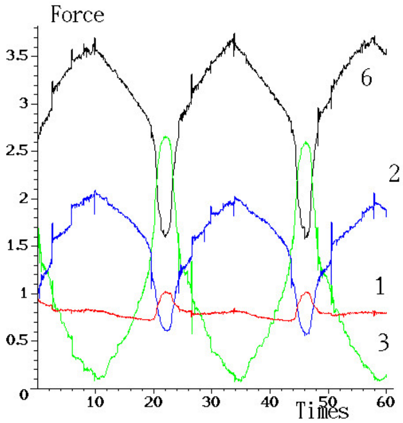

Fig. 4. The cable tension (N) as measured by the strain gages during a repetitive motion. Cable 2, initially slack, become under tension while cable 3, initially under tension, becomes slack. Cable 1 and 6 remain under tension during the platform motion.

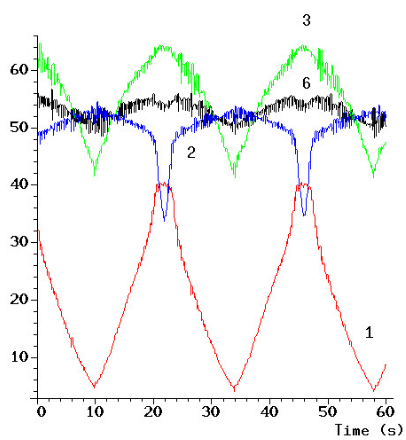

Fig. 5. The angle (in degrees) between the platform plane and the cable during a repetitive motion

by cables 3 and 6 with a minor contribution of cable 1. After the beginning of the motion the tension of cable 2 increases quickly while the tension of cable 1 decreases rapidly and the cable may be considered as slack (although this is not seen on the force measurements). The tension of cable 3 is also decreasing and it may be estimated that at time 35 the platform is supported only by the cables 2 and 6.

We have then studied the coherence between the measurements and the ground truth, deduced from a simulated model with non deformable cables, on the motion between time 0 and 10s of figure 4. We have first compared the angle between the platform normal and gravity as measured by the accelerometer on the platform and the simulated one (figure 6). It may be seen that there is significant difference between the simulation and the measurement (roughly from 3.5 to 5 degrees in this experiment but we have noted that the error may go up to 15 degrees). This difference may be explained: the accelerometer measures both the gravity

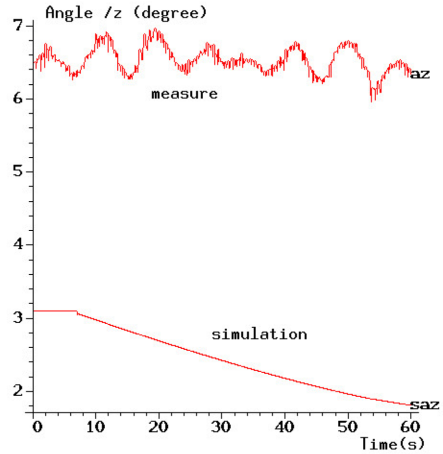

Fig. 6. Angle between the platform normal and the vertical axis as measured by the platform accelerometer and the simulated value

acceleration and the motion acceleration. We have tried to use an observer to filter out the motion acceleration but without any significant improvement in the error. Hence orientation of the platform during a motion may only be roughly estimated from the accelerometer.

We have then considered the cables tensions and angles presented on figure 7 for the tension and on figure 8 for the angles between the platform plane and the cables (a slack cable is assumed to have a 0 angle).

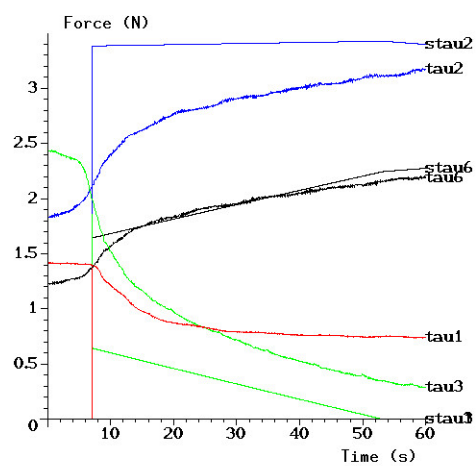

Fig. 7. Measured and simulated cable tensions during a motion. $stau$ is the simulation and tau the measurements

As may be seen on these figures we have relatively small errors both for the tension and the angle for cables with a significant tension and large errors for cables with low tension. These differences may be due to the non deformable cable model used to establish the ground truth, which is only approximate even for cable with a low linear density and no elasticity. Hence we have decided to investigate a more complex cable model.

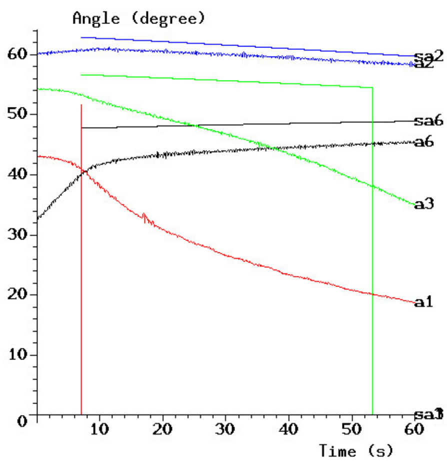

Fig. 8. Measured and simulated angles between the platform plane and the cables during a motion. sa is the simulation and a the measurements

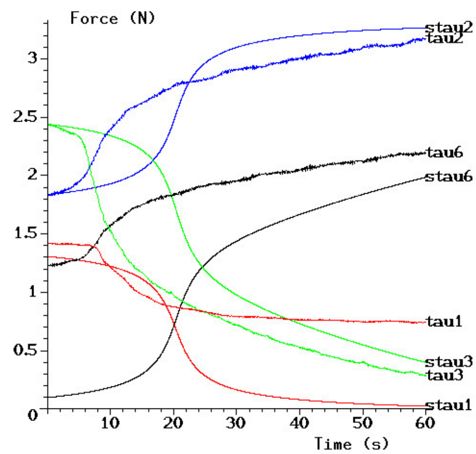

Fig. 9. Measured and simulated cable tensions during a motion with deformable cable model. $stau$ is the simulation and tau the measurements

A. Cable model

In this paper we will use the Irvine sagging cable model that is valid for elastic and deformable cable with mass [32]. Experimental works have shown a very good agreement between this model and the behavior of cables classically used for CDPR [33]. This model is established in the cable plane in which we have $A_i = (0, 0)$ and $B_i = (x_b \geq 0, z_b)$. Vertical and horizontal forces $F_z, F_x > 0$ are exerted on the cable at point B_i. We will assume that the Young modulus of the cable material is very large so that we neglect the elastic terms.

For a cable with length at rest L_0 the coordinates of B are given by [32]:

$$x_b = F_x\left(\frac{sinh^{-1}(F_z) - sinh^{-1}((F_z - \frac{\mu g L_0}{F_x})}{\mu g}\right) \tag{3}$$

$$z_b = \frac{\sqrt{F_x^2 + F_z^2} - \sqrt{F_x^2 + (F_z - \mu g L_0)^2}}{\mu g} \tag{4}$$

where μ is the cable linear density.

This model has been integrated in the DK solver that is used to determine the pose and to calculate both the cable tensions and the angle of the cables at the cable accelerometer position.

B. Results

Figures 9 presents the measurement and simulated tension while figure 10 shows the angles.

Comparing figures 7 and 9 it may be seen that there is a small improvement for the tensions in the cable with low tension while for the cables with a significant tension the results are unclear (improvement for cable 2 and degradation for cable 6). On the other hand the comparison between figures 8 and 10 shows a significant improvement on the cable orientation for the cables with significant tension (cable 2 and 6) and an improvement for the cable with low tension (1 and 3). This comforts us in our opinion that measuring, even partly, the cable orientation angle with accelerometer is promising.

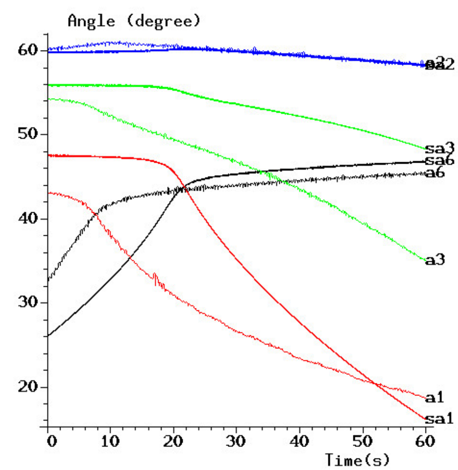

Fig. 10. Measured and simulated angles between the platform plane and the cables during a motion with a deformable cable model. sa is the simulation and a the measurements

As for the angle between the platform normal and the vertical deduced from the platform accelerometer, the deformable cable model does not provide any improvement.

VI. CONCLUSIONS

The primary objective of this paper was to determine what extra sensor types may be used on a CDPR in order to speed-up the direct kinematics and to provide as solution only the current pose of the platform. The second objective was to determine how reliable were the different types of sensors. It appears that measuring the cable tensions is not a viable option (but may still may be used for control purposes with some precautions). Accelerometers on the platform is a good option for still pose but is less useful when the platform is moving. The best option is measuring the cable orientation angles and accelerometer may be used for that purpose. We have also determined bounds on the measurement errors.

The problem at hand is now to improve the positioning accuracy of the CDPR using these extra sensors being given their accuracy and to determine if the results are dependent upon the scale of the robot. As mentioned by one reviewer errors on the Young modulus and linear density may in that case have a drastic influence on the ground truth measurements. However we believe that including these errors in the DK solver is possible, thereby leading to guaranteed DK regions that will still allow for comparing ground truth and measurements. For given measurements it is certainly possible to determine the region in the pose parameters space that include the current pose. A real-time approach is also required to calculate an estimation of the current pose based on all the available measurements, taking into account that their uncertainty is bounded and that their distribution is uniform. Kalman filter specific for uniform noise have been proposed for that purpose. Another approach that may be worth investigating will be deep learning. It will certainly be worth to quantify the quality of the estimation using the approach proposed in [34] which looks at the interval residual of the equations.

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