On the redundancy of cable-driven parallel robots

J-P. Merlet [1]

[1] *INRIA Sophia-Antipolis,France, e-mail: Jean-Pierre.Merlet@inria.fr*

Abstract. This paper addresses the concept of redundancy for cable-driven parallel robot (CDPR). We show that although CDPR may be considered as kinematically redundant, they constitute a special class for which the self-motion manifold is 0-dimensional and that they are not not statically redundant (i.e. the tension distribution cannot be changed continuously while keeping the platform at a given pose). A direct consequence is that a CDPR with more than 6 cables is always in a configuration where at most 6 cables are simultaneously under tension. However for a given pose there may be several set of 6 cables that are valid, which allow us to define the concept of *weak statical redundancy*. We show how the possible valid configuration(s) may be determined on a trajectory. All these concepts are illustrated on a real robot.

Key words: cable-driven parallel robots, kinematics, redundancy

1 Introduction

Cable-driven parallel robot (CDPR) have the mechanical structure of the Gough platform with UPS rigid legs except that the UPS rigid structure is substituted by a cable whose length may be controlled. In practice we may assume that the output of the coiling system for cable i is a single point A_i while the cable is connected at point B_i on the platform (figure 1). The flexibility of the cable at A_i, B_i allows to consider that we have a U and S joint at these points. We denote by d_i the distance $||\mathbf{A_iB_i}||$ and by l_i the length of the cable between the coiling system and B_i. The length l_i may be written as $l_i = a_i + \rho_i$ where a_i is a constant length corresponding to the amount of cable between the coiling system and A_i and ρ_i is the cable length between A_i, B_i. In this paper we will assume that the weight of the cable is negligible so that there is no sagging. If cable i is under tension, then it exerts a force $\mathbf{f_i}$ on the platform such that

$$\mathbf{f_i} = -\frac{\mathbf{A_iB_i}}{\rho_i}\tau_i \quad (1)$$

where τ_i is the positive tension in the cable. As a cable cannot exert a pushing force, if the cable is not under tension then $\mathbf{f_i}$ is 0. Consider a CDPR with n cables and let τ

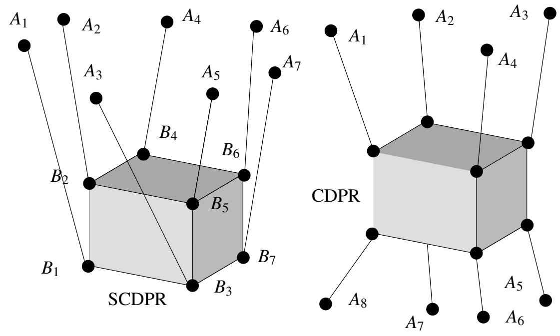

Fig. 1 Cable driven parallel robots: on the left the suspended version

denotes the set of f_i which are not 0 while $\mathscr{F}$ will be the external wrench applied on the platform with the torques applied around a point C. The mechanical equilibrium imposes

$$\mathscr{F} = \mathbf{J^{-T}}(\mathbf{X})\tau \tag{2}$$

where $\mathbf{J^{-T}}$ is the transpose of the inverse kinematic jacobian matrix of the robot whose j-th column is $((\mathbf{A_jB_j}/\rho_j \;\; \mathbf{CB_j} \times \mathbf{A_jB_j}/\rho_j))$. This matrix is dependent upon the pose $\mathbf{X}$ of the platform. A CDPR will be called *pure force submitted* (PFS) if the wrench $\mathscr{F}$ at is center of mass G is reduced to a force while the torque components are 0. A pose will be called *suspended* if

1. the robot is PFS and the platform is only submitted to a force $\mathbf{g}$ composed of the gravity force and of small non vertical disturbances
2. at this pose we have $\mathbf{g}.\mathbf{A_iB_i} > 0$ for all i such that cable i is under tension

A CDPR will be called *suspended* (SCDPR) if all poses of its workspace are suspended. Hence the cables of a SCDPR cannot exert a downward force and the mechanical equilibrium of the robot relies on the gravity force.

A CDPR with a platform having m dof is also called *fully constrained* if all the dof can be controlled. A well known result is that a fully constrained robot must have at least $m+1$ cables, except in the case of a SCDPR where only m are required. However a strong argument for designing CDPR with more than the strict minimum of cables (and hence usually called *redundant* CDPR) is that additional cables, if appropriately located, may drastically increase the size of the workspace of the robot. Hence several authors have addressed the problem of calculating the reachable workspace with positive tensions in the cables [1, 4, 5, 7, 11, 13, 15, 16]. Another reason to have more cables is to to be able to change the distribution of the cable tensions. Numerous works have addressed the problem of computing an appropriate set of cable tensions, for a given platform pose, see [2, 3, 10, 12, 14] to name a few. In this paper we will consider CDPR with more than 6 cables and examine the notion of redundancy of such a robot.

2 Redundancy of CDPR

In this paper we assume that the cables of a CDPR are not elastic (elasticity of the cables induce other difficulties that has been partly addressed in [8]). The concepts of redundancy of parallel robots has been properly addressed recently by Müller [9] but needs to be refined for CDPRs. Let $\mathbf{q}$ be the set of the n joint variables of the robot and let us define the loop closure constraints by $\mathbf{h}(\mathbf{q}) = \mathbf{0}$. Time differentiating this equation leads to $\mathbf{J}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0}$ where $\mathbf{J}$ is the constraint jacobian. The local dof of the robot is defined as $n - \text{rank}(\mathbf{J})$ while the maximal value of this quantity over a given workspace W is referred as the *global dof* δ. The robot will be denoted *kinematically redundant* if $\delta > \text{Dim}(W)$ while the *degree of kinematic redundancy* is defined as $\delta - \text{Dim}(W)$. The *self-motion* of a kinematically redundant robot is defined as the set of joint variables that may be obtained for a fixed pose of the end-effector. The dimension of this manifold may be equal to the degree of kinematic redundancy (e.g. for a 7R serial robot) but this is not always the case. For example for a 6 dof CDPR with $m > 6$ cables at a given pose the joint variables ρ have fixed values: hence although the degree of kinematic redundancy is $m - 6 > 0$ the self-motion manifold is zero-dimensional. Hence we may refine the concept of kinematics redundancy by asserting that a robot is kinematically redundant iff the dimension of its self-motion manifold is not equal to 0: with that definition **CDPR are not kinematically redundant**.

Now let's look at the actuation scheme and define m as the number of actuated joint variables, under the assumption that the CDPR have at least 7 cables (or 6 for SCDPR). The *degree of redundancy of the actuation* is defined as $m - \delta$ and a parallel robot is called *redundantly actuated* if $m - \delta > 0$. With that definition a CDPR with at least 8 cables (7 for SCDPR) is redundantly actuated. But we have to to determine if this redundancy can be used to change the distribution of the tension in the cables while maintaining the pose of the platform: in other words we have to examine if actuation redundancy implies *statical redundancy*.

We must first note that a cable coiling mechanism is a single input- single output system (SISO) which implies that, for example, we may control the length of the cable or its tension but not both. As for preserving the pose of the platform we have to control the lengths of the cables we consequently cannot change their tensions.

Another point in that direction is to look at the mechanical equilibrium condition (2) which is used as the basic equation for workspace and tension distribution algorithm. There is a constraint for using this equation that is seldom mentioned: they are valid only if $\rho_j = ||\mathbf{A_j B_j}||$. Even if we assume that we have an exact measurement of ρ_j managing the distribution of all cable tensions implies that the CDPR controller is able to satisfy the constraint $\rho_j = ||\mathbf{A_j B_j}||$ **at any time**. Such an assumption is unrealistic (especially with the discrete time controller that is used) and with the unavoidable uncertainties in the measurements and control. In reality a CDPR at each time will have at most six cables under tension, while the other cables will be such that $\rho_j > ||\mathbf{A_j B_j}||$ and consequently are slack. This allows us to claim that **redundantly actuated CDPR are not statically redundant**.

We define the *cable configuration* of a CDPR at a given pose as the set of cable number that are under tension at this pose. A *m cable configuration* is a cable configuration with *m* cables under tension, the others being slack. We consider CDPR with non elastic cables with $m > 7$ cables ($m > 6$ for SCDPR). In that case the number of cables under tension is at most 6. But for a given pose the number of cables sextuplets such that the cable tension are positive and satisfy the mechanical equilibrium condition (2) may not be unique. If this is the case let us define m_6 as the number of valid cables sextuplets. For each of them we may calculate the cables tensions by solving (2) which is a set of 6 linear equations in the 6 unknown cable tensions. These tensions will differ for each of the m_6 cable configurations. Hence a pose will be called **weakly statically redundant** if $m_6 > 1$: at such a pose we may adjust the cable tension distribution by selecting one of the cable configuration in the set of m_6 cable configurations. In practice this means that the cables that are not part of the selected cable configuration should be made slack by setting their control variable ρ_j to a value that is significantly larger than $||\mathbf{A_j B_j}||$ for this pose. We will illustrate these concepts on an example.

We consider the large scale robot developed by LIRMM and Tecnalia as part of the ANR project Cogiro [6] This robot is a SCDPR with 8 cables whose A_i coordinates are given in the following table will all dimensions in meters:

x	y	z	x	y	z
-7.175	-5.244	5.462	-7.316	-5.1	5.47
-7.3	5.2	5.476	-7.161	5.3	5.485
7.182	5.3	5.488	7.323	5.2	5.499
7.3	-5.1	5.489	7.161	-5.27	5.497

The platform pose is defined by the coordinates x_g, y_g, z_g of its center of mass in a given reference frame. The orientation is defined by the three Euler's angles ψ, θ, ϕ. We consider a circular trajectory for this robot defined by

$$x_g = \cos(\lambda\pi) \quad y_g = \sin(\lambda\pi) \quad z_g = 2 \quad \psi = \theta = \phi = 0$$

where λ is a parameter in the range [0,1]. As mentioned previously although this CDPR has 8 cables only 6 of them at most will be in tension simultaneously. Our purpose is to determine what are the cable configurations that satisfy the constraint (2) on the trajectory. For that purpose we will consider all combinations of 6 cables among the possible 8. It is then relatively easy to determine on which part of the trajectory a given set of 6 cables allows to satisfy the constraints.

Figure 2 shows the result for all the trajectory. The analysis of these curves shows that there is not a single 6 cables configuration that remain valid on all the trajectory. Furthermore at any pose on the trajectory there are at least 3 valid cable configurations and up to 7 configurations. A possible strategy to perform this trajectory while minimizing the number of configuration changes will be to start with $\lambda = 0$ under configuration 345678 until λ reaches the value 0.39239, switch to configuration 123456 at this pose until λ is 0.589, move to configuration 123478 at this point until λ is 0.9043 and then move again to configuration 345678 until $\lambda = 1$.

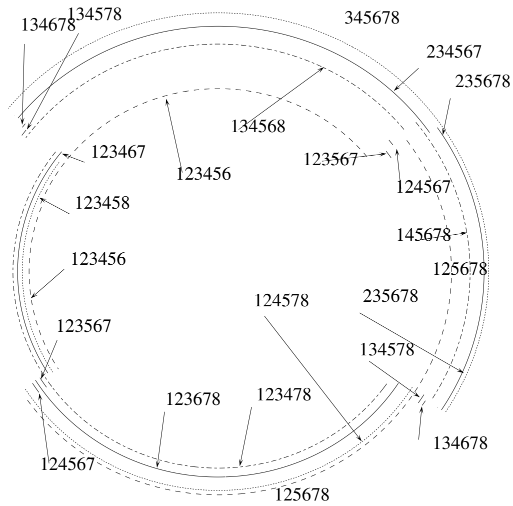

Fig. 2 The possible cable configuration on the circular trajectory. The arcs have been translated to show the valid cable configuration.

The circular trajectory has been tested by LIRMM on the robot without taking into account the cable configurations. An analysis of the measured motor torques (which does not exactly reflect the cable tensions but give a rough idea) have shown that indeed the robot switches between 6 cables configurations during its motion at time that are compatible with our calculation.

This trajectory is hence weakly statically redundant and we may use this property to manage the tension distribution. For example for $\lambda = 0$ there are 4 valid cable configurations with $||\tau||_\infty$ and $||\tau||_2$ for a mass of 1N: 125678 (0.78425, 1.48345), 145678 (0.81685, 1.51699), 235678 (0.76469, 1.4422), 345678 (0.76623, 1.4639).

3 Cable configuration and uncertainties

Up to now we have examined the cable configuration at a given pose but we have also to consider that the pose is obtained from the uncertain cable lengths measurements. Hence to reach a nominal pose $\mathbf{X_0}$ we apply as control ρ_0 but the real lengths of the j-th cable lies in the range $[\rho_0^j - \Delta\rho, \rho_0^j + \Delta\rho]$ where $\Delta\rho$ represents the limits of the control and measurement errors. The problem we want to solve may be stated as follows:

Problem: determine all possible valid 6-cables configurations for all values of the ρ's in their ranges

An assumption is made in our algorithm: the actual pose of the platform is close to $\mathbf{X_0}$, i.e. the platform has not moved close to another solution $\mathbf{X_1}$ of the forward kinematic problem $\rho_0 = \mathbf{f}(\mathbf{X})$. This is a weak sssumption as as we are able to calculate all the forward kinematics solutions and therefore the full set of valid cable configurations. We will call *dominant cables* the one under tension in a given cable configuration.

As the ρ's have interval values we consider solving this problem with interval analysis. Under that assumption the mechanical equilibrium constraints become a interval linear system $\mathscr{F} = \mathbf{A}\tau$ for which there are methods that allows one to determine if this system admits only positive solutions in τ or has at least one of the τ that is always negative or maybe either positive or negative according to the real values of the ρ's. In the samme manner interval analysis allows to determine if for given ranges for the ρ's constraints such as $||\mathbf{A_j B_j}|| < \rho_j$ is always valid, is always violated or may be valid for some some ρ_j and violated for some others..

For a given cable configuration with n cables under tension the unknowns are the pose parameters, the real values of the ρ of the CDPR with $m \geq 6$ cables and the n tensions. With the minimal representation of a pose we end up with $6+m+n$ unknowns. If $n = 6$ the constraints we have can be decomposed into three sets

- a system of 6 equations composed of the 6 kinematic equations in the pose parameters and the 6 ρ
- the 6 equations (2) that is easily solvable as soon as the pose is known
- a set of $m-6$ inequalities $\rho_j > ||\mathbf{A_j B_j}||$

Note that all the unknowns may be bounded as we have assumed that the pose remains close to the nominal pose $\mathbf{X_0}$ so that all solutions shall lie within a set of ranges $\mathscr{I}_0$. The interval analysis algorithm we are using is a classical branch and bound algorithm whose principle has been explained in several papers. It basically consider a list $\mathscr{L}$ of possible set of intervals for the unknowns, called a *box*, that is initialized with $\mathscr{I}_0$.

For each box in $\mathscr{L}$ we start the algorithm by fixing the pose parameters to the mid-points of their intervals. For this pose we have a unique value for the cable lengths and we check if the lengths of the dominating cables all lie in the ρ ranges and for the non dominating cables k that we have a value ρ_k^l in the ρ_k ranges such that $\rho_k^l > ||\mathbf{A_k B_k}||$. If the cable lengths satisfy these properties we then check the positiveness of the τ of the dominating cables by solving the linear system (2). If this is the case the current cable configuration is valid and the algorithm completes.

If this test fails the algorithm checks if at least one of the constraints is always violated, in which case the box is eliminated from $\mathscr{L}$. If this cannot be asserted the box is bisected (i.e. one unknown is selected and its range ist bisected into two ranges) and the 2 boxes resulting from this bisection are placed at the end of $\mathscr{L}$ and the algorithm moves to the next box in $\mathscr{L}$.

This algorithm is guaranteed to complete because of the bisection process except in 2 cases: there is a singularity around $\mathbf{X_0}$ or a box is reduced to a single point and the numerical round-off errors do not enable to assert the constraints In both cases we perform a local analysis that is not described here for lack of space.

We have implemented the algorithm described in the previous section to determine all 6-cables configurations. We use as test an analysis of the possible cable configurations for the pose obtained for $\lambda = 0$ of the circular trajectory for the robot presented in section 2. At this pose if we have no uncertainty on the cable lengths, then we have 4 possible cable configurations: 125678, 145678 ,235678 , 345678. If we have an uncertainty of $\pm$ 3 cm on the ρ measurement the cable configuration 124578 becomes possible. The 5 cable configuration are determined in a computation time of 43 mn. If we extend $\Delta\rho$ to $\pm$ 4 cm, then the cable configuration 123678 becomes also possible and it takes about 3h to determine all cable configurations. For a $\Delta\rho$ of 1 cm the maximal deviation of the platform pose $||\mathbf{X} - \mathbf{X_0}||$ is 0.019307+ [0,0001] meter for the Euclidean distance and the maximal absolute deviation for each components of C are 0.008064, 0.007066, 0.011388 meters with an error in the range [0,0.0001].

4 Conclusions

This first major contribution of this paper is to clarify the concept of redundancy for CDPR. Having more cables than strictly necessary to control the 6 dof of the platform has a large influence on the robot workspace but the redundancy level is much weaker than is usually believed: the kinematics redundancy and static redundancy manifolds are 0 dimensional (meaning that the cable tensions cannot be continuously controlled). Hence it is unclear if a cable tension control scheme will really improve the tensions distribution while certainly will leads to poor positioning accuracy. On the other hand position and velocity control are much less sensitive to parameters errors, which explain the surprisingly good performances of CDPR prototypes. Note that the concept of cable configurations with 6 cables under tension while the other one are slack has been observed experimentally on a 8 cables CDPR.

The performances of CDPR may possibly be improved by using a cable configuration planning algorithm that will

- select the best set of 6 dominating cables among the possible cable configurations. For example the best set may be the one leading to the lowest positioning errors for given bound of the ρ measurement errors or the one satisfying an optimality criterion for the cable tensions
- deliberately let the non dominating cables become slack.

This strategy of voluntary letting cables become slack is clearly counter-intuitive and has to be confirmed experimentally but may be the best one for CDPR.

To conclude we have examined cable configuration having 6 dominant cables at a given pose but we have already extended the cable configuration research to less than 6 dominant cables and to a trajectory. This result will be presented in another paper.

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