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Analysis of wire elasticity for wire-driven parallel robots

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This paper presents an experimental study of the elasticity of wires that may be used for wire-driven parallel robot. Although wire behavior have been studied since a long time (e.g. in civil engineering) their use in parallel robots are quite different as they are submitted to large changes in length and tension while being usually much lighter than the one used in civil engineering. These experiments show that the tension in a wire submitted to a given change in length ΔL is a function of ΔL but also of time and we propose an empiric model to characterize this deformation. However we show that an appropriate elastic model may still be used for control purposes, provided that the zero-tension wire lengths are regularly updated through calibration.

KEYWORDS: elasticity, wire-driven parallel robot

1 Introduction

1.1 Wire-driven parallel robots

Among the category of parallel robot (i.e. robot in which the end-effector is connected to the ground through multiple independent kinematic chains) there is a special class of robot which uses wire as the connecting element between the end-effector and the ground. The pose of the end-effector is controlled by changing the lengths of the connecting wires. Numerous such robots have been proposed in the past (1), (2),(4),(5),(6),(7),(9),(10),(11),(12). Most of them use the coiling of the wires on a drum to allow for wire lengths changes but we have proposed recently another approach, namely the use of a linear actuator combined with a pulley system (8).

Compared to their counterpart with rigid links, wire-driven parallel robots have the advantages of

- allowing large changes in link lengths and consequently a larger workspace
- having a very reduced mobile mass, thereby allowing a higher dynamic

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On the other hand they have the drawback that the control must ensure that the tension in the wires is always positive as it is not possible to push a wire.

Another important point is the potential deformation of the wires under the tension to which they are submitted. Such deformation will indeed modify the wire length and consequently the location of the end-effector.

1.2 Wire deformation and forward kinematics

It is usually assumed that for a wire of initial length L_0 submitted to a tension τ there will a change in length that follows Hooke's law

$$\tau = k\frac{(L - L_0)}{L_0} \tag{1}$$

where L is the new wire length and k a constant. It is also mentioned in the literature that the wire may experiment a *creep* phenomenon, which consists in a permanent deformation of the wire even if it is used in its elastic domain (3). It is usually assumed that the amount of permanent deformation ϵ is given by

$$\epsilon = B\tau^U \tag{2}$$

where B, U are constants. But this law is established under the assumption that the amount of tension and the application time are sufficient for the permanent deformation to take place, an hypothesis that is not valid for our robot. Additionally the wires that are used in parallel robot have usually a low mass/unit length ratio and we may therefore assume that there is no deformation of the wire due to its own mass. Surprisingly the material literature is extremely sparse regarding the elasticity of "light" wires while Hooke's law (1) is usually used in the robotics literature (5),(6). Our motivation is therefore to experimentally determine if Hooke's law is indeed appropriate for the type of wire usually used for parallel robots and, otherwise, to propose a better model.

As will be shown in the experimental results the amount of wire length changes due to the tension is significant even for relatively small tension values. Hence these deformations will play an important role for the positioning accuracy of our robot. We have recently shown[1] that if the wire deformation follows Hooke's law (1) or the variant (3) that will be proposed in this paper, then it is possible to take the wire deformations into account into the forward kinematics problem (finding the location of the end-effector being given the measured value of the joint variables), even under real-time constraint, by using an interval analysis-based solver. This was a major issue as solving this problem amounts to be able to solve the classical rigid links forward kinematics problem (an already difficult issue) to which are added the mechanical equilibrium equations. We have also shown that our solver is also able to manage the modified Hooke's law

$$\tau = k\frac{(L - L_0)}{L} \tag{3}$$

under the same real-time constraint.

Still we have to validate that our wires follows a deformation law that is reasonably close to (1) or (3) and this paper addresses this issue.

[1] the corresponding paper will be presented at ICRA 2008

2 Experimental set-up

We are currently using three different wires for our parallel robot: one Roc Line fishing lines with 2.2 kg max load, a nylon wire with 15kg maximal load while the third one is a wire that is used for small kites.

Our purpose was to examine the relation between the tension in the wire and its deformation. For that purpose we have rigidly connected one extremity of a wire sample to a fixed point while the other extremity was connected to a force sensor, that allows one to measure the tension in the wire. The force sensor was fixed on the moving part of a milling machine whose displacement was accurately measured. The initial position of the wire extremity was such that a small but significant tension in the wire was measured. Then the force sensor was incrementally moved by a step of 1 mm and in this position the measured tension in the wire was recorded every 30 seconds for a period of 2 minutes.

2.1 Experimental results

For the Roc Line, 2.2 kg the experiment was performed for two samples with an initial length of 1m and 1.71 m. The experimental results are presented in Table 1.

displacement (mm)	τ	τ (30s)	τ (1mn)	τ (1mn30s)	τ (2mn)
1 ($L_0 = 1m$)	0.95	0.88	0.86	0.85	0.85
2	1.5	1.4	1.37	1.35	1.34
3	2	1.91	1.88	1.86	1.84
4	2.63	2.52	2.47	2.44	2.42
6	3.79	3.66	3.61	3.57	3.55
7	4.31	4.2	4.15	4.11	4.08
8	4.92	4.79	4.73	4.69	4.65
9	5.44	5.32	5.27	5.22	5.18
10	6.05	5.91	5.85	5.80	5.76
11	6.55	6.42	6.36	6.31	6.26
12	7.14	6.99	6.93	6.87	6.83
13	7.65	7.49	7.42	7.36	7.32
1 ($L_0 = 1.71m$)	0.63	0.62	0.61	0.61	0.61
2	1.04	1.01	0.98	0.97	0.96
3	1.37	1.33	1.30	1.28	1.27
4	1.73	1.67	1.64	1.62	.161
5	2.06	1.99	1.95	1.93	1.91
6	2.38	2.31	2.28	2.25	2.23
7	2.68	2.61	2.58	2.55	2.53
8	3.02	2.94	2.91	2.88	2.86
9	3.33	3.24	3.20	3.17	3.15
10	3.64	3.55	3.51	3.48	3.45
11	3.91	3.84	3.80	3.76	3.74
12	4.24	4.15	4.11	4.08	4.05

Table 1: Measurement for the Roc Line, 2.2 kg, initial length 1m and 1.71m

For the Nylon line the experiment was performed using a sample of length 1.5m and the results are presented in Table 2. For the kite line the experiment was performed using a sample of length 1.5m and the results are presented in Table 3.

displacement (mm)	τ	τ (30s)	τ (1mn)	τ (1mn30s)	τ (2mn)
1	2.12	1.91	1.82	1.76	1.71
2	2.50	2.30	2.23	2.18	2.14
3	3.01	2.83	2.76	2.71	2.67
4	3.53	3.34	3.27	3.22	3.18
5	4.10	3.91	3.84	3.77	3.72
6	4.61	4.43	4.35	4.29	4.23
7	5.21	5.00	4.92	4.86	4.81
8	5.74	5.54	5.46	5.38	5.32
9	6.33	6.11	6.02	5.95	5.89
10	6.86	6.63	6.55	6.48	6.42
11	7.46	7.21	7.12	7.06	7.00

Table 2: Measurement for the Nylon line, initial length 1.5m

displacement (mm)	τ	τ (30s)	τ (1mn)	τ (1mn30s)	τ (2mn)
1	2.21	1.66	1.43	1.30	1.21
2	3.84	3.04	2.77	2.62	2.50
3	5.43	4.29	3.98	3.76	3.64
5	8.55	7.02	6.62	6.38	6.15
6	10.09	8.63	8.25	7.95	7.72
7	11.88	10.36	9.87	9.56	9.34
8	14.04	12.32	11.87	11.48	11.23
9	15.92	14.35	13.81	13.43	13.15

Table 3: Measurement for the kite line initial length 1.5m

3 Analysis

As may be seen in the measurements tables, the wire tensions significantly change in time for a given change in the length of the wire. The analysis of the experimental result has led us to propose a deformation model in which a wire of initial length L_0 that is submitted to a tension τ for a sufficient amount of time will have a change in length ϵ given by:

$$\epsilon = B\tau^U \tag{4}$$

in which B, U are constants that depends on L_0 and on the wire material. The time law for this deformation is obtained as:

$$L_0(t) = L_0 + \epsilon(1 - e^{-k_2 t^N}) \tag{5}$$

where k_2, N are constants. The tension in the wire may be calculated as

$$\tau(t) = k_1((L_1 + \Delta L)/L_0(t) - 1) \tag{6}$$

where $L_1 = L_0(0)$ is the length of the wire at time 0. In summary the parameters of this model are B, U, k_1, k_2, N. Intuitively it may be thought that the parameter k_1 is dependent on the material but not on the L_0, while the other parameters should be length dependent.

To determine these parameters for each wire we have run a least square fitting algorithm with the first five measurements, the remaining measurements being used to test the model. The parameter identification results are presented in Table (4) and some examples of comparisons between the model and unused

Wire	k_1	B	U	k_2	N
Roc Line,2.2kg,L_0=1m,	699.152	0.2068	1/2.5	0.02138	1
Roc Line,2.2kg,L_0=1.71m	689.136	0.271	1/2	0.016	1
Nylon line,15kg,L_0=1.5m	1545.245	2.047	0.1355	0.0147	0.5
Kite line,L_0=1.5m	7049.85	0.4681	0.5123	0.05855	0.41

Table 4: Parameters identification results

experiments are presented in figures 1, 2,3, 4. It may be seen that there is a very good match between the model and the experimental results.

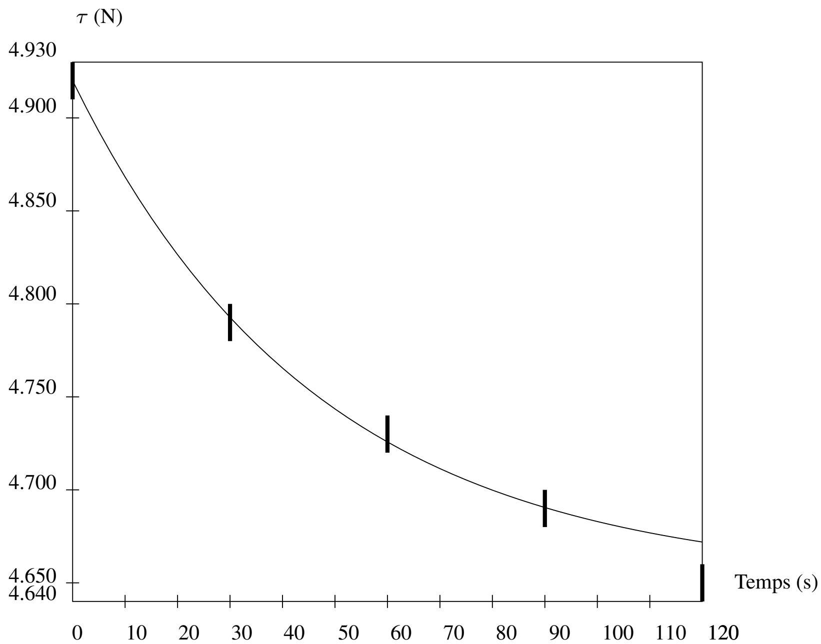

Figure 1: Model result for the last experiment for the Roc Line, L_0=1m. The experimental results are indicated with thick marks

4 Influence of the deformations on the control

We have introduced the model of wire deformation in the inverse kinematics, using equation (6) as a relation between the tension and the wire length and assuming that the total length of the wires with no tension L_0 was known. Being given the mass of the end-effector this module will calculate the active joints values in such way that both the kinematics and mechanical equilibrium equations are satisfied.

For a desired pose where the end-effector normal should be vertical our simulation have shown that after 60 seconds the change in the location of the center of the end-effector will be small (typically less than 0.3mm) while the angle between the normal and the vertical will be less than 0.055 degrees.

Such deviation in time is satisfactory but implies that we must calibrate the wire length L_0 on a regular

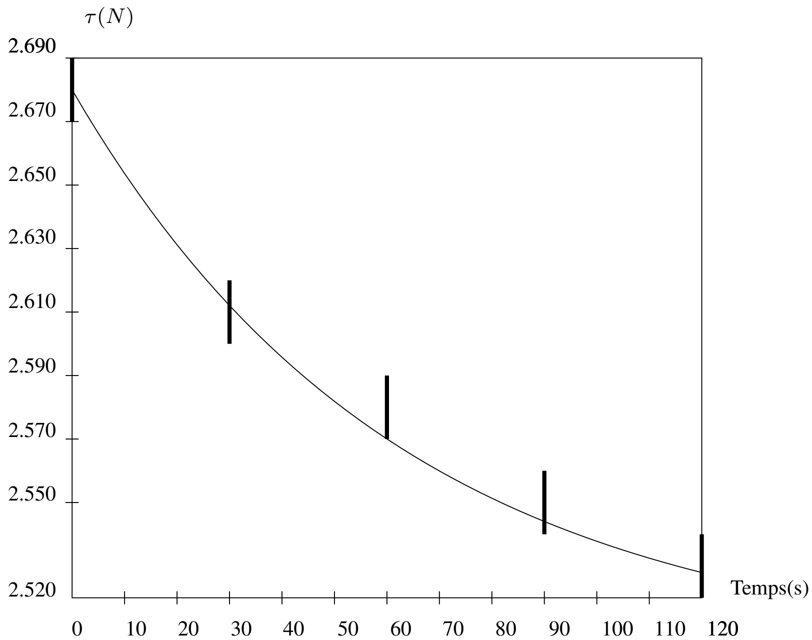

Figure 2: Model result for the last experiment for the Roc Line, L_0=1m. The experimental results are indicated with thick marks

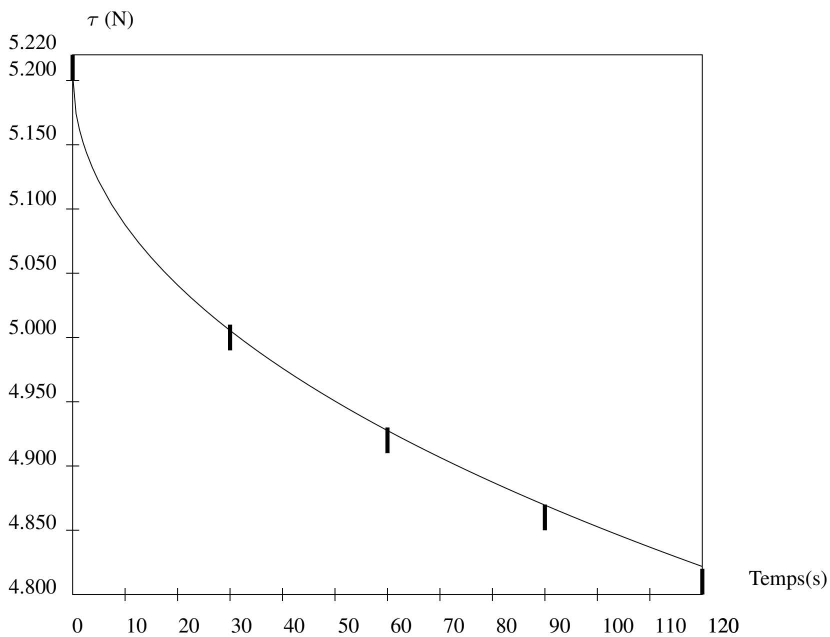

Figure 3: Model result for the last experiment for the Nylon line. The experimental results are indicated with thick marks

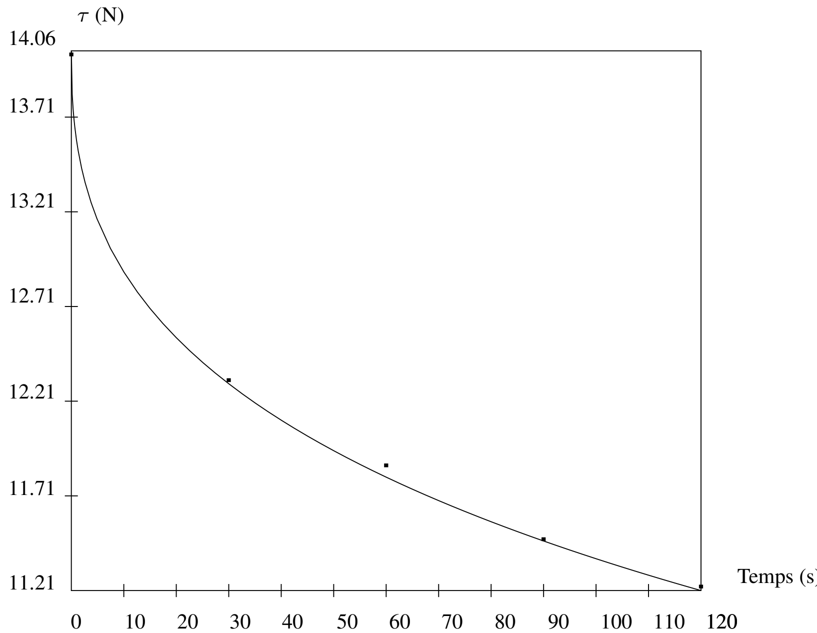

Figure 4: Model result for the last experiment for the kite line. The experimental results are indicated with thick marks

basis. Such calibration should be possible on-line without requiring external sensor to measure the pose of the end-effector as our robot is redundant. Introducing the wire deformation model in the calibration process is an open issue.

5 Conclusion

We have presented a full wire deformation model that may be used for wire-driven parallel robots. The model has been tested on 3 different wires and provide the necessary information for introducing wire deformations in the inverse and direct kinematics of the robot. Such experimentally determined deformation model was curiously lacking in the robotics literature. Our simulation have shown that if the total wire lengths with zero tension is known, then the deviation of the end-effector pose as a function of time is low. However we plan to confirm these simulations through measurements on our current platform.

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