Abstract: *Parallel structures have inherent advantages for many applications: rigidity, accurate positioning, high velocities. Obtaining high performances require to choose the right mechanism dimensions especially as there is a much more larger variation in the performances of parallel structures according to the dimensions than for classical, serial structures.*

Choosing the best dimensions according to the task at hand is a difficult problem as many different performances criterion are to be taken into account. Furthermore these criterion are difficult to evaluate as they are dependent upon the pose of the moving platform and therefore need to be evaluated over the whole workspace of the robot. The purpose of this paper is to present the outlines of some recently developed algorithms which enable to determine efficiently some important performances of parallel robots: workspaces, singularities, stiffness, extremum of the articular coordinates and articular velocities.

1 Introduction

1.1 The design process

Parallel structures have inherent advantages for many applications: rigidity, accurate positioning, high velocities. Obtaining high performances require two steps during the design process:

1. choosing the appropriate mechanical structure
2. choosing the right dimensions

The second point is very important as there is a much more larger variation in the performances of parallel structures according to the dimensions than for classical, serial structure. In fact for a given application a poorly appropriate mechanical structure with "optimal" dimensions may exhibit better performances than a more appropriate structure with poorly chosen dimensions.

Choosing the best dimensions according to the task at hand is a difficult problem for many reasons:

1. there is usually many different performances criterion that are to be taken into account (see for example [1, 7, 9]) and these criteria are often antagonistic with respect to the design parameters (for example changing one dimension of the structure will increase one of the performances but at the same time decrease another). A good example of this fact is given by Stoughton [73] who has shown that dexterity and workspace volume were varying in an opposite manner.

2. most of the performances that are to be taken into account are of the type "best case-worst case" over the whole 6D workspace of the robot. For example if we consider the stiffness of the structure the performance criteria will be to determine what will be the lowest and highest stiffness of the structure. But in most cases we are able to compute the performance criterion only for a given posture of the robot, hence the need of algorithms to compute efficiently this type of performances over the whole 6D workspace of the robot.

In the sequel we will address the problem of the efficient determination of the performances of a given parallel structure. We will more specifically address this problem for the classical Gough platform type of parallel robot (figure 1) although most of the algorithm that will be outlined may be applicable to other type of mechanical architecture.

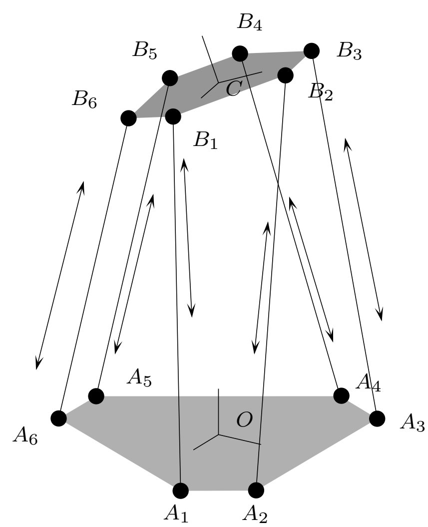

Figure 1: The classical Gough platform

A pose of this robot will be determinated by the coordinate in the reference frame of some point C of the moving platform and by the rotation matrix that define the rotation between a frame attached to the moving platform and the reference frame.

To stress the importance of dimensioning for parallel robots we will give simply one example. Consider the three parallel structures depicted in figure 2, which are simple derivations of a Gough platform. They differ only by the locations of the attachment points which are or are not regrouped by pair either on the base and on the platform. The MSSM has triangular base and platform, the TSSM has a triangular platform but an hexagonal base and the SSM has hexagonal base and platform. Consider manipulators of the three types with similar size: the attachment points will lie on circle with same radius for the three mechanisms and we will assume that the limits on the leg lengths will be identical for the three robots. The constraint on the leg lengths implies that there are limitations on the location that can can be reached by the center of the platform. We will now compare the workspace volume for a fixed orientation of the platform, calculated with the algorithm described in [20, 51] for mechanisms having all the attachment point on a circle of radius 13 for the base and 7 for the platform while the leg length limits are [55,60] and the angle of separation between two successive attachment points being either 0 (i.e. the points share the same location) or 20 degrees. It appears that the TSSM will have a workspace volume 25%

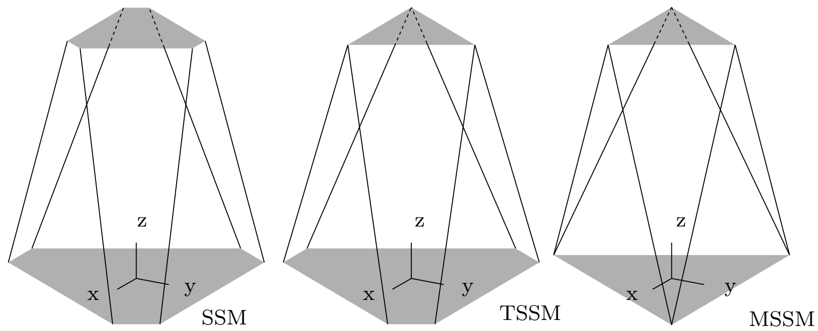

Figure 2: Three variations of a Gough platform

larger than the MSSM while the SSM workspace volume will be 70% larger, this percentage being approximatively constant whatever the orientation of the platform is... This exemplify clearly the importance of dimensioning.

1.2 Determining the characteristics of a parallel robot

In general we are able to calculate the value of the main features of a given mechanism for a given pose of the platform, at least numerically. For example we are able to evaluate the leg lengths, stiffness, positioning accuracy, articular forces for a given load on the platform, ... as soon as the pose of the platform is fixed. But in the design process we are interested in evaluating the extremum of these features for a set of pose. Indeed a general user requirement will be that its machine should perform correctly in some workspace W (for example C should be able to be located within a cube of given dimensions for any orientation of the platform within some set usually given by some intervals on the orientation angles). Consequently it is necessary to evaluate the features of a given machine over the workspace W in order to determine if its performances are correct. For example it may be essential:

- to determine the extremum of the leg lengths over W to check if the machine can really reach any pose within W
- to determine the extremum of the forces in the legs (being given a load on the platform) in order to verify if the mechanism can sustain this load

To address this problem most of the authors in the literature use a discretisation method: the 6D workspace W is reduced to a discrete set of poses which are the nodes of a grid and whose parameters are defined by regularly spaced values along the 6 axis of W, the difference between two successive values being the step size of the grid. But this approach has some drawbacks:

- there is no guarantee on the result: according to the size of the 6D grid there may be large differences between the real extremum and the values given by the algorithm (we have observed in some cases a discrepancy of 10% to 40% between the two values although the computation time of the discretisation method was much more larger).
- to deal with the above-mentioned problem we may be tempted to reduce the step size of the algorithm but the computation time will increase as the power of 6 of the inverse of the step size, therefore becoming quickly prohibitive

- in many cases the designer may accept approximate values for the extremum as soon as there is a guarantee on the result, especially if this enable to reduce the computation time. This is difficult to do with a discretisation method.

The following section will give some examples of algorithms which can evaluate some important features of parallel robots without relying on discretisation.

2 Some algorithms for performance evaluation of parallel robots

2.1 Workspaces

Workspace determination is clearly very important during the design process. It is also a difficult subject as position and orientation are coupled: the translation abilities of a parallel robot depends upon the orientation of the moving platform. Therefore most of the time the authors will just compute the translation workspace for a fixed orientation of the platform. It is also one of the subject for which discretisation has been the most used [14, 45]. We will define different types of workspaces:

- the *constant orientation workspace:* all the locations of C than can be reached for a given orientation of the platform
- the *total orientation workspace:* all the locations of C than can be reached for any orientation within a set defined by three intervals on the orientation angles
- the *inclusive orientation workspace:* all the locations of C than can be reached for at least one orientation within a set defined by three intervals on the orientation angles
- the *orientation workspace:* all the orientations that can be reached for a fixed position of C

For the constant orientation workspace the key point is: `think geometry!`. Indeed the border of this workspace can be easily determined from geometrical considerations. For example Gosselin[20] has shown that, when considering only the leg lengths limits, any planar cross-section of this workspace has a border which is obtained as the intersection of 6 annular regions, which can be computed easily. Similarly the border of the 3D workspace can be obtained geometrically as the intersection of 12 spheres [24]. These algorithms has been extended in [51] to deal with limits on the passive joints and interference between the legs.

The total orientation workspace is clearly quite important during the design process. It describes one important feature of a given parallel robot as it will give the useful workspace of the structure. Surprisingly this problem has been addressed only by Kim [33] which compute a rough approximation of this workspace. We have recently presented an algorithm which compute the exact border of this workspace for planar robot [55] and more recently for 6 d.o.f. robot [50]. The later algorithm enables also to compute the inclusive orientation workspace and to verify if a given 6D workspace is included in the reachable workspace of the structure (this verification is fast: from a few seconds to a few minutes). The total orientation workspace is computed approximately with an accuracy which is given as input to the program: the result is a set of cartesian boxes which are either fully included in the total orientation workspace or are only partially included. Hence the quality of the approximation is determined by the number and size of the later boxes.

A key point in this algorithm is a procedure which compute the extremum of the leg lengths for a given workspace or at least a lower and an upper bound of the leg length over the workspace. If the orientation of the platform is constant or if the base and moving platform are planar this problem can be solved exactly while for the most general structure (not planar base or platform) we use interval analysis to compute bounds on the extremum of the leg lengths.

The determination of the orientation workspace has been addressed only in [2, 4, 57, 67]. We have presented an algorithm to solve partially this problem in [47]: the main difficulty is to find a way to represent the result in an understandable way. We have chosen to attach an unit vector to the moving platform and to represent the possible orientation as the region of the unit sphere that can be reached by the extremity of the vector. This enable to represent 2 rotational d.o.f. of the structure and our algorithm take into account the limits on the leg lengths and passive joints and interference between the legs[1].

2.2 Singularity analysis

Singularity is a very important factor in the design of a parallel structure, a fact that has been often ignored recently. Indeed in this special configuration the articular forces may go to infinity causing the breakdown of the structure. Singular configurations are defined as the pose of the platform in which the determinant of the inverse jacobian matrix of the robot goes to 0. Although the inverse jacobian matrix can be determined in analytical form the expression of its determinant is huge and therefore difficult to analyze. There has been a lot of works on this subject [8, 17, 23, 36, 42, 43, 46, 61, 68, 71, 75, 76, 78]. The first problem was to get some insight on how to deal with the size of the singularity condition: here also a geometrical approach was the most efficient. The use of Grassmann line geometry was a key point: singular configurations are obtained when specific, geometrical conditions are fulfilled by the lines attached to the legs of the structure (and this method can be applied on any type of parallel robots: planar with 6 legs or less ...). Finding the relations between the pose parameters of the platform such that these conditions are satisfied has proven to lead to a set of rather simple singularity conditions [53].

Plotting these conditions on the workspace representation of a given structure is a possible way to determine if there is any singularity within the workspace. But due to the difficulty of representing the full 6D workspace of the robot this method is not practical except for planar robot [69].

Recently we have proposed a new algorithm which enable to determine if there any singularity within a 6D workspace W which may be specified either as a geometrical 3D object for the possible locations of C and three ranges for the orientation angles or as any pose that can be reached by the platform with leg lengths within some given limits [52]. The principle of this algorithm is quite simple. We start by taking a random pose within the given 6D workspace and compute the value of the determinant for this pose and more specifically its sign, that we call the *initial sign*. We use then another procedure which is able, using interval analysis, to determine bounds on the determinant for any pose within a 6D workspace defined as a cartesian box and three ranges on the orientation parameters. This procedure is applied on W and lead to three possible cases:

1. the two bounds have same sign, the same than the initial sign: there is no singularity within W

[1] An implementation of the constant orientation workspace and orientation workspace calculation is available via anonymous ftp on `zenon.inria.fr`, directory `saga/Workspace`

2. the two bounds have same sign, opposite to the initial sign: there is a singularity within W
3. the two bounds have opposite sign: we bisect the location part of W and get 8 new sub-boxes. We then repeat the procedure for each of these new boxes. So we end up with a list of workspaces and the algorithm will stop either when one of theses workspaces satisfy condition 2 (singularity in W) or when all the workspaces satisfy condition 1 (no singularity in W)

Although this algorithm is not very fast (it takes about 2 days of computation time to verify that there is no singularity within the reachable workspace of a given robot), it gives a formal answer about singularity.

2.3 Force analysis

When designing the legs of a parallel structure it is important to determine what will be the extremum of the leg forces τ for a given load $\mathcal{F}$ on the moving platform, whatever is the pose of the platform within some given 6D workspace W. The articular forces are dependent upon the pose of the platform as:

$$\tau = J^T \mathcal{F}$$

where J is the jacobian matrix of the robot, which is pose dependent. Hence finding the extremum of τ over W is a difficult problem as first the analytical form of J is difficult to obtain and then we have here a large constrained optimization problem.

The mapping between articular force and forces/torques applied on the moving platform has been studied in the past [35, 59] especially as parallel robot may be also used as force sensor. For a given pose of the platform the articular hypercube defined by $\tau \leq 1$ maps into a generalized ellipsoid (the *force ellipsoid*) in generalized cartesian force space. But the variation of the force ellipsoid main dimensions have not been examined.

Recently we have designed an algorithm which enable to determine the extremum of the articular forces for a given load on the platform over a translation workspace (the location of C is defined by a geometrical 3D object and the orientation of the platform is constant) [49]. This algorithm computes an approximation of the extremum of the articular forces with an accuracy given as input of the program. Although being largely faster than a discretisation method we have noticed that there may be large discrepancy between the outputs of our algorithm and the result obtained with a discretisation method (from 10 to 30%).

2.4 Stiffness analysis

A similar problem as for force analysis occur when considering the stiffness of a parallel structure, which is an important problem for many applications. Let us assume that the stiffness of each leg is k. Then the overall stiffness matrix of the parallel structure will be given by:

$$K = kJ^{-T}J^{-1}$$

where J^{-1} is the inverse jacobian matrix of the structure [12]. Hence the stiffness of the structure is pose dependent. An important problem is therefore to determine what will be the extremum of the stiffness matrix components (especially their minimum values) over a 6D workspace W. Optimization of the stiffness matrix at a given pose has been studied [3, 5, 79] and iso-stiffness curves within constant orientation workspace has been plotted by Gosselin [21].

But determining the extremum of the stiffness over a 6D workspace is an open problem. Recently we have proposed an algorithm which enable to determine the extremum of the stiffness over a translation workspace (the orientation of the platform is kept constant) [54]. Here also we have noticed that large variations of the stiffness may occur even for small changes in the structure dimensions: for example a change of less than 2% in the radii of the platforms may lead to a change of stiffness of more than 5%.

2.5 Velocity and positioning error analysis

Another important problem when designing a parallel structure is to determine what should be the extremum of the articular velocities $\dot{\boldsymbol{\rho}}$ enabling to reach some given platform velocity $\mathcal{V}$ over a 6D workspace W. An even more general problem will be to determine the extremum of $\dot{\boldsymbol{\rho}}$ so that the platform velocity amplitude over W has at least a lower bound V (i.e. the platform may have velocity V in any direction over W).

For a given pose the articular velocity vector is related to $\mathcal{V}$ by:

$$\dot{\boldsymbol{\rho}} = J^{-1}\mathcal{V}$$

where J^{-1} is the inverse jacobian matrix (hence the amplitude of each component of $\dot{\boldsymbol{\rho}}$ is pose dependent). The problem of finding the extremum of $\dot{\boldsymbol{\rho}}$ over W has received few attention, most authors dealing only with very special case [30, 39, 41, 44, 63].

Recently we have proposed an algorithm which enable to calculate efficiently these extremum for a given platform velocity vector $\mathcal{V}$ over a translation workspace with a guaranteed accuracy [48]. Even for complex translation workspace the computation time does not exceed a few minutes[2]. Still determining the extremum over a full 6D workspace and/or for any velocity vector of fixed amplitude is an open problem.

A related problem is to determine what will be the extremum of the positioning error $\boldsymbol{\Delta X}$ for a given error $\boldsymbol{\Delta\rho}$ on the measurement of the leg lengths. These errors are related by:

$$\boldsymbol{\Delta X} = J\boldsymbol{\Delta\rho}$$

Although this problem seems to be similar to the previous one note that in fact it is quite complex as we have to deal with the jacobian matrix J which has a complex analytical form. Another related problem is to determine the overall dexterity of the platform which may be measured by the inverse of the condition number κ of the inverse jacobian matrix. The value of κ will be 0 at a singularity and 1 in an isotropic configuration in which case the positioning error will be the same in any direction [64, 80]. The condition number is obtained as the ratio of the minimal singular value of J^{-1} over the maximal singular value of this matrix. As these singular values are obtained as the roots of the sixth degree characteristic polynomial of J^{-1} it may be seen that finding its extremum over a set of pose is a difficult problem.

3 Others open problems

In this section we will mention some open problems in this field.

[2] An implementation of the algorithm is available via anonymous ftp on `zenon.inria.fr`, directory `saga/Speed`

3.1 Passive joints motions

Clearly the determination of the extremum of the passive joint motions over a 6D workspace is very important to design these joints. To the best of our knowledge this problem has not yet been addressed in the literature. A partial answer is a side result of the algorithm for velocity analysis we have described in section 2.5. Indeed this algorithm enable to determine the maximum value of the angle of the leg unit vector with any fixed direction for any pose within a translation workspace.

3.2 Static balancing

Recently there has been a lot of interest for the application of parallel structures in the field of machining or payload pointing [1, 6, 11, 37, 40, 58, 70]. This type of application involves generally the manipulation of large load and in some cases the general direction of the parallel structure will be horizontal. Hence the articular forces may be quite large and the actuators may have to exert large forces just to equilibrate the load. The idea of passive static balancing is to add counterweights or springs to the structure so that it is balanced in any pose. If that condition can be realized there is a drastic decrease on the need on the actuator forces.

Some result on static balancing can be found in [13, 25, 31] but can be applied only on very special kind of parallel structure.

3.3 Vibration and Dynamic analysis

Vibration damping of parallel structure may play an important role for some application [11, 29] and in some cases parallel structure have been proposed to be used as vibration isolator [15, 19, 27, 72]. Similarly all the features we have mentioned in the previous section may have also to be analyzed by taking into account the dynamic of the structure, in which case the problem is much more complicated. Although dynamic modeling has been addressed in the literature [10, 16, 18, 22, 26, 28, 32, 34, 38, 56, 60, 62, 65, 66, 74, 77, 81] it is still a difficult field.

4 Conclusion

When analyzing a parallel structure it is important to determine the values of its main characteristics in order to verify if the structure will perform correctly in a given application. This is in general a difficult problem as first these values are pose dependent over a 6D workspace and, second as the computation time should be low. We have presented some recently developed algorithms which enable, at least partially, to determine some of the important characteristics of parallel structures.

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