The influence of discrete-time control on the kinematico-static behavior of cable-driven parallel robot with elastic cables

J-P. Merlet

Abstract Robots are controlled with a discrete-time controller that includes a high-level loop for motion control and a faster internal loop that controls the actuators. We intend to simulate the behavior of the whole chain for a cable-driven parallel robot (CDPR) with linear elastic cables and we will show that such a simulation cannot be performed using classical simulation tools. We exhibit a simulation algorithm which computes exactly the pose and cable tensions on a given trajectory. As an example we consider a redundantly actuated robot with 8 cables. We show that the discrete-time control has a moderate influence on the accuracy of the positioning but a very large influence on the cable tensions.

Key words: cable-driven parallel robot, elastic cable, discrete-time control

1 Introduction

The study of CDPR has started about 30 years ago with the pioneering work of Albus [2] and Landsberger [13] but there has been recently a renewed interest in such a robot, both from a theoretical and application viewpoint. For example kinematics analysis of CDPR is much more complex than the one of parallel robot with rigid legs as static equilibrium has to be taken into account [5, 11, 21] and is still an open issue especially as not all cables of a robot with m cables may be under tension [1, 3, 6, 8, 16] and that only stable solutions have to be determined [7]. This analysis is even more complex if we consider that the cables may be elastic and/or deformable [9, 10, 12, 18].

Numerous applications of CDPRs have been mentioned e.g. large scale maintenance studied in the European project Cablebot [17], rescue robot [15, 19] and transfer robot for elderly people [14] to name a few.

INRIA Sophia-Antipolis e-mail: Jean-Pierre.Merlet@inria.fr

However a problem has never been addressed when simulating CDPRs: the influence of the use of discrete-time control on the kinematic and static behavior of the robot. Only Borgstrom [4] has presented a motion planning algorithm for CDPRs that takes into account the discrete-time nature of the controller. The purpose of this paper is to study the controller influence on the kinematic and static behavior of a CDPR with linear elastic cables when performing a given trajectory.

2 Discrete-time control

A robot controller is basically constituted of two discrete-time control loops:

- a high-level loop with a sampling frequency Δt_1: at time $k\Delta t_1$, k being an integer, this loop get sensory information from the robot, process it and send a new command for the actuators at time $(k+1)\Delta t_1$
- an inner loop with a sampling frequency $\Delta t_2 < \Delta t_1$: at time $l\Delta t_2$, l being an integer, this loop get sensory information from the actuators and process it for sending new voltages or currents to the actuators at time $(l+1)\Delta t_2$. For the sake of simplicity we will assume that Δt_1 is a multiple of Δt_2.

A consequence of this scheme is that during the time interval $[l\Delta t_2, (l+1)\Delta t_2]$ the actuators are submitted to a constant voltage/current V and the CDPR state evolves according to only the actuator state. We will assume that a time-model of the actuator is available i.e. the output θ of the motor at any time T in the range $[t, t+\Delta t_2]$ may be obtained as $\theta(T) = H(T, \theta(t), V)$.

3 Cable configurations and kinematico-static equations

We consider a CDPR with m elastic cables, numbered from 1 to m, whose extremities are located on the robot base at point A_i and attached to the platform at point B_i. If ρ is the cable real length and l_0 its length at rest, then the tension τ in the cable is $\tau = K(\rho - l_0)$, where K is the stiffness constant, provided that l_0 is larger than the distance $||\mathbf{A_i B_i}||$, otherwise the cable is slack and $\tau = 0$. As a cable may become slack we introduce the concept of *cable configuration* (CC): a cable configuration M_i at a pose is a set of $i \leq m$ integers which are the numbers of the cables that are currently under tension, all other cables being slack. Note that at a given pose several CC may be possible, the current one depending on the history of the system.

We may now investigate the equations that are involved in the forward kinematics (FK) problem for a given CC M_j. The unknowns are the 6 parameters $\mathbf{X}$ that describe the pose of the platform and the i tensions τ or, equivalently, the i cable lengths ρ, for a total of $6+i$ unknowns. The equations are the i inverse kinematic equations and the 6 equation from the mechanical equilibrium:

$$\rho = G(\mathbf{X}) \qquad \mathbf{F} = \mathbf{J^{-T}}\tau \tag{1}$$

where $\mathbf{F}$ is the external force applied on the platform. In this paper we will assume that the only external force is gravity that is applied at the center of mass C of the platform. The matrix $\mathbf{J^{-T}}$ is the $6 \times i$ transpose of the inverse kinematic jacobian. The j-th row J_j of $\mathbf{J^{-T}}$ is given by

$$J_j = ((\frac{\mathbf{A_jB_j}}{\rho_j} \quad \mathbf{CB_j} \times \frac{\mathbf{A_jB_j}}{\rho_j}))$$

Hence we end up with a square system of $6+i$ equations that may be written as

$$\mathbf{F}(\mathbf{X}, \rho, l_0) = 0 \tag{2}$$

4 Kinematico-static simulation on a trajectory

We are interested in determining the kinematic and static behavior of a CDPR under a discrete-time controller when it has to move along a given trajectory. We will assume that when the CDPR starts its task the current CC M_l is known. We will also assume that the pose at the start point is known.

4.1 Finding a certified end-pose

We consider a time interval $\mathscr{T} = [l\Delta t_2, l\Delta t_2 + \Delta t]$, with $\Delta t \leq \Delta t_2$, and we assume that at time t the CC M_j is known, together with the pose $\mathbf{X_t}$. Our objective is to determine what is the pose at time $l\Delta t_2 + \Delta t$, under the assumption that the CC does not change on the whole time interval. If such result can be obtained the time interval $\mathscr{T}$ will be called *valid*. For checking the validity of $\mathscr{T}$ we consider the system of equations (1) at time T, i.e. for fixed values of the l_0. This system may admit several solutions i.e. the pose $\mathbf{X}$ may lie on different *kinematic branches* $\mathscr{S}_i$ and our objectives are to show 1) that for any time T the pose lies on the same branch $\mathscr{S}_t$ than $\mathbf{X_t}$ and 2) to calculate the pose at time $l\Delta t_2 + \Delta t$. Assume that we are able to show that for any time T system (1) admits a single solution in a ball centered at $\mathbf{X_t}$: this implies that during the time interval the kinematic branch on which lies $\mathbf{X_t}$ does not cross any other branch and that the pose always lies on the branch $\mathscr{S}_t$, hence fulfilling 1). For showing the unicity of the solution in a ball centered at $\mathbf{X_t}$ we will use *Kantorovitch theorem* [20], that is presented now. Let an arbitrary system of n equations in n unknowns $\mathbf{F} = \{F_i(x_1, \ldots, x_n) = 0, i \in [1, n]\}$ and $\mathbf{x_0}$ be a point and U a ball centered at $\mathbf{x_0}$ with radius B_0. Assume that $\mathbf{x_0}$ is such that:

1. the Jacobian matrix $\mathbf{J_0}$ of the system has an inverse Γ_0 at $\mathbf{x_0}$ such that $||\Gamma_0|| \leq A_0$
2. $||\Gamma_0 \mathbf{F}(\mathbf{x_0})|| \leq 2B_0$

3. $\sum_{k=1}^{n} |\frac{\partial^2 F_i(\mathbf{x})}{\partial x_j \partial x_k}| \leq C$ for $i, j = 1, \ldots, n$ and $\mathbf{x} \in U$
4. the constants A_0, B_0, C satisfy $2nA_0B_0C \leq 1$ (A)

Then there is an unique solution of $\mathbf{F} = 0$ in U and Newton iterative scheme used with $\mathbf{x_0}$ as estimate of the solution will converge toward this solution.

In our case however as we consider any time in the time interval, equations (1) is not a single system but a family of systems because l_0 vary over time. We may however assume that the time model of the actuator allow us to determine an interval $\mathscr{I}^i = [l^i_{min}, l^i_{max}]$ such that for all cables we have $l^i_0 \in \mathscr{I}^i$. Note that the width of $\mathscr{I}^i$ will decrease with Δt. Assuming that the reader is familiar with interval analysis (IA) we may now apply Kantorovitch theorem to the system (2) using $\mathbf{X_t}$ as $\mathbf{x_0}$ with the following modifications:

- $\mathbf{F}(\mathbf{x_0})$ has now an interval value
- the matrix $\mathbf{J_0}$ is an interval matrix. Classical method allows to obtain its inverse but may fail if the width of the intervals in $\mathbf{J_0}$ is too large
- the Hessian matrix appearing in item 3 of the theorem is also an interval matrix but its norm can be calculated with IA methods

We start by setting $\Delta t = \Delta t_2$. If the interval matrix $\mathbf{J_0}$ cannot be inverted or condition (A) of the theorem is not satisfied, then we set $\Delta t = \Delta t/2$, update the ranges $\mathscr{I}^i$ and starts again until a valid Δt is determined. This approach may fail only in two cases: (a) system (2) is close to a singularity (in which case we cannot predict the behavior of the robot) or (b) in case of insufficient computer accuracy (this issue will be addressed in a later section). If a valid Δt is found we are able to calculate the pose at time $l\Delta t_2 + \Delta t$ unless a CC change occurs in the time interval.

4.2 Finding cable configuration changes

Assume that a valid interval $[l\Delta t_2, l\Delta t_2 + \Delta t]$ has been determined in the previous step. If no CC change occur in this time interval, then we are able to calculate the pose at time $l\Delta t_2 + \Delta t$. Necessary conditions for a CC change are

1. there is a time T in the time interval at which the tension of a cable $i \in M_j$ is exactly equal to 0, i.e. $\rho_i = l^i_0 = ||\mathbf{A_i B_i}||$
2. there is a time T in the time interval at which the length l^i_0 a cable $i \notin M_j$ is such that $l^i_0 = ||\mathbf{A_i B_i}||$.

In both cases equations (2) for the CC M_j are still valid but we have an additional unknown, T, while one of the unknown, ρ_i, has now a known value. Hence this new version of (2), denoted $\mathbf{F_{mod}}^i$, is still a square system. As we have to consider that all the m cable may possibly satisfy in turn $\rho_i = l^i_0$, we have therefore m system $\mathbf{F_{mod}}^i$. Note that we have bounds on all unknowns of the new system: $\mathbf{X}$ and the ρ_i have to lie in the ball provided by the Kantorovitvh theorem. Hence it is quite natural to use IA to determine **all** possible solutions of all m systems $\mathbf{F_{mod}}$. All the n solutions are then ordered by increasing value $T_1^{k_1}, T_2^{k_2}, \ldots, T_n^{k_n}$ where the superscript k_i denotes the cable number for which $\rho_{k_i} = l_0^{k_i}$. In the time interval $[l\Delta t_2, l\Delta t_2 + T_1^{k_1}]$ we are sure that the platform lie on the kinematic branch $\mathscr{S}_t$. A possible CC change may occur at time $T_1^{k_1}$, where we have $\rho_{k_1} = l_0^{k_1}$, but is not certain. Indeed if $k_1 \in M_j$ the tension may decrease before $T_1^{k_1}$, cancel at $T_1^{k_1}$ but may then increase. In the same manner if $k_1 \notin M_j$ the distance $||\mathbf{A_{k_1}B_{k_1}}||$ may increase before $T_1^{k_1}$, reach $l_0^{k_1}$ at $T_1^{k_1}$ but may then decrease so that cable k_1 remains slack. To determine if such case occurs we consider a new CC M_{j+1} obtained by adding k_1 to the CC M_j. We then apply the Kantorovitch theorem on the equations (2 valid for this new CC, using as $\mathbf{x_0}$ the pose obtained for the time $T_1^{k_1}$. We then calculate times $T_1', T_2', \ldots, T_u'$ at which a CC change may occur. We then solve the system obtained for time $(T_1^{k_1} + T_1')/2$. If at this time we have both $k_1 \in M_j$ and $||\mathbf{A_{k_1}B_{k_1}}|| < l_0^{k_1}$ or both $k_1 \notin M_j$ and $\rho_{k_1} > l_0^{k_1}$, then a CC change occurs at $T_1^{k_1}$, cable k_1 becoming slack in the first case and under tension in the second one. If this not the case we repeat the procedure for time $T_2^{k_2}$, $\ldots$, $T_n^{k_n}$ until either a CC change occur or there is no CC change at $T_n^{k_n}$, which implies that at time $l\Delta t_2 + \Delta t$ the platform still lies on the branch $\mathscr{S}_t$.

4.3 Trajectory checking

The two previous sections allow us to determine the pose, cable configuration and cable tensions at any time when the CDPR performs a trajectory. As soon as the high level loop has sent an order to the inner one we will determine the pose and tension during the time intervals $[k\Delta t_1 + l\Delta t_2, k\Delta t_1 + (l+1)\Delta t_2]$ until $k\Delta t_1 + (l+1)\Delta t_2 = (k+1)\Delta t_1$. We store the CDPR status at times $k\Delta t_1 + l\Delta t_2, k\Delta t_1 + (l+1)\Delta t_2$ and possible at intermediate times. Note that uncertainties may be taken into account: for example we may use in the simulation a different value of the cable stiffness K_i than the one used in the high level loop or we may introduce arbitrary random errors in the measurements of the l_0 that is used by the inner loop.

5 Implementation and example

The previous algorithm has been implemented assuming a first order time model for the velocity of the actuator. Let V_c be the desired velocity of the actuator and $V(t_0)$ the known velocity at time t_0. Then the actuator velocity $V(t)$ at time $t \geq t_0$ is

$$V(t) = V_c + (V(t_0) - V_c)e^{-\frac{t-t_0}{U}}$$

where U is a known constant. The inner loop is a simple P controller that send to the actuator at time $(l+1)\Delta t_2$ a velocity order $V^l = V_c + k_p(V_c - V_m)$, where V_m is the measured velocity at time $l\Delta t_2$, V_c the velocity sent by the high level loop and k_p a constant gain.

The IA part of the algorithm has been implemented using our IA library `ALIAS`, that takes into account round-off errors. But even with this library we have encountered numerical problems, especially with the convergence of the Newton scheme. The satisfaction of Kantorovitch theorem requires that the pose computed for a given time T is accurate enough. Indeed a minimum condition for the theorem to provide a positive answer is that it is satisfied at time T. Roughly this means that the absolute value of the components of $\mathbf{F}$ at this time have a value less than $1/(2kA_0^2C_0)$, where k is the number of equations in $\mathbf{F}$. In some cases we have noticed that this value is very low and well below the accuracy of floating-point calculation. Hence the floating-point version of the Newton scheme oscillates around the solution without ever producing a value of $\mathbf{F}$ which is small enough. Fortunately we have a mean to solve this issue: `ALIAS` has a Maple interface that includes a multi-precision Newton scheme, allowing to calculate a solution with an arbitrary accuracy. Consequently when Kantorovitch theorem is satisfied but the floating-point Newton does not converge, then we use the Maple version.

As a complex example we consider the 8-cables large scale robot developed by LIRMM and Tecnalia as part of the ANR project Cogiro and the FP7 project CABLEBOT. This robot is a suspended CDPR whose dimensions have been given in several papers [10]. The platform is assumed to have a mass of 1/9.81 kg. We consider a planar circular trajectory centered at (0,0,2) with radius 1. The sampling times were fixed to $\Delta t_1 = 0.005s$, $\Delta t_2 = 0.001s$ and the motor constant U to 0.1s. The high level loop computes at time $k\Delta t_1$ what should be the pose at time $(k+2)\Delta t_1$ and calculates the l_0^c for this pose that minimize $\sum \tau_j^2$. The inner loop generates a velocity order for the actuators as $K_p(l_0^c - l_0^m)$ where K_p is a constant gain and l_0^m the l_0 measured at time $k\Delta t_1$. We have considered two simulation cases. In the first one there is no error on the measurement of the l_0 and the stiffness of the cables was set to 1000 N/m (which correspond roughly to the stiffness of nylon). In the second case we add a random error on the l_0^m in the range $[-0.01, 0.1]$ (the average value of the l_0 on this trajectory is about 800), the high level loop assume a cable stiffness of 1000 but the real cable stiffness was set to $1050, 900, 950, 1020, 1010, 1000, 1040, 980$.

In the first case the maximal positioning error on the trajectory is 0.00002275 with a mean value of 0.36610^{-5}. In the second case the maximal error is 0.00575 with a mean value of 0.00104 (figure 1). Hence it may be seen that the uncertainties on the stiffness and length measurement has a relatively low influence on the positioning accuracy, The situation is quite different for the tensions in the cables. Without uncertainties the maximal difference between the cable tensions and the optimal one over all cables is 0.000221N with a mean value of 0.0001 N. With uncertainties the maximal difference is 0.4844 N with a mean value of 0.28097 N. In percentage of the optimal tension the maximal difference is 140.13% and the mean value is 72.85%. Figure 2 presents tension of cable 1 together with its optimal tension during the first 3 seconds of the trajectory. It may be seen that a perfect knowledge of the cable stiffness allows to follow accurately the optimal tension. But as soon that as the real stiffness differ by a small amount from the assumed one the cable tension

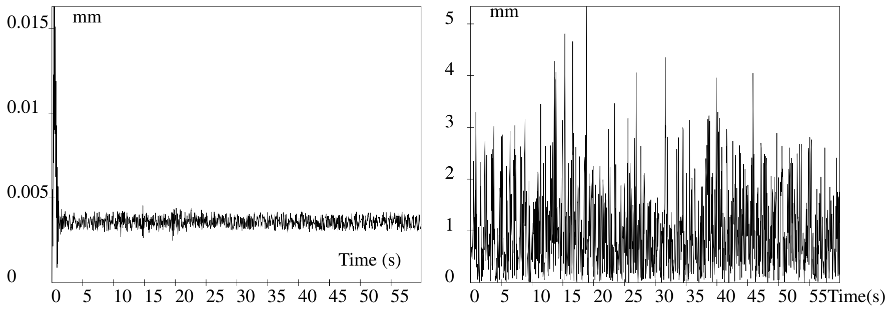

Fig. 1 Positioning error without and with uncertainties (mm)

oscillates between slack state and under tension. This analysis confirms that the use

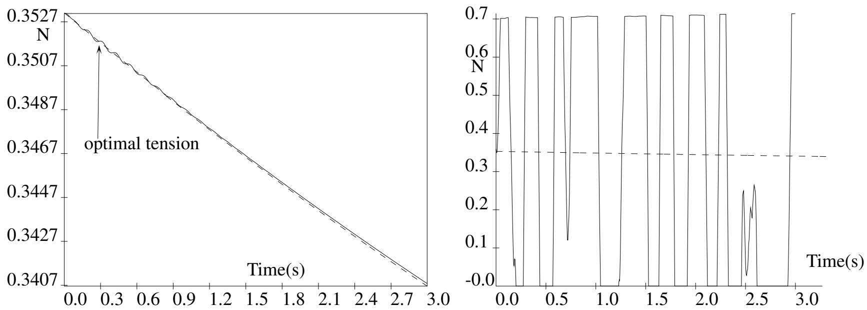

Fig. 2 Tension of cable 1 without uncertainty and with uncertainty (optimal tension is the dashed line)

of a discrete-time controller prohibits tension control in CDPRs.

6 Conclusion

To the best of our knowledge this paper has presented for the first time a simulation of CDPRs that takes into account the discrete-time nature of current controller. Implementing this simulation is a complex task because it involves solving the FK but also because the necessary accuracy for obtaining this simulation may be lower than the one obtained with floating point calculation. Results on an example shows that positioning accuracy is not that much influenced by the controller but that on the other hand cable tensions are drastically influenced. Our next objective will be to take into account the dynamics of the robots in this simulation.

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