Dynamic Interference Avoidance of 2-DOF Robot Arms Using Interval Analysis

H. Fang
LASMEA
24, av. des Landais
63177 Aubiere Cedex France
hao@lasmea.univ-bpclermont.fr

J-P Merlet
INRIA Sophia-Antipolis
BP 93
06902 Sophia-Antipolis Cedex, France
Jean-Pierre.Merlet@sophia.inria.fr

***Abstract*— In this paper the problem of interference avoidance for robots subject to dynamic constraints is investigated. First computed-torque method is used to obtain a linearized closed-loop system. For this linearized system the desired state that the robot is going to take at the next sampling period is checked by phase plane analysis to ensure the robot can be stopped without interferences, dynamic constraints are taken into account by calculating the bounds of the drive torques with interval evaluation. When the desired next state is not valid for interference avoidance, a new state is scheduled by optimizing the next velocity, interval analysis is used again which allows to partition the complex constrained optimization problem into a simple two-stage problem. The resulting optimal state not only secures the robot against interference but also leads the robot to trace the desired path closely. Simulation results of a 2-dof robot arm show the effectiveness of the proposed approach.**

I. Introduction

Interference checking and avoidance are classical problems in robotics research. They are key issues for analyzing robot maneuver, mobile robot safety and multi-robot collaborative tasks. Classical approaches dealing with interference avoidance are divided into two categories [1]. The first one is kinematic algorithm, which only considers the robot kinematics. In this approach interference avoidance is realized based on geometric relationships and velocity constraints, from kinematics point of view, the robot can avoid the interference with a interference-free path [2]-[6]. But the kinematic algorithms ignore the dynamic constraints, for example inertia and drive capability, the kinematic interference-free path perhaps cannot be realized in practice. Another category is kinodynamic or dynamic algorithm in which dynamic constraints are considered [7]-[10]. Dynamic algorithms are more realistic but the difficulty in this category is how to deal with dynamic models of robots which are non-linear, coupled and second order differential equations. To avoid solving dynamic equations, some approaches assume that the robots move along a straight line and can be stopped with a fixed acceleration, in this way a simplified dynamic model is obtained [11]. But in practical implementations, it is not the only case. Another assumption is that the robot dynamics is integrable so that an analytical function between the robot movement states and the drive forces can be obtained, then using this function the canonical or near-canonical solution is determined which can reject the interference and satisfy the dynamic constraints [1]. But generally most of dynamic models are too complex to be integrated. In addition some artificial intelligence methods are also used. In [12] the randomized approach to kinodynamic planning is presented using Rapidly-exploring Random Trees (RRTs). In [13] the techniques of time-square of joint torques are devised to smooth the controls and improve the tracking accuracy. In [14] a smooth trajectory is constructed in the $s - \dot{s}$ phase plane using parametrized cubic splines, the dynamics of the manipulator together with limits on the actuator torques and torque rates are considered, but those two algorithms cost too much computation.

The main idea of this paper is regarding the interference position as the target state of a set-point control, then critical damped control is used to guarantee the closed-loop system converge to zero without any overshoots which means that the robot can stop before interference occurs, the dynamic constraints are fulfilled in this process by checking the bounds of the control torques using interval analysis. This paper is organized as follow, in section 2 a linearized closed-loop system is obtained by using computed-torque method. In section 3 the state of the closed-loop system is checked with phase-plane analysis to avoid the interference, interval analysis is used to ensure the dynamic constraints are always satisfied. In section 4 interval analysis is applied to schedule a valid state for the next sampling period by solving a constrained optimization problem. In section 5 some simulation results are presented to validate the algorithms.

II. Interference avoidance with exact robot dynamics

A. *Closed-loop system of set-point control*

The dynamic model of a 2-DOF rigid robot arm is given by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{1}$$

where $q \in R^{\mathbf{2}}$ is the vector of the joint angles, $M(q) \in R^{\mathbf{2\times 2}}$ is the inertia matrix, $C(q,\dot{q})\dot{q}$ represents the Coriolis and centrifugal forces, $G(q)$ is the torque vector due to the gravitational force, τ is the vector of the drive torque.

The dynamic constraints considered in this paper are drive capability

$$|\tau_i| \leq |\tau_{max}| \tag{2}$$

and the velocity limitation

$$|v_i| \leq |v_{max}| \tag{3}$$

where τ_{max} and v_{max} are determined by commercially available actuators.

To avoid solving the complex dynamic model, the computed-torque method is used to achieve a linearized closed-loop system, the control input is

$$\tau = \hat{M}(K_p e + K_v \dot{e} + \ddot{q}_d) + \hat{C}(q, \dot{q})\dot{q} + \hat{G}(q) \tag{4}$$

where ˆ indicates the estimated values of the dynamic parameters, then the following closed-loop system is obtained:

$$\begin{aligned} &\hat{M}(K_p e + K_v \dot{e} + \ddot{e}) + \hat{C}(q, \dot{q})\dot{q} \\ &-C(q, \dot{q})\dot{q} + \hat{G}(q) - G(q) - (M - \hat{M})\ddot{q} = 0 \end{aligned} \tag{5}$$

where K_v and K_p are the derivative and proportional gains of the controller. If the dynamic model is constructed exactly, that is $\hat{M} = M$, $\hat{C}(q, \dot{q})\dot{q} = C(q, \dot{q})\dot{q}$, $\hat{G}(q) = G(q)$, then the linearized closed loop system is

$$\ddot{e} + K_v \dot{e} + K_p e = 0 \tag{6}$$

B. Next states of the robot arms

In practical implementations the sampling period can be set small enough that a linear relationship between the current state and next state exists:

$$q(t + \delta t) = q(t) + \frac{\dot{q}(t) + \dot{q}(t + \delta t)}{2}\delta t \tag{7}$$

$$\dot{q}(t + \delta t) = \dot{q}(t) + \ddot{q}(t)\delta t \tag{8}$$

The joint acceleration can be evaluated from inverse dynamics:

$$\ddot{q} = M^{-1}(\tau - G(q) - C(q, \dot{q})\dot{q}) \tag{9}$$

The dynamic constraint is the limited drive capability of the actuator

$$\tau \in [-\tau_{max}, \tau_{max}] \tag{10}$$

so the joint acceleration is constrained by

$$M^{-1}(-\tau_{max} - f) \leq \ddot{q} \leq M^{-1}(\tau_{max} - f) \tag{11}$$

f is the corresponding function. From (8) the range of the next velocity is determined by

$$\dot{q}(t + \delta t) \in [\underline{\dot{q}}(i + 1), \overline{\dot{q}}(i + 1)] \tag{12}$$

where

$$\underline{\dot{q}}(i + 1) = \dot{q}(t) + M^{-1}(-\tau_{max} - f)\delta t \tag{13}$$

$$\overline{\dot{q}}(i + 1) = \dot{q}(t) + M^{-1}(\tau_{max} - f)\delta t \tag{14}$$

III. Interference avoidance using phase plane analysis

For a 2-dof robot arm whose links are connected with revolute joints, the collisions between the link and the revolute joint are only possible interferences considered in this paper. To protect the mechanism against the interferences, the movement ranges of the revolute joints are constrained (which is a reasonable assumption in most cases). The maximum allowed angles of the revolute joints are defined as q_o, if all the joints can stop before $q = q_o$, then it is thought that no interference occurs. The joint position and velocity can be measured with multi-sensors.

A. Critical damped system

From classical control theory point of view, if the extreme position q_o is regarded as the target position of a set-point control (in which $\dot{q}_d = 0, \ddot{q}_d = 0$), then the position error $e = q_o - q$ equals to the surplus joint angle which allows the robot arm to move without interference. Consequently $e \geq 0$ means the robot arm runs safely, while $e < 0$ means that the links of the robot arm have collided with other components, so the key problem of interference avoidance has been transformed into how to guarantee the position error of the set-point control non-negative.

In (6) if we choose the controller gains $K_p = diag\{k_p\}$ and $K_v = diag\{k_v\}$ such that their elements satisfy the critical damping condition

$$k_v = 2\sqrt{k_p} \tag{15}$$

then the closed-loop system (6) becomes

$$\ddot{e} + k_v \dot{e} + \frac{k_v^2}{4} e = 0 \tag{16}$$

which is critical damping. Figure (1) shows phase portraits

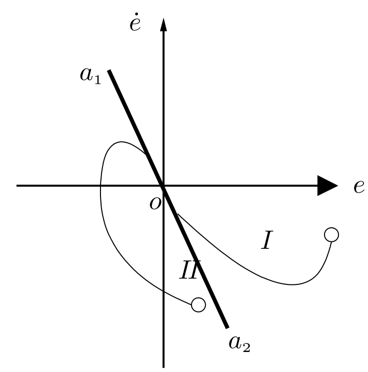

Fig. 1. Phase portrait of the critical damped system

of a critical damped system. From figure(1), it is clear that for a critical damped system all the states $(e, \dot{e})$ lying in zone I will finally converge to the origin asymptotically along the line a_1a_2 without overshoots, keeping the position error e non-negative. a_1a_2 is determined by

$$\dot{e} + \frac{k_v}{2} e = 0 \tag{17}$$

So for a given k_v every point in zone I yields an interference-free path without considering the dynamic constraints. But for the states out of zone I, no interference-free path can be guaranteed. The border between zone I and zone II is determined by (17).

Although each point in zone I may yield an interference-free path, but not all of them are valid for interference avoidance because all the states on the phase portrait must satisfy the dynamic constraints when they converge to the origin

$$-\tau_{max} \leq M(\frac{k_v^2}{4}e + k_v\dot{e} + \ddot{q}_d) + C(q,\dot{q})\dot{q} + G(q) \leq \tau_{max} \quad (18)$$

Noting that for a set-point control, $\dot{q}_d = 0$, $\ddot{q}_d = 0$, (18) can be simplified as

$$-\tau_{max} \leq M(\frac{k_v^2}{4}e - k_v\dot{q}) + C(q,\dot{q})\dot{q} + G(q) \leq \tau_{max} \quad (19)$$

In practical implementations it is always expected that the robot arm can be stopped with the shortest time, so for a specified state $p = (e(t_p), \dot{e}(t_p))$, we choose $k_{vp} = -\frac{2\dot{e}(t_p)}{e(t_p)}$, the control torque becomes

$$\tau_p(q,\dot{q}) = M(q)(\frac{k_{vp}^2}{4}e + k_{vp}\dot{e} + \ddot{q}_d) + C(q,\dot{q})\dot{q} + G(q) \quad (20)$$

then the states of the closed-loop system will converge to the origin exponentially along the line po defined by $\dot{e} + \frac{k_{vp}}{2}e = 0$ without overshoots (see figure 2). If every point on po satisfies the dynamic constraints such that

$$-\tau_{max} \leq \tau_p(q,\dot{q}) \leq \tau_{max} \qquad \forall(q,\dot{q}) \in po \quad (21)$$

then we think $(e(t_p), \dot{e}(t_p))$ is valid for interference avoidance guaranteeing a *realistic* interference-free path.

B. Interference avoidance by checking desired next states

(20) shows the analytical expression of the control torque is complex and nonlinear, how to verify the dynamic constraints for each point on po is a big problem. In classical methods usually some points have to be sampled to ensure the whole phase portrait satisfy the dynamic constraints which may be not reasonable in practice, but using interval analysis, we can solve this problem effectively.

When the robot arm moves along the desired path, the state that the robot arm is going to take at the next sampling period is checked to avoid all the interferences. Here $(e_d(t+\delta t), \dot{e}_d(t+\delta t))$ is defined as the desired next state of the closed-loop system. It is clear that if we choose $k_{v1} = -\frac{2\dot{e}_d(t+\delta t)}{e_d(t+\delta t)}$, then the phase portrait of the closed-loop system will converge to the origin along the line $\dot{e} + \frac{k_{v1}}{2}e = 0$ without overshoots. For each point on this line the control torque is expressed by letting $k_v = k_{v1}$ and substituting $\dot{e} = -\frac{k_{v1}}{2}e$ in (18), so we get

$$\tau(e) = M(q_o, e)(-\frac{k_{v1}^2}{4}e + \ddot{q}_d) + C(q_o, e) + G(q_o, e) \quad (22)$$

When the phase portrait converges from the desired next state to the origin, the position error varies in the interval $[e]$ (see figure 2).

To verify the dynamic constraints for all the points on the phase portrait, the interval evaluation is used. Given the desired next state $(q_d(t+\delta t), \dot{q}_d(t+\delta t))$, the interval evaluation of the control torque (22) is

$$[\tau] = M(q_o, [e])(-\frac{k_{v1}^2}{4}[e] + \ddot{q}_d) + C(q_o, [e]) + G(q_o, [e]) \quad (23)$$

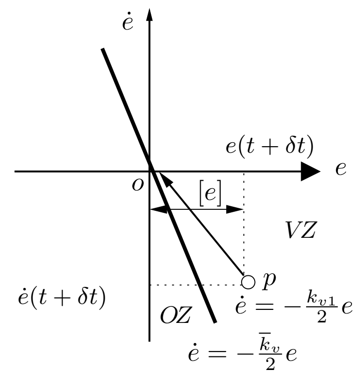

Fig. 2. Phase portrait converging to the origin exponentially

where

$$[e] = [0, e(t+\delta t)] \quad (24)$$

$$k_{v1} = \frac{-2\dot{e}_d(t+\delta t)}{e_d(t+\delta t)} \quad (25)$$

$$e(t+\delta t) = q_o - q_d(t+\delta t) \quad (26)$$

As explained in [15][16], $[\tau]$ derived from (23) may be overestimated, but we may reduce the overestimation by using other powerful methods of interval analysis, for example recognizing monotonicity of the function by interval evaluation of the derivative.

$[\tau]$ determines the lower bound $\underline{\tau}$ and upper bound $\overline{\tau}$ of the control torques for all the points on the phase portrait when it converges to the origin without overshoots, so if

$$-\tau_{max} \leq \underline{\tau} \;\; and \;\; \overline{\tau} \leq \tau_{max} \quad (27)$$

then we think that the whole phase portrait satisfies the dynamic constraints, the desired next state is valid for interference avoidance yielding a *realistic* interference-free path. Otherwise we cannot decide whether the desired next state is valid or not. To ensure the robot arm run safely the next state should be re-scheduled. In this paper the area in which every point can guarantee a *realistic* interference-free path is defined as VZ, while the area not guaranteeing a *realistic* interference-free path is defined as OZ. The border between VZ and OZ is defined by $\dot{e} = -\frac{\overline{k}_v}{2}e$, where $\overline{k}_v$ is determined by the dynamic constraints.

This method is conservative in some degrees since we do not investigate the condition of

$$-\tau_{max} > \underline{\tau} \;\; or \;\; \overline{\tau} > \tau_{max} \quad (28)$$

maybe the overestimation of the interval evaluation instead of breakthrough of the dynamic constraints is the right reason. But as the compromise between the accuracy and the computing time, the desired next state who causes the condition (28) or the condition of

$$\tau_{max} < \underline{\tau} \;\; or \;\; \overline{\tau} < -\tau_{max} \quad (29)$$

is thought infeasible for avoiding interference, both conditions will trigger another process to re-schedule a new state for the next sampling period.

IV. Optimization of interference-free trajectory by using interval analysis

A. Determination of interference-free trajectory by using interval analysis

When the desired next state is not feasible, it is necessary to schedule a new state for the next sampling period which not only yields a realistic collision-free path but also leads the robot arm to trace the desired path as close as possible. So it is a constrained optimization problem which can be modeled as follow:

$$\min_{\dot{q}(t+\delta t)}(\|q(t)+\frac{\dot{q}(t+\delta t)+\dot{q}(t)}{2}\delta t-q_d(t+\delta t)\|^2) \quad (30)$$

S.T.

$$\dot{q}(t+\delta t)\in[\underline{\dot{q}}(i+1),\overline{\dot{q}}(i+1)] \quad (31)$$

$$q(t)+\frac{\dot{q}(t+\delta t)+\dot{q}(t)}{2}\delta t\le q_o \quad (32)$$

$$\{e(t+\delta t),\ \dot{e}(t+\delta t)\}\in VZ \quad (33)$$

In this model the objective function is the distance between the actual position and the desired position at the next sampling period, the variable to be optimized is the next velocity. The objective function is subject to some constraints, the first constraint requires that the next velocity is optimized only within the possible range derived from (12). The second constraint guarantees the robot arm not to interfere with other components at the next sampling period. The last constraint indicates that the next state must yield a realistic interference-free path.

Sometimes classical real-valued methods for example extended cost-function approaches with weighted sum of constraints are used to find an optimal solution, but it has some drawbacks:

- As explained in section (III-A), the expression of the control torque (20) is a complex nonlinear function, how to verify the dynamic constraints for all the states from the next sampling period until the robot arm stops is a big problem for classical real-valued approaches.
- In cost-function approaches, the cost function is composed of the weighted sum of the constraints, but it is difficult to determine the weights.
- Integrating all the constraints together with the objective function will result in a huge nonlinear cost function which may lead to the numerical optimization procedure converge towards a local minimum.

But using interval analysis we can partition this complex constrained optimization problem into a two-stage problem and solve it effectively.

To deal with the constraint (33), the control torque is expressed in a similar way as (23)

$$[\tau]=M(q_o,[e])(-\frac{[k_{v1}]^2}{4}[e]+\ddot{q}_d)+C(q_o,[e])+G(q_o,[e]) \quad (34)$$

where

$$[e]=[0,\overline{e}(t+\delta t)] \quad (35)$$

Noticing that the difference between (23) and (34) is that

- In (23) the desired next state has been known and used as the starting point of the phase portrait to verify the dynamic constraints. But when the desired next state is not feasible, only the allowed range of the next state is available, so in (34) we use the range of the next state to verify the dynamic constraints, in (35) $[e]$ describes the *maximum* possible varying range of the phase portrait when it converges to the origin from the next state, $\overline{e}(t+\delta t)$ is the upper bound of $[e(t+\delta t)]$ which is determined by

$$[e(t+\delta t)]=e(t)-\frac{\dot{q}(t)+[\dot{q}(t+\delta t)]}{2}\delta t \quad (36)$$

- Instead of a real value in (23), k_{v1} is substituted by an interval variable $[k_{v1}]$ which is the varying range of k_{v1} when every point on p_1p_2 (see figure 3) is assumed to converge to the origin exponentially.

All the possible next states evolved from the current state $(e_0,\dot{e}_0)$ only lie on the segment p_1p_2 which follows the equation

$$\dot{e}=\frac{2}{\delta t}(e-e_0)-\dot{e}_0 \quad (37)$$

so $[k_{v1}]$ is determined by

$$[k_{v1}]=\frac{-2}{\frac{\delta t}{2}+\frac{1}{[\dot{e}(t+\delta t)]}(\frac{\delta t}{2}\dot{e}(t)+e(t))} \quad (38)$$

Substituting (36)(38) into (34), we get

$$\begin{aligned}[\tau]=M([\dot{q}(t+\delta t)])(F[\dot{q}(t+\delta t)]+\ddot{q}_d)\\+C([\dot{q}(t+\delta t)])+G([\dot{q}(t+\delta t)])\end{aligned} \quad (39)$$

$F(\cdot),C(\cdot),G(\cdot)$ are corresponding functions. Similarly when

$$-\tau_{max}\le\underline{\tau}\ \ and\ \ \overline{\tau}\le\tau_{max} \quad (40)$$

we think that $\{q(t+\delta t),\ \dot{q}(t+\delta t)\}\in VZ$

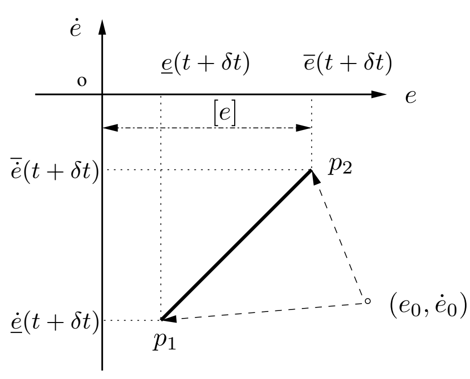

Fig. 3. Range of the next state in phase plane

B. Contractors

To schedule a new state for the next sampling period, an optimal velocity is going to be determined from the segment p_1p_2. Based on the interval analysis an algorithm $F(\mathcal{Q})$ is designed. For a 2-dof robot arm $\mathcal{Q}$ is a 2-D interval box $\mathcal{Q} = \{[\underline{\dot{q}_1}(t+1), \overline{\dot{q}_1}(t+1)], [\underline{\dot{q}_2}(t+1), \overline{\dot{q}_2}(t+1)\}$, in the algorithm $F(\mathcal{Q})$ the bounds of the control torques are evaluated from (39), $F(\mathcal{Q})$ will return:

- 1 if $-\tau_{max} \leq \underline{\tau}$ *and* $\overline{\tau} \leq \tau_{max}$ for all the actuators
- -1 if $-\tau_{max} > \overline{\tau}$ *or* $\underline{\tau} > \tau_{max}$ for at least one actuator
- 0 if $-\tau_{max} > \underline{\tau}$ *or* $\overline{\tau} > \tau_{max}$ for at least one actuator

If $F(\mathcal{Q})$ returns 1, then any velocities included in $\mathcal{Q}$ guarantee that the robot arm can stop at the extreme position q_o and throughout this process the dynamic constraints are always satisfied. If $F(\mathcal{Q})$ returns -1, then the points in $\mathcal{Q}$ cannot yield a realistic interference-free path subject to the dynamic constraints, the robot arm who takes the point in $\mathcal{Q}$ as the next velocity will collide with other components. If $F(\mathcal{Q})$ returns 0 then we cannot decide whether the points in $\mathcal{Q}$ (or in some parts of $\mathcal{Q}$) are valid or not, as the overestimation of interval arithmetics may be the reason why $-\tau_{max} > \underline{\tau}$ *or* $\overline{\tau} > \tau_{max}$.

The algorithm that determines all the valid velocities for the next sampling period is designed based on $F(\mathcal{Q})$. It uses a list of boxes $\mathcal{Q}_i$, the constraints (31)(32) are used to initialize $\mathcal{Q}_0$. To take the velocity measurement accuracy and actuator performances into account, the box whose width is smaller than a given threshold ϵ will be discarded from the list. The algorithm proceeds as follows:

1) if $i > n$, then EXIT
2) if $F(\mathcal{Q}_i)$= -1, then $i = i+1$, go to 1
3) if $F(\mathcal{Q}_i)$= 1, then store $\mathcal{Q}_i$ as the valid next state, $i = i+1$, go to 1
4) if $w(\mathcal{Q}_i) < \epsilon$, then store $\mathcal{Q}_i$ as a neglected box, $i = i+1$, go to 1
5) bisect $\mathcal{Q}_i$ and get two new boxes, add two new boxes to the end of list, $n = n+2$, $i = i+1$ and go to 1

C. Determination of an optimal state by optimizing next state

When the loop is finished we can obtain almost all the ranges of the valid velocity, any points in these ranges can define a valid state for the next sampling period which guarantee the robot arm stop at the extreme position without interference. Among them the best solution which tracks the desired path most closely can be found. It is a simple optimization problem only with the constraints on the varying ranges of the variables. After obtaining all the valid ranges of the velocity in the form of intervals, interval analysis has the most advantage in finding the best solution by minimizing the objective function which indicates the distance between the actual path and desired path shown as (30).

V. Simulation results

This section presents numerical examples of interference avoidance based on our approach. A 2-DOF robot arm is considered, the state variable q_1 is the angle between the axis x and the link I. q_2 is the angle between the axis of link I and link II. The maximum allowed angle q_o for q_1, q_2 is $\frac{2}{3}\pi$, when $q_1, q_2 > q_o$, we think that the interference between the mechanical components occurs. The mass of the links are 4.185 *kg* and 2.854 *kg*, the link lengths are 0.1605 *m* and 0.175 *m*. the nominal angular velocities are 2 *r/m* and 1.4 *r/m*, the bounds of the angular velocity are $[-3, 3]r/m$ which are determined by the motor nominal parameters. The desired motions of the joint angles with respect to t are defined as

$$\begin{cases} {q_1}^d = -0.5 + 2t \\ {q_2}^d = 0.4 + 1.4t \end{cases} \quad t < 1.6 \tag{41}$$

$$\begin{cases} {q_1}^d = 2.7 - \frac{1}{2}(t-1.6) \\ {q_2}^d = 2.64 - \frac{1}{3}(t-1.6) \end{cases} \quad t \geq 1.6 \tag{42}$$

In order to fully present the proposed approach, two different sets of drive torques are tested. In the first case the joints are driven with a high capability, the ranges of the drive torques are $[-2.5, 2.5]nm$ and $[-1.0, 1.0]nm$. In the second case the joints are driven with a relatively low capability which are $[-1, 1]nm$ and $[-0.3, 0.3]nm$.

The approach has been implemented using the high level interval analysis package `ALIAS`[1] which relies on the C++ interval arithmetics package BIAS/Profil. The function `Minimize_Maximize` which is used to achieve the global minimum is available in ALIAS. Figure (4) demonstrates the simulation results of the joint angles, the solid curve describes the desired value of the joint angle, the dashed curve presents the varying of the joint angle driven with a high capability, the dotted curve presents the result of the second case with a low drive capacity. It is obviously that in both cases the joint angles can stop before they arrive at the extreme angles, no interference occurs, when the desired trajectory steps out of the dangerous zone, the joint can track it again, but the robot arm with the higher drive capability can track the desired trajectory more precisely. Figure (5) shows the velocity curves. In this figure the robot arm with the higher drive capacity begins to slow the velocity later than another because its velocity can be decreased with a larger acceleration, but as a result it will cause worse vibrations. Figure (6) shows the desired and actual path of the end effector in $x - y$ plane. From this figure it is proved that the actual path is not only interference-free but also tracks the desired path as close as possible.

VI. Conclusion

A new approach of interference avoidance is proposed for a class of robots whose interference positions have been

[1] www.inria-sop.fr/coprin/logiciel/ALIAS/ALIAS.html

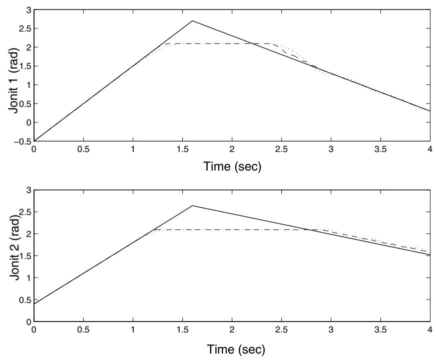

Fig. 4. Motion of the joint angles

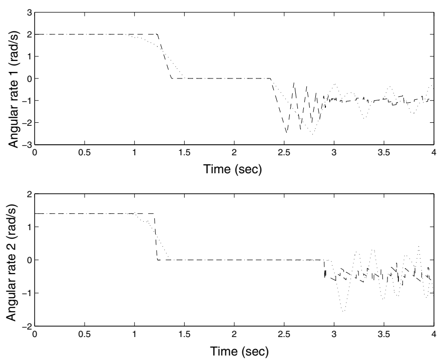

Fig. 5. Velocity of the joint angles

known, all the interferences can be avoided by checking the next state based on the principle of critical damping control. The problem of how to verify the dynamic constraints for a series of states is solved by using interval arithmetics with phase plane analysis, the dynamic constraints are considered by computing the bounds of the control torques. Interval analysis is also used to optimize the next state when it is not valid, the merit of interval-based optimization is that it allows to partition the complicated constrained optimization problem into a simple two-stage problem. Simulation results show the effectiveness of the proposed approach.

The prospective works are:

1) The assumptions of the dynamic model being exactly known and system without disturbances are too restricted, robust sliding mode control is an interesting candidate to ensure the phase portrait converge to the origin without overshoots when the assumptions are not fulfilled.
2) Extending the methodology to process other occasions. For example, integrating with vision systems to avoid interferences for multi-robot collaborative tasks where the interference positions are unknown.

REFERENCES

[1] A.M.Shkel, V.J.Lumelsky, "The Jogger's Problem: Control of Dynamics in Real-time Motion Planning", *Automatica*, 33(7): 1219-1233, 1997.

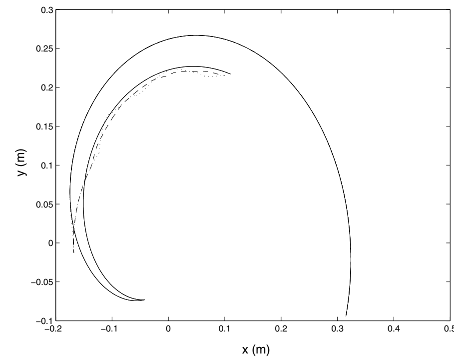

Fig. 6. Path of the end effector

[2] Kevin M. LYNCH, Naoji SHIROMA "Collision-Free Trajectory Planning for a 3-DOF Robot with a Passive Joint", *The International Journal of Robotics Research*, 19(12): 1171-1184, December 2000.

[3] R.K.Culley, K.G.Kempf. "A collision detection algorithm based on velocity and distance bounds", *IEEE International Conference of Robotics and Automation*, pages 1064-1068, 1986.

[4] N. Rahmanian-Shari, I. Troch, "Collision-avoidance control for redundant articulated robots", *Robotica* 13(2), 159-168,1995.

[5] J. Baillieul, "Avoiding obstacles and resolving kinematic redundancy", *IEEE Int. Conf. on Robotics and Automation* 1698-1704, 1986.

[6] O. Khatib, "Real-time obstacle avoidance for manipulators and mobile robots", *Int. J. Robotics Research*, 5(1), 90-98, 1986.

[7] B. R. Donald and P. Xavier, "Provably Good Approximation Algorithms for Optimal Kinodynamic Planning: Robots with Decoupled Dynamics Bounds ", *Algorithmica* ,Vol. 14, no 6, pp. 443-479,1995

[8] S.Ma, S.Hirose, "A dynamic approach to real-time obstacle avoidance control of redundant manipulators", *JSME Int. J., series C,* 39(2), 317-322, 1996.

[9] S.I Choi,B.K Kim, "Obstacle avoidance control for redundant manipulators using collidability measure", *Robotica* 18,143-151, 2000.

[10] Pradeep K.Khosla, Takeo Kanade, "Experimental evaluation of nonlinear feedback and feedforward control schemes for manipulators", *The International Journal of Robotics Research*, 7(1): 18-28, Feb 1988

[11] S. Bobyr, Vladimir J. Lumelsky, "Control of Dynamics and Sensor Based Motion Planning for a Differential Drive Robot", *IEEE International Conference on Robotics and Automation*,Volume 1, 157-162,1999

[12] S.M. LaValle and J.J. Kuffner, "Randomized kinodynamic planning", *International Journal of Robotics Research*, 20(5):378-400, May 2001.

[13] Shiller, Z., "Time-Energy Optimal Control of Articulated Systems with Geometric Path Constraints", *ASME Journal of Dynamic Systems, Measurement and Control.*, Vol. 118 No. 1, pp. 139-143, March 1996,

[14] Constantinescu, D. and Croft, E.A., "Smooth and Time-Optimal Trajectory Planning for Industrial Manipulators along Specified Paths", *Journal of Robotic Systems*, Vol. 17, No. 5, pp. 233-249, 2000,

[15] Moore R.E. *Methods and Applications of Interval Analysis*, SIAM Studies in Applied Mathematics, 1979.

[16] Jaulin Luc, Kieffer Michel. *Applied interval analysis : with examples in parameter and state estimation, robust control and robotics*, Springer, 2001.