Managing the redundancy of N-1 wire-driven parallel robots

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Abstract We consider wire-driven parallel robot with $N \geq 4$ wires that are connected at the same point on the platform. Such robot has 3 d.o.f. but it is non redundant (e.g. we cannot control the tension in the wires) as there will always be only at most 3 wires under tension simultaneously. We consider in this paper three approaches that make this robot really redundant: elasticity in the wires, using counterweights in the wires or attaching the redundant wires to a fixed point on the other wires. We show that these methods may be effective but still require further studies.

Key words: cable robot,wire-driven parallel robot, redundancy

1 The $N-1$ wire driven parallel robot

A wire-driven parallel robot has the same mechanical structure as a parallel robot with rigid extensible legs but the linear actuators are substituted by wires that can be coiled and uncoiled. Such robot has the advantages of being mechanically simple and to allow for large workspace (the leg length variations being much larger than with rigid legs). Their main drawback is that wire can be pulled but cannot be pushed: hence kinematics cannot be decoupled from statics, especially for robot having less than 6 d.o.f., and this added complexity explains why the kinematics of such robot is still an open issue [5]. A large number of potential applications has led to a renewal of interest for wire-driven parallel robots for example for rescue crane [2, 9, 11], assistance robots and rehabilitation [3] or haptic devices [1, 6].

In this paper we are considering a special class of wire-driven parallel robot, called the $N-1$ robot, in which the N wires are all connected at the same point C on the platform. If $N \geq 3$, then the robot has 3 d.o.f., namely it allows to control the position of C but not the orientation of the platform. As soon as $N \geq 4$ such robot is called *redundant*, whatever the definition of redundancy is [7, 10]. Redundant robot will be the topic of this paper, starting by an examination of the reality of this redundancy.

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2 Is the $N-1$ robot redundant?

Using more than 3 non-elastic wires for a $N-1$ robot is a natural idea to improve the performances of this robot. Let us denote by A_i the fixed output points of the wires on the base and consider the workspace of this robot which is the volume V_ε spanned by moving the convex hull of the A_is along the downward vertical (with an upper limit which is the base plane and a lower limit determined by the maximal wire lengths). Hence adding a wire allows one to increase the workspace volume as soon as the added A_i is not located within the convex hull of the previous A_is.

It may also be sought that having redundant wire(s) allows one to control the tension distribution in the wires [4] and hence to improve the carrying capacity of the robot. Unfortunately we have shown both theoretically and experimentally in our ICRA paper from 2012 that this is not true if the wires are not elastic: **whatever the number of wires is, at a given pose there will always be at most 3 wires under tension while the other wires will be slack**. Without going into the proof let us explain intuitively this result. Consider a 3-1 robot in a pose C_3 that lies in its V_ε: the mechanical equilibrium at C_3 is satisfied and the tension in the 3 wires are uniquely determined. If we add a 4th wire its length is uniquely determined as the distance between A_4 and C_3. A wire system allows one to control either the wire length or the wire tension (but not both). Hence:

- if we impose the length, then the mechanical equilibrium will be satisfied with a 0 tension in the 4th wire
- if we impose a tension (i.e. the wire length is smaller than the distance between A_4, C_3), then C will move in a location different from C_3

Being unable to control the tensions in the wires is a disappointing result because this is typically one of the most obvious advantage of redundancy. We will propose in the next sections different ways to exploit the availability of additional wires for tension management while preserving the pose of the platform.

3 Tension management

3.1 Elastic wires

As mentioned previously a drawback of non-elastic wires is that tension control is difficult. This may be changed if we assume elasticity in the wires. Let τ_i be the tension in wire i, l_i its length at rest and ρ_i its length when under tension. If we assume that the wire is a perfect linear spring we have

$$\tau_i = k(\rho_i - l_i) \tag{1}$$

where k is the stiffness of the wire (assumed to be identical for all wires). Consider now a 3-1 robot submitted to a load of mass m and having all 3 wires connected at the center of mass of the load. The robot is submitted to a pure force $\mathscr{F} = (0,0,-mg)^T$ and the tension τ in the wires may be calculated as

$$\tau = \mathbf{J^T}\mathscr{F} \tag{2}$$

where $\mathbf{J^T}$ is the transpose of the jacobian matrix of the robot. For the inverse kinematics (IK) the coordinates x,y,z of C are known, which allows one to calculate $\mathbf{J^T}$ and then ρ_i as the distance between A_i, C. Using equation (2) we may then determine the τ. The control length of the wire may thus be calculated as

$$l_i = \rho_i - \frac{\tau_i}{k}$$

But there are sources of uncertainties in the modeling: on k, on l_i, on the location of the A_i and due to the fact that the wires are not exactly attached at the same point on the platform. We will focus on the influence of the uncertainities on k and l_i, assuming that the A_i have been calibrated while the influence of the colocation of the wire attachment points will be addressed in another paper. It is therefore necessary to investigate what is the influence of the stiffness on the pose of the robot for a given control input. We have thus solved the forward kinematics (FK) problem i.e. determine what are the possible coordinates of C for given l_is. Equation (2) allows one to calculate τ as functions of the coordinates of C while equation (1) has now as unknowns x,y,z,ρ_i. The geometrical IK of the robot provides an additional equation

$$\rho_i^2 = ||\mathbf{A_i C}||^2 \tag{3}$$

Equation (1) is linear in ρ_i and the result is reported in equation (3) to get a constraint equation in x,y,z. Repeating this process for all 3 wires leads to 3 constraint equations. Using resultant on these equations allows for successive elimination of x,y, leading to an univariate polynomial in z. This polynomial may be factored out as the product of 2 polynomials of degree 22, 34. Note that this approach is less efficient than the one proposed by Dietmaier [8] but has the advantage of providing directly the x,y,z. With this tool we may investigate the influence of uncertainties on k and l_i on the positioning.

As an example we consider the 3-1 wires robot with anchor points $A_1 = (0,0,0)$, $A_2 = (0,400,0)$, $A_3 = (400,0,0)$. The wire control values are given as $l_1 = 200, l_2 = 350, l_3 = 300$ which leads to the pose $x = 137.5$, $y = 96.875$, $z = -108.208$ for wires without elasticity. To take into account the uncertainty on the control l_i and on the stiffness k we have considered a possible ± 3 error on the l_i and a $\pm 0.1k$ error on the k. We have then solved the FK for a random sampling of 1000 sets of k,l within these ranges. For a nominal value of $k = 100$ we have found that the variations on x,y,z were in the ranges [-3.86,2.7], [-3.24,3.93], [-5.16,3.74], while the τ variations were [-15.49,14.23], [26.9,28.96], [-26.11,24.18]. For a nominal value of $k = 3000$ we found out that the variations on x,y,z were in the ranges [-3.64,3.63], [-4.3,3.47], [-6.32,6.14], while the τ variations were [-20.58,19.46], [-40.1,36.2], [-44.21,39.36].

Hence even small uncertainties on the values of the l,k lead to significant positioning errors for the robot.

We consider now a 4-1 robot with the purpose of using the redundancy to adjust the wire tension e.g. to minimize the criteria $H = \sum_{j=1}^{j=4} \tau_j^2$. Using equation (2) one can obtain three wire tensions as a linear function of the remaining one. Without lack of exhaustivity we may calculate $\tau_2, \tau_3 \tau_4$ as a function of τ_1. H is then a quadratic function in τ_1 and it is therefore trivial to determine τ_1 that leads to the minimum of H. For the IK, being given the pose of the load, the τ and equation (1) we may determine the four l_is. To determine the influence on the positioning of the uncertainties on k and on l_is we have to solve the FK problem.

In the FK problem the l_i's are given and we have to determine the pose of the load. For that purpose we note that the first equation of (1) allows to determine τ_1 as function of ρ_1, while equation (2) is used to determine τ_2, τ_3, τ_4. The three remaining equations of (1) are then linear in x,y,z. After solving this system we report the result in the IK equations (3) after subtracting the equation for wire 1 to the equations for wire 2, 3, 4. Together with (3) for wire 1 these equations constitutes a system of 4 equations in the unknowns $\rho_1, \rho_2, \rho_3, \rho_4$. One of this equation is linear in ρ_4 and is solved for this variable. The 3 remaining equations, denoted a_1, a_2, a_3, are of degree (6,6,2), (3,3,3), (9,9,3) in ρ_1, ρ_2, ρ_3. The four equations $a_1, \rho_3 a_1, a_2, a_3$ are linear in the monomials $1, \rho_3, \rho_3^2, \rho_3^3$ and hence the determinant of the matrix of this linear system should be 0, which leads to a polynomial $\mathscr{P}_1$ of degree 15 in ρ_1, ρ_2. Taking the resultant of a_1, a_2 in ρ_3 leads to a polynomial $\mathscr{P}_2$ of degree 18 in ρ_1, ρ_2. The resultant of $\mathscr{P}_1, \mathscr{P}_2$ factors out in 2 polynomials of degree 76 and 96 in ρ_1. Although this complete the theoretical solution, the degree of the involved polynomial is too high to be used in practice and consequently we have to resort to a numerical procedure. For that purpose we solve the linear system (2) to get τ_2, τ_3, τ_4 as function of τ_1. Then the first equation of (1) is used to determine τ_1 as a function of ρ_1. The three remaining equations of (1) together with the IK equations (3) constitute a system of 7 equations in the 7 unknowns $x,y,z,\rho_1,\rho_2,\rho_3,\rho_4$. As all unknowns may easily be bounded we have used an interval analysis approach to solve this system, all solutions being found in less than one second.

We have considered the 4-1 robot derived from the previous 3-1 by adding a 4th wire whose exit point on the base is $A_4 = (400, 400, 0)$. We have then used the IK to determine what should be the l_i to reach the pose $x = 100, y = 200, z = -200$ while minimizing $\sum_{j=1}^{j=4} \tau_i^2$ for $k = 1000$. The nominal values were determined as $l_1 = l_2 = 299.558$, $l_3 = l_4 = 412.1083$ which leads to $\tau_1 = \tau_2 = 441.45$, $\tau_3 = \tau_4 = 202.238$. Using the FK with these values of the ls leads also to solutions in which only 3 wires are under tension, namely wires (1,2,3) or (1,2,4), both cases leading to the same pose of the load with $x = 99.6834$, $y = 200.2192$, $z = -200.1581$. It should be noted that already elasticity does not allow for precise positioning as we are unable to determine the final pose of the platform for given control inputs.

We have then assumed similar errors on the ls and on k than in the previous example and have computed the FK solutions for 1000 sets of (l,k) chosen randomly. We found out that the variations on x,y,z were in the ranges [-7.532, 4.579], [-6.868, 5.75], [-7.6, 2.875] for a nominal value of $k = 100$ and [-5.488, 4.7948],

[-4.79, 4.262],[-5.1429, 4.247] for $k = 1000$. Over the test set the mean values of the τ_i's for $k = 1000$ were 467.606, 463.06, 278.35, 256.35 with a variation of [-192, 133.86], [-187.45, 138.09], [-274.185, 146.07], [-241.75, 168.745]. Here again we observe significant positioning errors and very large change in the tensions. As a conclusion adding elasticity in the wires for managing redundancy will require a very good wire length control together with a perfect stiffness calibration.

3.2 Using counterweights

The principle here is to attach known weight(s) on some wire(s), close to the platform in order not to disturb the coiling of the wires. The purpose of the counterweight is to change the direction of the tension(s) applied on the platform in order to possibly control the value of the wire tensions (in this section a wire is under tension if its τ is negative).

Consider for example a 4-1 robot with a counterweight of mass m_4 on wire 4 located at point M at a distance d of C (figure 2). For a given pose of C we are able to calculate the values of ρ_1, ρ_2, ρ_3. Wire 4 exerts on the platform a force τ_{14} that is directed along $\mathbf{MC}$ while it exerts a force τ_{24} on the counterweight that is directed along $\mathbf{A_4M}$. The mechanical equilibrium of the platform may be written as:

$$\sum_{j=1}^{j=3} \tau_j \mathbf{A_j C}/\rho_j + \tau_{14}\mathbf{MC}/d + (0,0,-mg)^T = \mathbf{0} \tag{4}$$

The mechanical equilibrium of the counterweight may be written as:

$$-\tau_{14}\mathbf{MC}/d + \tau_{24}\mathbf{A_4M}/(\rho_4 - d) + (0,0,-m_1 g)^T = \mathbf{0} \tag{5}$$

A direct consequence of equations (5) is that M must lie in the vertical plane that includes A_4, C. This constraint, together with the equations $||MC||^2 = d^2$, $||MA_4||^2 = (\rho_4 - d)^2$, allows one to determine the unique location of M as a function of ρ_4. Substituting the values of the coordinates of M into equations (4, 5) leads to a linear system of 5 equations in the unknowns $\tau_1, \tau_2, \tau_3, \tau_{14}, \tau_{24}$. Hence the wire tensions may be established as functions of ρ_4: their generic form is $\tau_i = P_i/W$, where P_i is a polynomial of degree 8 in ρ_4 while W is quadratic in this variable. Figure 1 shows the values of the tension τ_1, τ_2, τ_3 as a function of ρ_4 at the pose (25,125,-300) for a load of 80 kg and a counterweight of 5 kg located at a distance 50 from the platform together with the values of the tensions for the 3-1 robot with wires 1, 2, 3. It may be seen that even for a relatively low counterweight mass the tensions in the wire 2,3 are substantially lower while the tension in wire 1 increases. We may consider the problem of determining the value of ρ_4 that maximizes $H = \sum_{j=1}^{j=3} \tau_j$, all tensions being negative or equal to 0. The derivative of H with respect to ρ_4 is a 14th order polynomial in ρ_4 and determining its positive roots allows one to find ρ_4 that maximizes H. We have then considered the cases where counterweights were attached to a single wire at different location or a counterweight was added to

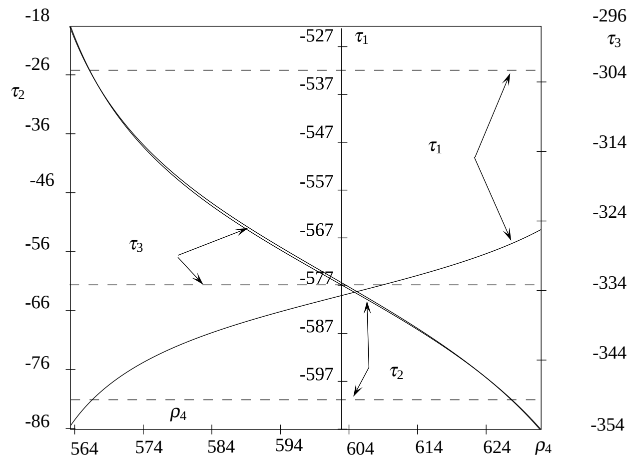

Fig. 1 The tensions in the wires of a 4-1 robot at the pose (25,125,-300) for a load of 80 kg and a counterweight of 5 kg on wire 4 located at a distance of 50 from the platform are shown as a function of ρ_4. The horizontal dashed lines show the value of the tensions in the wires for the 3-1 robot without wire 4

several wires. Extensive numerical tests confirmed that by adding counterweight(s) tension in some wires may be significantly reduced but at the cost of a large increase for the other wires and altogether no improvement for H. Note that, as mentioned by a reviewer, we may study the the problem by assuming that the platform is a rigid line MC having 2R3T motion and this will lead to the same equations and results.

In conclusion adding counterweight is a possibility to deal with specific cases (e.g. decreasing the tension in one wire so that it can be disconnected) but is not a solution for overall improvement of the tensions in the wires.

3.3 Attaching wires to wires

The idea here is to have some wires that are not connected to C but to fixed location on other wires. As an example we will consider a 4-1 robot in which the 4th wire is attached at point M_1 on wire 1 so that the distance between M_1 and C when wire 1 is under tension is l_1 (figure 2). The unknowns for the IK are the 3 coordinates of M_1 and the 5 tensions τ_1 to τ_5. First note that the mechanical equilibrium at M_1 imposes that M_1, A_1, A_4, C are coplanar and that for given M_1, τ_1 the tensions τ_4, τ_5 may be derived from the mechanical equilibrium. Hence we may consider only the following 6 unknowns: the coordinates x_1, y_1, z_1 of M_1 and the tensions τ_1, τ_2, τ_3. But for a given M_1 the equilibrium condition of the load (2) is a linear system in τ_1, τ_2, τ_3 that may be solved independently. Hence we may focus only on the constraints on M_1 i.e. $||\mathbf{M_1C}|| = l_1$ and the coplanarity condition between M_1, A_1, A_4, C. We end up with a system of 2 equations in 3 unknowns with one linear equation and one

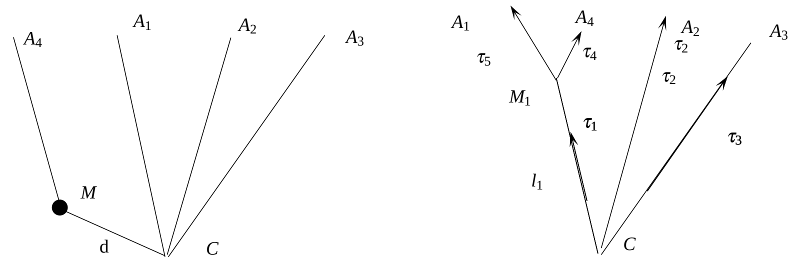

Fig. 2 A 4-1 robot with a counterweight attached on wire 4 and a 4-1 robot with wire 4 attached to wire 1. This robot reaches some pose under the influence of gravity

quadratic equation. Consequently we are able to express all tensions τ_1 to τ_5 as functions of any single variable in the set x_1, y_1, z_1. As we have a free variable we may choose it to optimize different criterion such as minimizing the sum H of the force exerted by the motors or minimizing the maximum H_{min} of the wire tensions. Note that finding the optimal choice for these criterion is easy: although the tensions are not algebraic expression of the free variable, the derivatives of H, H_{min} with respect to this variable are polynomials of degree 12 in the free variable.

As an example we have considered the pose $x = 110, y = 150, z = -200$ with $l_1 = 100$. If only 3 wires were used the criteria H was optimal when using wires 1, 2, 4 with a value of 1268.83 and wire tensions 669.85, 132.83, 466.14, while for H_{min} the best configuration is obtained when using wires 1, 2, 3 with a criteria value of 498.14 and wire tensions 375.116, 498.14, 413.17. We have then considered the 4-1 redundant case, examining all possible wire configurations. Surprisingly although we have tested numerous poses it appears that H is never improved when using redundancy, although we have not be able to figure out a theoretical explanation.

On the other hand H_{min} has been improved for the test pose with a value of 430.082 with as main wires 1,2,4 and wire 3 attached to wire 4. This corresponds to a gain of 13.66% compared to the non redundant case. For this pose we have tested all values of l_1 between 10 and 130 with a step increment of 10 without observing any significant change in H_{min}. Finally we have performed 100 test with random values for the coordinates of C within the ranges [60,340], [60,340], [-300,-50] and random values for l_1 in the range [10,130]. The mean value for the improvement on H_{min} was 13.27% with a minimum value of 0 and a maximal value of 37.714087%. Hence attaching wires to other wires seems to be a feasible solution to manage tension distribution in the wires.

4 Conclusion

Although apparently redundant a $N-1$ robot with $N \geq 4$ does not allow to manage tension distribution in the wires if they are not elastic. Tension management using the elasticity of the wires is quite difficult as the positioning of the platform is very sensitive to the stiffness of the wires and to wire lengths control. We have then investigated the use of adding counterweights in the wires, showing that the overall tension distribution is not improved, although this solution may lead to a decrease in some tensions. Then we have examined attaching redundant wires to fixed location on other wires: this simple solution is efficient to decrease the value of the maximal tension although the sum of the wire tensions is not improved. Management of redundancy opens numerous kinematics issues that are worth being investigated.

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