Passive joints and application examples

Thibault Gayral

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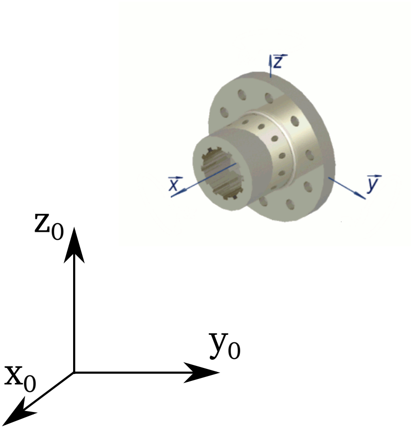
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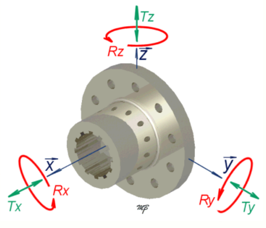
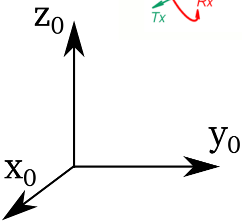
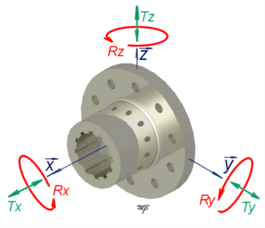
x
y
z0
y0
x0

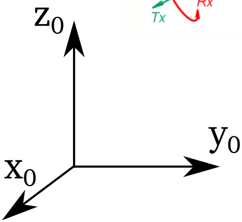




6 DOF

- 3 rotations Rx, Ry, Rz
- 3 translations Tx, Ty, Tz



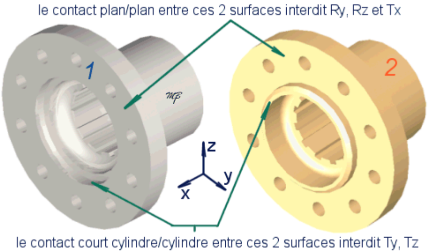


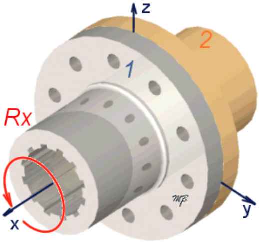
6 DOF

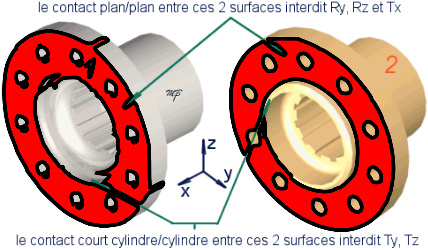
- 3 rotations Rx, Ry, Rz
- 3 translations Tx, Ty, Tz

Kinematic torsor

$$\mathcal{V} = \begin{bmatrix} Rx & Tx \\ Ry & Ty \\ Rz & Tz \end{bmatrix}$$


le contact plan/plan entre ces 2 surfaces interdit Ry, Rz et Tx
1
2
z
y
x
le contact court cylindre/cylindre entre ces 2 surfaces interdit Ty, Tz


z
2
1
Rx
x
y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2
z
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x
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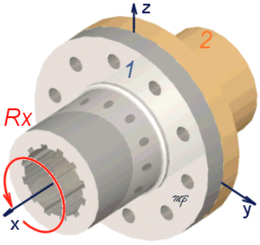
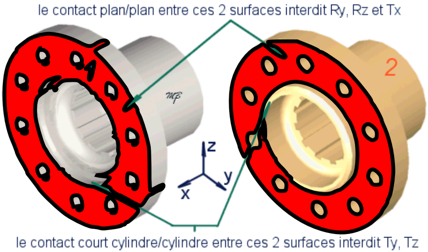
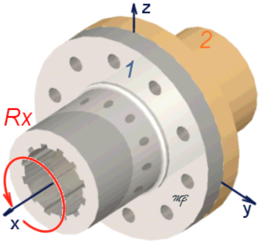
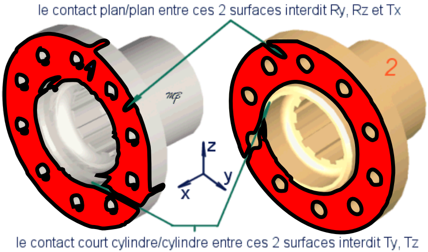
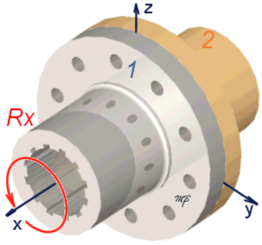

z
2
1
Rx
y
x





$$\mathcal{V}_{1/2} = \begin{bmatrix} & \\ & \end{bmatrix}$$

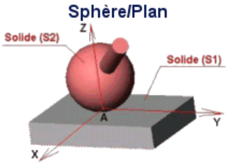


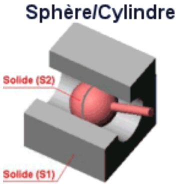


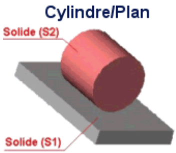
$$\mathcal{V}_{1/2} = \begin{bmatrix} Rx & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

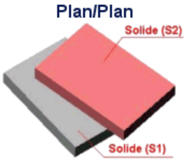
Type of contact

Point	Line	Surface









Name	DOF	2D Representation	3D Representation
Rigid			
Revolute			
Prismatic			

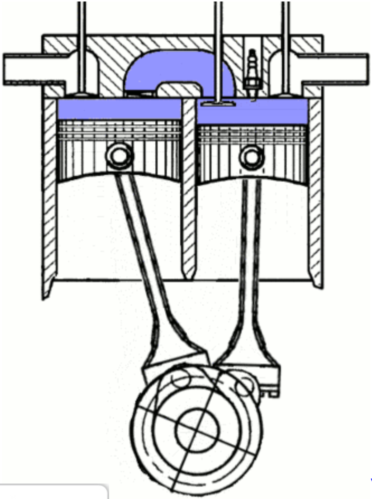
TAB.: Usual Kinematic Joints (1)

Name	DOF	2D Representation	3D Representation
Cylindrical			
Spherical			
Planar			

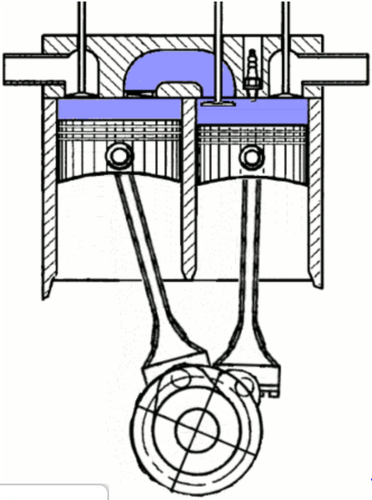
TAB.: Usual Kinematic Joints (2)

Name	DOF	2D Representation	3D Representation
Cylindrical Slider			
Spherical Slider			
Helicoid			

TAB.: Usual Kinematic Joints (3)



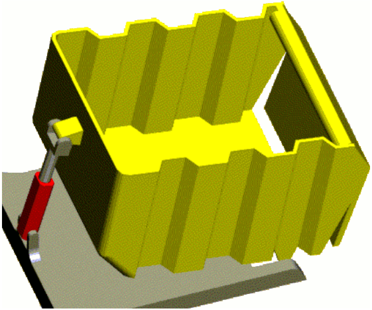
Video

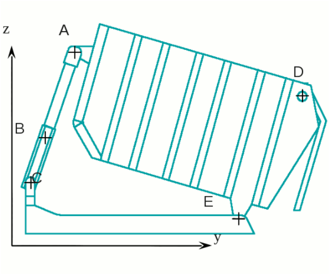


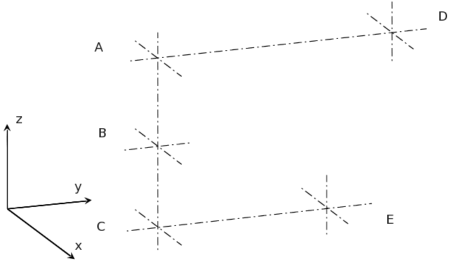
Video

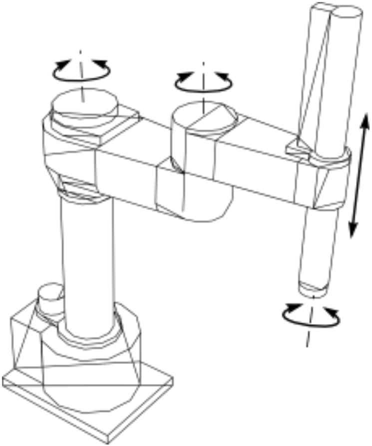
Hyperstaticity

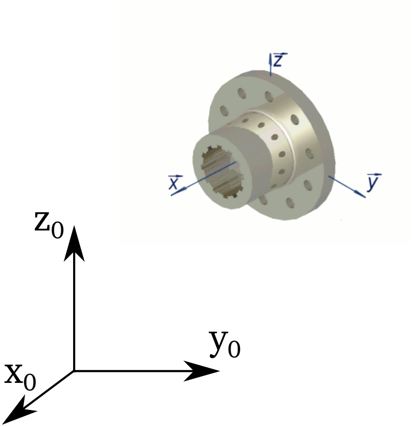

SO.VA.MAT
SOCIÉTÉ DE TRANSPORT ET DE VALORISATION DE MATÉRIAUX

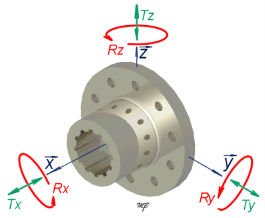


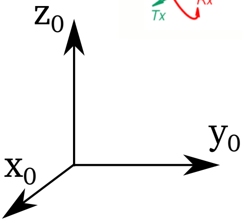

z
A
B
C
D
E
y


A
D
B
z
y
x
C
E




$\vec{z}$
$\vec{x}$
$\vec{y}$
z_0
y_0
x_0





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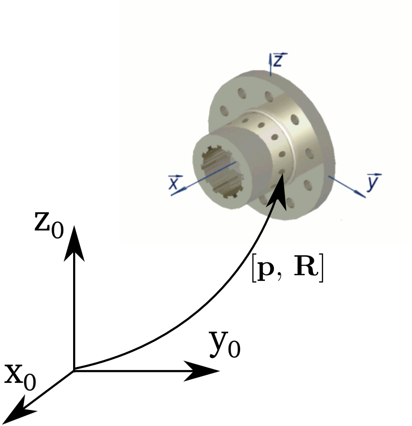

6 DOF

- 3 rotations Rx, Ry, Rz
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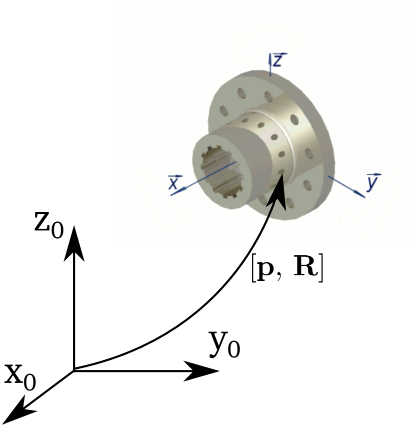
6 Positioning parameters

- 1 rotation matrix $\mathbf{R}_{3\times3}$
- 1 position vector $\mathbf{p}_{3\times1}$



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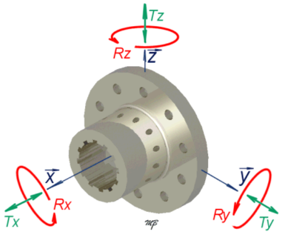
- 1 position vector $\mathbf{p}_{3\times1}$

Orientation parametrization

- Euler angles
- Rodrigues parameters
- Quaternion
- etc...

Euler angles

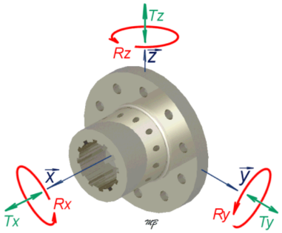


$$R_x(\theta_x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{pmatrix}$$

$$R_y(\theta_y) = \begin{pmatrix} \cos\theta_x & 0 & \sin\theta_x \\ 0 & 1 & 0 \\ -\sin\theta_x & 0 & \cos\theta_x \end{pmatrix}$$

$$R_z(\theta_z) = \begin{pmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_x & \cos\theta_x & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Euler angles



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$$R_y(\theta_y) = \begin{pmatrix} \cos\theta_x & 0 & \sin\theta_x \\ 0 & 1 & 0 \\ -\sin\theta_x & 0 & \cos\theta_x \end{pmatrix}$$

$$R_z(\theta_z) = \begin{pmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_x & \cos\theta_x & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R = R_x(\phi).R_y(\theta).R_z(\psi) \qquad \text{(Bryant)}$$

$$= \begin{pmatrix} \cos\theta\cos\psi & -\cos\theta\sin\psi & \sin\theta \\ \sin\phi\sin\theta\cos\psi + \cos\phi\sin\psi & \cos\phi\cos\psi - \sin\phi\sin\theta\sin\psi & -\sin\phi\cos\theta \\ -\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi + \sin\phi\cos\psi & \cos\phi\cos\theta \end{pmatrix}$$

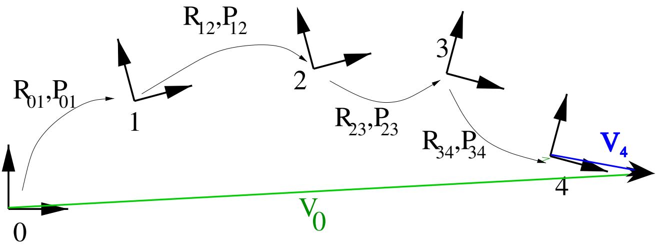
Rodrigues parameters

R : Rotation of an angle θ around the unit vector $\begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$.

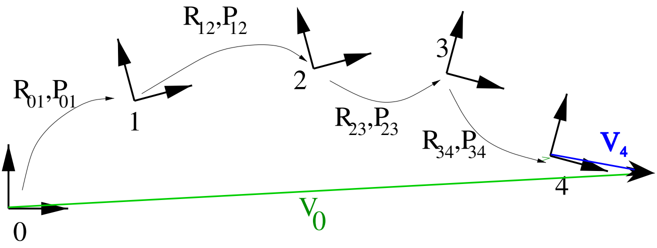
$$\begin{aligned} \mathcal{Q}_1 &= u_x \tan\frac{\theta}{2} \\ \mathcal{Q}_2 &= u_y \tan\frac{\theta}{2} \\ \mathcal{Q}_3 &= u_z \tan\frac{\theta}{2} \end{aligned}$$

$$R = \frac{1}{1+\mathcal{Q}_1^2+\mathcal{Q}_2^2+\mathcal{Q}_3^2}\begin{pmatrix} 1+\mathcal{Q}_1^2-\mathcal{Q}_2^2-\mathcal{Q}_3^2 & 2(\mathcal{Q}_1\mathcal{Q}_2-\mathcal{Q}_3) & 2(\mathcal{Q}_1\mathcal{Q}_3+\mathcal{Q}_2) \\ 2(\mathcal{Q}_1\mathcal{Q}_2+\mathcal{Q}_3) & 1-\mathcal{Q}_1^2+\mathcal{Q}_2^2-\mathcal{Q}_3^2 & 2(\mathcal{Q}_2\mathcal{Q}_3-\mathcal{Q}_1) \\ 2(\mathcal{Q}_3\mathcal{Q}_1-\mathcal{Q}_2) & 2(\mathcal{Q}_2\mathcal{Q}_3+\mathcal{Q}_1) & 1-\mathcal{Q}_1^2-\mathcal{Q}_2^2+\mathcal{Q}_3^2 \end{pmatrix}$$

Kinematic Chain



Kinematic Chain



$$V_3 = R_{34}.V_4 + P_{34}$$
$$V_2 = R_{23}.V_3 + P_{23} = R_{23}.(R_{34}.V + P_{34}) + P_{23}$$
$$V_1 = R_{12}.(R_{23}.(R_{34}.V + P_{34}) + P_{23}) + P_{12}$$
$$V_0 = R_{01}.(R_{12}.(R_{23}.(R_{34}.V + P_{34}) + P_{23}) + P_{12}) + P_{01}$$

Homogeneous Transformation Matrix

$$^{i}T_j = \left(\begin{array}{ccc|c} R_{1,1} & R_{1,2} & R_{1,3} & P_1 \\ R_{2,1} & R_{2,2} & R_{2,3} & P_2 \\ R_{3,1} & R_{3,2} & R_{3,3} & P_3 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)_{4\times4}$$

$$\begin{pmatrix} V_i \\ 1 \end{pmatrix}_{4\times1} = {}^{i}T_j . \begin{pmatrix} V_j \\ 1 \end{pmatrix}_{4\times1}$$

$$\begin{aligned} {}^{0}T_4 &= {}^{0}T_1 . {}^{1}T_2 . {}^{2}T_3 . {}^{3}T_4 \\ {}^{i}T_i &= I \end{aligned}$$

Homogeneous Transformation Matrix

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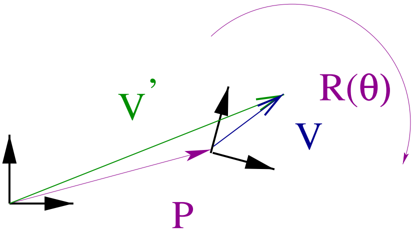
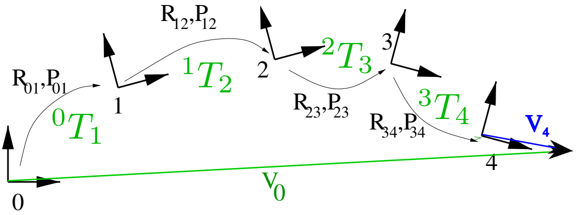


$$ {}^{i}T_{j} = \left(\begin{array}{cc|c} R_{1,1} & R_{1,2} & P_1 \\ R_{2,1} & R_{2,2} & P_2 \\ \hline 0 & 0 & 1 \end{array}\right)_{3\times 3} \rightarrow \begin{pmatrix} V' \\ 1 \end{pmatrix} = \left(\begin{array}{c|c} R & P \\ \hline 0 & 1 \end{array}\right) . \begin{pmatrix} V \\ 1 \end{pmatrix} = \begin{pmatrix} R.V + P \\ 1 \end{pmatrix} $$



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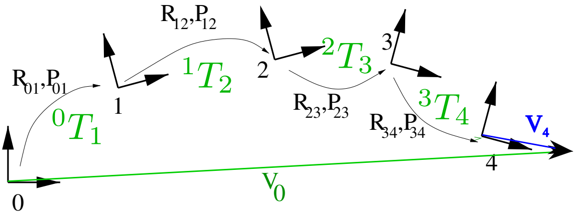
OR

$$\begin{pmatrix} V_0 \\ 1 \end{pmatrix} = {}^0T_1.{}^1T_2.{}^2T_3.{}^3T_4.\begin{pmatrix} V_4 \\ 1 \end{pmatrix}$$



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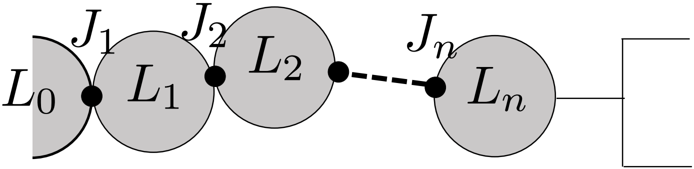
\end{aligned}
$$

OR

$$
\begin{pmatrix} V_0 \\ 1 \end{pmatrix} =^0 T_1.^1T_2.^2T_3.^3T_4.\begin{pmatrix} V_4 \\ 1 \end{pmatrix}
$$

DH Idea

- The kinematic chain of a robot can be modeled by rigid links and perfect joints P and R
- 6 $\longrightarrow$ 4 parameters



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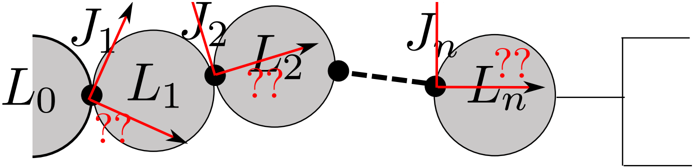
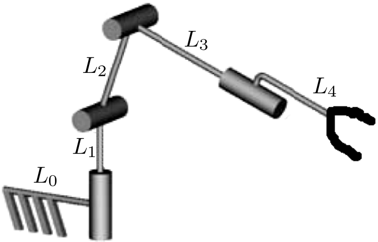


DH Conventions

- J_i connects L_{i-1} and L_i
- $(O_i, \mathbf{x}_i, \mathbf{y}_i, \mathbf{z}_i)$ the fixed frame with respect to L_i
- $\mathbf{z}_i$ the axis of J_i
- $\mathbf{x}_i$ the common perpendicular of $\mathbf{z}_i$ and $\mathbf{z}_{i+1}$



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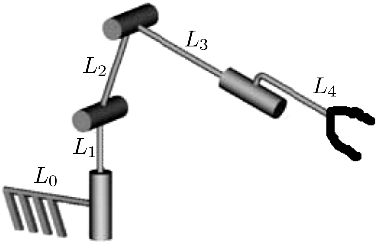

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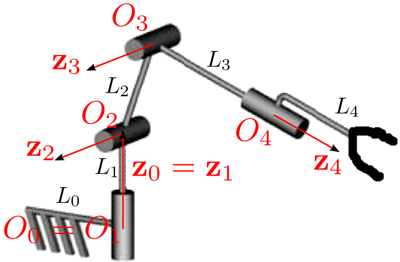
Special cases

- Frame 0 = Frame 1
- $\mathbf{x}_n$ can be taken along $\mathbf{x}_{n-1}$



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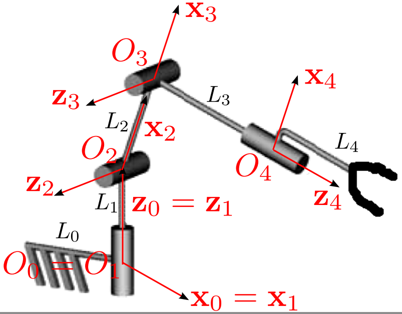
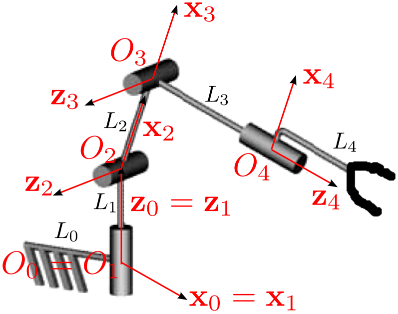


DH Parameters

- α_i angle between $\mathbf{z}_{i-1}$ and $\mathbf{z}_i$ about $\mathbf{x}_{i-1}$
- d_i distance between O_{i-1} and $\mathbf{z}_i$, along $\mathbf{x}_{i-1}$
- r_i distance between $\mathbf{x}_{i-1}$ and O_i, along $\mathbf{z}_i$
- θ_i angle between $\mathbf{x}_{i-1}$ and $\mathbf{x}_i$ about $\mathbf{z}_i$

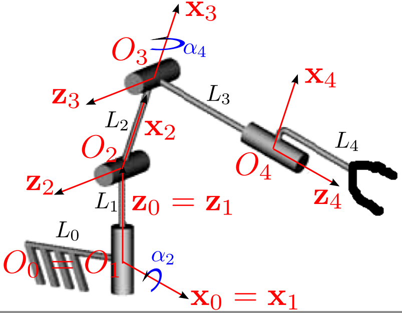
i	α	d	r	θ
1				
2				
3				
4				



DH Parameters

- α_i angle between $\mathbf{z}_{i-1}$ and $\mathbf{z}_i$ about $\mathbf{x}_{i-1}$
- d_i distance between O_{i-1} and $\mathbf{z}_i$, along $\mathbf{x}_{i-1}$
- r_i distance between $\mathbf{x}_{i-1}$ and O_i, along $\mathbf{z}_i$
- θ_i angle between $\mathbf{x}_{i-1}$ and $\mathbf{x}_i$ about $\mathbf{z}_i$

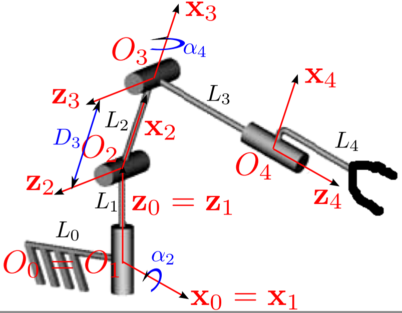
i	α	d	r	θ
1	0			
2	90°			
3	0			
4	90°			



DH Parameters

- α_i angle between $\mathbf{z}_{i-1}$ and $\mathbf{z}_i$ about $\mathbf{x}_{i-1}$
- d_i distance between O_{i-1} and $\mathbf{z}_i$, along $\mathbf{x}_{i-1}$
- r_i distance between $\mathbf{x}_{i-1}$ and O_i, along $\mathbf{z}_i$
- θ_i angle between $\mathbf{x}_{i-1}$ and $\mathbf{x}_i$ about $\mathbf{z}_i$

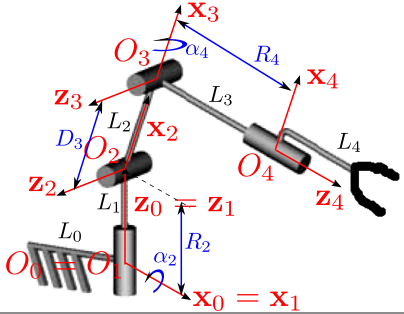
i	α	d	r	θ
1	0	0		
2	90°	0		
3	0	D_3		
4	90°	0		



DH Parameters

- α_i angle between $\mathbf{z}_{i-1}$ and $\mathbf{z}_i$ about $\mathbf{x}_{i-1}$
- d_i distance between O_{i-1} and $\mathbf{z}_i$, along $\mathbf{x}_{i-1}$
- r_i distance between $\mathbf{x}_{i-1}$ and O_i, along $\mathbf{z}_i$
- θ_i angle between $\mathbf{x}_{i-1}$ and $\mathbf{x}_i$ about $\mathbf{z}_i$

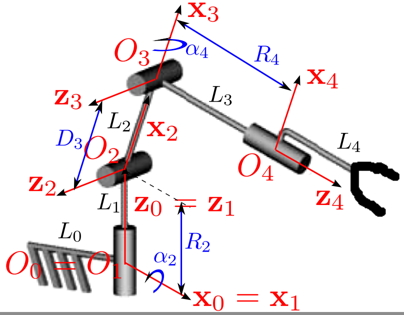
i	α	d	r	θ
1	0	0	0	
2	90°	0	R_2	
3	0	D_3	0	
4	90°	0	R_4	



DH Parameters

- α_i angle between $\mathbf{z}_{i-1}$ and $\mathbf{z}_i$ about $\mathbf{x}_{i-1}$
- d_i distance between O_{i-1} and $\mathbf{z}_i$, along $\mathbf{x}_{i-1}$
- r_i distance between $\mathbf{x}_{i-1}$ and O_i, along $\mathbf{z}_i$
- θ_i angle between $\mathbf{x}_{i-1}$ and $\mathbf{x}_i$ about $\mathbf{z}_i$

i	α	d	r	θ
1	0	0	0	θ_1
2	$90°$	0	R_2	θ_2
3	0	D_3	0	θ_3
4	$90°$	0	R_4	θ_4

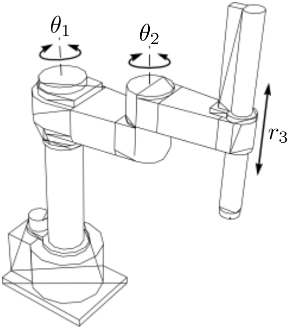


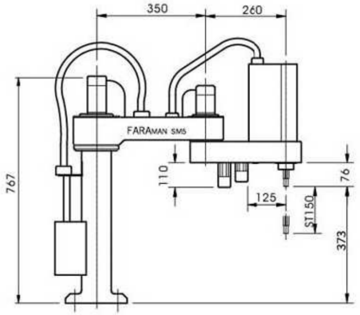
DH Transformation Matrix

- α_i angle between $\mathbf{z}_{i-1}$ and $\mathbf{z}_i$ about $\mathbf{x}_{i-1}$
- d_i distance between O_{i-1} and $\mathbf{z}_i$, along $\mathbf{x}_{i-1}$
- r_i distance between $\mathbf{x}_{i-1}$ and O_i, along $\mathbf{z}_i$
- θ_i angle between $\mathbf{x}_{i-1}$ and $\mathbf{x}_i$ about $\mathbf{z}_i$

$$
\begin{aligned}
{}^{i-1}\mathbf{T}_i &= \mathcal{R}(\mathbf{x}, \alpha_i).\mathcal{T}(\mathbf{x}, d_i).\mathcal{T}(\mathbf{z}, r_i).\mathcal{R}(\mathbf{z}, \theta_i) \\
&= \left(\begin{array}{ccc|c}
\cos(\theta_i) & -\sin(\theta_i) & 0 & d_i \\
\cos(\alpha_i).\sin(\theta_i) & \cos(\alpha_i).\cos(\theta_i) & -\sin(\alpha_i) & -r_i.\sin(\alpha_i) \\
\sin(\alpha_i).\sin(\theta_i) & \sin(\alpha_i).\cos(\theta_i) & \cos(\alpha_i) & r_i.\cos(\alpha_i) \\
\hline
0 & 0 & 0 & 1
\end{array}\right)
\end{aligned}
$$

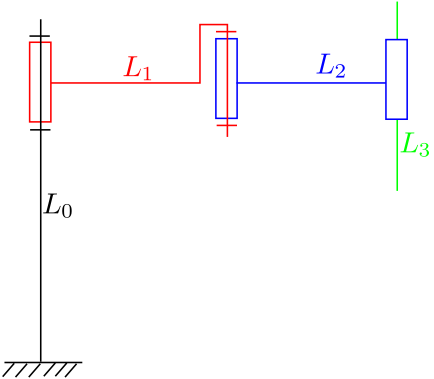
Exercise



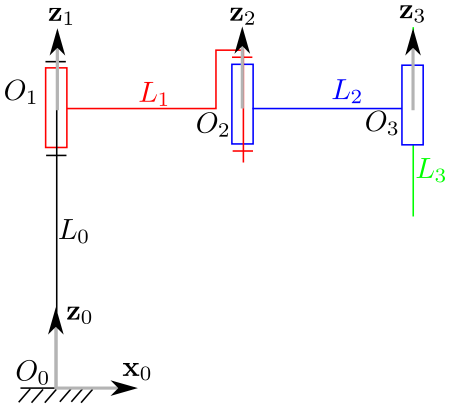


with $\theta_1 = 30°$, $\theta_2 = 12°$ and $r_3 = 80$ mm

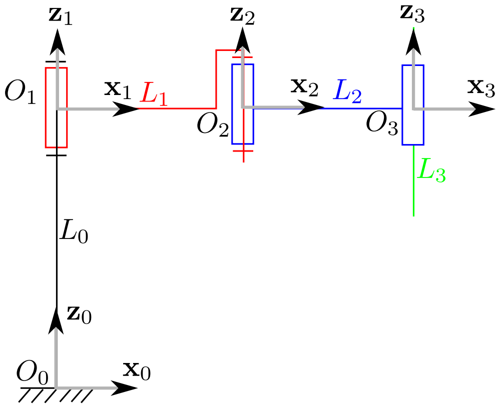
Exercise (Correction)



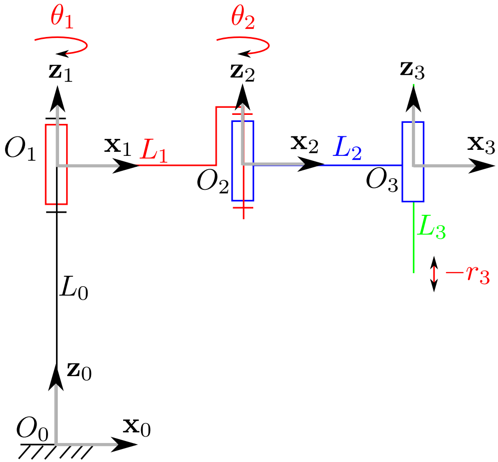
Exercise (Correction)



Exercise (Correction)

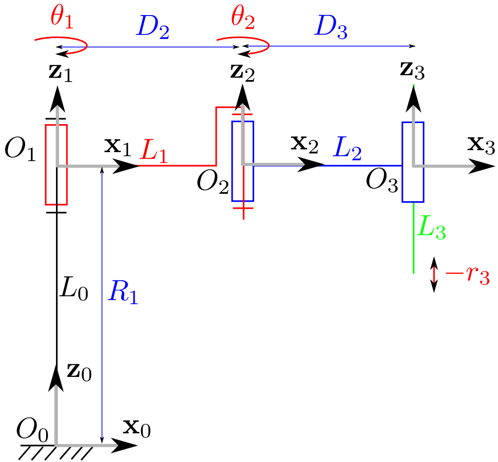


Exercise (Correction)



Exercise (Correction)

i	α	d	r	θ
1	0	0	R_1	θ_1
2	0	D_2	0	θ_2
3	0	D_3	-r_3	0



Exercise (Correction)

$$ {}^{0}\mathbf{T}_1 = \left(\begin{array}{ccc|c} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & R_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) $$

$$ {}^{2}\mathbf{T}_3 = \left(\begin{array}{ccc|c} 1 & 0 & 0 & D_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -r_3 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) $$

$$ {}^{1}\mathbf{T}_2 = \left(\begin{array}{ccc|c} \cos(\theta_2) & -\sin(\theta_2) & 0 & D_2 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) $$

$$ {}^{0}\mathbf{T}_1.{}^{1}\mathbf{T}_2.{}^{2}\mathbf{T}_3 = {}^{0}\mathbf{T}_3 = \begin{pmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 & D_2\cos(\theta_1)+D_3\cos(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 & D_2\sin(\theta_1)+D_3\sin(\theta_1+\theta_2) \\ 0 & 0 & 1 & R_1-r_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

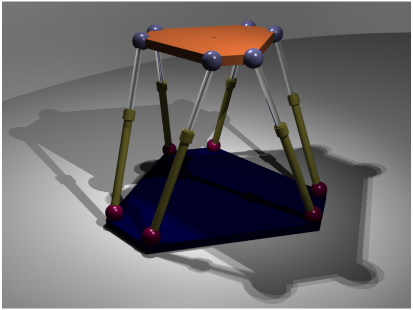
Exercise (Correction)

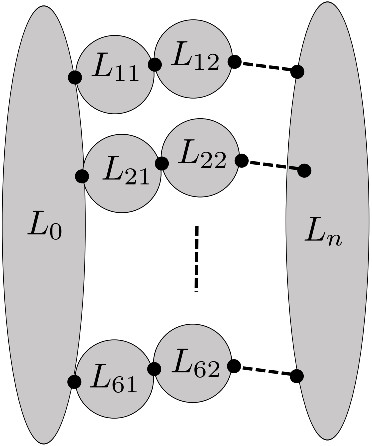
$$^{0}\mathbf{T}_1 = \left(\begin{array}{ccc|c} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & R_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$$^{2}\mathbf{T}_3 = \left(\begin{array}{ccc|c} 1 & 0 & 0 & D_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -r_3 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$$^{1}\mathbf{T}_2 = \left(\begin{array}{ccc|c} \cos(\theta_2) & -\sin(\theta_2) & 0 & D_2 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$$^{0}\mathbf{T}_1 . ^{1}\mathbf{T}_2 . ^{2}\mathbf{T}_3 = ^{0}\mathbf{T}_3 = \begin{pmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 & D_2\cos(\theta_1) + D_3\cos(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 & D_2\sin(\theta_1) + D_3\sin(\theta_1+\theta_2) \\ 0 & 0 & 1 & R_1 - r_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



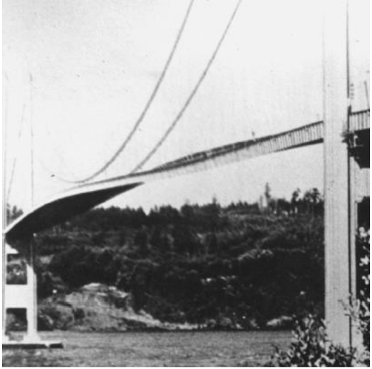

L_{11}
L_{12}
L_{21}
L_{22}
L_0
L_n
L_{61}
L_{62}

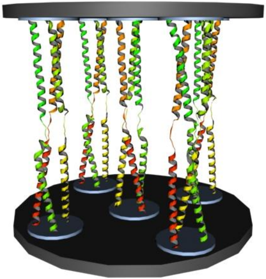
For more information...

A New Geometric Notation for Open and Closed-loop Robots
Khalil and Kleinfinger, in *Robotics and Automation*, 1986.

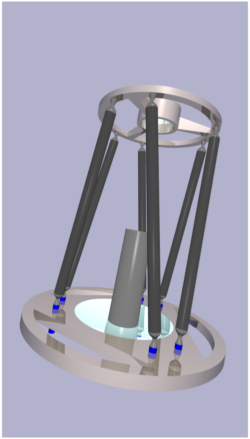
- Problems of DH parameters
- How to describe a tree-structure robot
- How to describe a closed-loop kinematic chain
- etc...

- Non-rigid links
- Backlash
- Important forces
- Stiffness
- etc ...

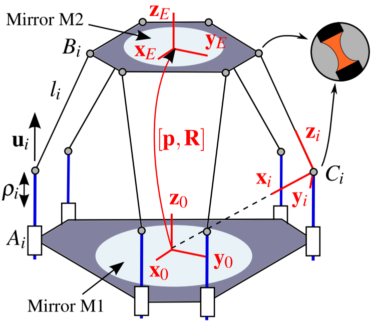


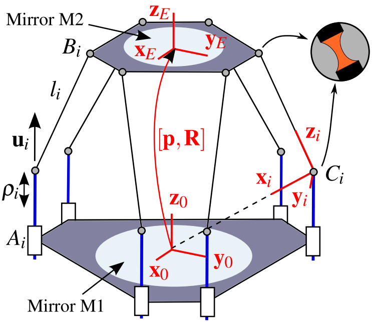


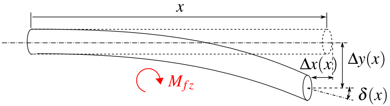
Nano-technology

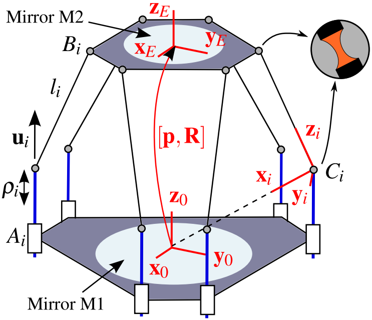


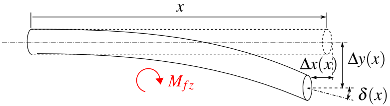
Difficult environmental conditions

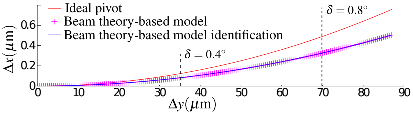

Mirror M2
$\mathbf{z}_E$
$\mathbf{y}_E$
$\mathbf{x}_E$
B_i
l_i
$\mathbf{u}_i$
ρ_i
$[\mathbf{p}, \mathbf{R}]$
$\mathbf{z}_i$
$\mathbf{x}_i$
$\mathbf{y}_i$
C_i
$\mathbf{z}_0$
A_i
$\mathbf{y}_0$
$\mathbf{x}_0$
Mirror M1

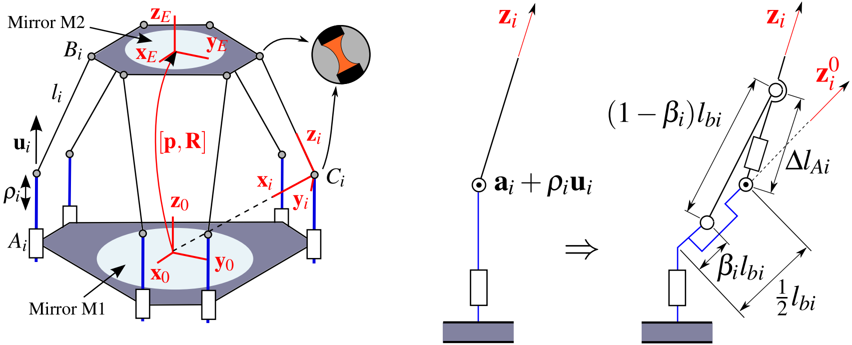

Mirror M2
$\mathbf{z}_E$
B_i
$\mathbf{y}_E$
$\mathbf{x}_E$
l_i
$\mathbf{u}_i$
$[\mathbf{p}, \mathbf{R}]$
$\mathbf{z}_i$
C_i
ρ_i
$\mathbf{x}_i$
$\mathbf{y}_i$
$\mathbf{z}_0$
A_i
$\mathbf{y}_0$
$\mathbf{x}_0$
Mirror M1

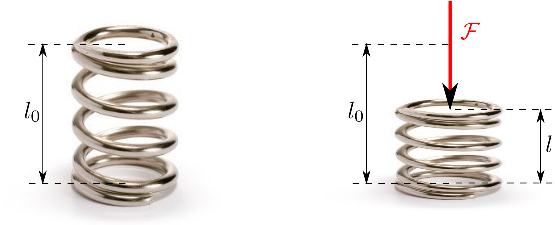

x
$\Delta y(x)$
$\Delta x(x)$
M_{fz}
$\delta(x)$


Mirror M2
$\mathbf{z}_E$
B_i
$\mathbf{y}_E$
$\mathbf{x}_E$
l_i
$\mathbf{u}_i$
$[\mathbf{p}, \mathbf{R}]$
$\mathbf{z}_i$
C_i
ρ_i
$\mathbf{x}_i$
$\mathbf{y}_i$
$\mathbf{z}_0$
A_i
$\mathbf{y}_0$
$\mathbf{x}_0$
Mirror M1

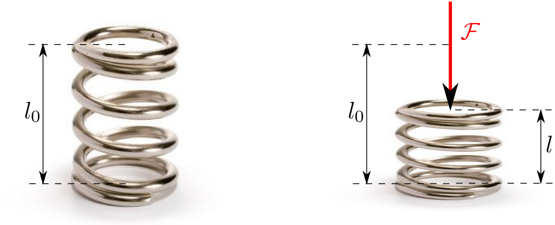

x
$\Delta y(x)$
$\Delta x(x)$
M_{fz}
$\delta(x)$


Ideal pivot
Beam theory-based model
Beam theory-based model identification
$\delta = 0.8°$
$\delta = 0.4°$
$\Delta x(\mu m)$
$\Delta y(\mu m)$
0
0.2
0.4
0.6
0
10
20
30
40
50
60
70
80
90

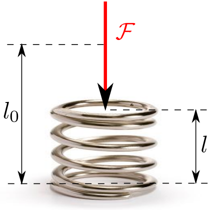

Mirror M2
$\mathbf{z}_E$
B_i
$\mathbf{x}_E$
$\mathbf{y}_E$
l_i
$\mathbf{u}_i$
ρ_i
A_i
$[\mathbf{p}, \mathbf{R}]$
$\mathbf{z}_i$
$\mathbf{x}_i$
$\mathbf{y}_i$
C_i
$\mathbf{z}_0$
$\mathbf{y}_0$
$\mathbf{x}_0$
Mirror M1
$\mathbf{z}_i$
$\mathbf{a}_i + \rho_i \mathbf{u}_i$
$\Rightarrow$
$\mathbf{z}_i$
$\mathbf{z}_i^0$
$(1-\beta_i)l_{bi}$
Δl_{Ai}
$\beta_i l_{bi}$
$\frac{1}{2} l_{bi}$



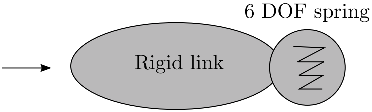
$$\mathcal{F} = k.(l - l_0)$$



$$\mathcal{F} = k.(l - l_0)$$

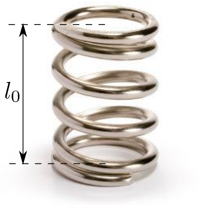


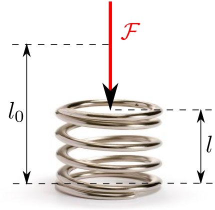
$$\mathcal{F} = k.(l - l_0)$$



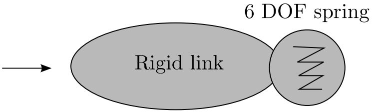
$$\mathbf{F} = \mathbf{K}.\Delta\mathbf{X}$$

with $\mathbf{K}$ a 6×6 matrix



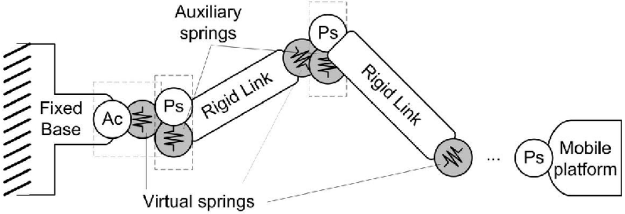


$$\mathcal{F} = k.(l - l_0)$$



$$\mathbf{F} = \mathbf{K}.\Delta\mathbf{X}$$

with $\mathbf{K}$ a 6×6 matrix



(b) VJM model of serial kinematic chain

Stiffness Mapping for Parallel Manipulators
Gosselin, in *IEEE Transactions on Robotics and Automation*, 1990.

Stiffness Analysis of Parallel Manipulators with Preloaded Passive Joints
Pashkevich, Klimchik and Chablat, in *Advances in Robot Kinematics*, 2010.

How to model a wire-driven parallel robot ? ?