Calibration of Parallel Robot

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A cable-driven Parallel robot

LIRMM

LASMEA

Université Blaise Pascal

tecnalia

Démonstrateur du projet CoGiRo

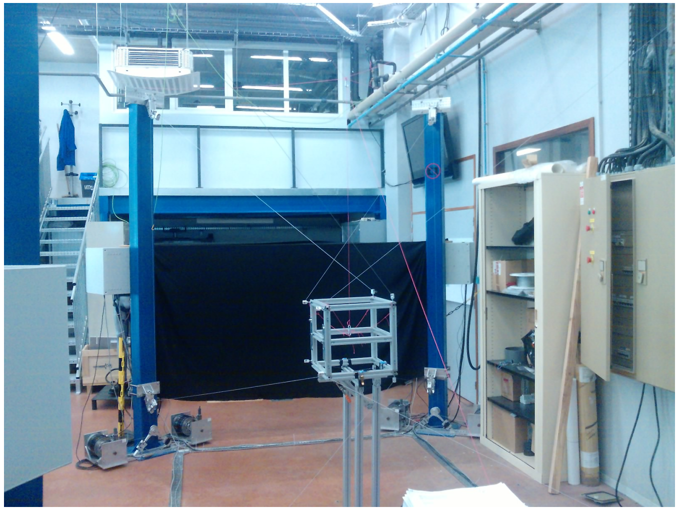
http://www2.lirmm.fr/cogiro/

PROJET FINANCÉ PAR L'ANR - PROJECT FUNDED BY THE ANR

ANR

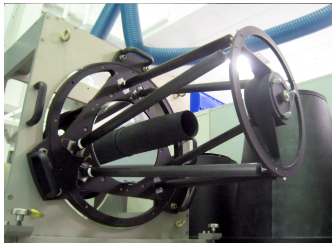
la Région Languedoc Roussillon

Robots



Prototype Cable robot

CMW milling machine



Thales

Errors decease the robot accuracy (Static)

Geometric errors

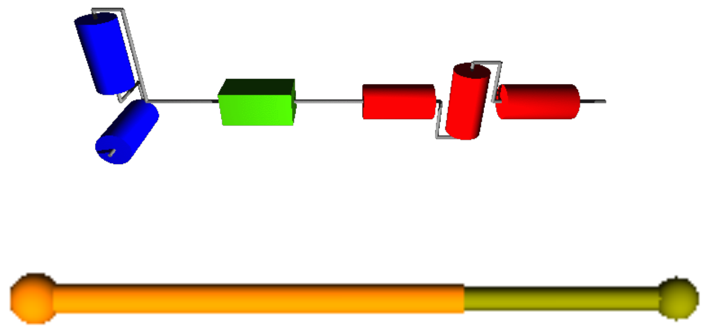
- Inaccuracy on Kinematic parameters values (i.e. position/orientation of articulations)

Nongeometric errors

- Sensors based parameters (gain, offset)
- Thermal effects
- Joint, base flexibilities

Model errors

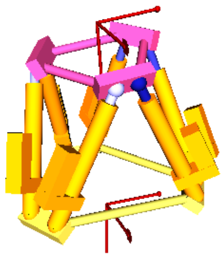
Exemple

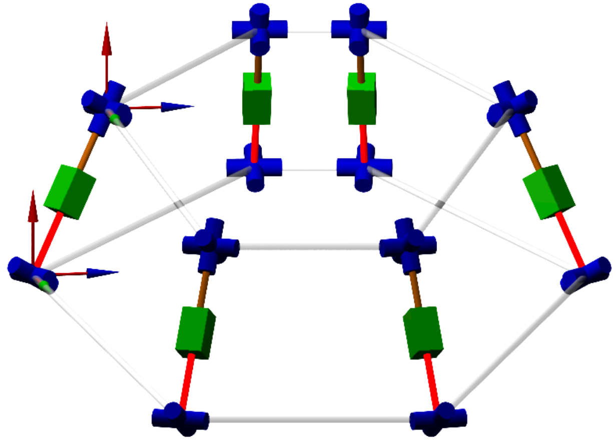


Aim of Calibration

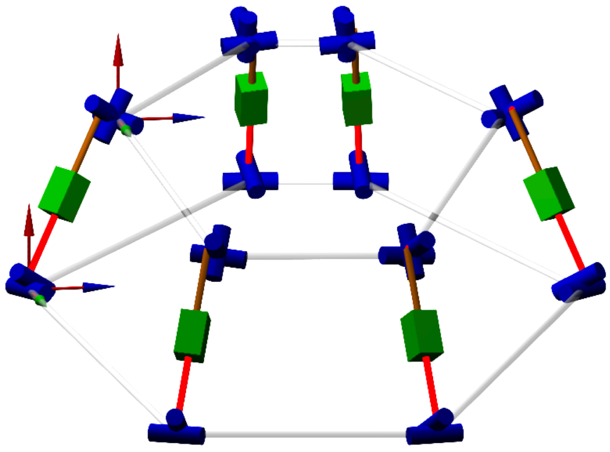
Aim: to improve the robot accuracy

- Identification of kinematic parameters by a Calibration procedure





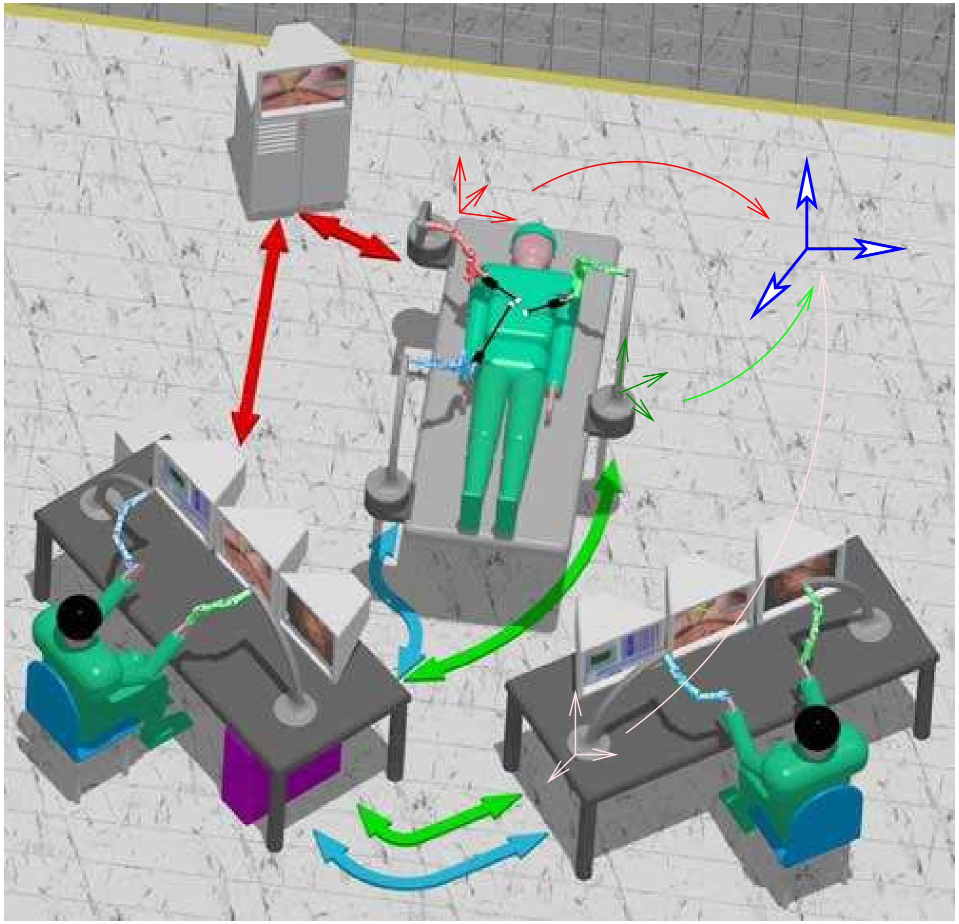
"Theoretical" Robot Model



Actual robot

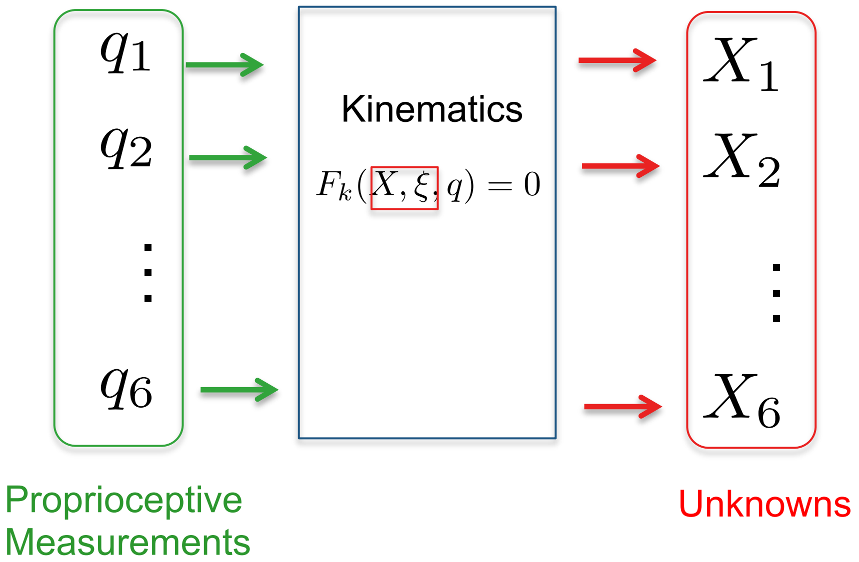
Type of Calibration

Internal / external calibration



Create a redundant information on the robot state

For one configuration of a robot k



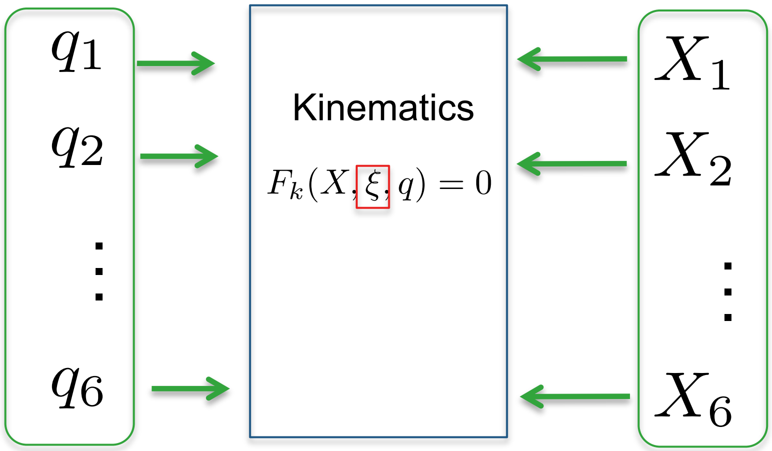
Unknowns : $k \times dim(X) + dim(\xi)$

Equations : $k \times dim(F_k)$

?!!!

Create a redundant information on the robot state

For one configuration of a robot k



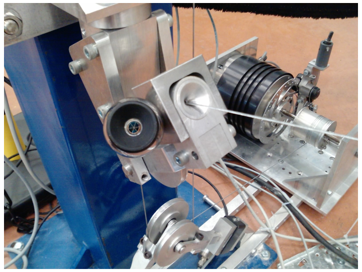
Internal Measurements

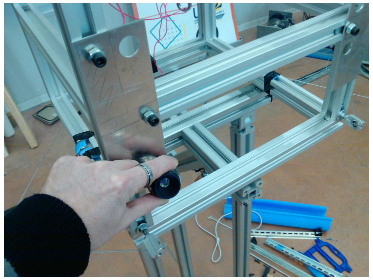
External Measurements

Unknowns : $dim(\xi)$

Equations : $k \times dim(F_k)$

Unknowns < Equations

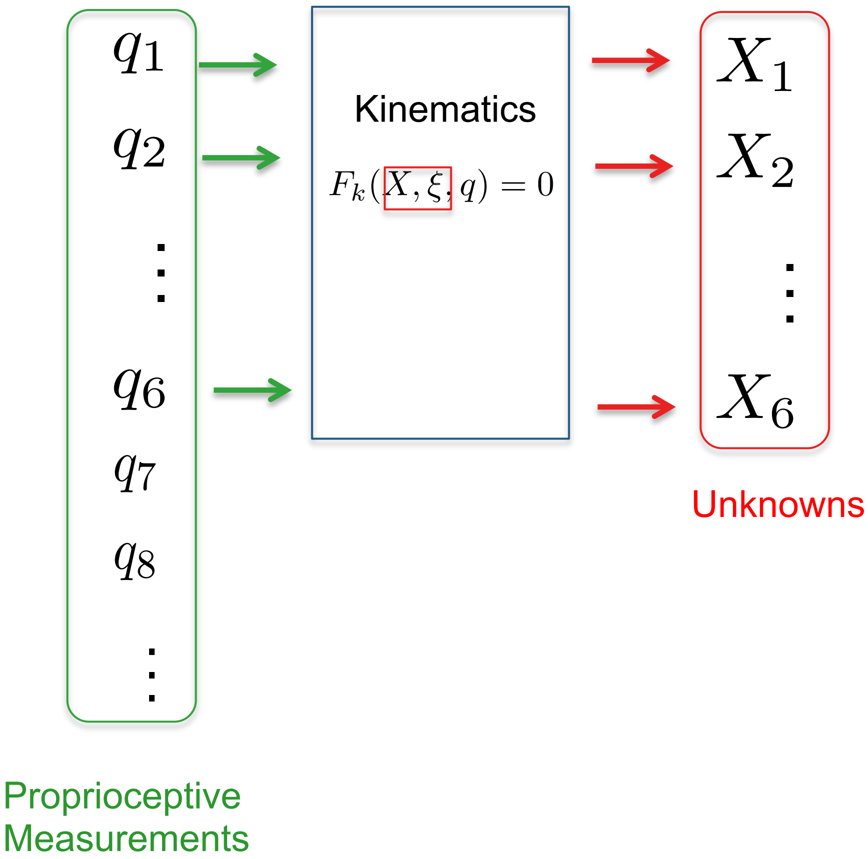




Additional exteroceptive measurement

Create a redundant information on the robot state

For one configuration of a robot k



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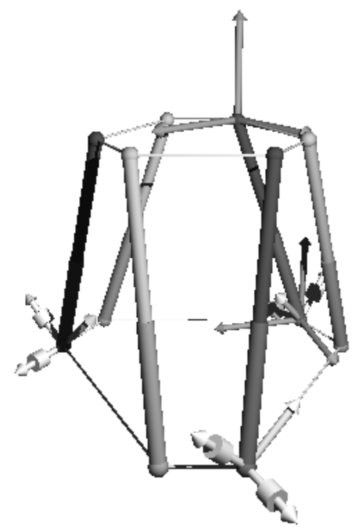
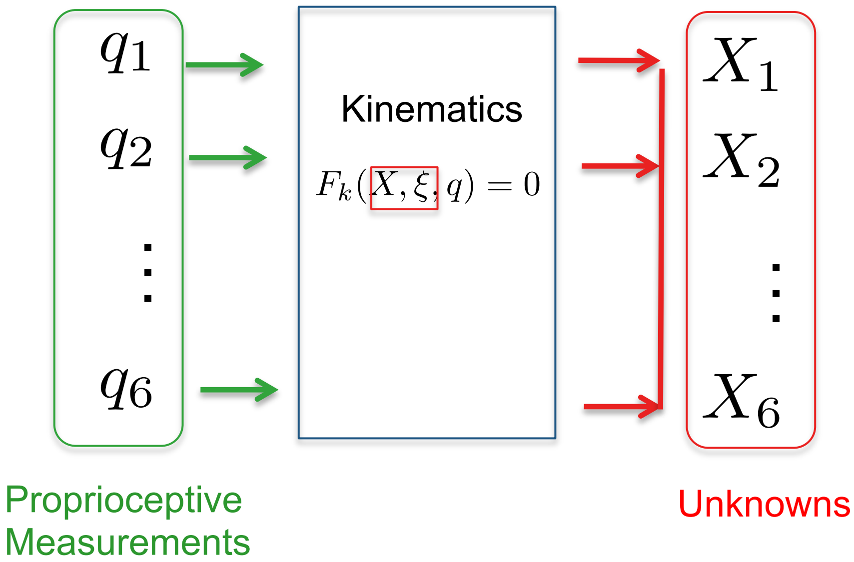


Create a redundant information on the robot state

For one configuration of a robot k

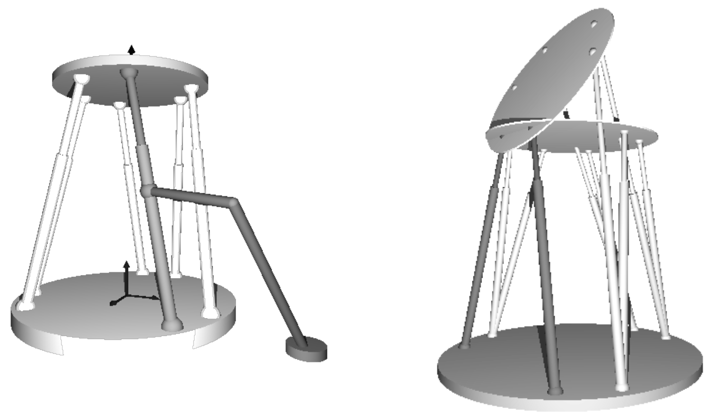


Unknowns : $dim(X) + dim(\xi)$

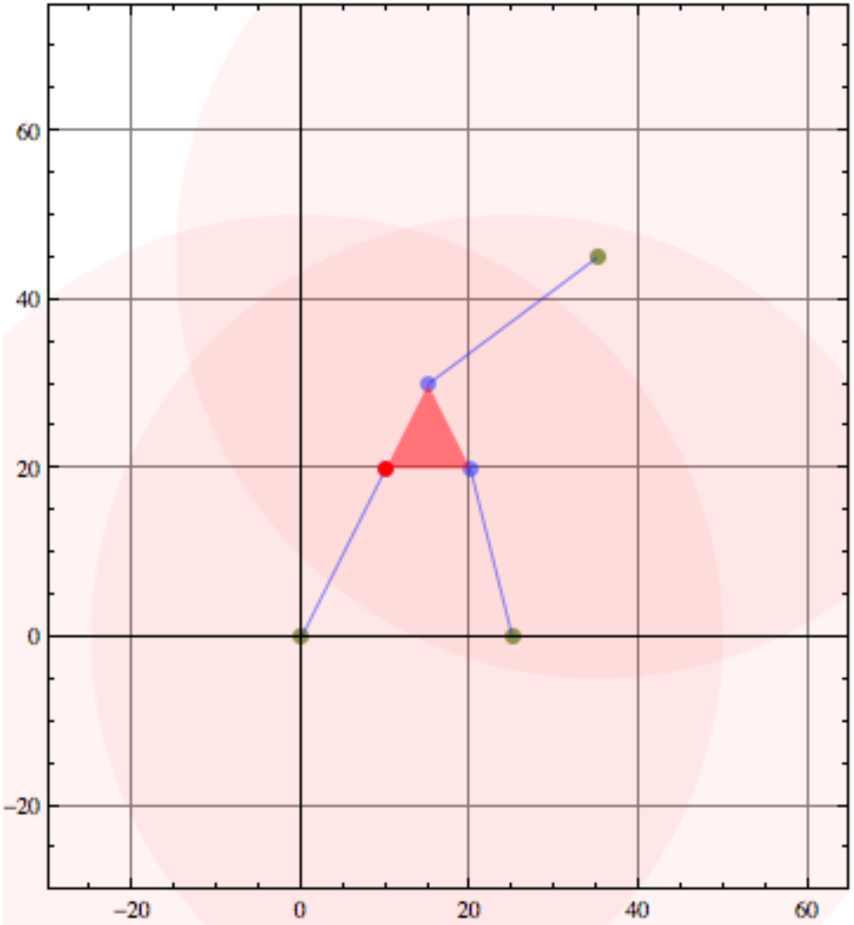
Equations : $k \times dim(F_k)$

Unknowns < Equations

Additional constraintes



Create a redundant information for a planar 3RPR



Frames:

Ω_0, Ω_c

Kinematics parameters:

$A_i[a_i]_{\Omega_0},\ B_i[b_i]_{\Omega_c},\ i = 1, 2, 3$

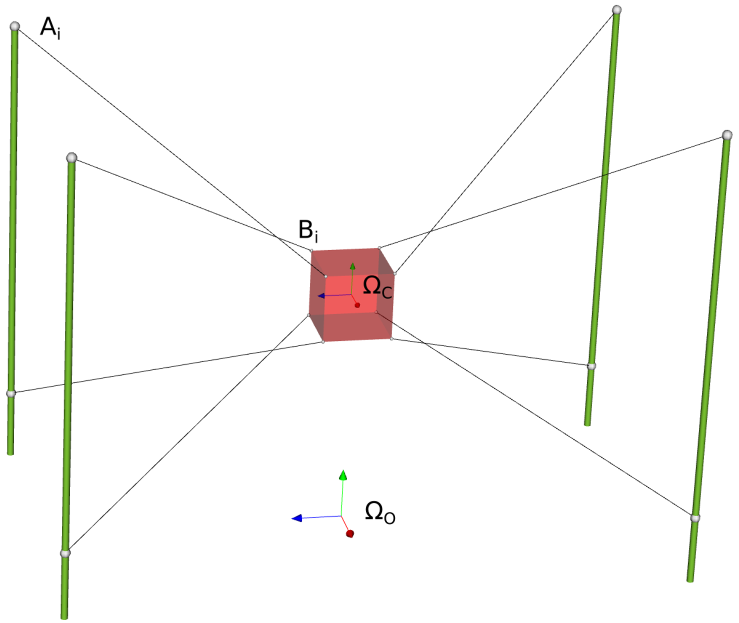
Generalized coordinates:

$P(x, y), R(\theta)$

Articular coordinates:

$l_i,\ i = 1, 2, 3$

Cable-driven Robot Model



Ω_o, Ω_creference/Mobile frame

$X_k[P_k, R_k]$, generalized coordinates of the k^{th} configuration

$L_{i,k}$articular coordinates, $i = 1 \dots 8$ cables

τ_i, Cable tension and $\mathcal{F}$, External wrenches

Kinematic parameters

$A_i[a_i]_{\Omega_o}$base points, $B_i[b_i]_{\Omega_c}$mobile points, $i = 1 \dots 8$ cables

ΔL_iOffset on the cable length

$\xi_i = [a_i, b_i, \Delta L_i]$, Kinematic parameters

Mesurements

$\hat{\rho}_{i,k}$, variation of cable length

$\hat{X}_k$ position, orientation for calibration case

$\hat{\tau}_{i,k}$ cable tensions

Models

Ω_o, Ω_creference/Mobile frame

$X_k[P_k, R_k]$, generalized coordinates of the k^{th} configuration

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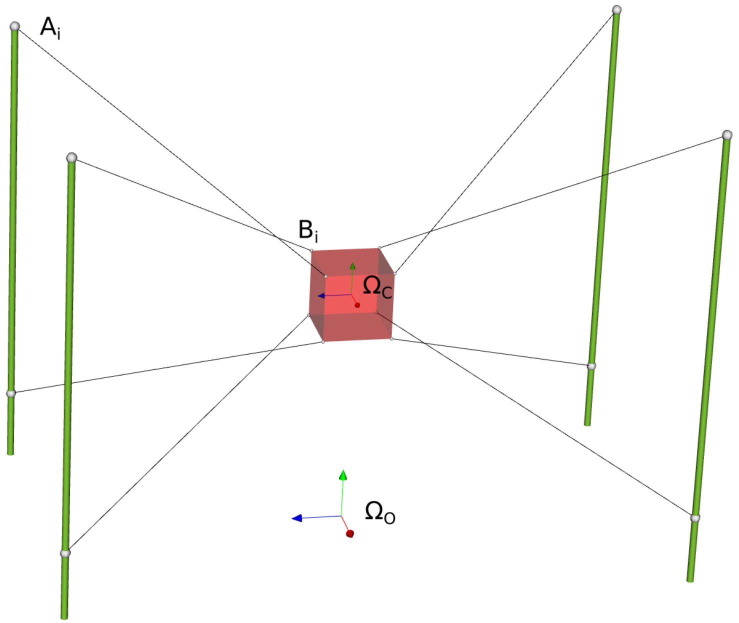

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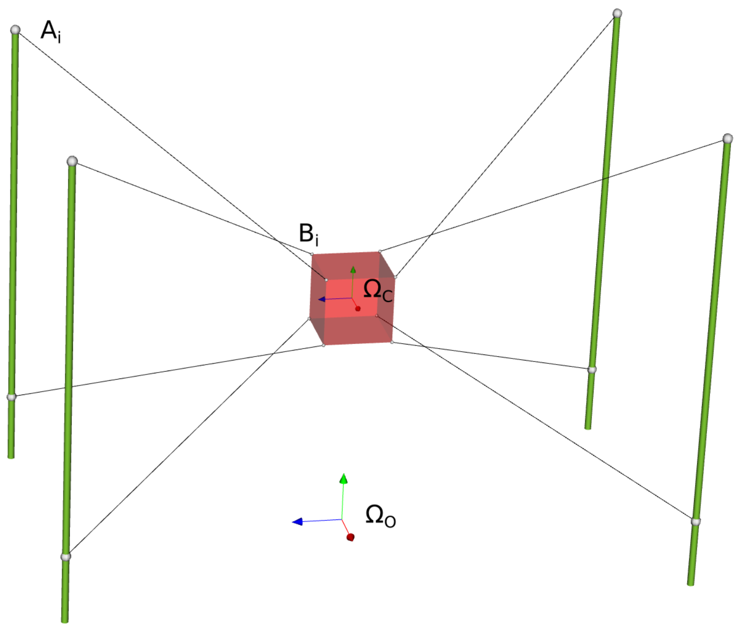
$\hat{\tau}_{i,k}$ cable tensions

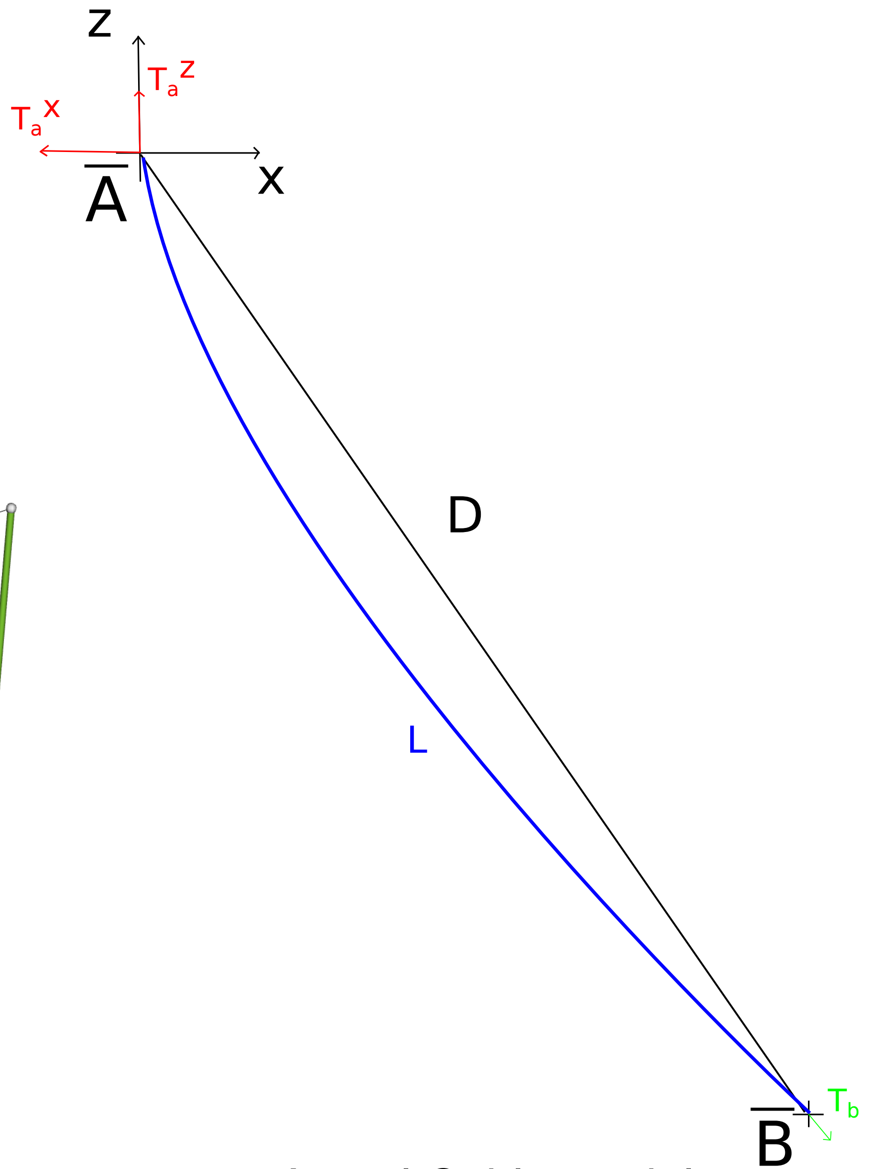
Implicit kinematic equations: $F = d[A_i, B_i(X_k)] - (\hat{\rho}_{i,k} + \Delta L_i) \sim 0$

Static relation: $F = \mathcal{F} + J^{-T}(X_k).\hat{\tau} \sim 0$, with $J^{-1}(X_k)$the inverse kinematic jacobian matrix

Model error

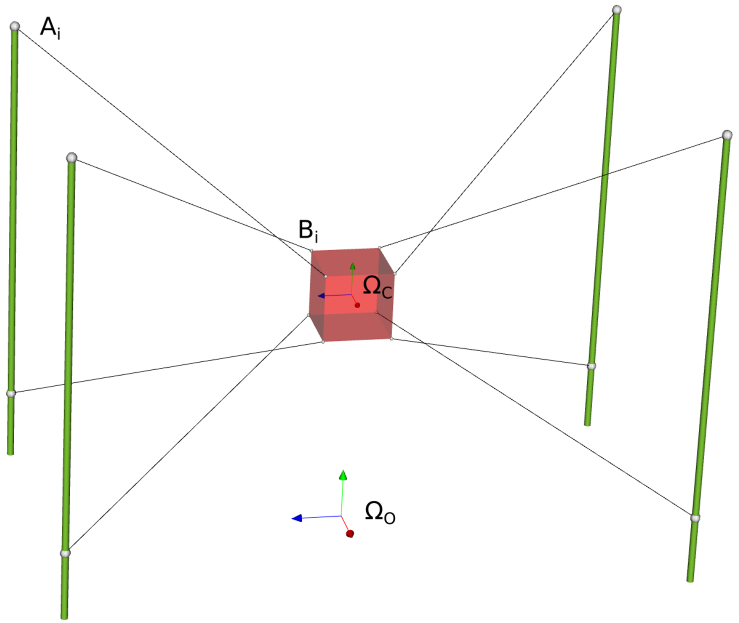


Simplified Cable model



Actual Cable model

Basic Calibration Equations (with external measurement)



Direct Method

$$\mathcal{C}_k = X_k^m - X_k^{fk}(q_k, \xi)$$

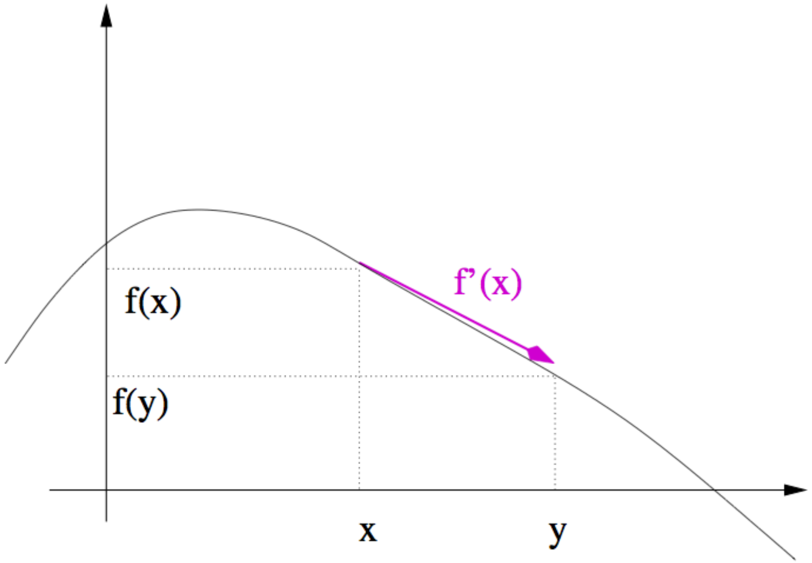
Inverse Method

$$\forall j, \mathcal{C}_{i,k} = q_{k,j}^{IK}(X_k^m, \xi_j) - q_{k,j}$$

Least square solution

$$\mathrm{argmin}_\xi\ \mathcal{C}^t\mathcal{C}$$

Newton Method



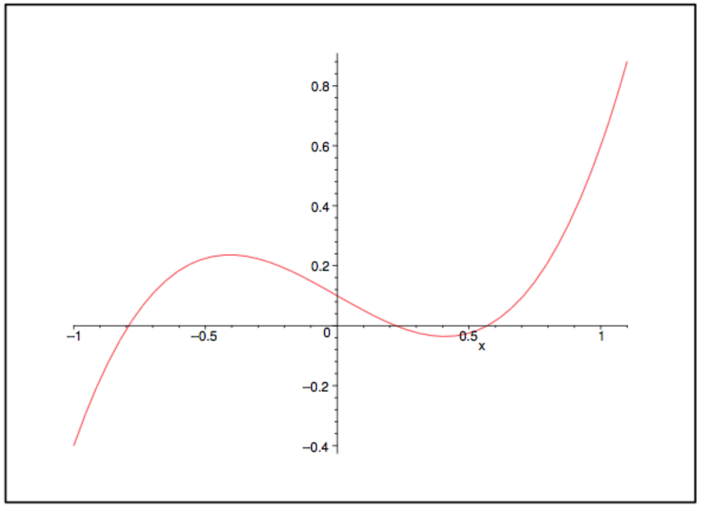
Solve $f(x) = 0$ with $x \sim x_0$

$$f(x_0) - f(x) \sim f^{'}(x_0).(x_0 - x)$$

$$x \sim x_0 - \frac{f(x_0)}{f'(x_0)}$$

Newton Scheme: $x_{k+1} - \frac{f(x_k)}{f'(x_k)}$

Newton Method: example



$$x^3 - 0.5 \times x + 0.1 = 0$$

- $f(x) = x^3 - 0.5 \times x + 0.1$
- $f'(x) = 3.x^2 - 0.5$
- $x_{k+1} = x_k - \frac{x^3 - 0.5 \times x + 0.1}{3 \times x^2 - 0.5}$

x_0	0	1	-0.5	-0.4
x_1	0.2	0.76	-1.4	11.4
x_2	0.2211	0.6310	-1.0387	7.6095
x_3	0.2218	0.5796	-0.8555	5.0871
x_4		0.5699	-0.7975	3.4121
x_5		0.5696	-0.7915	2.3048
x_6			-0.7914	1.5799
x_7				1.1143
x_8				0.8270
x_9				0.6645
x_{10}				0.5903
x_{11}				0.5710
x_{12}				0.5696

Newton Method: example

Compute: $\sqrt{3}$

Using: $\{+, \times, \div\},\ 5, 2$

Newton Method: example

Compute: $\sqrt{3}$

Using: $\{+, \times, \div\},\ 5, 2$

Solve: $x^2 - N = 0 \ \ (N = 3)$

- $x_{k+1} = x_k - \frac{x^2 - N}{2.x}$
- $x_{k+1} = \frac{1}{2}(x_k + \frac{N}{x_k})$
- $x_0 = 5$, $x_1 = 2.8$, $x_2 = 1.9357$, $x_3 = 1.7428$, $x_4 = 1.7321$.

Least Square Method (Linear)

Singular Value Decomposition SVD

$A_{m \times n} Matrix$

$A = U \Sigma V^T$

$U_{m \times m},$ Orthogonal matrices $U = [u_1, \ldots, u_m]$

$V_{n \times n}$ Orthogonal matrices $V = [v_1, \ldots, v_n]$

$\Sigma = diag(\sigma_1, \ldots, \sigma_p), p = min(m, n), \sigma_i$ a Singular Value

$r = rank(A)$

Least Square Method (Linear)

A Least Square Solution $Ax = b$

$$x_{LS} = \Sigma_{i=1}^{r} \frac{u_i^t.b}{\sigma_i}.v_i$$

Such that

$$\rho_{LS} = \|A.x_{LS} - b\|_2 = \min_x \|Ax - b\|_2$$

$$\rho_{LS}^2 = \|A.x_{LS} - b\|_2^2 = \Sigma_{i=r+1}^{m}(u_i^t.b)^2$$

Proof

Least Square Method (Linear)

Pseudo Inverse of A

$$A^+ = V\Sigma^+U^T$$

with $\Sigma^+_{n\times m} = diag(\frac{1}{\sigma_1}, \ldots, \frac{1}{\sigma_r}, 0, \ldots, 0)$

Over constraint system of equations, m>n

$$A^+ = (A^TA)^{-1}A^T$$

Note, Right pseudo-inverse for Under constraint system of equations, m<n

$$A^+ = A^T(AA^T)^{-1}$$

Non Linear Least Square Method

Unknowns p_n

$F(p) = [F_1(p), \ldots, F_m(p)]^T$

$m \geq n$

Determine p such that $Min_{\bar{p}=p}\ C(p)$

$$C(p) = \frac{1}{2}F(p)^T F(p) = \frac{1}{2}\Sigma_{i=1}^{m} F_i(p)^2$$

Non Linear Least Square Method

Identification Jacobian

$$J(p) = \begin{pmatrix} \frac{\partial F_1(p)}{\partial p_1} & \cdots & \frac{\partial F_1(p)}{\partial p_n} \\ \vdots & \cdots & \vdots \\ \frac{\partial F_m(p)}{\partial p_1} & \cdots & \frac{\partial F_m(p)}{\partial p_n} \end{pmatrix}$$

Taylor Series of $C(p) = \frac{1}{2} F(p)^T F(p) = \frac{1}{2} \Sigma_{i=1}^{m} F_i(p)^2$

- First order
- Second Order

Non Linear Least Square Method

- Gradient Method
- Newton Method
- Gauss-Newton Method
- Levenberg-Marquart Method

Identifiatiblity an Numerical Conditionning

Problem of parameters identifiability

Check the number of identifiable parameters by QR decomposition

$$J(p) = Q_{m \times m} \begin{pmatrix} R_{n_p \times n_p} \\ 0_{m-n_p} \end{pmatrix}$$

Q: Orthogonal matrix
R: upper-triangular matrix

non Identifiable parameters number, number of $R_{i,i} = 0$

What is 0 ?

$$\xi = m.\epsilon.\max_i \|R_{i,i}\|$$

Open problem …

Identifiatiblity an Numerical Conditionning

- Incorporating A priori Parameter Estimates

- Confidence Measure of Parameter Estimates

Identifiatiblity an Numerical Conditionning

Observability

Observability Indices …
Optimal Experimental Design

Scaling

Task Variable Scaling
Parameter Scaling

Calibration examples