

# A Numerical Evaluation of the Stiffness of CaHyMan (Cassino Hybrid Manipulator)

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## Abstract

*In this paper the novel hybrid manipulator, named as CaHyMan (Cassino Hybrid Manipulator), is analyzed in term of stiffness characteristics. A formulation is presented to deduce the stiffness matrix as a function of the most important stiffness parameters of the mechanical design. The specific design of CaHyMan, which has been designed and built at the Laboratory of Robotics in Cassino, Italy, helps to obtain closed-form expressions. A formulation for a stiffness performance index is proposed by using the obtained stiffness matrix. A numerical investigation has been carried out on the effects of design parameters and results are discussed in the paper.*

## 1. Introduction

In the last decade hybrid manipulators addressed great attention as a combination of serial and parallel chain architectures. Some significant examples are: ARTISAN from Stanford University (USA), [1; 2]; HRM from Korea Institute of Machinery and Materials (Korea), [3]; GEORGV from Institute of Production Engineering and Machine Tools (Germany), [4]; UPSarm from University of California at Davis (USA), [5].

A novel hybrid manipulator, named as CaHyMan (Cassino Hybrid Manipulator) has been designed and built at the Laboratory of Robotics and Mechatronics in Cassino, Italy. This prototype, Figs.1 and 2, is based on the mechanical design of a built prototype of CaPaMan (Cassino Parallel Manipulator), [6; 7], by adding to it a telescopic arm.

## 2. The design of CaHyMan

The proposed hybrid manipulator CaHyMan is shown in the kinematic sketch of Fig.2 as a combination of the parallel chain of CaPaMan with a telescopic arm architecture. In particular, the telescopic arm is installed on the mobile plate MP of CaPaMan. The aim of this assembly is that the parallel architecture will work as an intelligent compliant base for the telescopic arm, which will operate in static or quasi- static state for a-priori determined task.

The CaHyMan prototype has five dofs: three dofs are given by CaPaMan, and two more are given by the serial chain. It is composed of a movable plate MP which is connected to a fixed plate FP by means of three leg mechanisms. Each leg mechanism is composed of an articulated parallelogram AP whose coupler carries a prismatic joint SJ, a connecting bar CB which transmits the motion from AP to MP through SJ, and a spherical joint BJ which is installed on MP. The size of MP and FP are given by

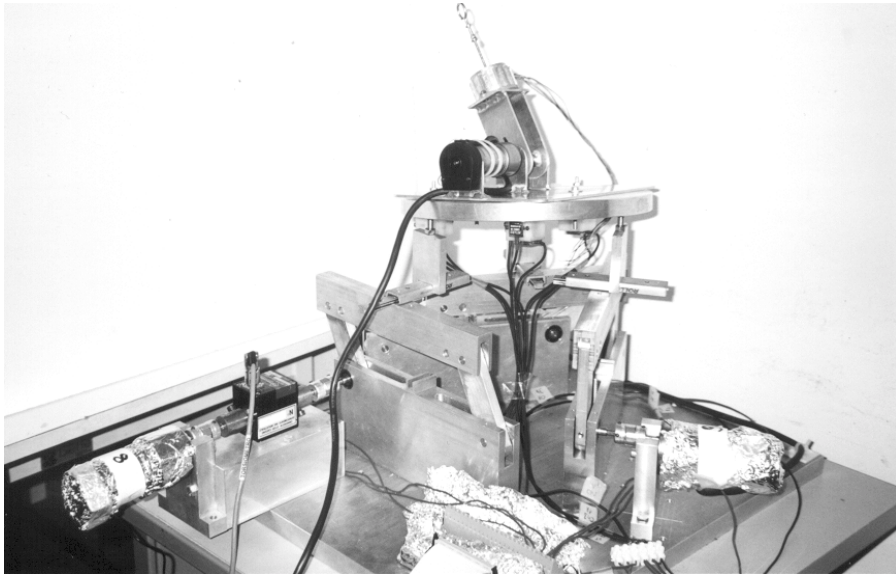


Fig.1 The built prototype of CaHyMan (Cassino Hybrid Manipulator) at Laboratory of Robotics and Mechatronics in Cassino.

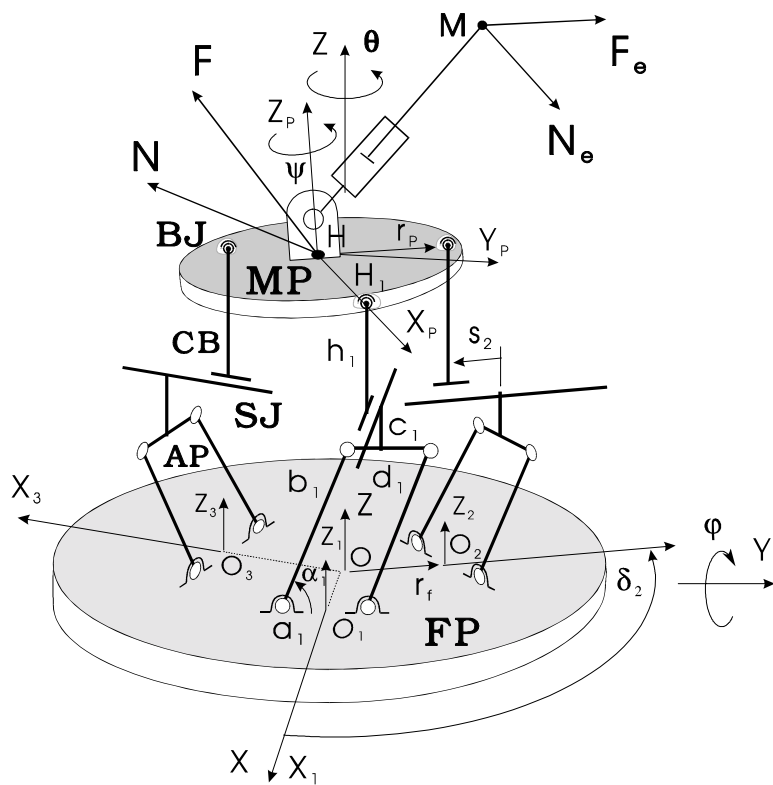


Fig.2 Kinematic chain and parameters for the parallel–serial hybrid manipulator named CaHyMan.

$r_p$  and  $r_p'$  respectively, Fig.2.

The design parameters for the parallel-serial manipulator CaHyMan are ( $k=1,2,3$ ):  $a_k=c_k$ ,  $b_k=d_k$ , links of the  $k$ -th leg mechanism;  $h_k$ , the length of the connecting bar;  $\alpha_k$ , the input crank angle;  $s_k$ , the stroke of the prismatic joint; HM, the length of the telescopic arm;  $s_k$  the stroke of the prismatic joint of the telescopic arm;  $\alpha_k$  the revolute joint angle; and  $\lambda$ , the angle, that locates the telescopic arm frame with respect to the mobile frame  $X_p Y_p Z_p$ .

The manipulative capability of the parallel-serial chain can be described by the position of the extremity point M, and the orientation of the telescopic link through the orientation angles which take into account the angles  $\varphi$ ,  $\vartheta$ ,  $\psi$ ,  $\lambda$  and  $\alpha_4$ . Similarly, the static behavior of the CaHyMan can be described by the actions exerted by the extremity link at M for given actions of the actuators, or vice versa.

### 3. CaHyMan Stiffness Properties

The manipulating performances of a robot are strictly related to stiffness properties. If the stiffness of links and joints are inadequate, external forces and moments may cause large deflections in the links bodies, which are undesirable from the viewpoint of both accuracy and payload performances.

The stiffness properties of the hybrid manipulator can be deduced using a quality index such as the determinant of the stiffness matrix. The stiffness matrix can be obtained in a closed-form expression as a function of the most important stiffness parameters of the links of the serial and parallel chain.

The displacement  $\Delta \mathbf{X}_{\text{CaHyMan}}$  of the hybrid manipulator CaHyMan is the sum of parallel and serial structure displacements  $\Delta \mathbf{X}_{\text{PAR}}$  and  $\Delta \mathbf{X}_{\text{SER}}$ , respectively, which are due to the action  $\mathbf{F}_M=(\mathbf{F}_e, \mathbf{N}_e)$  of external force  $\mathbf{F}_e$  and torque  $\mathbf{N}_e$ . This can be expressed as

$$\Delta \mathbf{X}_{\text{CaHyMan}} = \Delta \mathbf{X}_{\text{PAR}} + \Delta \mathbf{X}_{\text{SER}} \quad (1)$$

where

$$\Delta \mathbf{X}_{\text{PAR}} = \mathbf{K}_{\text{PAR}}^{-1} \mathbf{F}_M \quad (2)$$

$$\Delta \mathbf{X}_{\text{SER}} = \mathbf{K}_{\text{SER}}^{-1} \mathbf{F}_M$$

in which  $\mathbf{K}_{\text{PAR}}$  and  $\mathbf{K}_{\text{SER}}$  are the 6x6 stiffness matrix for the parallel and serial chain. described with respect to  $X_p Y_p Z_p$  frame.

Thus, the stiffness matrix of CaHyMan can be written as

$$\mathbf{K}_{\text{CaHyMan}}^{-1} = \mathbf{K}_{\text{PAR}}^{-1} + \mathbf{K}_{\text{SER}}^{-1} \quad (3)$$

#### 3.1. The stiffness matrix of the serial chain

The stiffness behavior of the serial chain depends on the following parameters, Fig.3:  $K_4$ , the stiffness of the  $QM_2$  link;  $K_S$ , the stiffness of the linear motor and  $MM_2$  link;  $K_{T4}$ , the stiffness of motor located in Q.

The scheme of Fig.3 can be considered to give

$$\mathbf{K}_{\text{SER}} = \mathbf{R}_{\text{SP}} \quad \mathbf{K}_{\text{SS}} \quad \mathbf{A}_S^{-1} \quad (4)$$

whose elements can be computed by considering

-  $R_{SP}$ , which is the 6x6 matrix given by

$$R_{SP} = \begin{bmatrix} R_{PAR} & 0 \\ 0 & I \end{bmatrix} \quad (5)$$

with  $I$  as identity 3x3 matrix and  $R_{PAR}$ , which describes the orientation of the mobile frame  $X_p Y_p Z_p$  with respect to the fixed frame  $XYZ$  in term of the Euler angles  $\varphi$ ,  $\theta$  and  $\psi$  for the parallel chain, given by

$$R_{PAR} = \begin{bmatrix} -s\theta s\psi + s\varphi c\theta c\psi & -s\theta c\psi - s\varphi c\theta s\psi & c\theta c\varphi \\ c\theta s\psi + s\varphi s\theta c\psi & c\theta c\psi - s\varphi s\theta s\psi & s\theta c\varphi \\ -c\varphi c\psi & c\varphi s\psi & s\varphi \end{bmatrix} \quad (6)$$

in which  $c\theta = \cos\theta$ ,  $s\theta = \sin\theta$  and so on.

-  $K_{SS}$ , which is expressed as

$$K_{SS} = \begin{bmatrix} K_4 c\alpha_4 & K_s c\alpha_4 & -[K_{T4}/(L+s_4)]s\alpha_4 \\ K_4 s\alpha_4 & K_s s\alpha_4 & [K_{T4}/(L+s_4)]c\alpha_4 \\ 0 & 0 & K_{T4} \end{bmatrix} \quad (7)$$

-  $A_S$ , which is the matrix converting  $\Delta L$ ,  $\Delta s_4$ ,  $\Delta\alpha_4$ , in the links to displacements coordinates, in the form

$$A_S = \begin{bmatrix} c\alpha_4 & c\alpha_4 & -(L+s_4)s\alpha_4 \\ s\alpha_4 & s\alpha_4 & (L+s_4)c\alpha_4 \\ 0 & 0 & 1 \end{bmatrix} \quad (8)$$

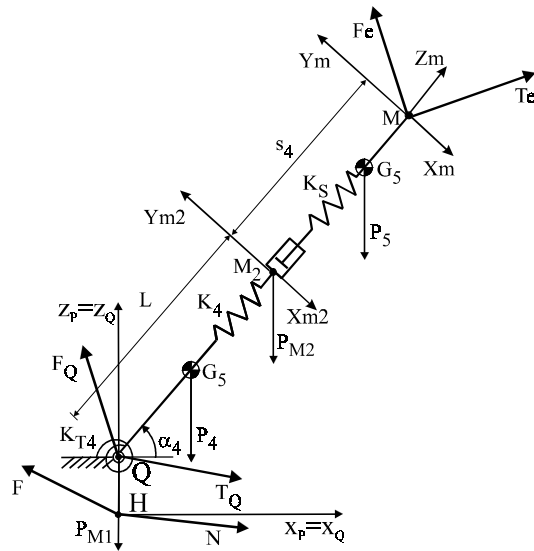


Fig.3 A scheme for the evaluation of statics equilibrium and stiffness matrix in the serial sub-chain of CaHyMan.

### 3.2. The stiffness matrix of the parallel chain

As regards the parallel chain the stiffness matrix  $K_{PAR}$  can be deduced as proposed in [8], for the CaPaMan prototype. Nevertheless, it is necessary to point out that for CaHyMan the application point of external force  $\mathbf{F}_e$  and torque  $\mathbf{T}_e$  is in the point M, Fig.2. Therefore, the reactions at the frame joint of the serial chain  $\mathbf{F}$  and  $\mathbf{N}$  are considered such as external force and torque for the parallel chain.

The stiffness behavior of the parallel chain depends on the following parameters, Fig.4:  $K_{bk}$ ,  $K_{ck}$ ,  $K_{dk}$ ,  $K_{hk}$ , which are the stiffness of the links  $b_k$ ,  $c_k$ ,  $d_k$ ,  $h_k$ , respectively;  $K_{Tk}$ , which is the stiffness of the motors. These stiffness parameters have been defined, by using mathematical models as in [9] and [10].

By using a suitable analysis of the static equilibrium of the deformed architecture through the model of Fig.4 the stiffness matrix  $K_{PAR}$  of CaPaMan module of CaHyMan can be formulated as

$$K_{PAR} = M_{FT}^{-1} \quad M_{FN} \quad K_P \quad C_P^{-1} \quad A_d^{-1} \quad (9)$$

in which

- $M_{FT}$  is the matrix giving  $\mathbf{F}_H=(\mathbf{F},\mathbf{N})$  as function of  $\mathbf{F}_M=(\mathbf{F}_e,\mathbf{T}_e)$  in the form

$$M_{FT} = \begin{bmatrix} -s\alpha_4 & 0 & -c\alpha_4 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ c\alpha_4 + C_{1x} & C_{1y} & -s\alpha_4 + C_{1z} & C_{2x} & C_{2y} & C_{2z} \\ 0 & (L+s_4)s\alpha_4 + HQ & 0 & -s\alpha_4 & 0 & -c\alpha_4 \\ -(L+s_4) - HQs\alpha_4 + C_{3x} & C_{3y} & -HQc\alpha_4 + C_{3z} & C_{4x} & -1 + C_{4y} & C_{4z} \\ 0 & -(L+s_4)c\alpha_4 & 0 & c\alpha_4 & 0 & -s\alpha_4 \end{bmatrix} \quad (10)$$

where the terms  $C_{1j}$ ,  $C_{2j}$ , and  $C_{3j}$  ( $j=x,y,z$ ) are introduced to consider the weights of links,  $m_{HQ}$ ,  $m_{QM}$ ,  $m_{MM2}$ , and motors,  $m_{M4}$  and  $m_{M5}$ , respectively. The above-mentioned parameters can be evaluated as

$$\begin{aligned} C_{1j} &= [9.81 (m_{HQ} + m_{QM} + m_{MM2} + m_{M4} + m_{M5})/6]/F_{ej} \\ C_{2j} &= [9.81 (m_{HQ} + m_{QM} + m_{MM2} + m_{M4} + m_{M5})/6]/T_{ej} \\ C_{3j} &= 1.635 [-m_{M5}(0.5s_4 + L) \quad c\alpha_4 - (0.5m_{M4} L \quad c\alpha_4) - (m_{MM2} \quad L \quad c\alpha_4)]/F_{ej} \\ C_{4j} &= 1.635 [-m_{M5}(0.5s_4 + L) \quad c\alpha_4 - (0.5m_{M4} L \quad c\alpha_4) - (m_{MM2} \quad L \quad c\alpha_4)]/T_{ej} \end{aligned} \quad (11)$$

- $M_{FN}$  is the matrix giving the forces acting on the legs  $\mathbf{F}_R=[R_{x1}, R_{x2}, R_{x3}, R_{z1}, R_{z2}, R_{z3}]^t$  as a function of  $\mathbf{F}_H$  in the form

$$M_{FN} = \begin{bmatrix} s\delta_1 & 0 & s\delta_2 & 0 & s\delta_3 & 0 \\ c\delta_1 & 0 & c\delta_2 & 0 & c\delta_3 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & r_1s\delta_1 & 0 & r_2s\delta_2 & 0 & r_3s\delta_3 \\ 0 & r_1c\delta_1 & 0 & r_2c\delta_2 & 0 & r_3c\delta_3 \\ r_1 & 0 & r_2 & 0 & r_3 & 0 \end{bmatrix} \quad (12)$$

- $K_P$  is the overall stiffness matrix for the CaPaMan legs in the form

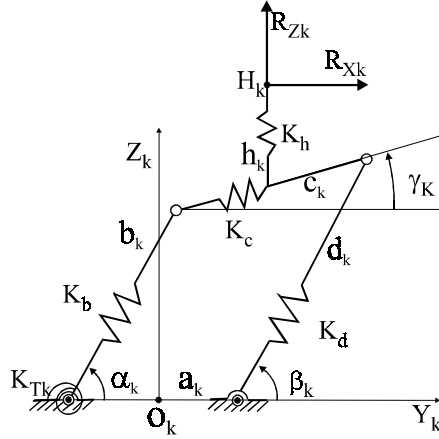


Fig.4 A scheme for the stiffness evaluation of the CaPaMan leg.

$$K_p = \begin{bmatrix} K_{p1} & 0 & 0 \\ 0 & K_{p2} & 0 \\ 0 & 0 & K_{p3} \end{bmatrix} \quad (13)$$

where each  $K_{pk}$  ( $k=1,2,3$ ) can be computed by using the first 2x2 submatrix of  $K_{pkk}$  given by

$$K_{pkk} = \begin{bmatrix} k_{bk} & c\alpha_k & k_{dk} & c\beta_k & -(k_{Tk}/b_k) s\alpha_k \\ k_{bk} & s\alpha_k & k_{dk} & s\beta_k & (k_{Tk}/b_k) c\alpha_k \\ k_{bk} & r_{bk} & -k_{dk} & r_{dk} & (k_{Tk}/b_k) r_{Tk} \end{bmatrix} \quad (14)$$

with

$$\begin{aligned} r_{bk} &= (c_k/2) \sin(\alpha_k + \gamma_k) + h_k \cos(\alpha_k + \gamma_k) \\ r_{dk} &= -(c_k/2) \sin(\alpha_k + \beta_k) + h_k \cos(\alpha_k + \beta_k) \\ r_{Tk} &= (c_k/2) \cos(\alpha_k - \gamma_k) + h_k \sin(\alpha_k - \gamma_k) \end{aligned} \quad (15)$$

-  $C_p$  is the matrix giving the coordinate variations as a function of deformed link parameters for the CaPaMan legs in the form

$$C_p = \begin{bmatrix} C_{p1} & 0 & 0 \\ 0 & C_{p2} & 0 \\ 0 & 0 & C_{p3} \end{bmatrix} \quad (16)$$

where each  $C_{pk}$  ( $k=1,2,3$ ) can be computed by using the first 2x2 submatrix of  $C_{pkk}$  given by

$$C_{pkk} = \begin{bmatrix} c\alpha_k - [(c_k - 2h_k)/2c_k] s\alpha_k & [(c_k - 2h_k)/2c_k] s\alpha_k & -b_k [(3c_k - 2h_k)/2c_k] \\ [(3c_k - 2h_k)/2c_k] s\alpha_k & -[(c_k - 2h_k)/2c_k] s\alpha_k & b_k [(3c_k - 2h_k)/2c_k] \\ s\alpha_k/c_k & -s\alpha_k/c_k & b_k/c_k \end{bmatrix} \quad (17)$$

- $A_d$  is the matrix converting the displacements  $\Delta \mathbf{v} = [\Delta y_1, \Delta z_1, \Delta y_2, \Delta z_2, \Delta y_3, \Delta z_3]^t$  of the articulation points to the displacement coordinates  $\Delta \mathbf{X}_{PAR} = [\Delta x_H, \Delta y_H, \Delta z_H, \Delta \phi, \Delta \theta, \Delta \psi]^t$  of the movable plate in the form

$$A_d = \begin{bmatrix} C_x & 0 & -1/\sqrt{3} & 0 & 1/\sqrt{3} & 0 \\ 1 & 0 & C_y & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 1/3 & 0 & 1/3 \\ C_\phi & -2/3r_p & 0 & 2/3r_p & 0 & 2/3r_p \\ 0 & 0 & 0 & -\sqrt{3}/C_\psi & 0 & -\sqrt{3}/C_\psi \\ 1/C_\theta & 0 & 1/C_\theta & \sqrt{3}/C_\psi & 1/C_\theta & -\sqrt{3}/C_\psi \end{bmatrix} \quad (18)$$

in which  $C_x, C_y, C_z, C_\phi, C_\psi$  and  $C_\theta$  can be evaluated, in a recursive way, as

$$\begin{aligned} C_x &= -(r_p/2\Delta\Delta_1)(1 - \sin \Delta\phi) \cos(\Delta\psi - \Delta\theta) \\ C_y &= -(r_p/\Delta y_2)(\sin \Delta\phi \cos \Delta\phi + \cos \Delta\psi \sin \Delta\phi \sin \Delta\theta) \\ C_\phi &= 1/\Delta y_1 \\ C_\psi &= 2\Delta\Delta_1 - \Delta z_2 - \Delta z_3 \\ C_\theta &= (3r_p/2)(1 + \sin \Delta\phi) \end{aligned} \quad (19)$$

The matrix  $A_d$  can be also computed by using a linearized solution of the equations converting the coordinates  $\Delta \mathbf{X}_{PAR} = [\Delta x_H, \Delta y_H, \Delta z_H, \Delta \phi, \Delta \theta, \Delta \psi]^t$  to the coordinates  $\Delta \mathbf{v} = [\Delta y_1, \Delta z_1, \Delta y_2, \Delta z_2, \Delta y_3, \Delta z_3]^t$ .

The details of the derivation and formulation are described in, [8], and [10] which the reader may refer to.

#### 4. Effect of design parameters

By using the proposed formulation the stiffness matrix of CaHyMan can be numerically computed. In addition, the determinant of  $K_{CaHyMan}$ , which is easy computable and particularly significant for stiffness singularity properties, can be used as a performance index to investigate synthetically the effect of the design parameters on the stiffness behavior of CaHyMan. In fact, no-null determinant of  $K_{CaHyMan}$  is needed to perform the computation of Eqs.(2), but even to ensure stiff behavior for given force applied to the extremity. Additionally, a numerical evaluation of eigenvalues of  $K_{CaHyMan}$  can be of interest for stability considerations, as pointed out in [11], but insight of matrix characteristics can be even obtained only by a numerical evaluation of the matrix determinant.

The built prototype of CaHyMan in Fig.1 has been analyzed and it is described by the stiffness parameters  $K_{bk}=K_{dk}=2.625 \times 10^6$  N/m,  $K_{Tk}=4.672 \times 10^3$  Nm/rad,  $K_4=2.625 \times 10^6$  N/m,  $K_s=0.697$  N/m,  $K_{T1}=0.876 \times 10^3$  Nm/rad, [12]. Figure 5 shows diagrams of the computed determinant of  $K_{CaHyMan}$  as a function of design parameters  $\alpha_k, \alpha_4, s_4, b_k, d_k, s_{kMAX}$  and  $L$ , ( $k=1,2,3$ ).

By using the proposed formulation the displacement of CaHyMan given by Eq.(1), which are due to the external force  $\mathbf{F}_e$  and torque  $\mathbf{N}_e$ , have been numerically computed through Eqs.(2) to (19). Tables 1, 2 and 3 show the computed values of the components the displacement of CaHyMan in three different configurations. The components of the displacement have been also computed as a function of the external force  $\mathbf{F}_e$  and torque  $\mathbf{N}_e$ , in the Figs.6 and 7. The proposed formulation can be also used to compute the extreme values of the stiffness over the workspace of the manipulator.

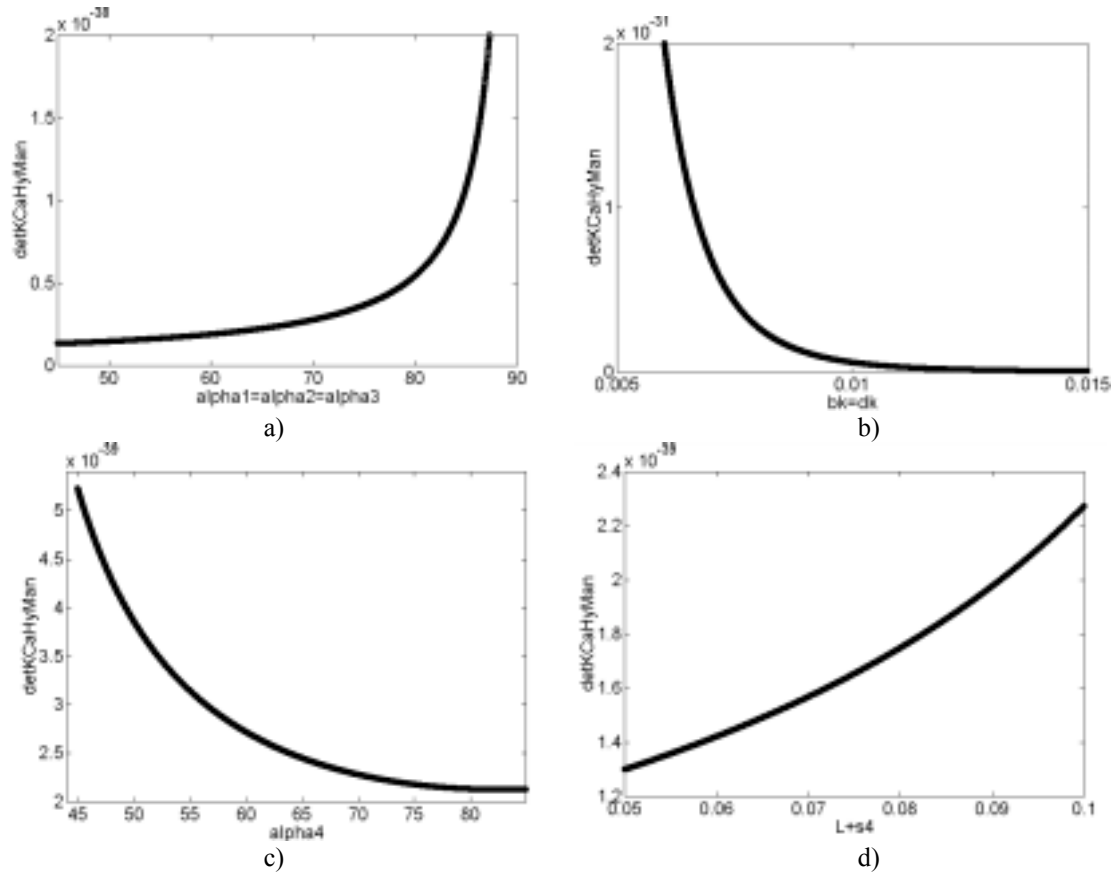


Fig.5 Diagrams of the determinant of  $K_{CaHyMan}$ : a) versus  $\alpha_1=\alpha_2=\alpha_3$  with  $\alpha_4=60$  deg. and  $s_4=25$  mm; b) versus  $b_k=d_k$  ( $k=1,2,3$ ) with  $\alpha_1=\alpha_2=\alpha_3=\alpha_4=60$  deg. and  $s_4=50$  mm; c) versus  $\alpha_4$  with  $\alpha_1=\alpha_2=\alpha_3=60$  deg. and  $s_4=50$  mm; d) versus  $L+s_4$  with  $\alpha_1=\alpha_2=\alpha_3=\alpha_4=60$  deg.

Tab.1 Displacements of CaHyMan when  $\alpha_1=\alpha_2=\alpha_3=\alpha_4=90$  deg. and  $s_4=50$ mm.

| $\Delta X_{CaHyMan}$   | $\mathbf{F}e=(1.0;1.0;1.0)^t$<br>$\mathbf{N}e=(0.0;0.0;0.0)^t$<br>[N; Nm] | $\mathbf{F}e=(0.0;0.0;0.0)^t$<br>$\mathbf{N}e=(1.0;1.0;1.0)^t$<br>[N; Nm] | $\mathbf{F}e=(1.0;1.0;1.0)^t$<br>$\mathbf{N}e=(1.0;1.0;1.0)^t$<br>[N; Nm] |
|------------------------|---|---|---|
| $\Delta x$ [mm]        | 0.10  | -0.50   | -0.50   |
| $\Delta y$ [mm]        | 0.01  | 0.01  | 0.01  |
| $\Delta z$ [mm]        | 0.37  | -3.07   | -2.70   |
| $\Delta \varphi$ [deg] | -0.92   | -0.85   | -0.74   |
| $\Delta \psi$ [deg]    | 0.84  | 1.52  | 1.33  |
| $\Delta \theta$ [deg]  | -0.57   | 0.99  | 0.66  |



Tab.2 Displacements of CaHyMan when  $\alpha_1=\alpha_2=\alpha_3=\alpha_4=30$  deg. and  $s_4=0$ mm.

| $\Delta \mathbf{X}_{\text{CaHyMan}}$ | $\mathbf{F}_e=(1.0;1.0;1.0)^t$<br>$\mathbf{N}_e=(0.0;0.0;0.0)^t$<br>[N; Nm] | $\mathbf{F}_e=(0.0;0.0;0.0)^t$<br>$\mathbf{N}_e=(1.0;1.0;1.0)^t$<br>[N; Nm] | $\mathbf{F}_e=(1.0;1.0;1.0)^t$<br>$\mathbf{N}_e=(1.0;1.0;1.0)^t$<br>[N; Nm] |
|--------------------------------------|---|---|---|
| $\Delta x$ [mm]                      | -0.50   | 7.60  | 7.10  |
| $\Delta y$ [mm]                      | -0.40   | 6.30  | 5.90  |
| $\Delta z$ [mm]                      | -0.40   | 5.70  | 5.30  |
| $\Delta \varphi$ [deg]               | -1.01   | 3.04  | 2.87  |
| $\Delta \psi$ [deg]                  | 0.97  | -2.49   | -2.35   |
| $\Delta \theta$ [deg]                | -0.08   | 0.21  | 0.65  |

Tab.3 Displacements of CaHyMan when  $\alpha_1=45$ deg.,  $\alpha_2=60$ deg.,  $\alpha_3=75$ deg.,  $\alpha_4=45$ deg. and  $s_4=0$ mm.

| $\Delta \mathbf{X}_{\text{CaHyMan}}$ | $\mathbf{F}_e=(1.0;1.0;1.0)^t$<br>$\mathbf{N}_e=(0.0;0.0;0.0)^t$<br>[N; Nm] | $\mathbf{F}_e=(0.0;0.0;0.0)^t$<br>$\mathbf{N}_e=(1.0;1.0;1.0)^t$<br>[N; Nm] | $\mathbf{F}_e=(1.0;1.0;1.0)^t$<br>$\mathbf{N}_e=(1.0;1.0;1.0)^t$<br>[N; Nm] |
|--------------------------------------|---|---|---|
| $\Delta x$ [mm]                      | 0.01  | -0.30   | -0.20   |
| $\Delta y$ [mm]                      | 0.01  | -0.10   | -0.10   |
| $\Delta z$ [mm]                      | 0.01  | -0.20   | -0.20   |
| $\Delta \varphi$ [deg]               | -0.79   | 0.22  | 0.34  |
| $\Delta \psi$ [deg]                  | 0.78  | -0.21   | -0.33   |
| $\Delta \theta$ [deg]                | 9.34  | 0.43  | -2.54   |

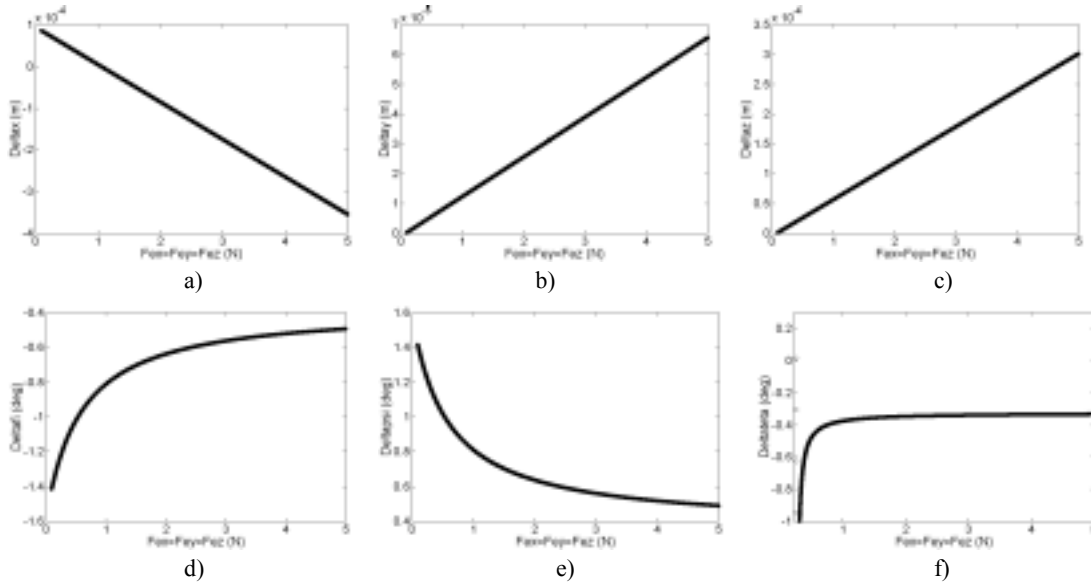


Fig.6 Displacements of CaHyMan as a function of  $F_{ex}=F_{ey}=F_{ez}$  when  $N_{ex}=N_{ey}=N_{ez}=0$  for  $\alpha_1=\alpha_2=\alpha_3=\alpha_4=60$ deg. and  $s_4=50$ mm: a)  $\Delta x$ ; b)  $\Delta y$ ; c)  $\Delta z$ ; d)  $\Delta \varphi$ ; e)  $\Delta \psi$ ; f)  $\Delta \theta$ .

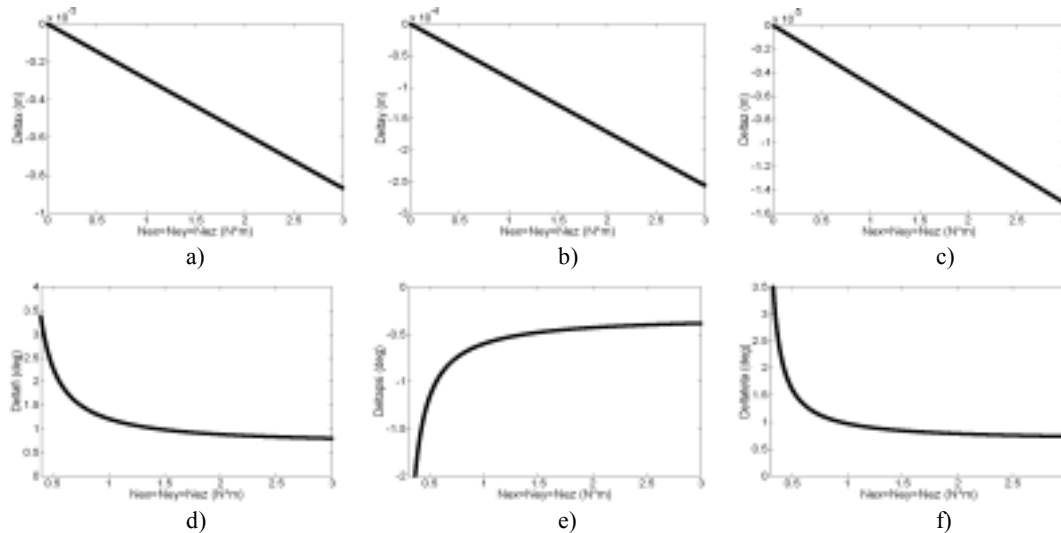


Fig.7 Displacements of CaHyMan as a function of  $N_{ex}=N_{ey}=N_{ez}$  when  $F_{ex}=F_{ey}=F_{ez}=0$  for  $\alpha_1=\alpha_2=\alpha_3=\alpha_4=60\text{deg}$ . and  $s_4=50\text{mm}$ : a)  $\Delta x$ ; b)  $\Delta y$ ; c)  $\Delta z$ ; d)  $\Delta\phi$ ; e)  $\Delta\psi$ ; f)  $\Delta\theta$ .

## 5. Conclusions

The stiffness behaviour of the hybrid parallel-serial manipulator CaHyMan has been investigated by using proper schemes in order to obtain a closed-form formulation. By using this formulation the stiffness matrix and displacement of CaHyMan has been numerically computed in different configurations of the manipulator. The satisfactory results confirm that CaHyMan is able to operate with high performances in static or quasi-static operations such as surgery applications.

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