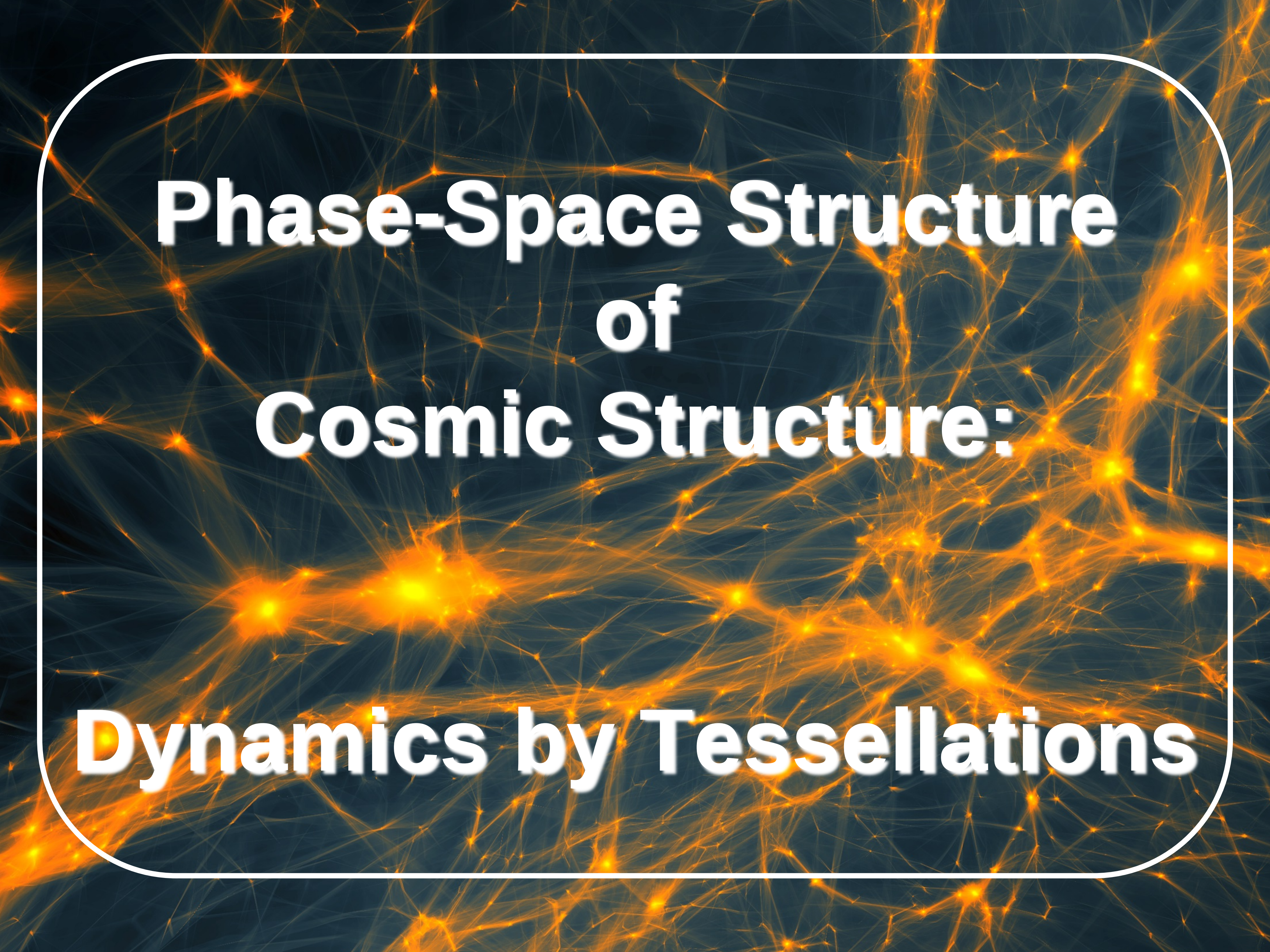


Phase-Space Structure of Cosmic Structure: Dynamics by Tessellations

Stochastic

Spatial

Pattern of

- Clusters,
- Filaments &
- Walls

around

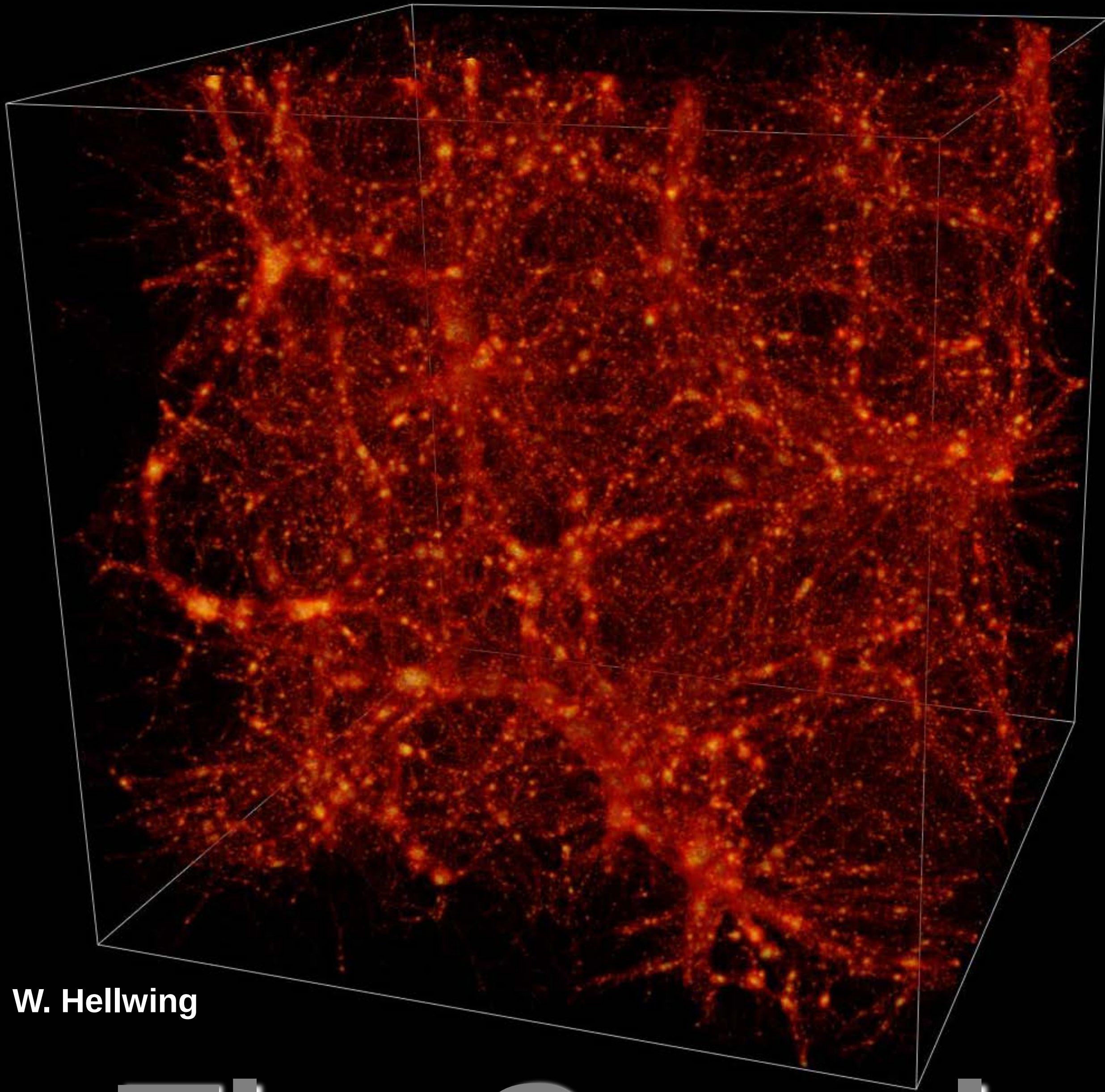
- Voids

in which

matter & galaxies

have agglomerated

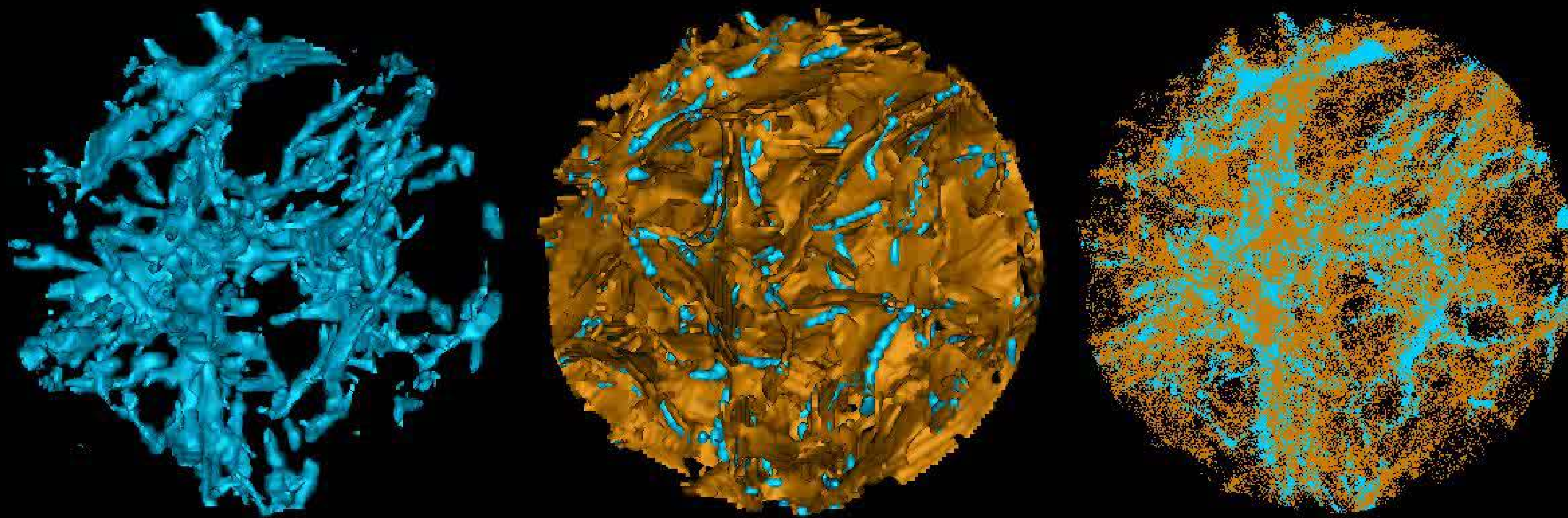
through gravity



W. Hellwing

The Cosmic Web

Nexus/MMF



MMF/Nexus
Cautun et al. 2013, 2014

Stochastic Spatial Pattern

- Clusters,
- Filaments &
- Walls

around

- Voids

in which matter & galaxies

have agglomerated

through gravity

Sergei Shandarin

Johan Hidding

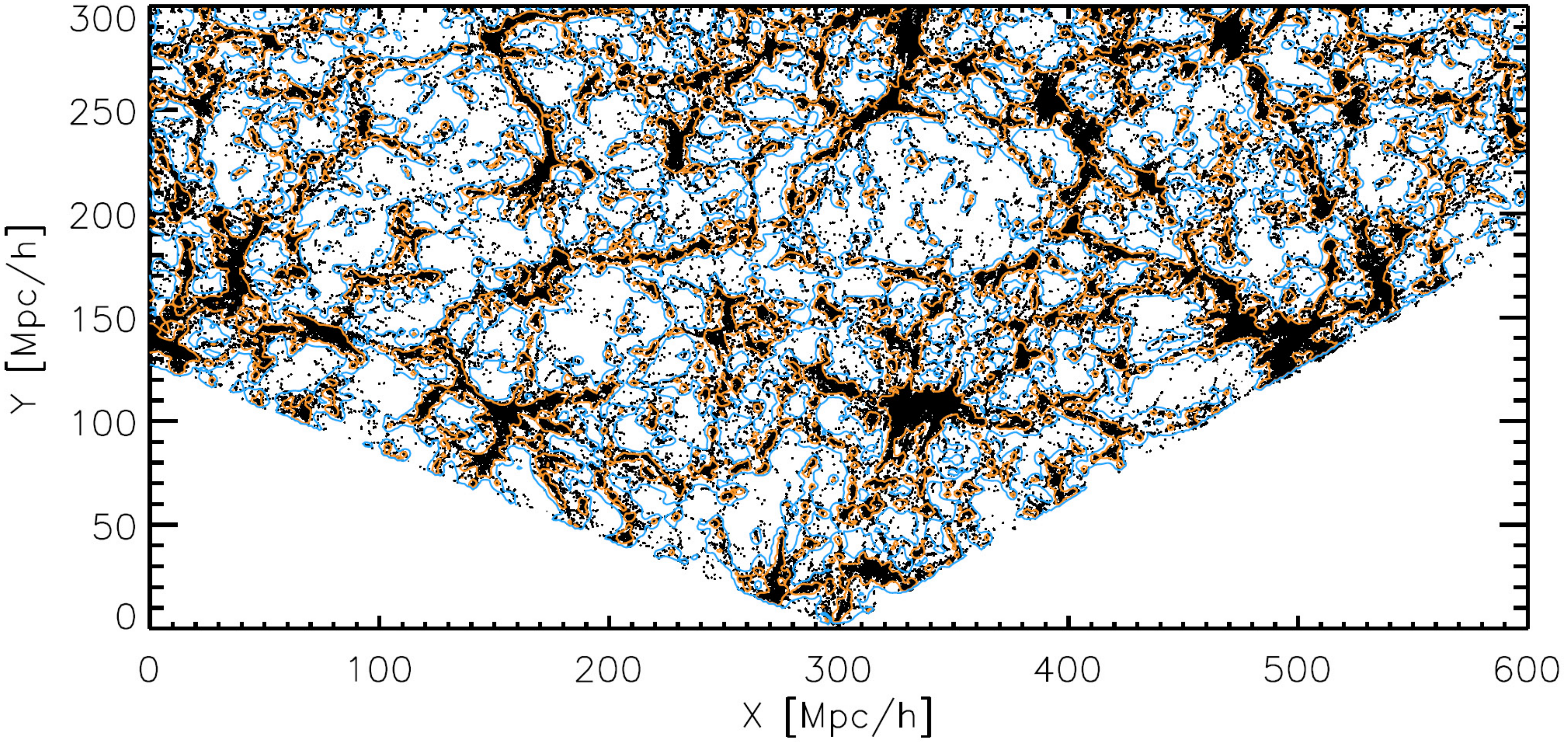
Job Feldbrugge

Cosmic Web:

Galaxies

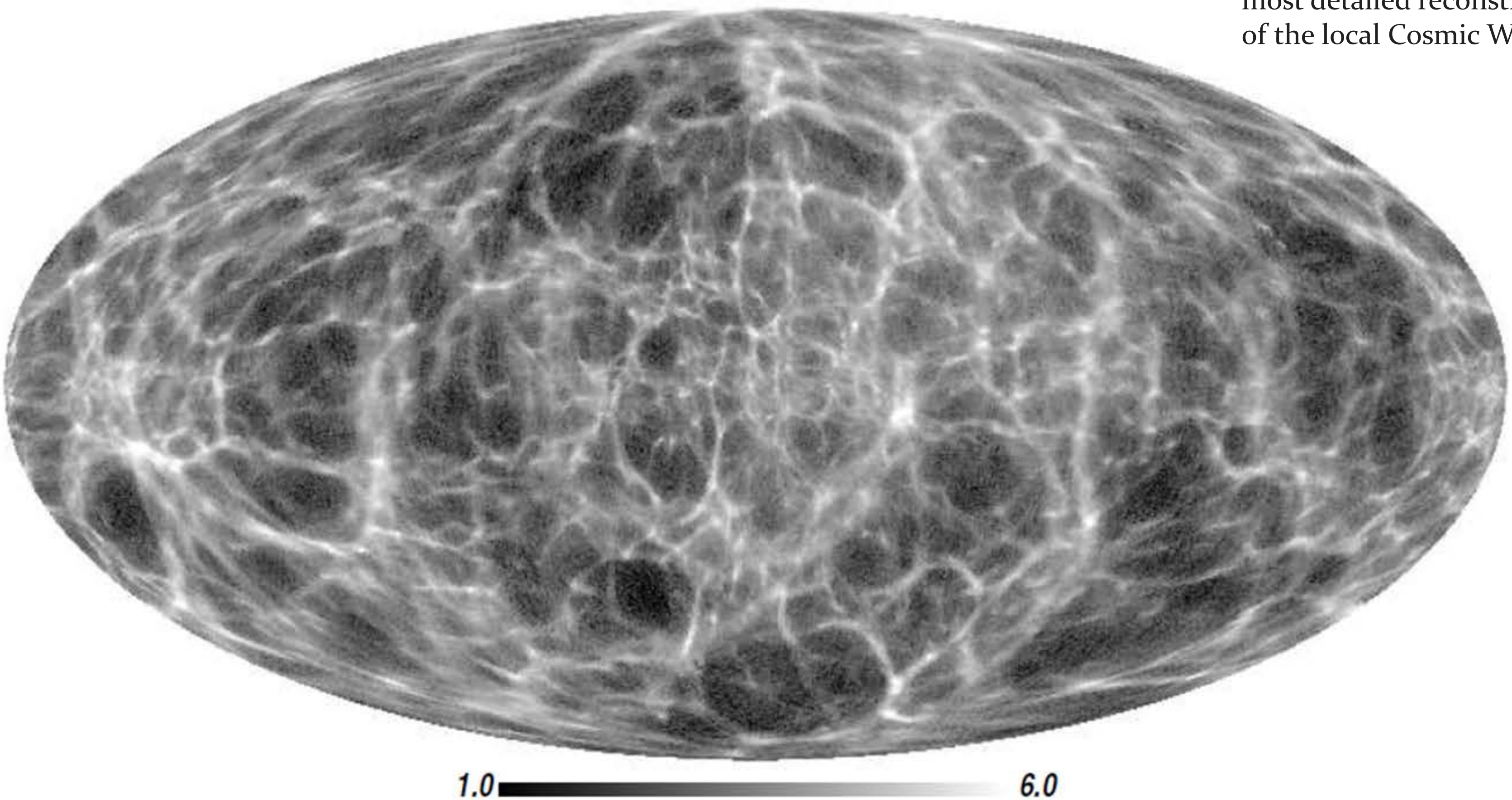
image courtesy:
Aragon-Calvo, Subbarao & Szalay

SDSS Galaxy Survey



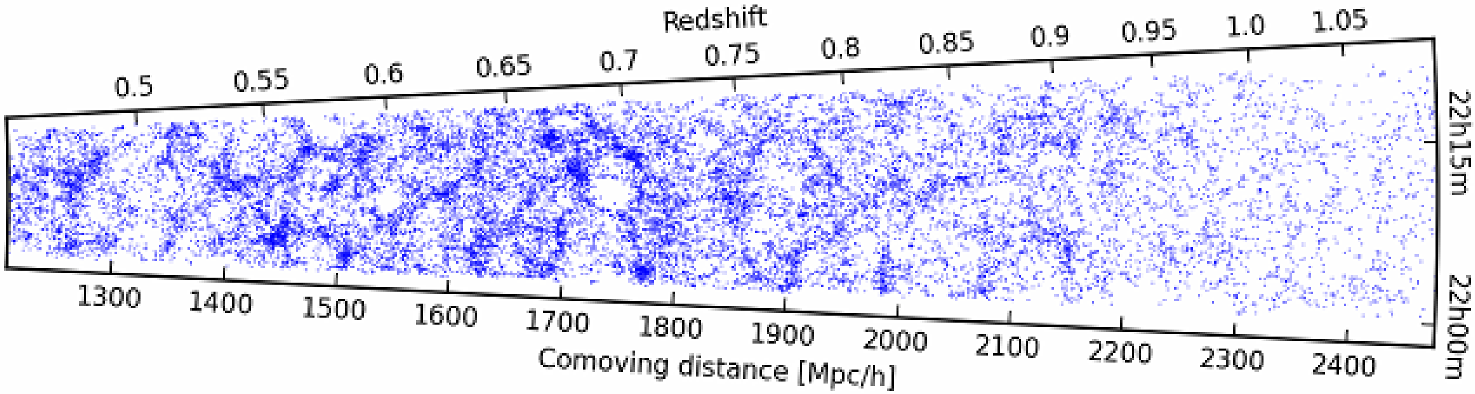
local Cosmic Web: 2MRS

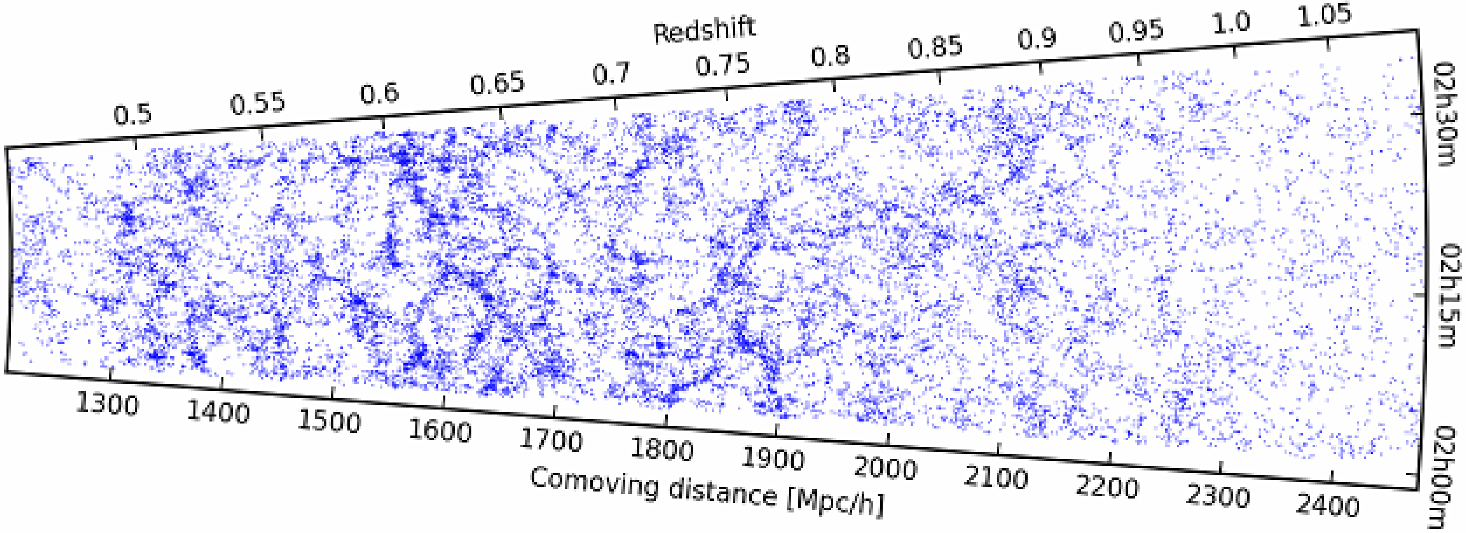
most detailed reconstruction
of the local Cosmic Web



Courtesy: Francisco Kitaura

Cosmic Web at High z

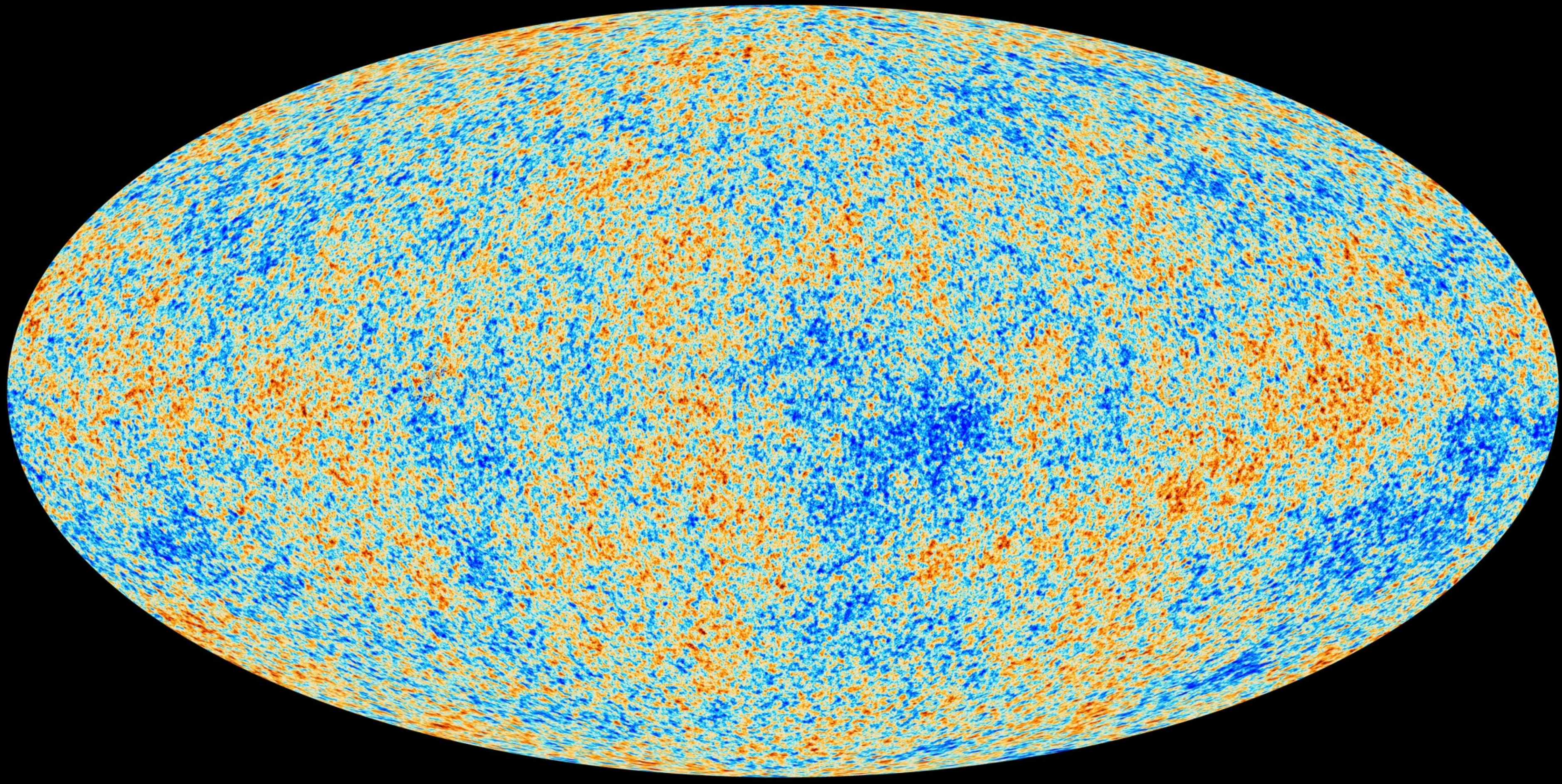




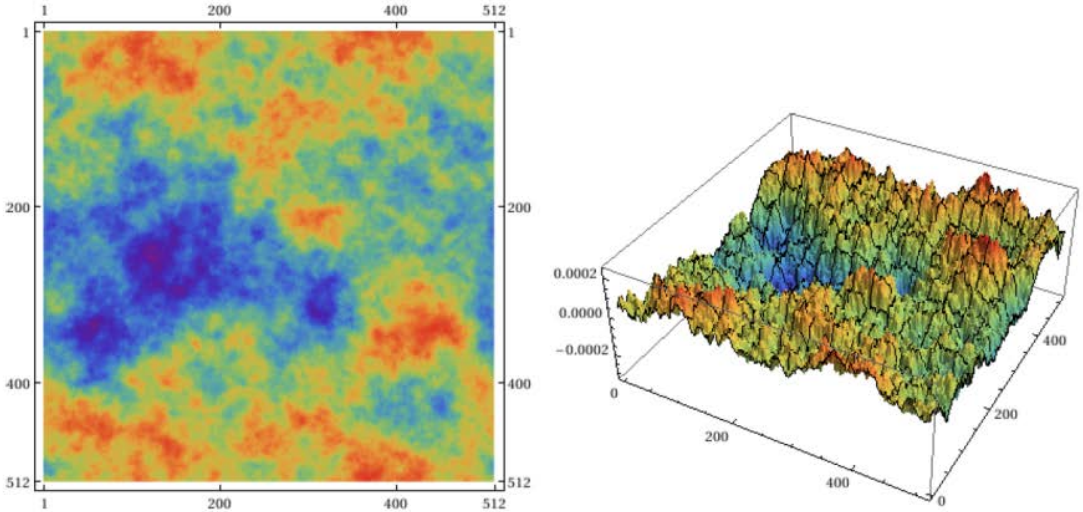
VIPERS deep redshift survey, z=0.4-1.2 (Guzzo et al.)

Dynamical Evolution of the Cosmic Web

Planck CMB map (2013)



the (Gaussian) primordial Universe
13.8 Gyrs ago

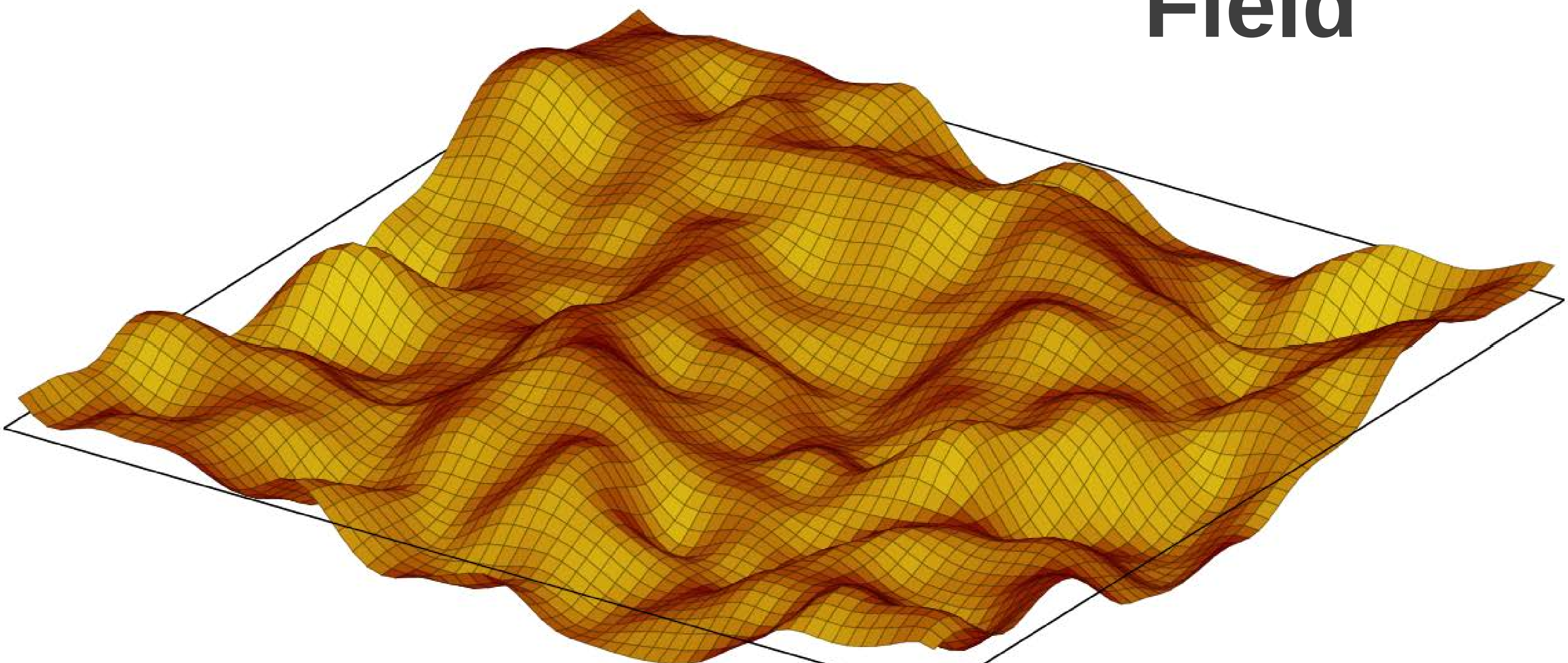


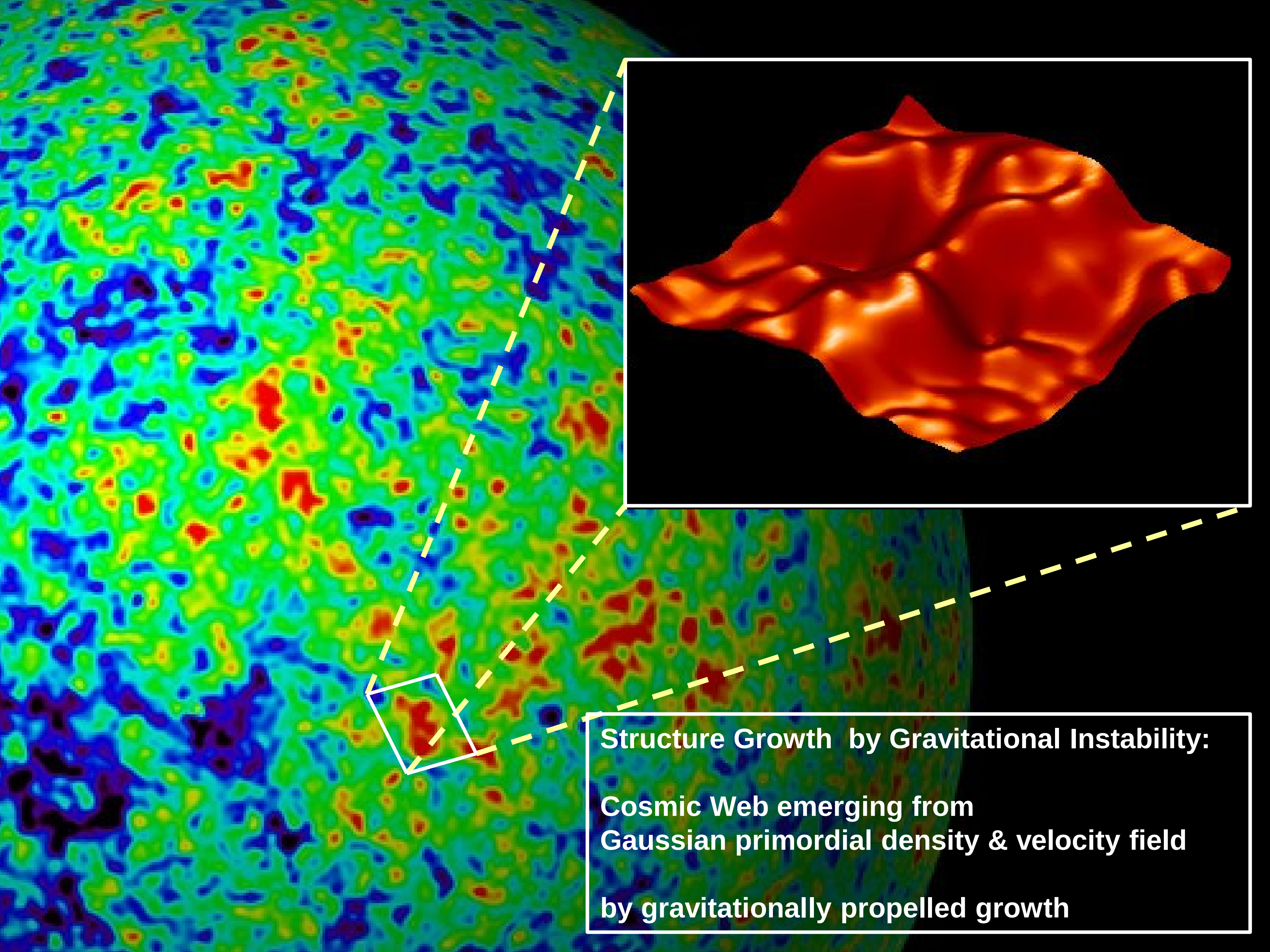
(a) A density plot of a realization of a Gaussian random field

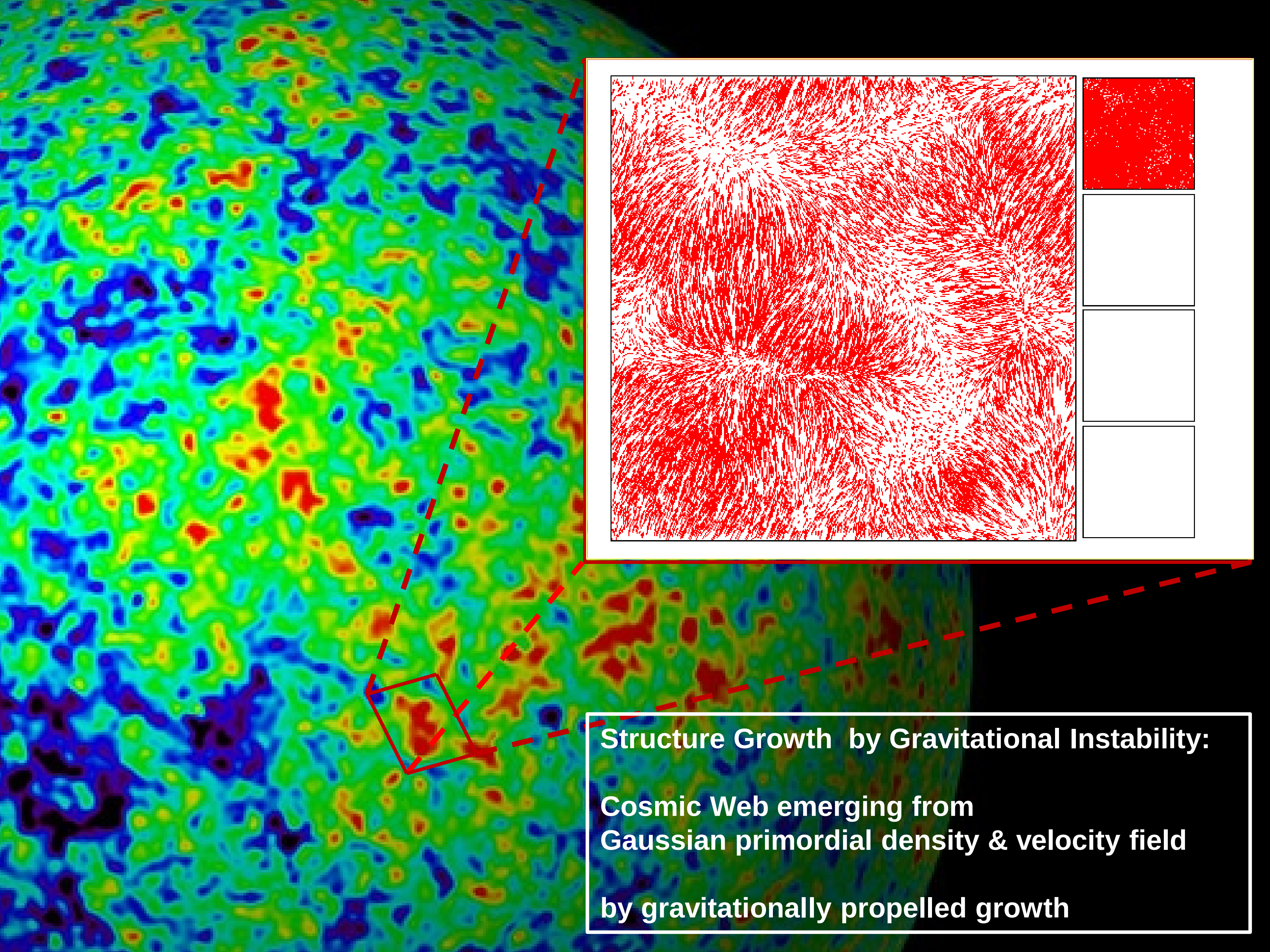
(b) A 3-dimensional plot of a realization of a Gaussian random field

Cosmic Initial Conditions:

Gaussian Random Field




Structure Growth by Gravitational Instability:
Cosmic Web emerging from
Gaussian primordial density & velocity field
by gravitationally propelled growth


Structure Growth by Gravitational Instability:
Cosmic Web emerging from
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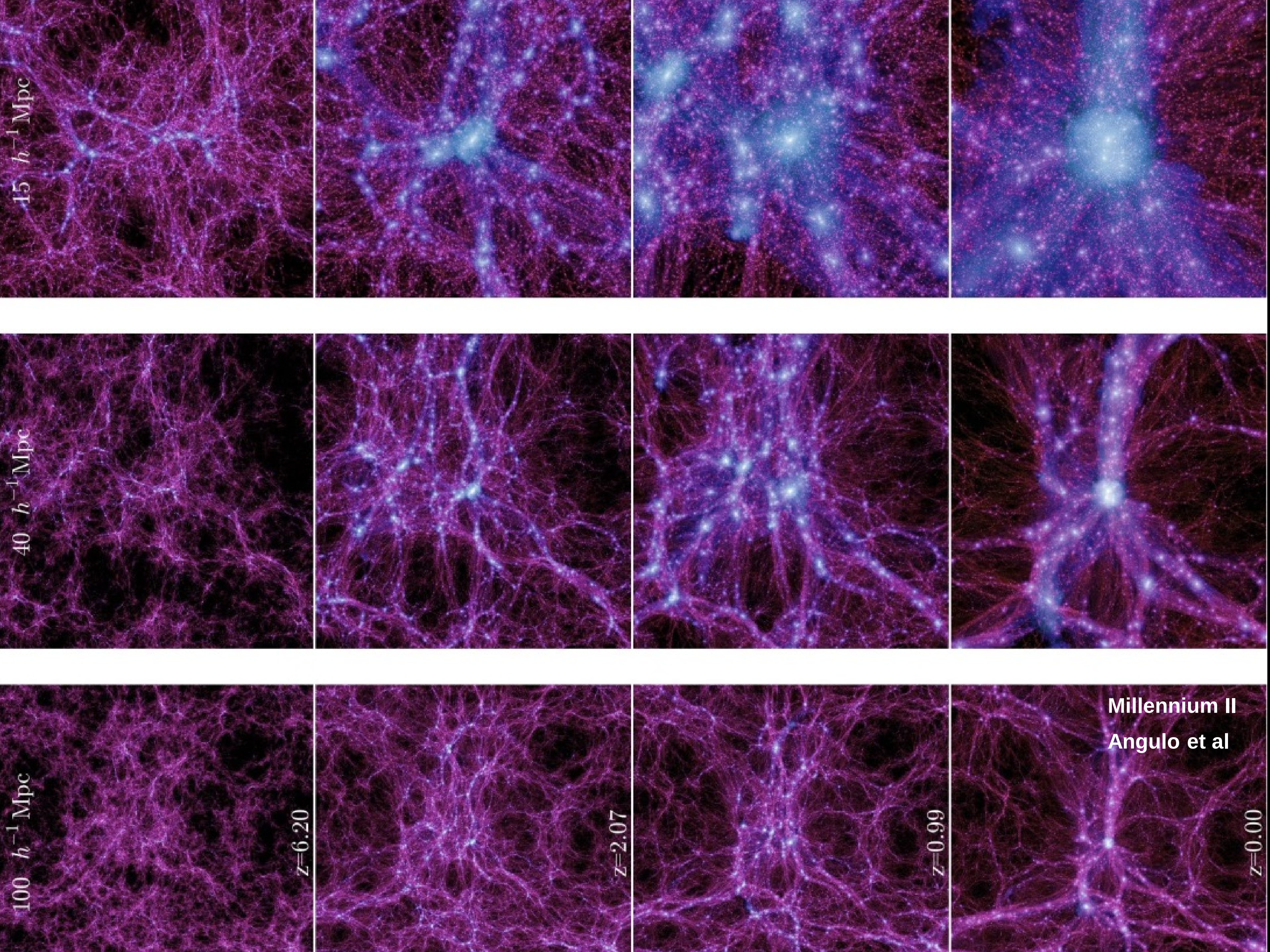
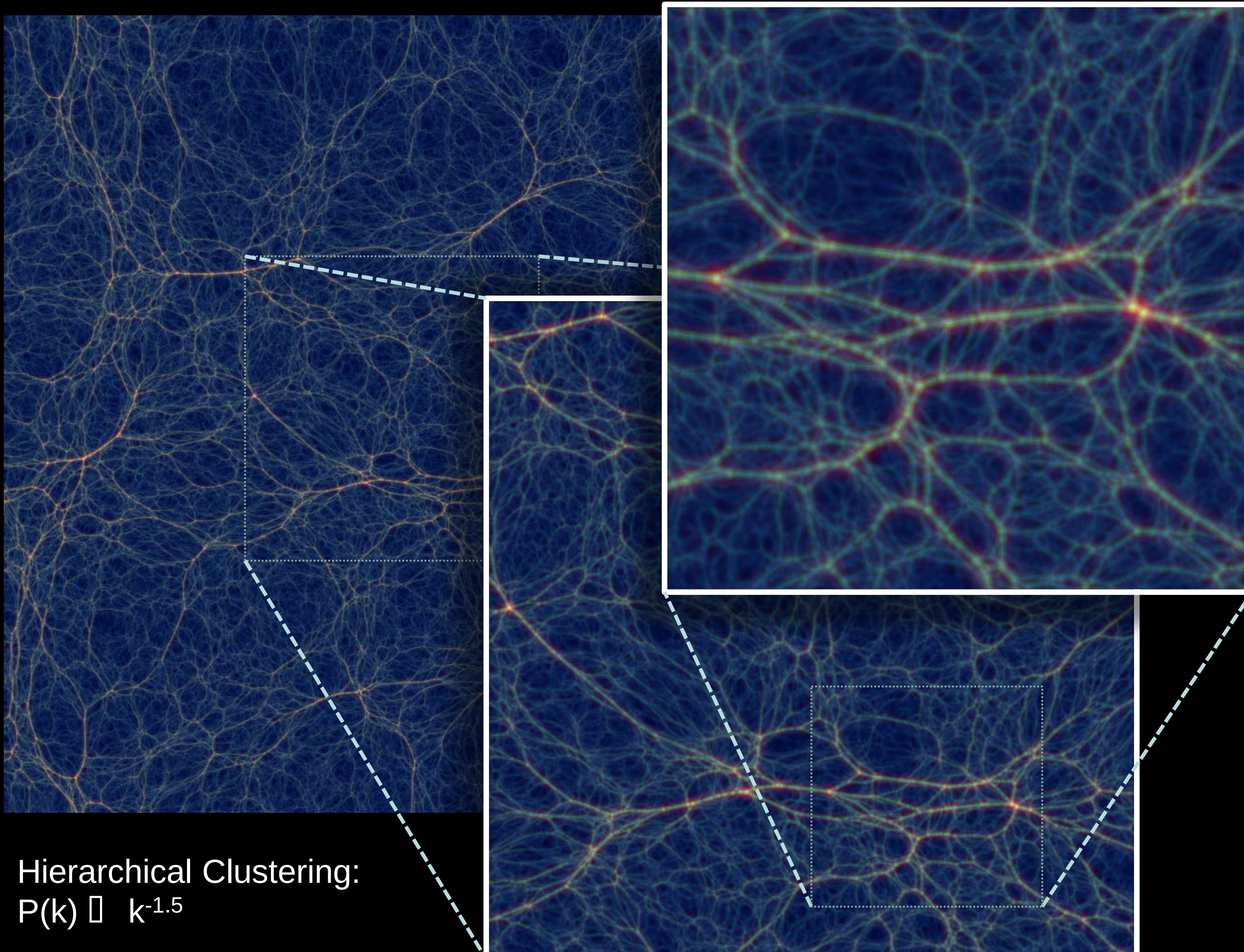

15 h^{-1} Mpc
40 h^{-1} Mpc
100 h^{-1} Mpc
z=6.20
z=2.07
z=0.99
z=0.00
Millennium II
Angulo et al

Dynamical Evolution Cosmic Web

- hierarchical structure formation
- anisotropic collapse
- void formation:
  asymmetry
  overdense vs. underdense



Hierarchical Clustering:
$P(k) \propto k^{-1.5}$

Movie:
J. Hidding

Phase-Space Dynamics

Phase-Space Dynamics

- 6D Phase-space consist of

  spatial coordinates $\vec{r}$

  velocity $\vec{v}$

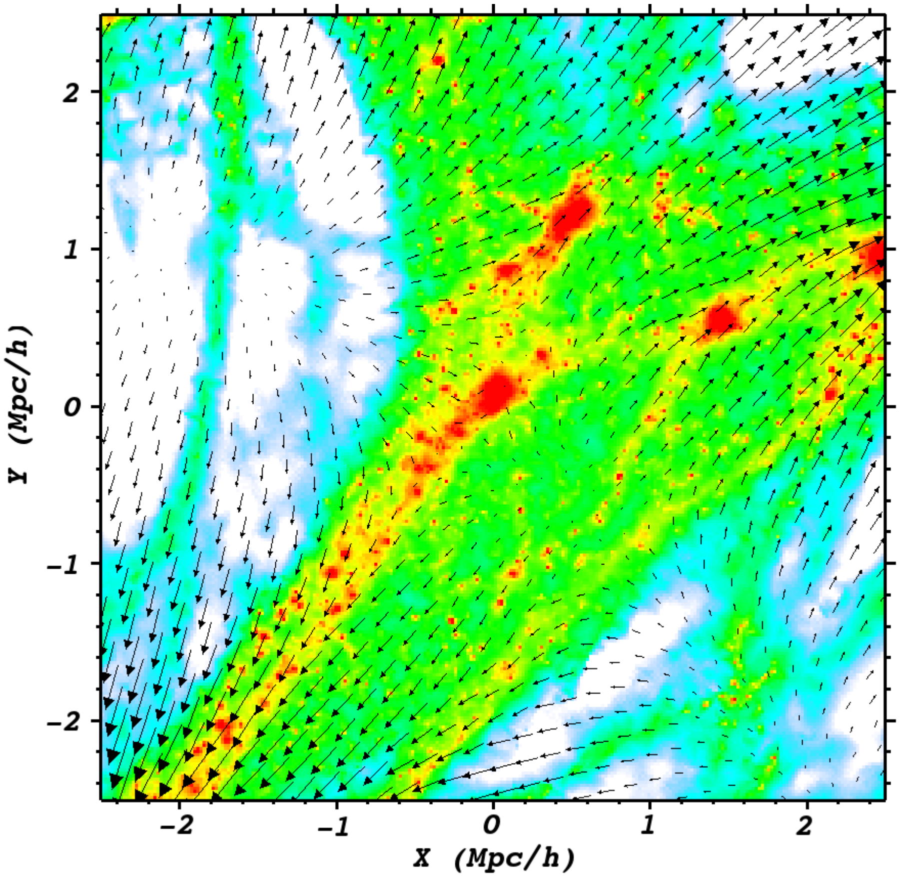
  of a mass element

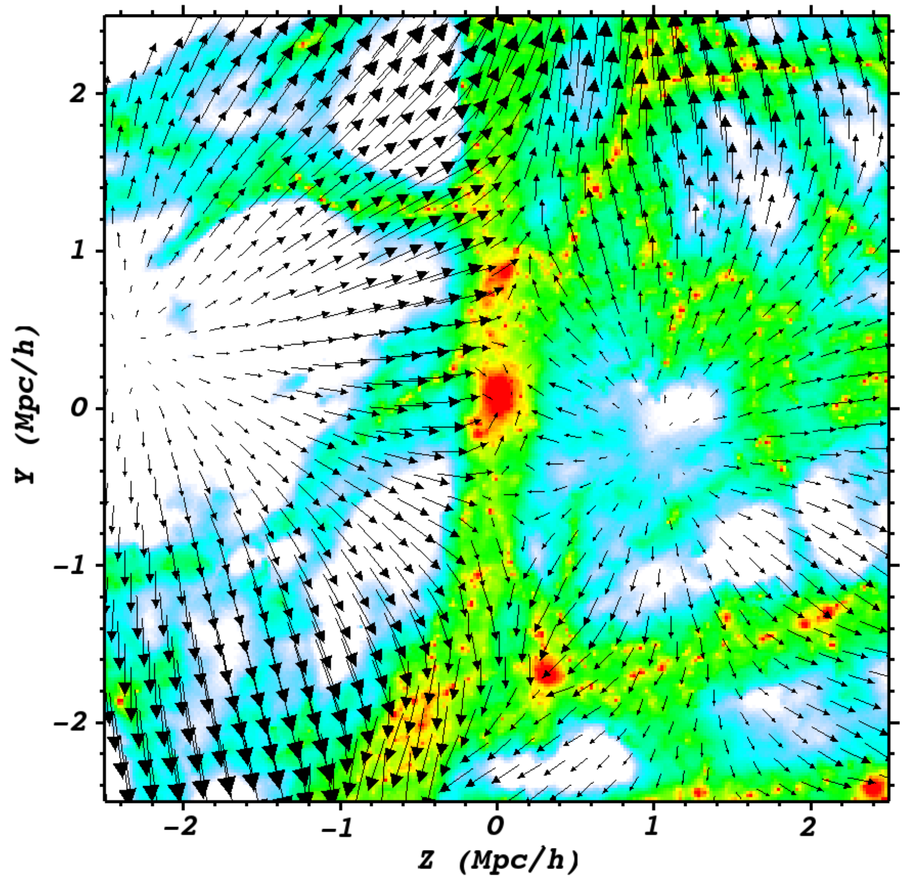
- To follow dynamics of a system, we need to know the phase-space density

$$f(\vec{r}, \vec{v})$$

describing the density of mass elements at location $\vec{r}$ and velocity $\vec{v}$

Cosmic Structure: Density & Velocity Fields





Rieder, vdW et al. 2013

Boltzmann Equation

- To follow dynamics of a system, we need to know the evolution of the phase-space density $f(\vec{r}, \vec{v})$

- The evolution of the system, embedded in a potential force field (eg. gravity) □ is described by the Boltzmann equation, one of the central equations of physics,

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla} f - \vec{\nabla}\Phi \cdot \vec{\nabla}_v f = \left(\frac{\delta f}{\delta t} \right)_c$$

Fluid Equations

- For the fluid approximation, in which the mass distribution is approximated by a continuous density & velocity field, the Boltzmann equation results in a set of

- equations of motion describing
- the evolution of the fluid in Eulerian space

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \vec{v} \right) = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \nabla \right) \vec{v} = -\nabla \Phi - \frac{1}{\rho} \nabla p$$

$$\nabla^2 \Phi = 4\pi G \rho$$

Weaving the Cosmic Web:

dynamics & tessellations

Hidding et al. 2012, Hidding et al. 2014

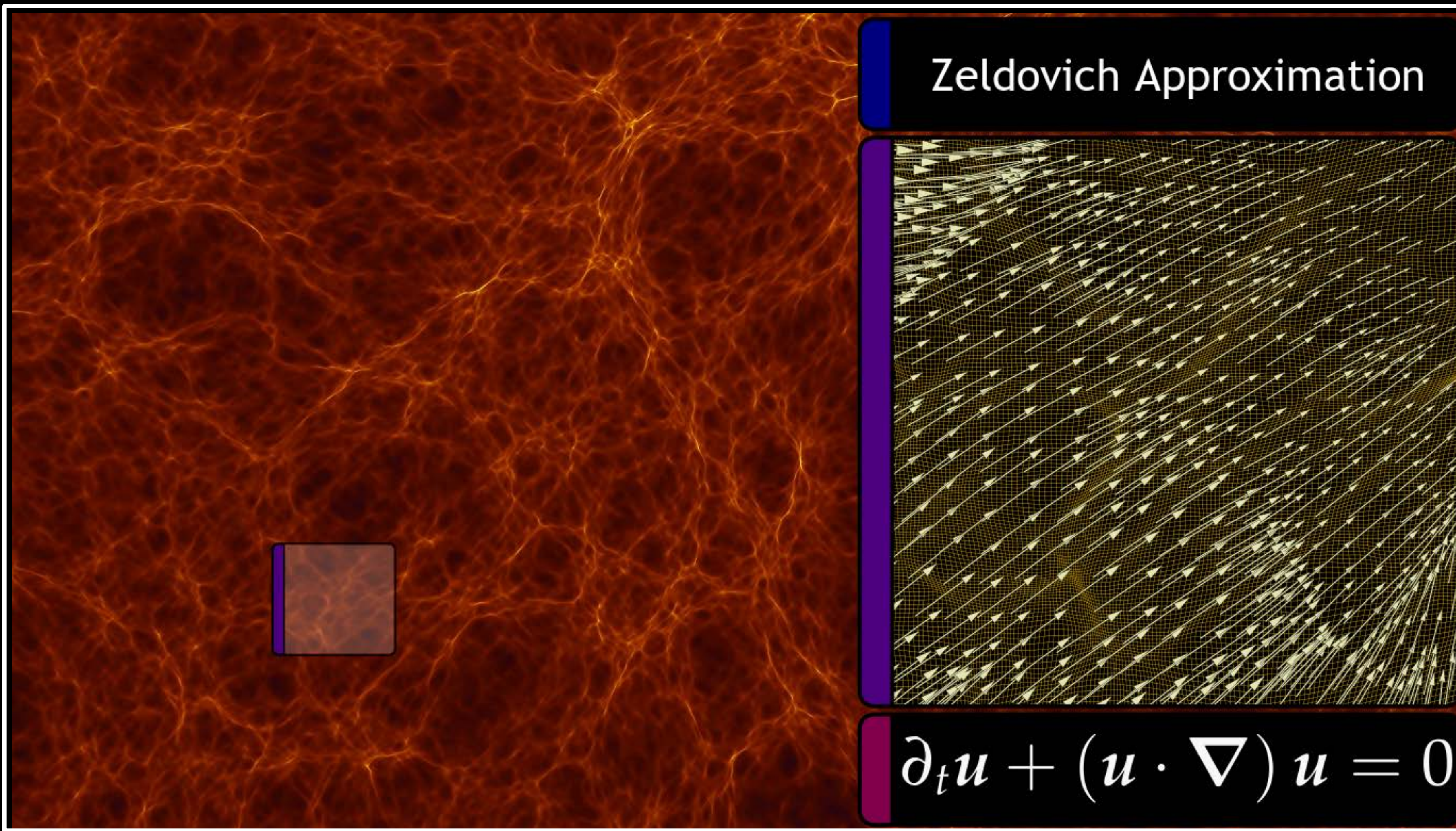
Zel’dovich Approximation

$$\vec{x} = \vec{q} + D(t)\,\vec{u}(\vec{q})$$

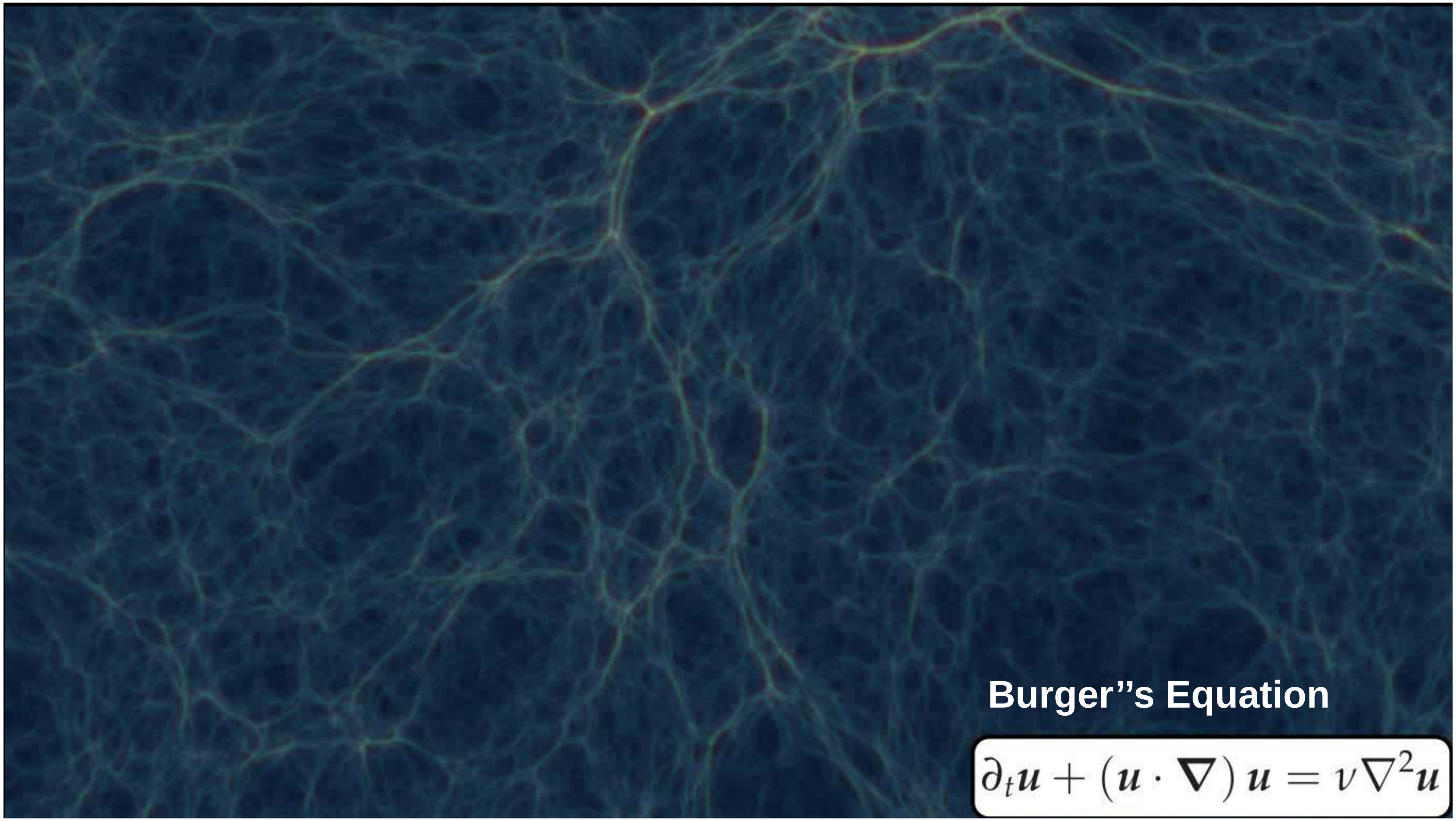
$$\vec{u}(\vec{q}) = -\vec{\nabla}\Phi(\vec{q})$$

$$\Phi(\vec{q}) = \frac{2}{3Da^2H^2\Omega}\,\phi_{lin}(\vec{q})$$

Zel’dovich Approximation



Adhesion Approximation

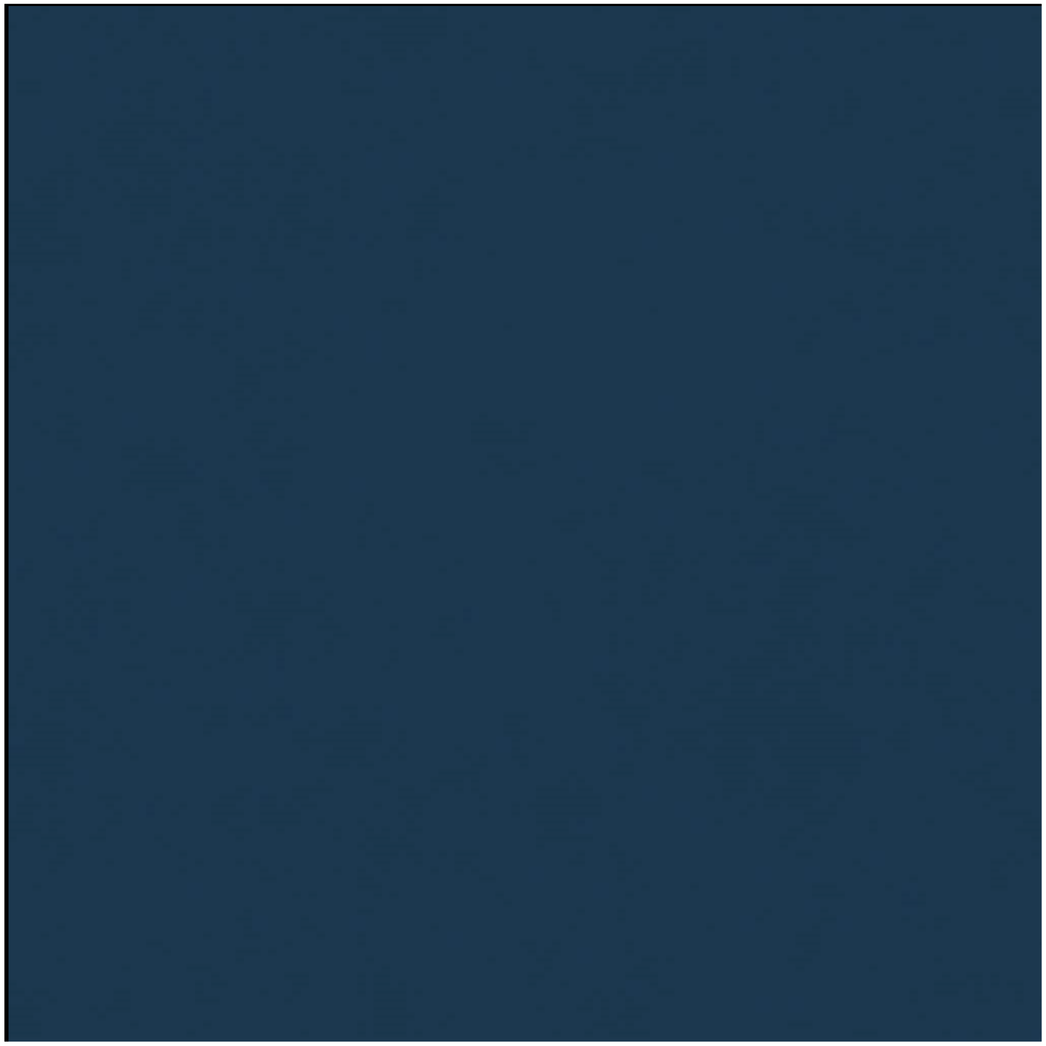


Burger''s Equation

$$\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\, \boldsymbol{u} = \nu \nabla^2 \boldsymbol{u}$$

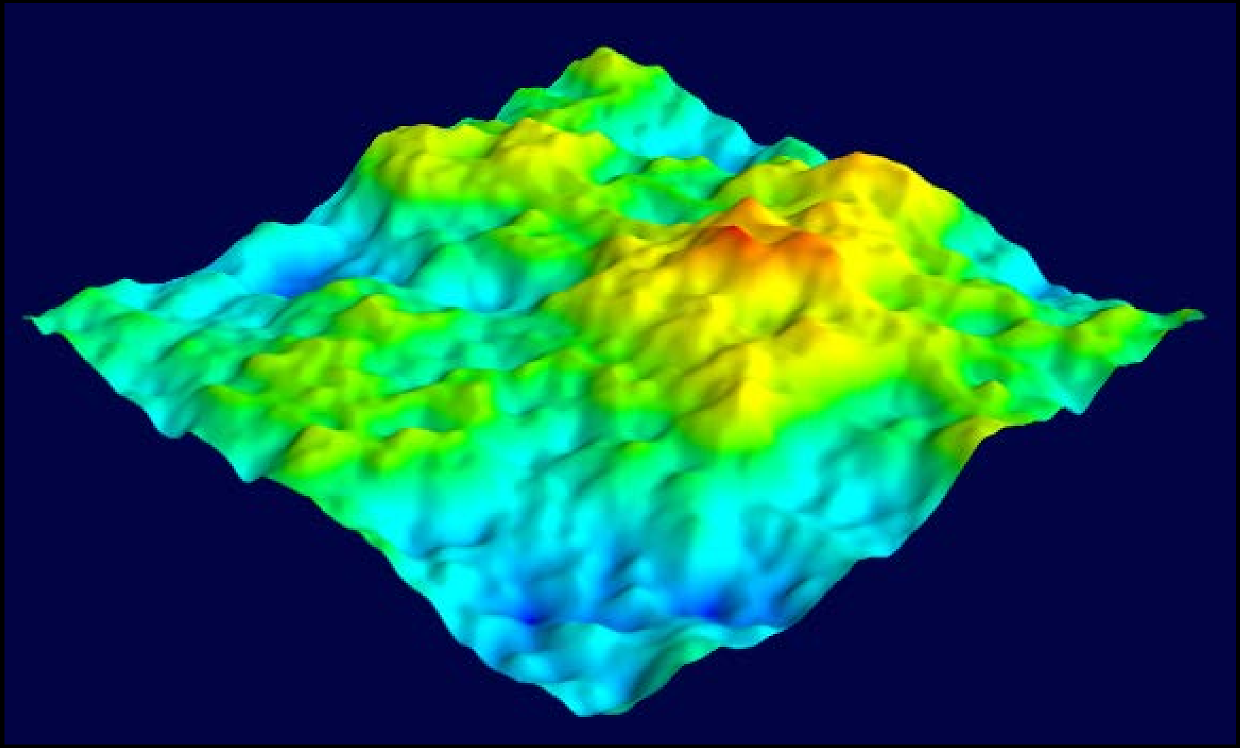
Adhesion Approximation

Gurbatov, Saichev & Shandarin 1987



Hidding 2012

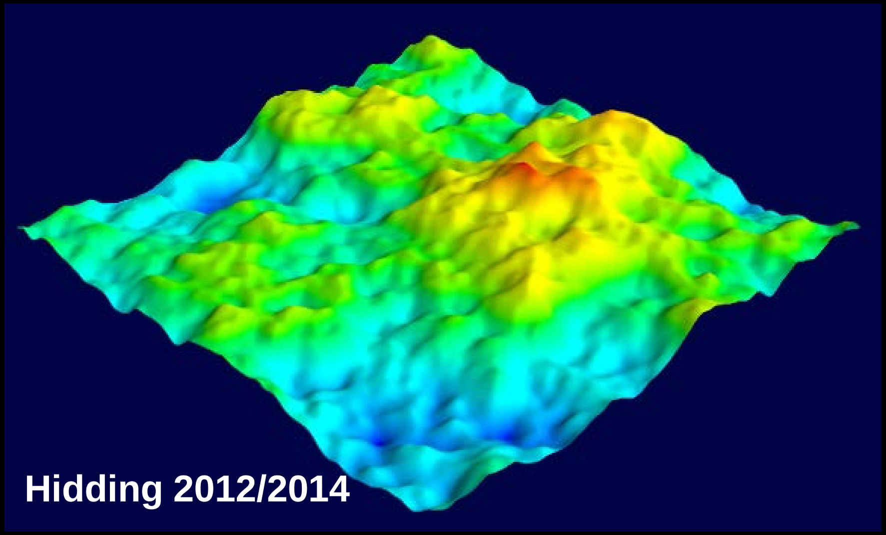
Velocity & Gravity Potential

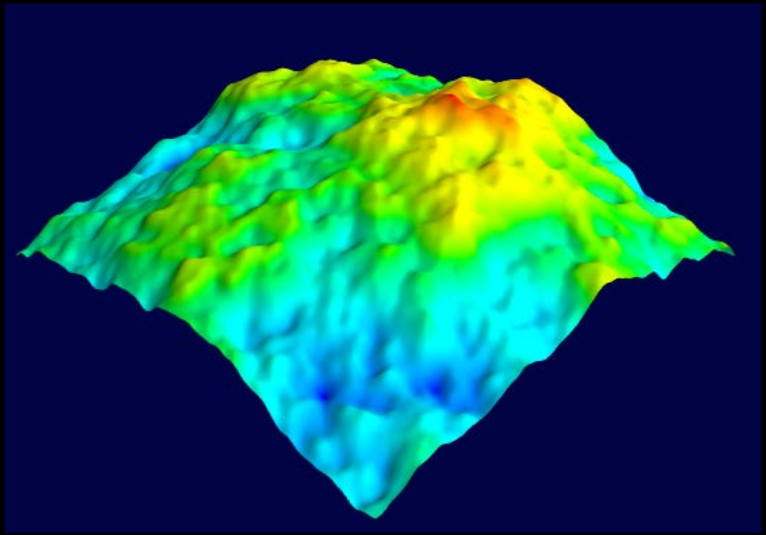


$$\vec{u}(\vec{q}) = \vec{\nabla}\Phi(\vec{q})$$

Burger's Equation: Hopf Solution

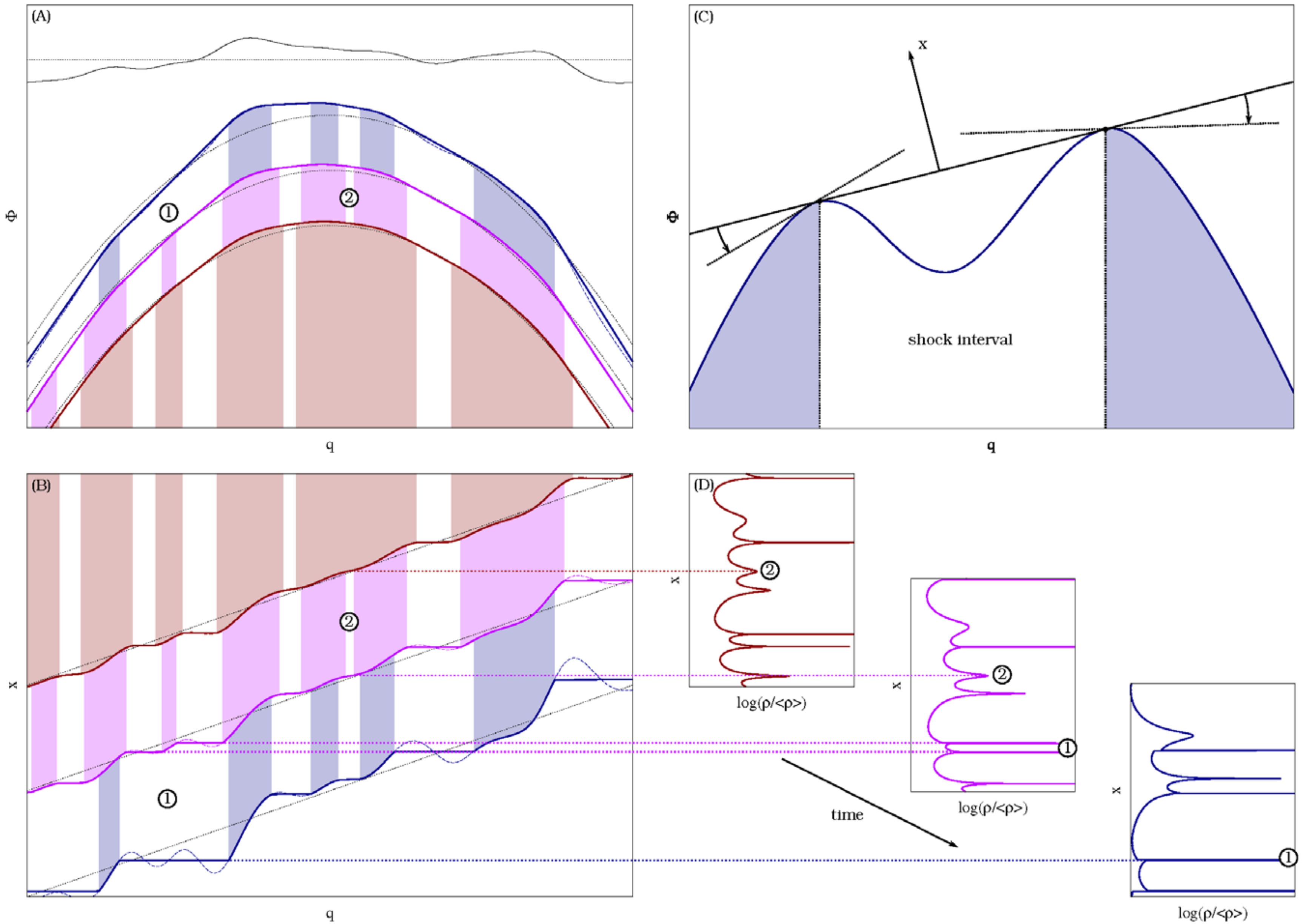
$$\frac{\partial \vec{u}}{\partial t} + \left(\vec{u} \cdot \vec{\nabla}\right)\vec{u} = \nu \nabla^2 \vec{u}$$

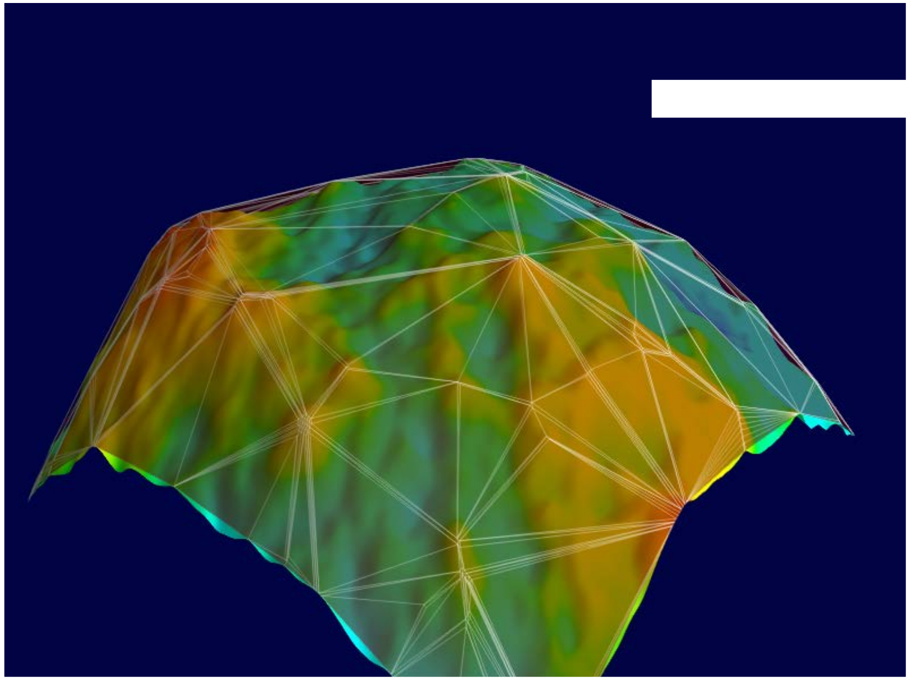


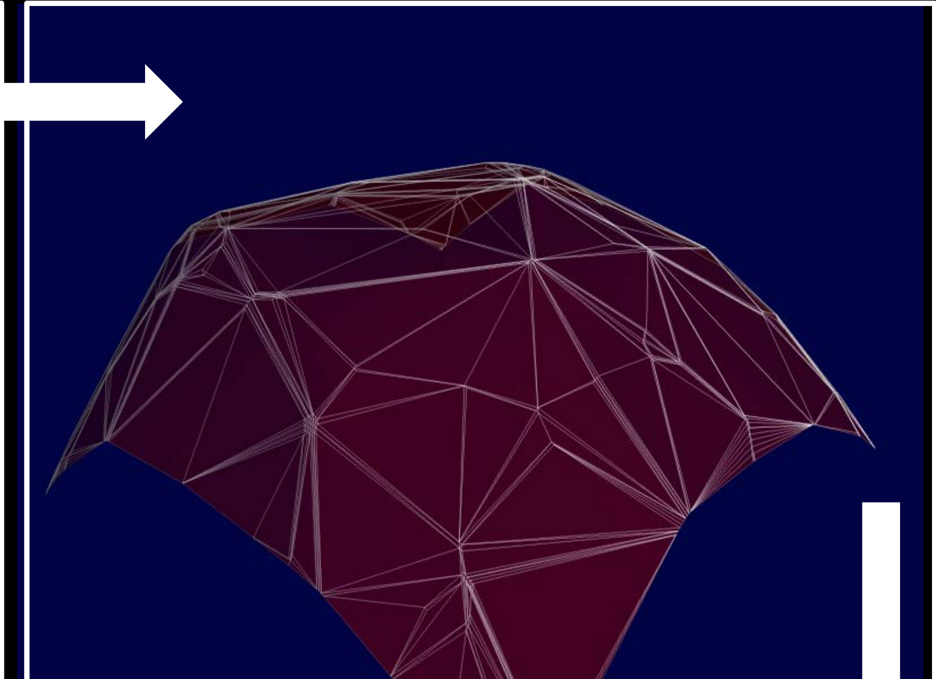


$$\Phi(\vec{x},t) + \frac{x^2}{2} = \max_q \left[\left(t\Phi_0(q) - \frac{q^2}{2} \right) + \vec{x} \cdot \vec{q} \right]$$

Burger's Equation: Hopf Solution



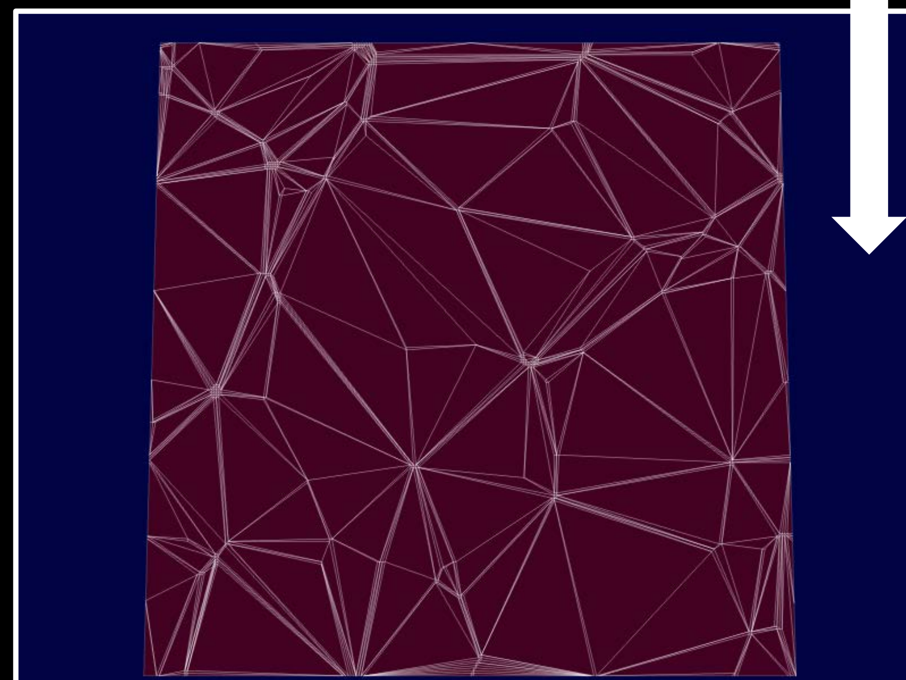




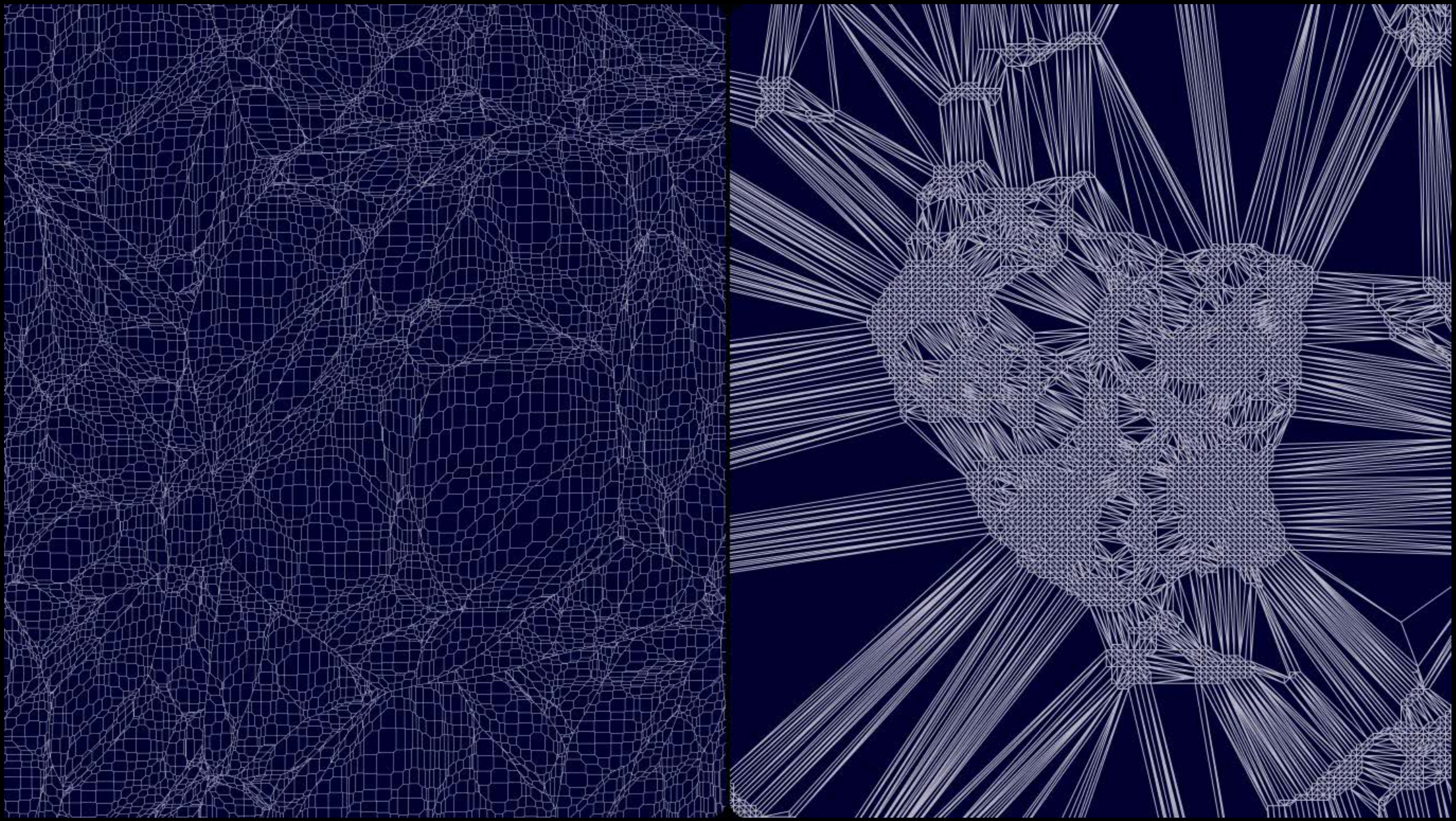
Hidding 2012/2014

Convex Hull
quadratically lifted potential field

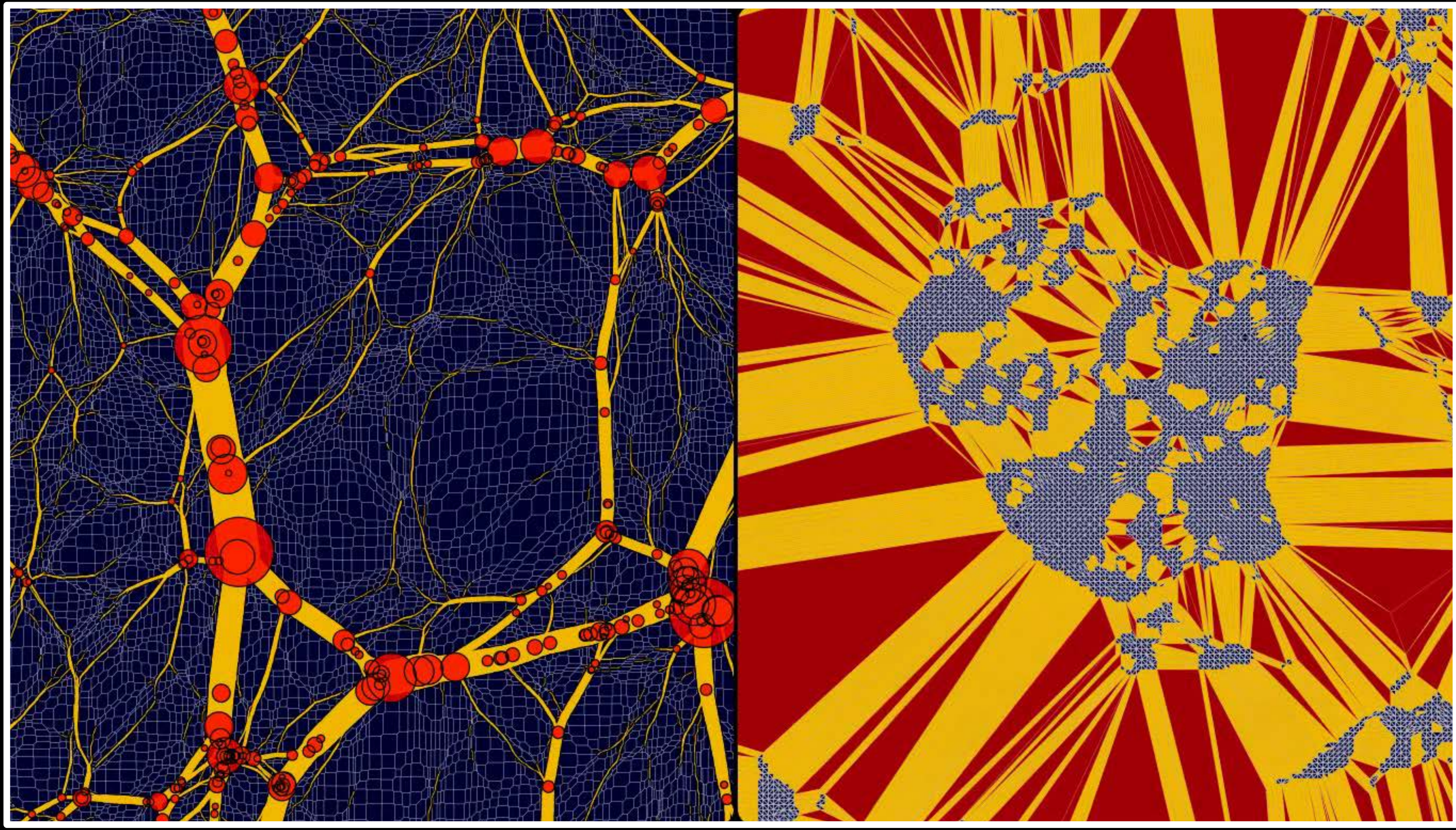
Delaunay tessellation
generated by maxima potential field



Eulerian – Lagrangian
Voronoi - Delaunay



Eulerian – Lagrangian
Voronoi - Delaunay



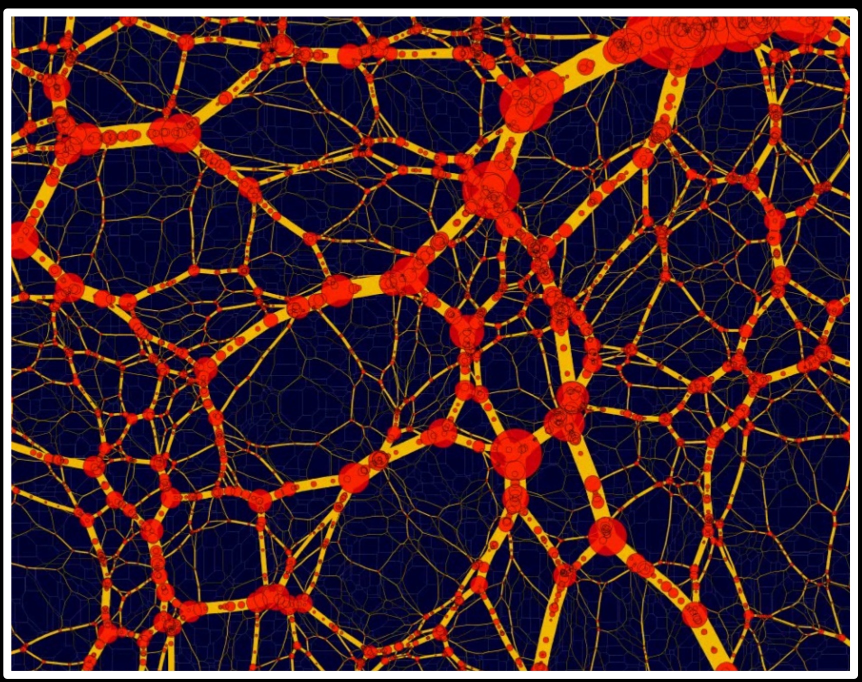
Cosmological Sensitivity

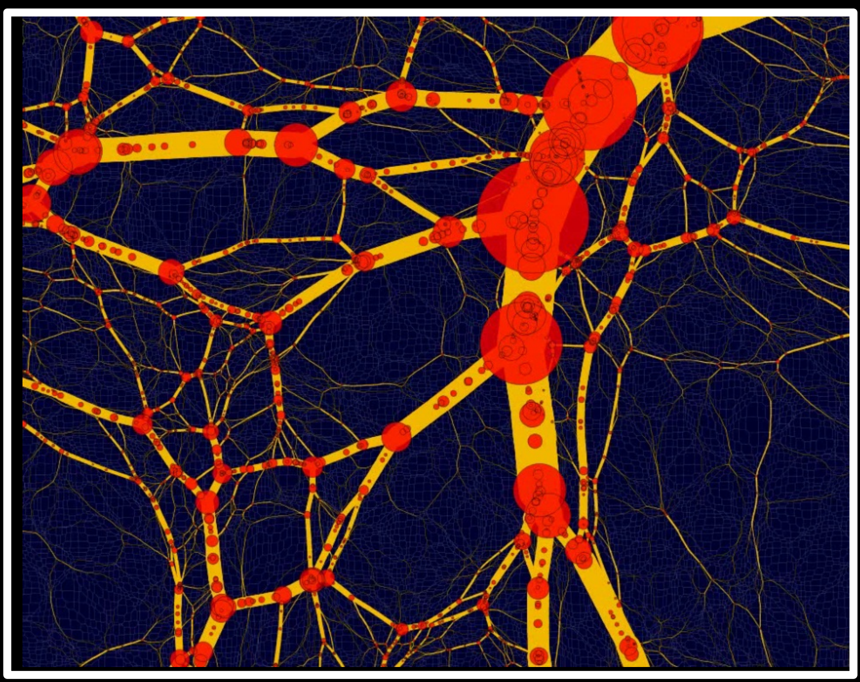
the morphology of the weblike network is
highly sensitive to the underlying cosmology

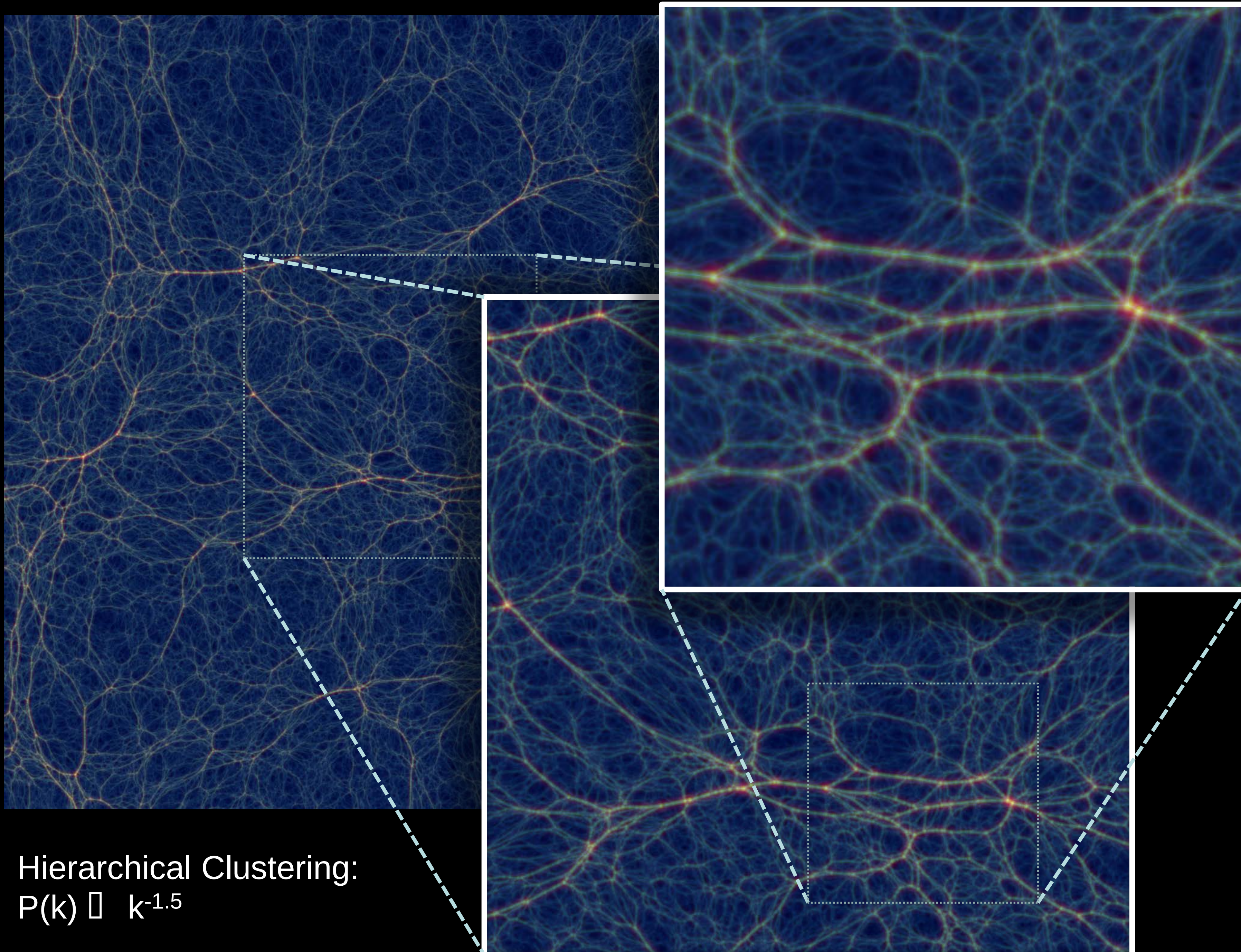
$P(k) \propto k^{-1.5}$

Hidding 2012/2014

$P(k) \propto k^{-2.0}$







Hierarchical Clustering:
$P(k) \propto k^{-1.5}$

The Dark Matter Sheet:

Phase-Space Dynamics
&
Tessellation Projections

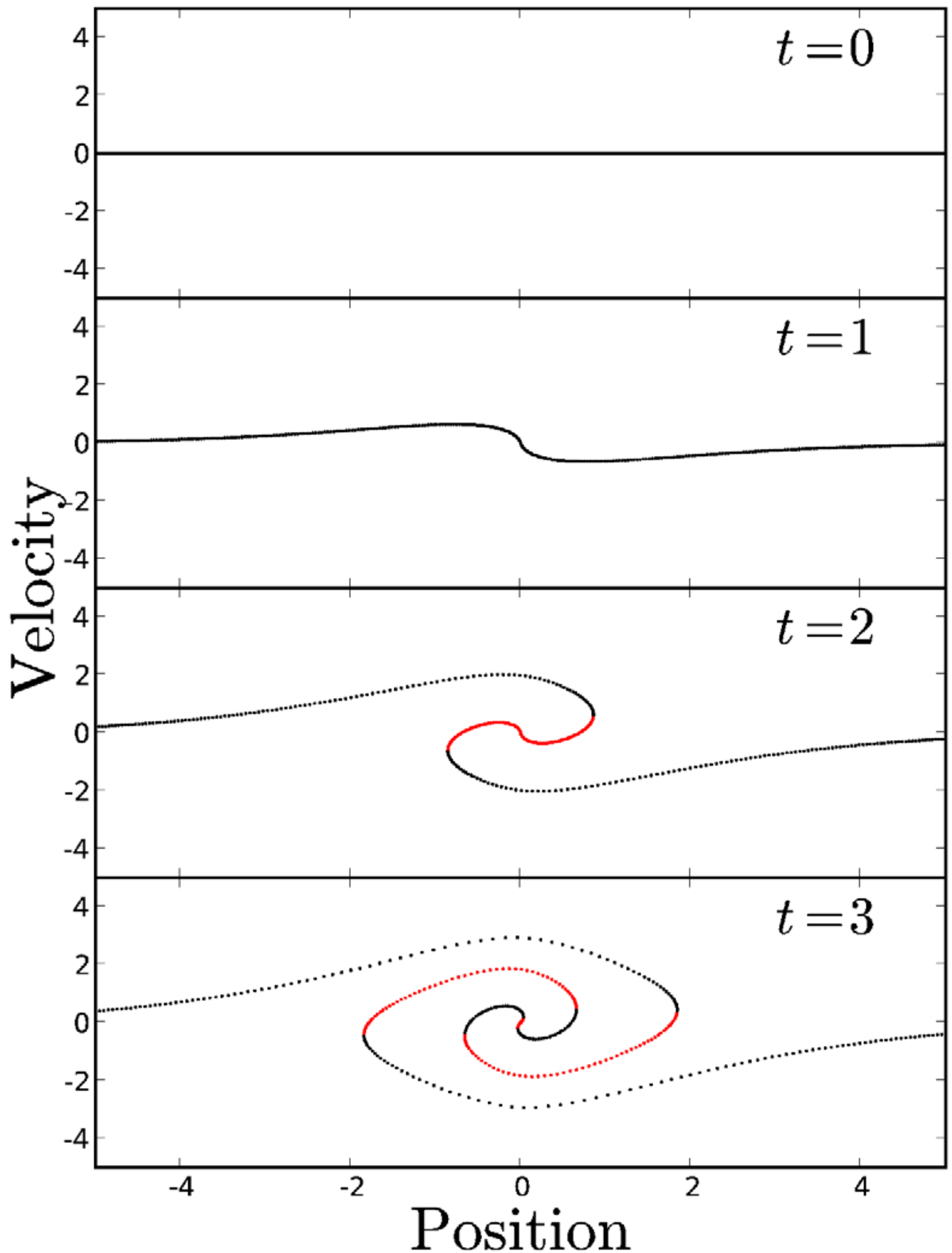
Phase Space Evolution

Dark Matter Phase Space sheet:

3-D structure projection of a folding DM phase space sheet
In 6-D phase space

- Shandarin 2010, 2011
- Neyrinck et al. 2011, 2012
  Origami
- Abel et al. 2011

Evolving matter distribution in position-velocity space – 1D



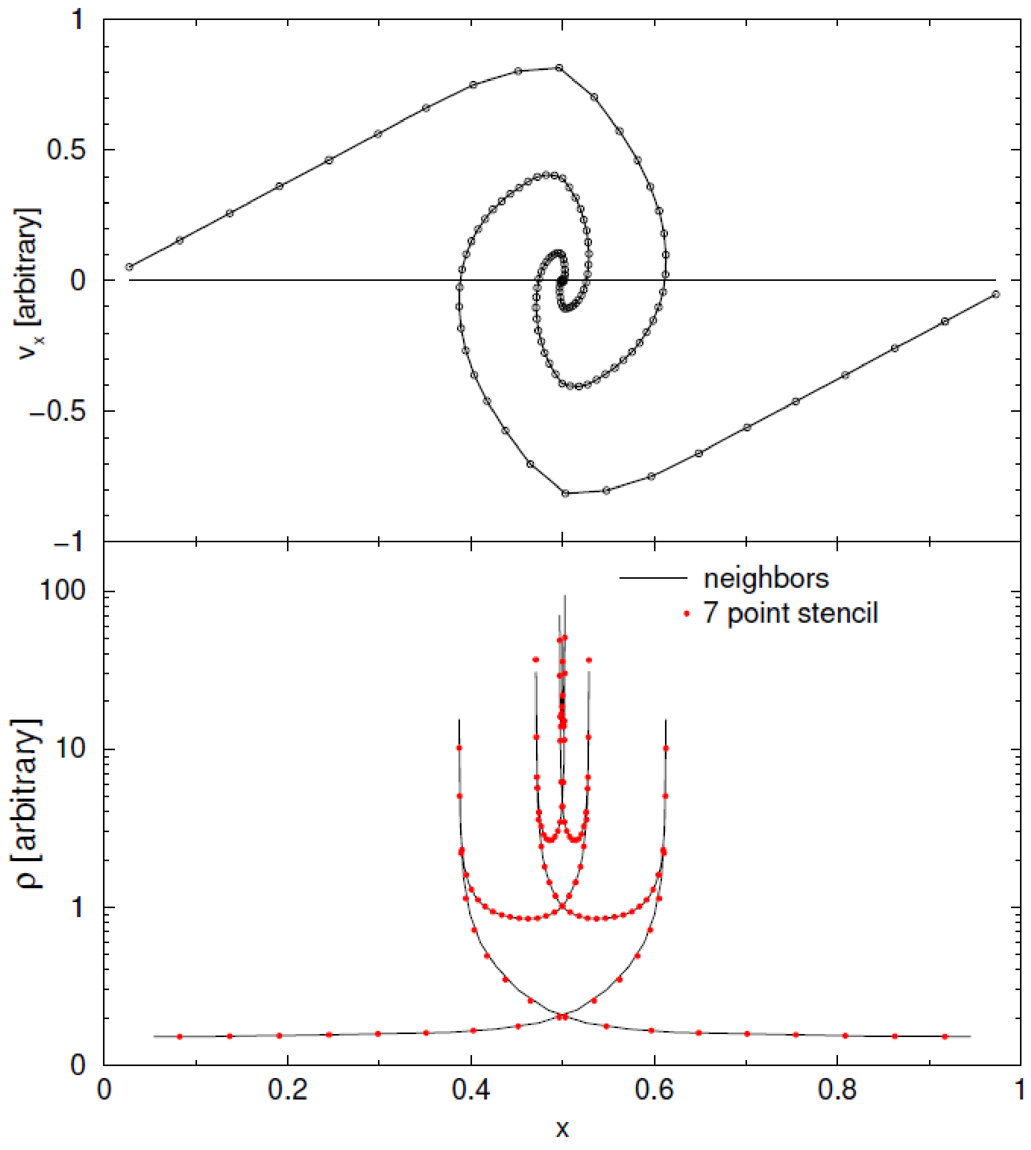
Phase Space Evolution

Phase space:

Velocity vs. Position

Density:

$$\rho(\vec{x},t) = \int f\left(\vec{x},\vec{v},t\right) d\vec{v}$$



Lagrangian-Eulerian Phase Space

To follow evolving phase-space of cosmic structure, it is sometimes insightful to consider a coordinate transformation of 6D phase-space:

Eulerian coordinates $\vec{x}$ and Eulerian coordinates $\vec{q}$ of a mass element:

$$f(\vec{x},\vec{q})$$

Note that in Zeldovich approximation, the velocity of a mass element is:

$$\vec{v}(\vec{q},t) = -a(t)D(t)f(\Omega)\ \vec{\nabla}\Phi(\vec{q})$$

Tessellation Deformation & Phase Space Projection

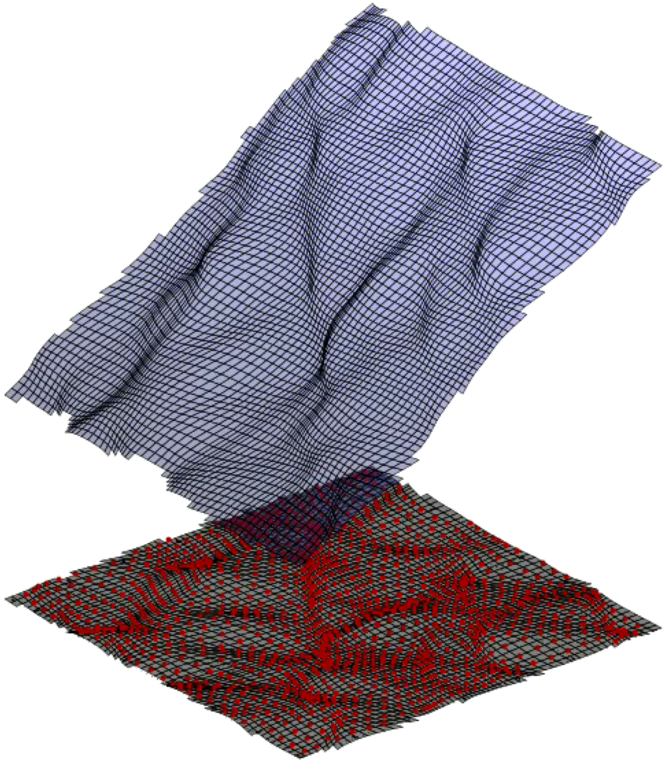
Translation towards Multi-D space:

- Look at deformation of initial tessellation

- each tessellation cell represents matter cell

- evolution deforms cell

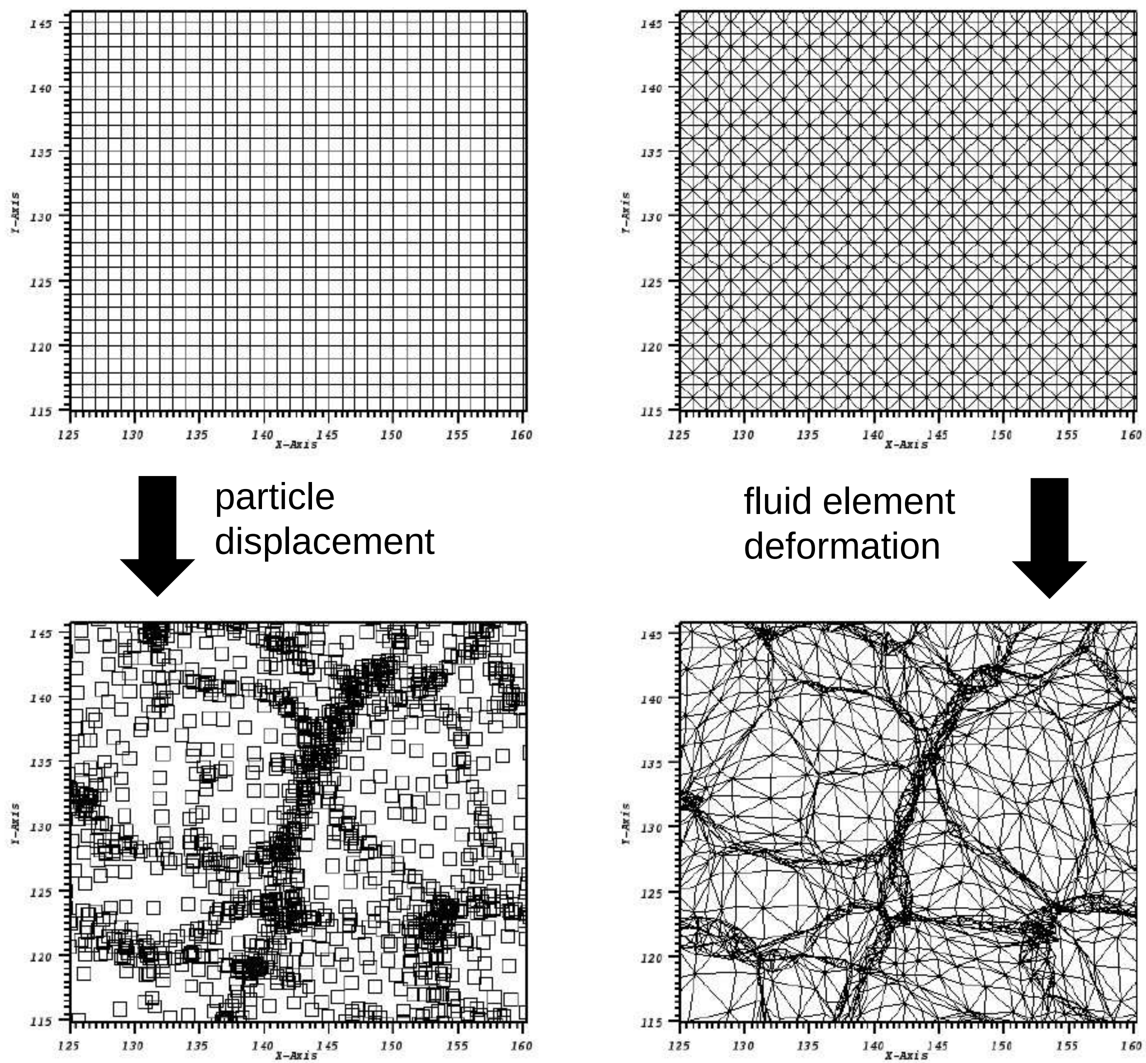
- once cells start to overlap, manifestation of different phase-space matter streams



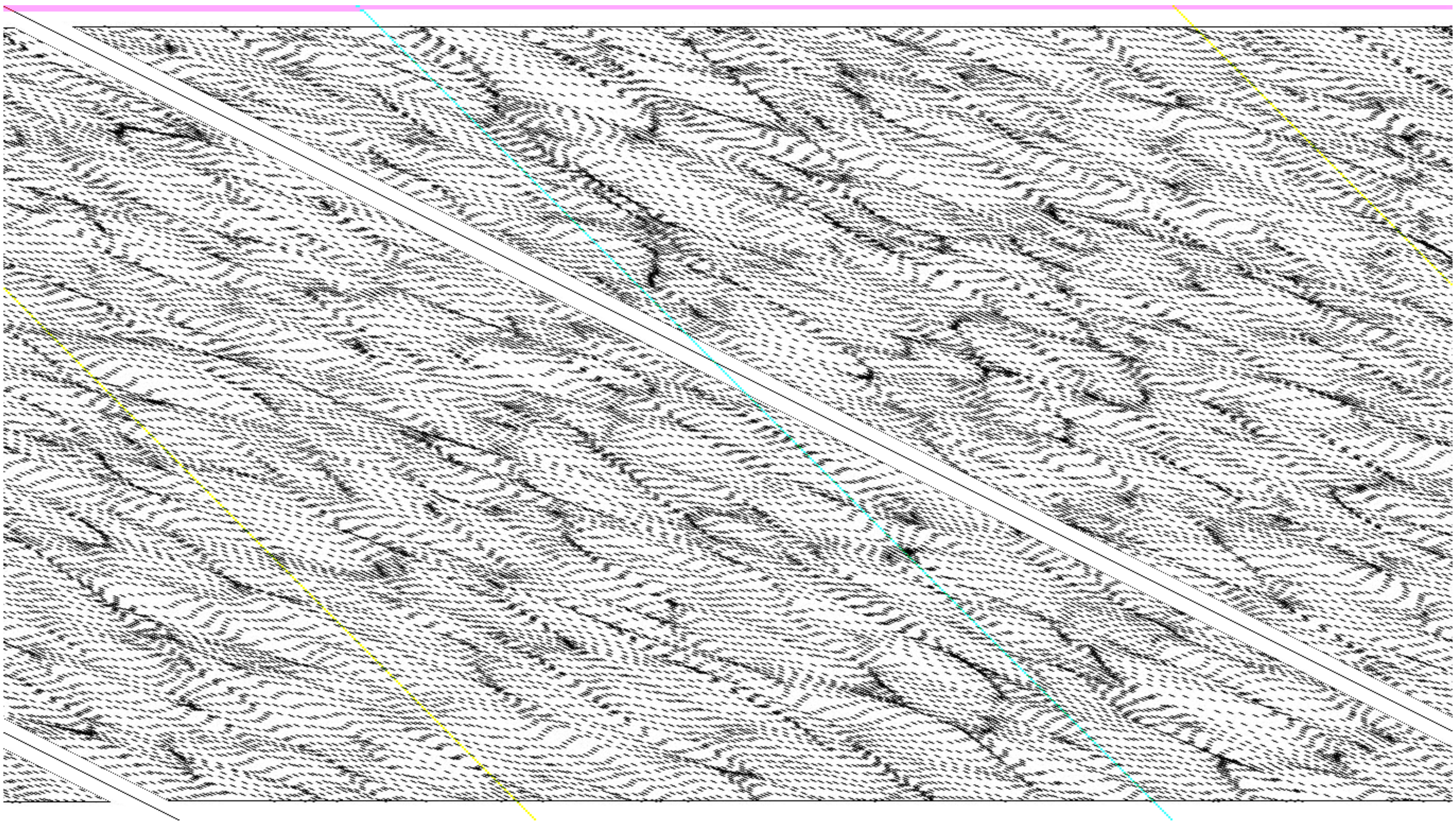
Tessellation Deformation & Phase Space Projection

Translation towards Multi-D space:

- Look at deformation of initial tessellation
- each tessellation cell represents matter cell
- evolution deforms cell
- once cells start to overlap, manifestation of different phase-space matter streams



Particle Simulation



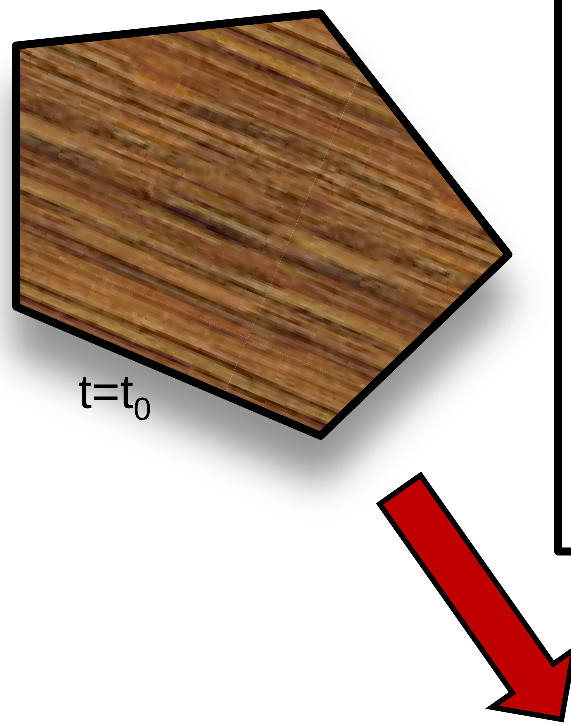
Mass Element Evolution

Tessellation Deformation & Phase Space Projection

Translation towards Multi-D space:

- Look at deformation of initial tessellation
- each tessellation cell represents matter cell
- evolution deforms cell

Monostream
Density Evolution



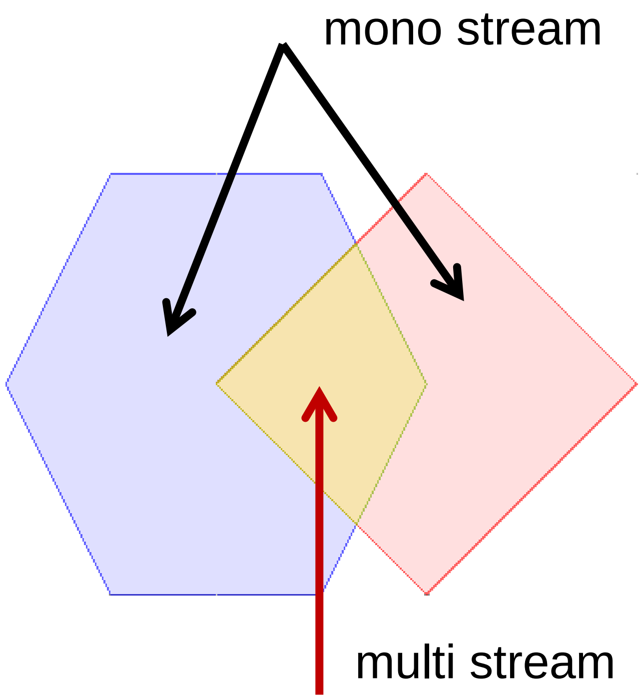
Conservation of mass (continuity eqn.):

$$\rho(\vec{x},t) = \left|J(\vec{x},\vec{q})\right|^{-1} \rho(\vec{q}) = \left|\frac{\partial \vec{x}}{\partial \vec{q}}\right|^{-1} \rho(\vec{q})$$

$$\downarrow$$

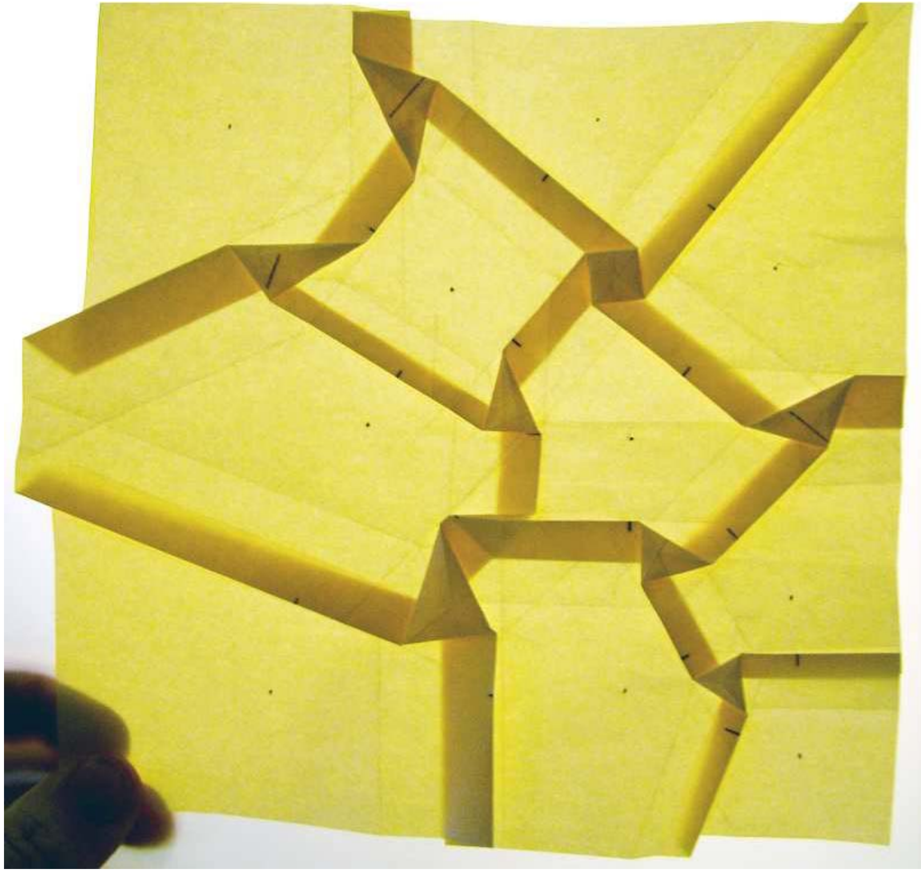
$$\rho(\vec{x},t_1) = \frac{V_0}{V_1}\rho(\vec{q},t_0)$$

(Cosmic) ORIGAMI



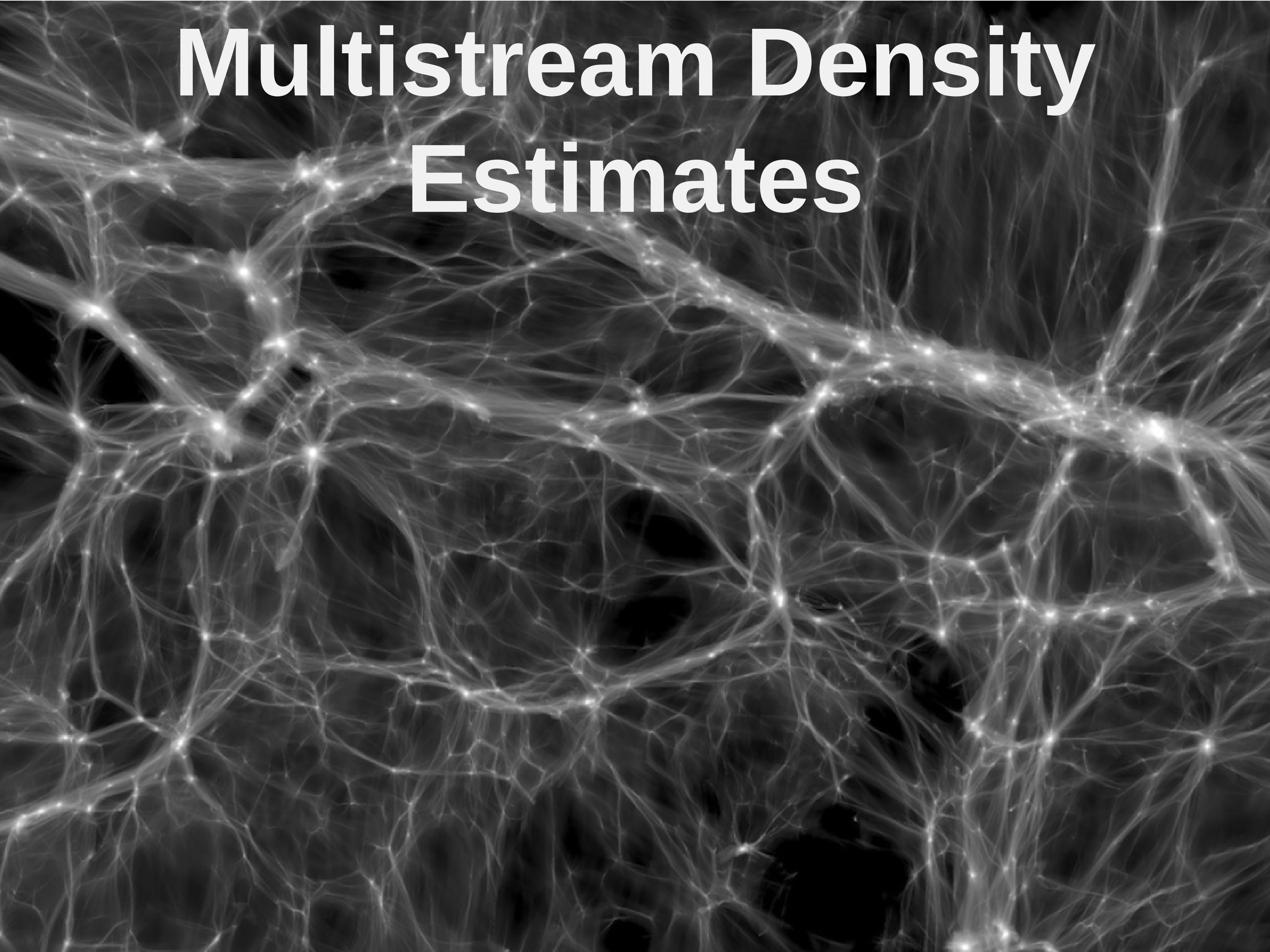
$$\rho_{total}(\vec{x},t_1) = \sum_i \frac{V_{0i}}{V_{1i}} \rho\left(\vec{q}_i, t_0\right)$$

Evolution of dynamical system:
Phase-space folding – Cosmic Origami

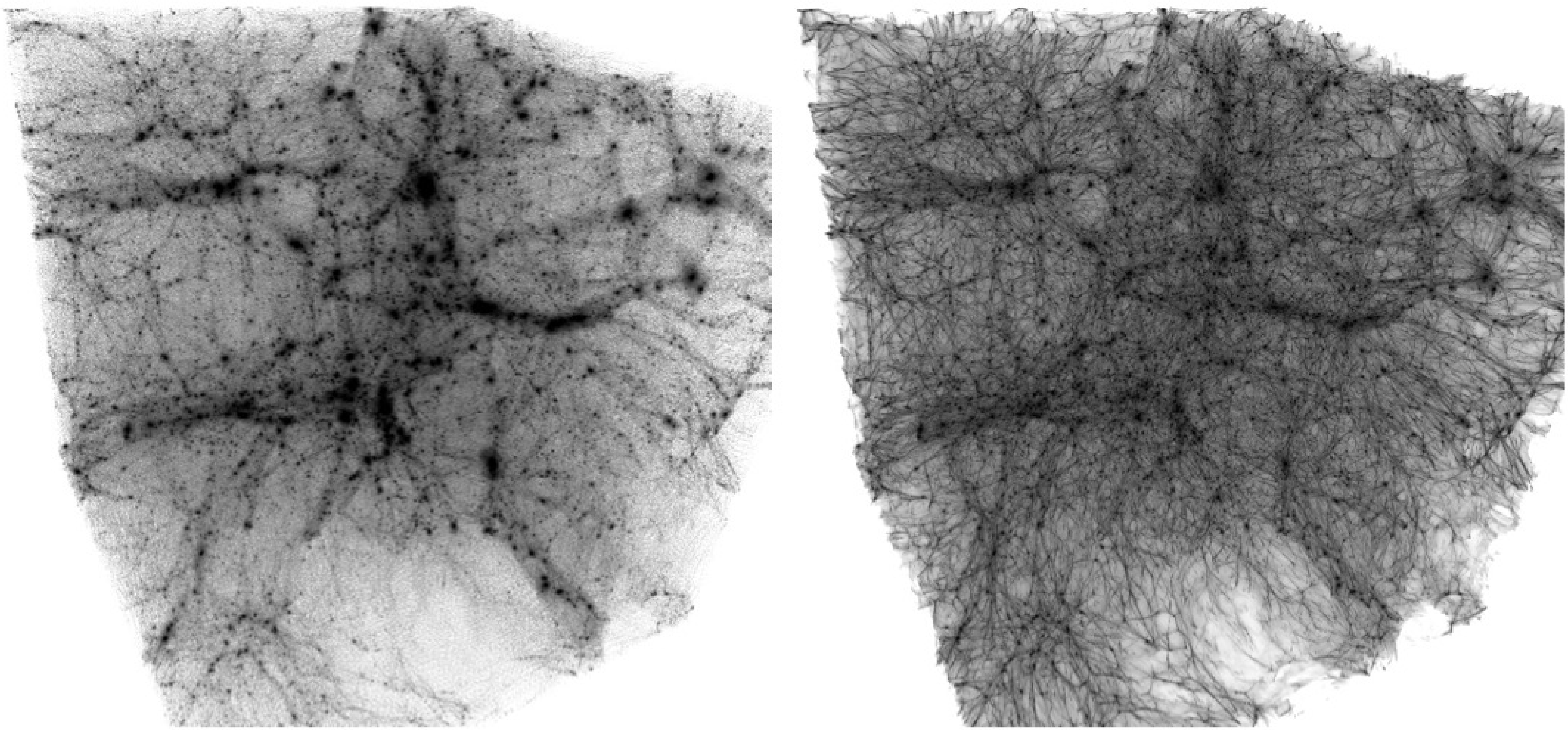


Mark Neyrinck

Multistream Density Estimates



Multistream Density Estimates



Cosmic Web Stream Density

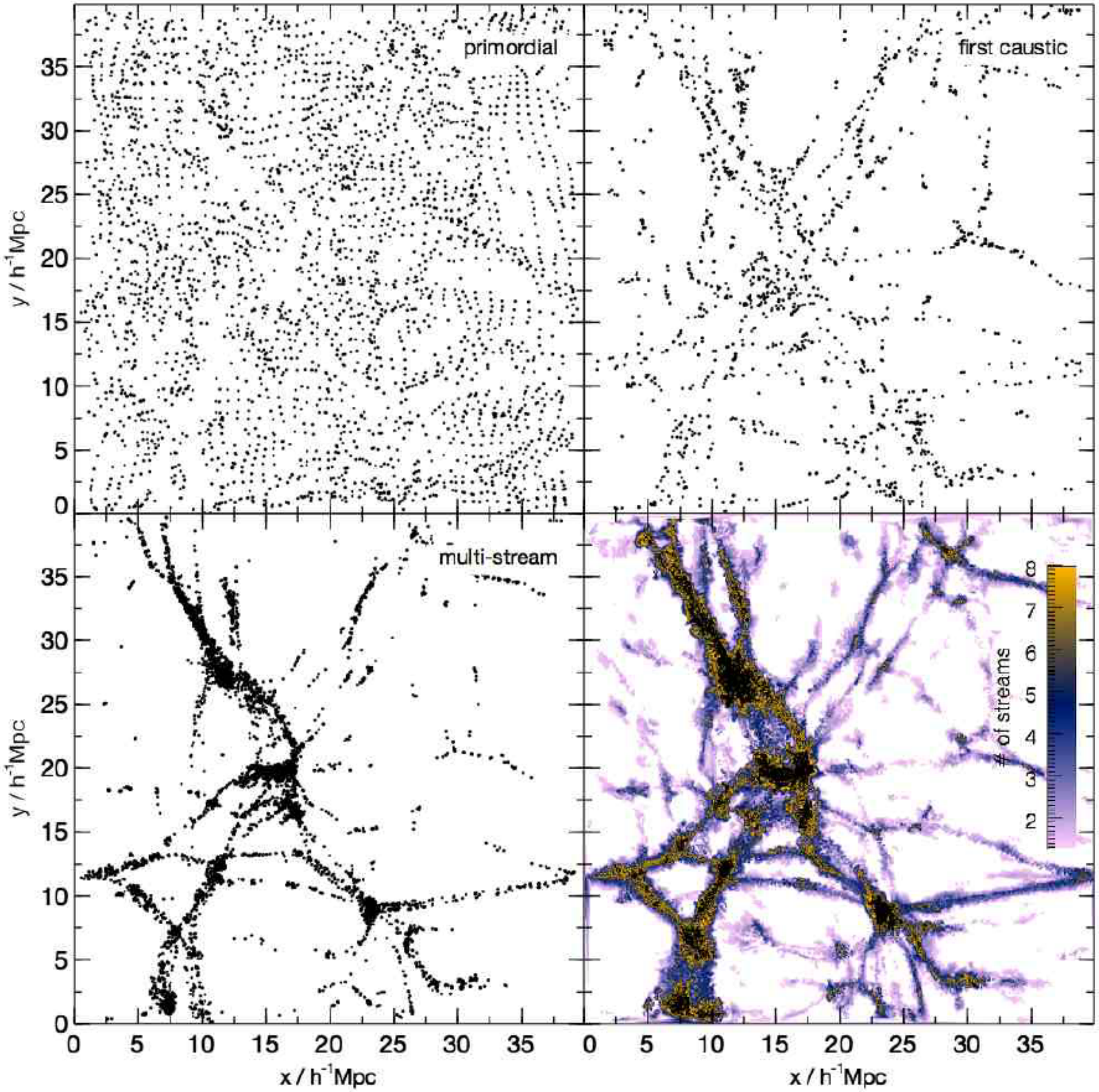
Translation towards Multi-D space:

Density of
dark matter streams:

- # phase space folds

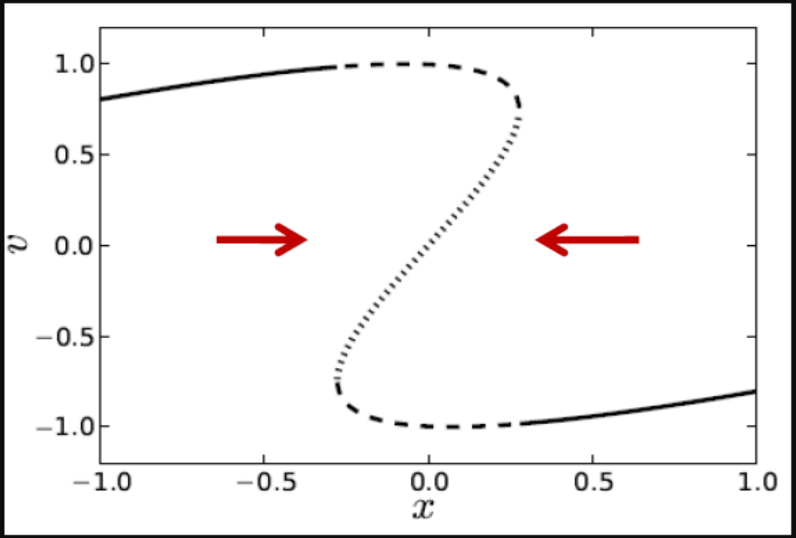
=

locally overlapping
tessellation cells



Origami Folding
&
Structural Morphology

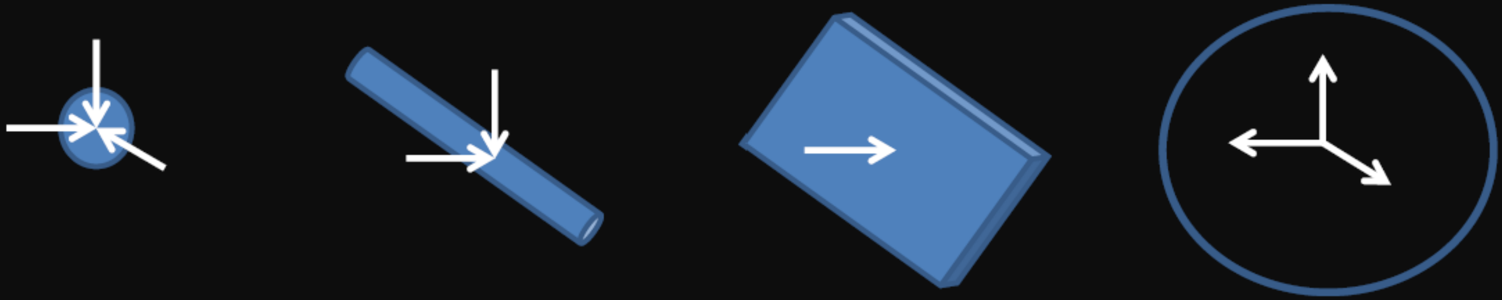
The ORIGAMI Cosmic Web



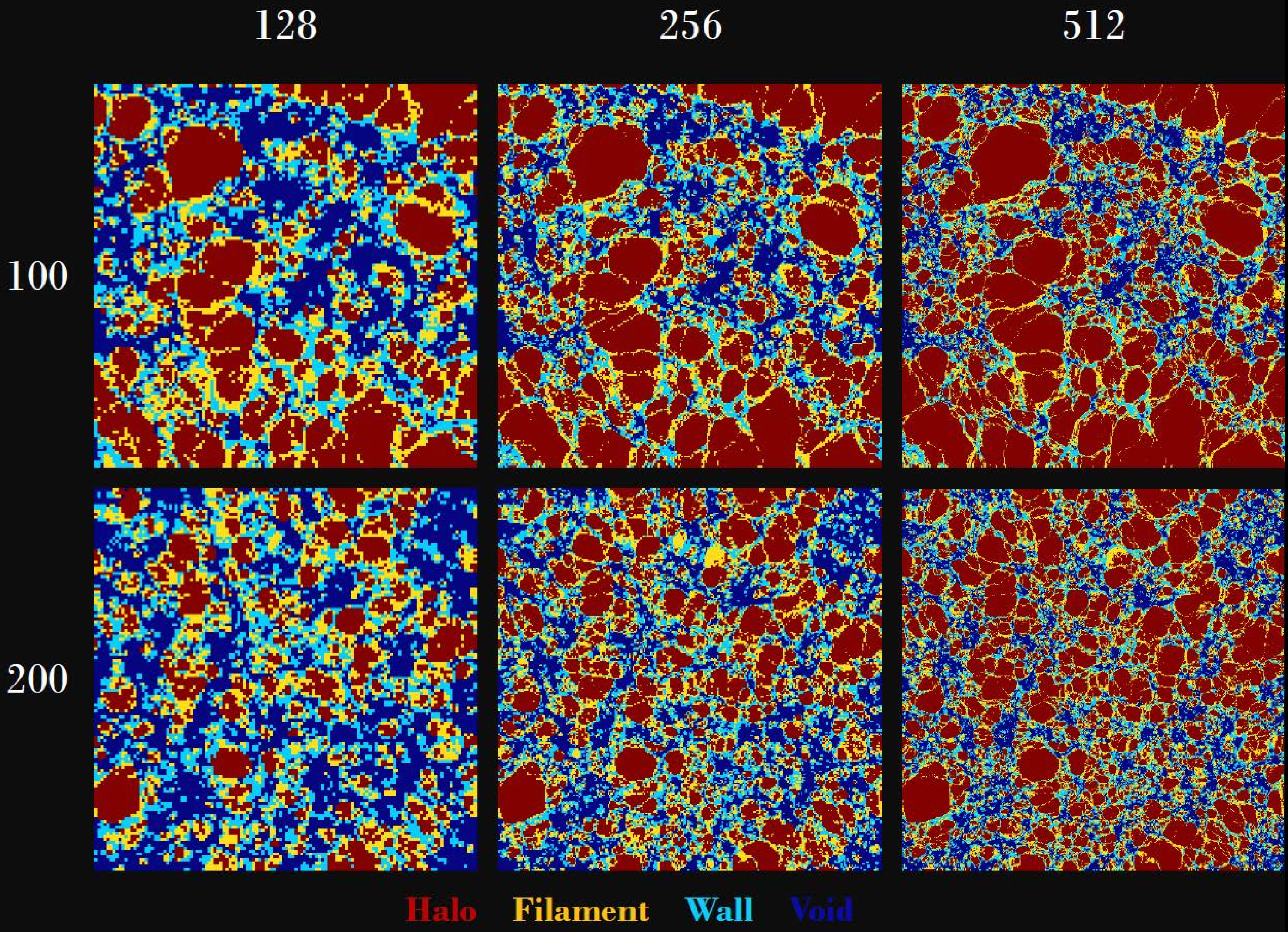
Find the phase-space folds by looking for simulation particles that are out of order along orthogonal axes

(Falck, Neyrinck, & Szalay 2012, 1201.2353)

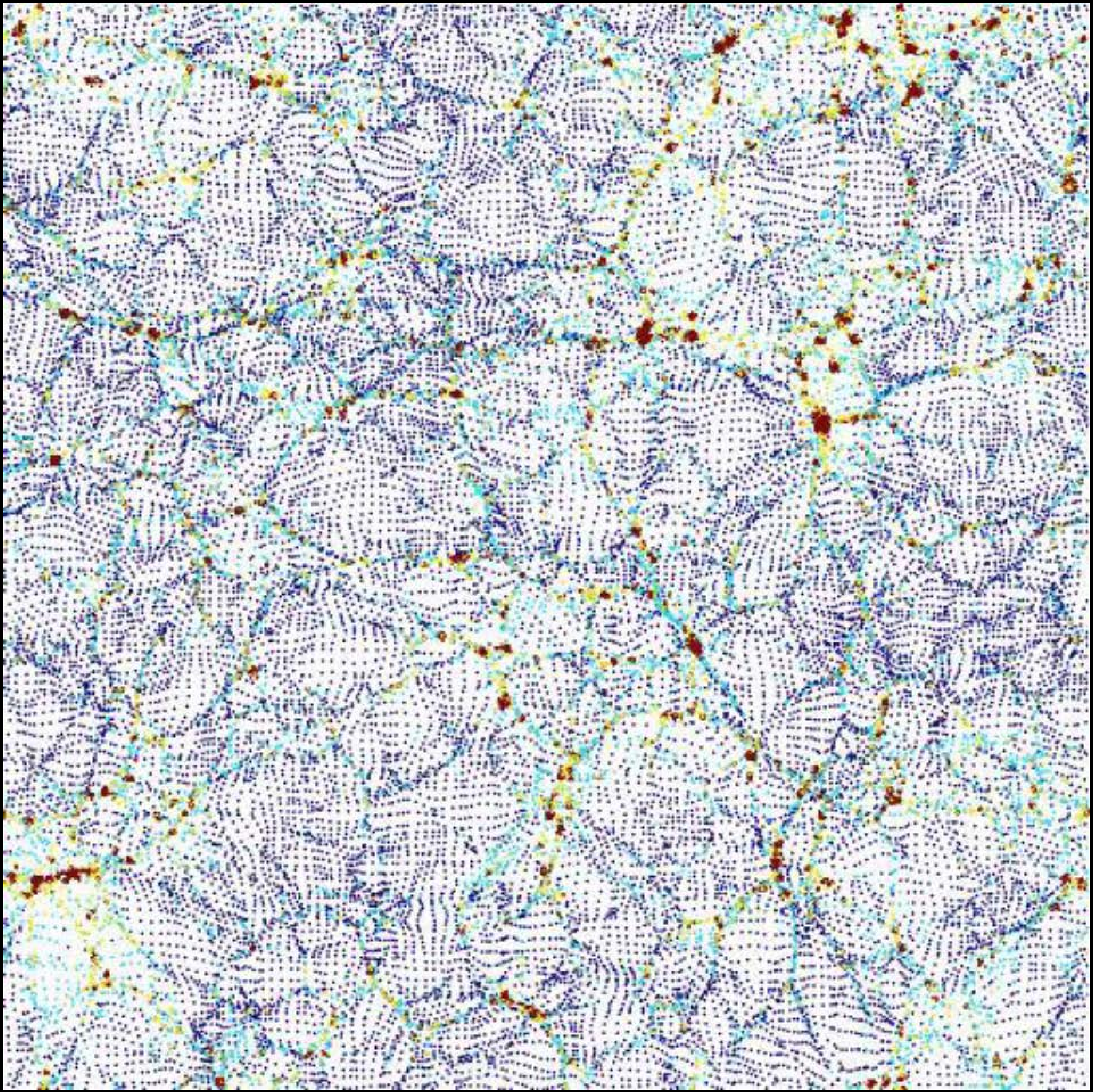
Halos collapse along 3 axes, Filaments 2, Walls 1, and Voids 0



Voronoi & Origami



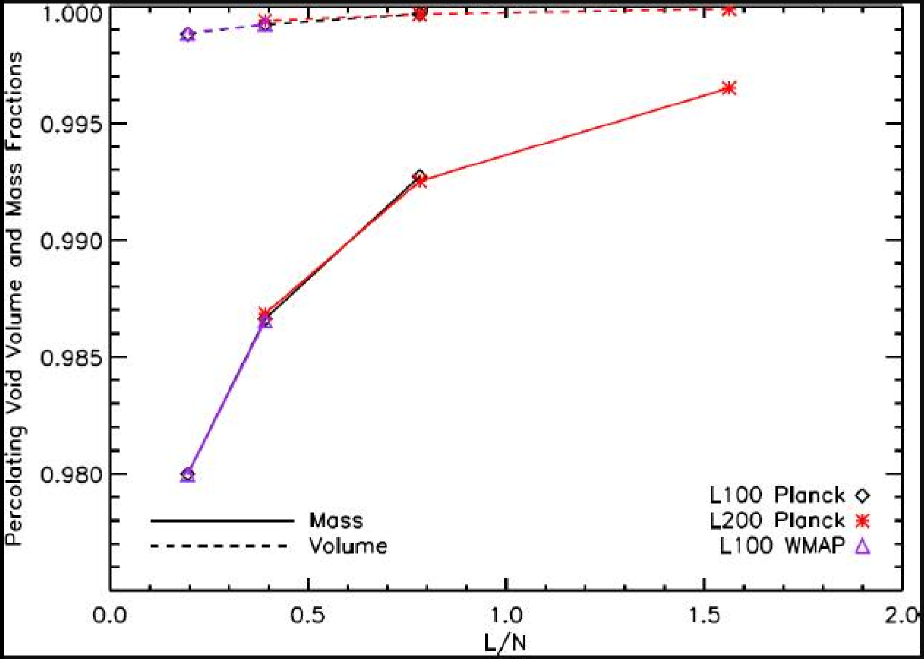
Falck, Neyrinck et al. 2012/2014



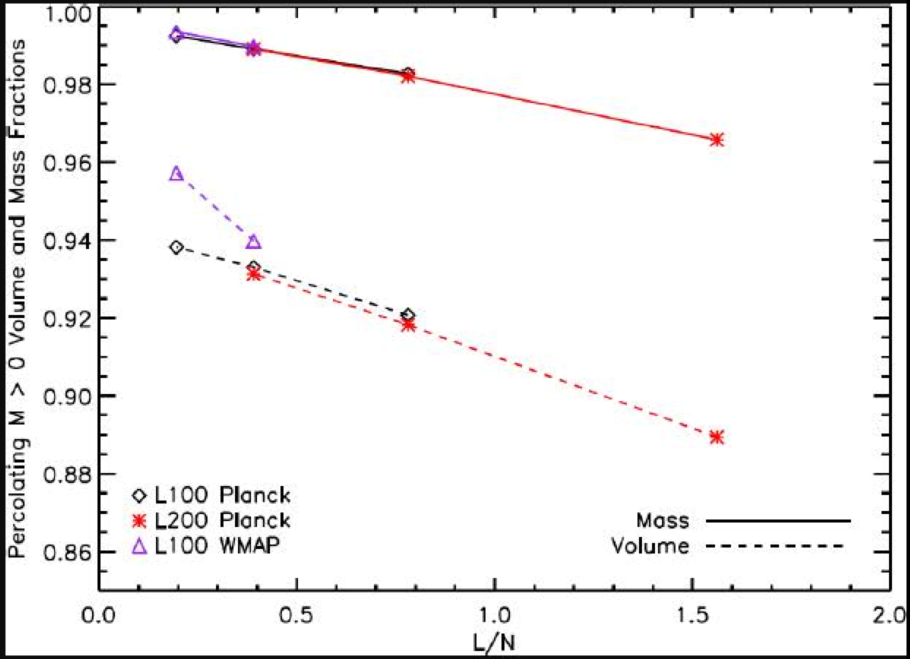
Falck, Neyrinck & Szalay
2012/2014

Volume & Mass fractions of largest (percolating) structures with respect to total particles in those structures:

Single-Stream



Multi-Stream



Percolating void mass and volume fractions depend on resolution, but vary slowly

Multi-stream regions also percolate, and volume/mass fractions vary more rapidly with resolution

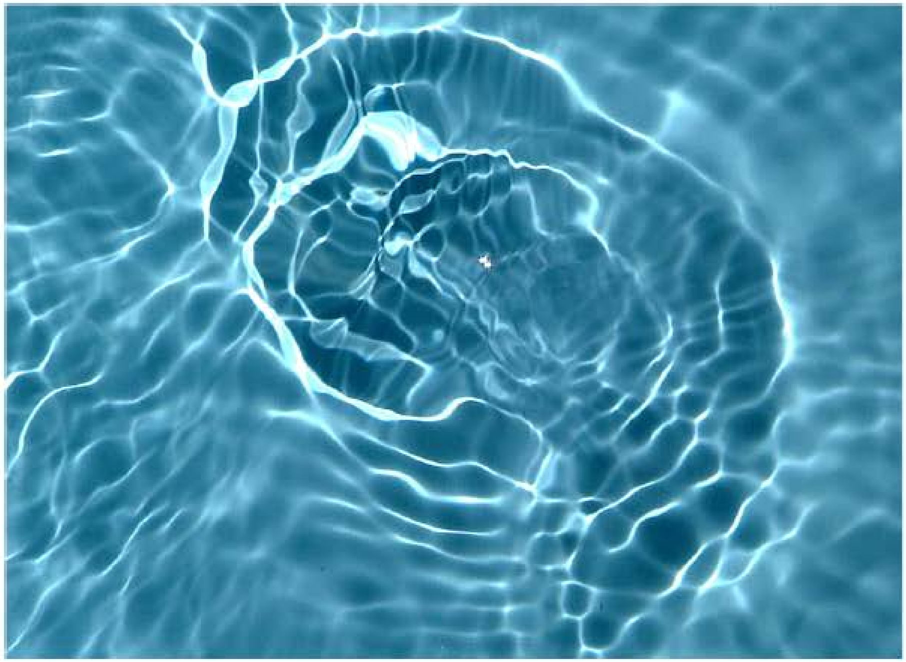
Falck, Neyrinck & Szalay 2012/2014

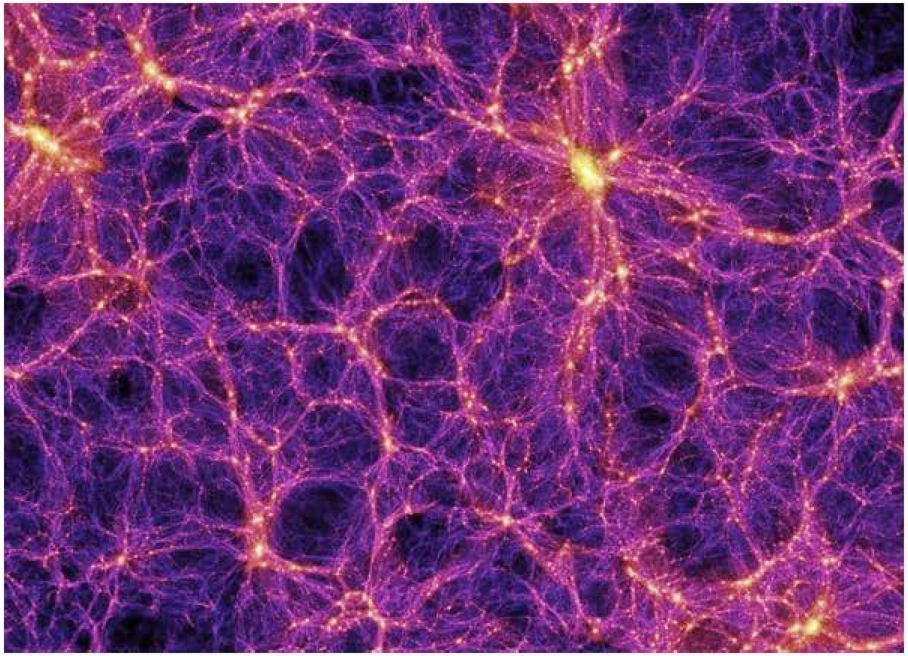
Cosmic Web:

from Zeldovich
to
catastrophes

Hidding, Shandarin & vdW 2014

Caustics in Cosmic Structure





Zel'dovich Approximation

$$\vec{x} = \vec{q} + D(t)\,\vec{u}(\vec{q})$$

$$\vec{u}(\vec{q}) = -\vec{\nabla}\Phi(\vec{q})$$

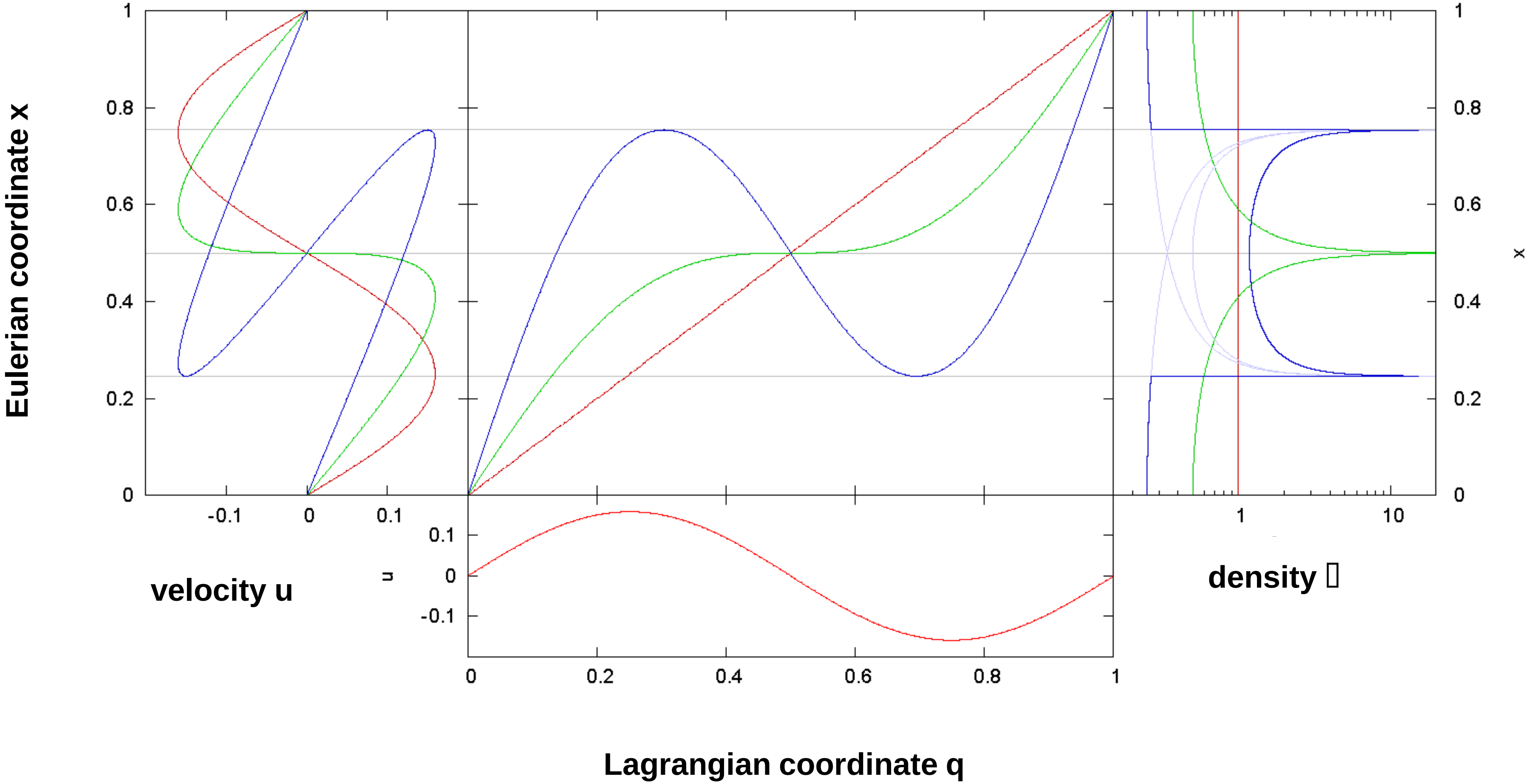
$$d_{ij} = -\frac{\partial u_i}{\partial q_j}$$

$$\rho(\vec{q},t) = \frac{\rho_u(t)}{\big(1 - D(t)\lambda_1(\vec{q})\big)\big(1 - D(t)\lambda_2(\vec{q})\big)\big(1 - D(t)\lambda_3(\vec{q})\big)}$$

structure of the cosmic web determined by the spatial field of eigenvalues

$$\lambda_1, \lambda_2, \lambda_3$$

Multistream Regions & Catastrophes



Cosmic Singularities

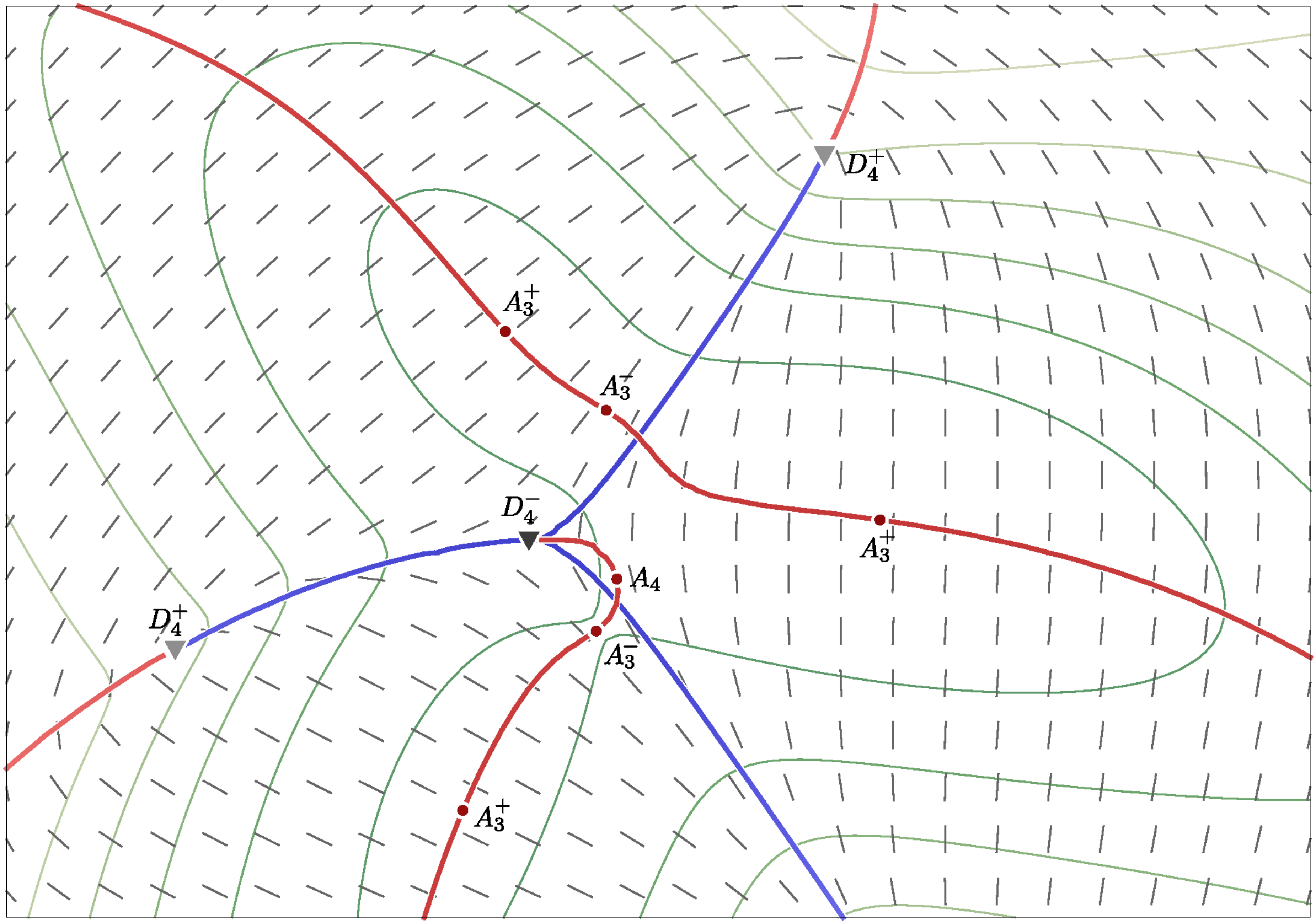
$$\rho\left(\vec{q},t\right)=\frac{\rho_u\left(t\right)}{\left(1-D(t)\lambda_1\left(\vec{q}\right)\right)\left(1-D(t)\lambda_2\left(\vec{q}\right)\right)\left(1-D(t)\lambda_3\left(\vec{q}\right)\right)}$$

- Evolution spatial structure determined by:

  spatial characteristics of deformation (tidal) tensor field in Lagrangian space

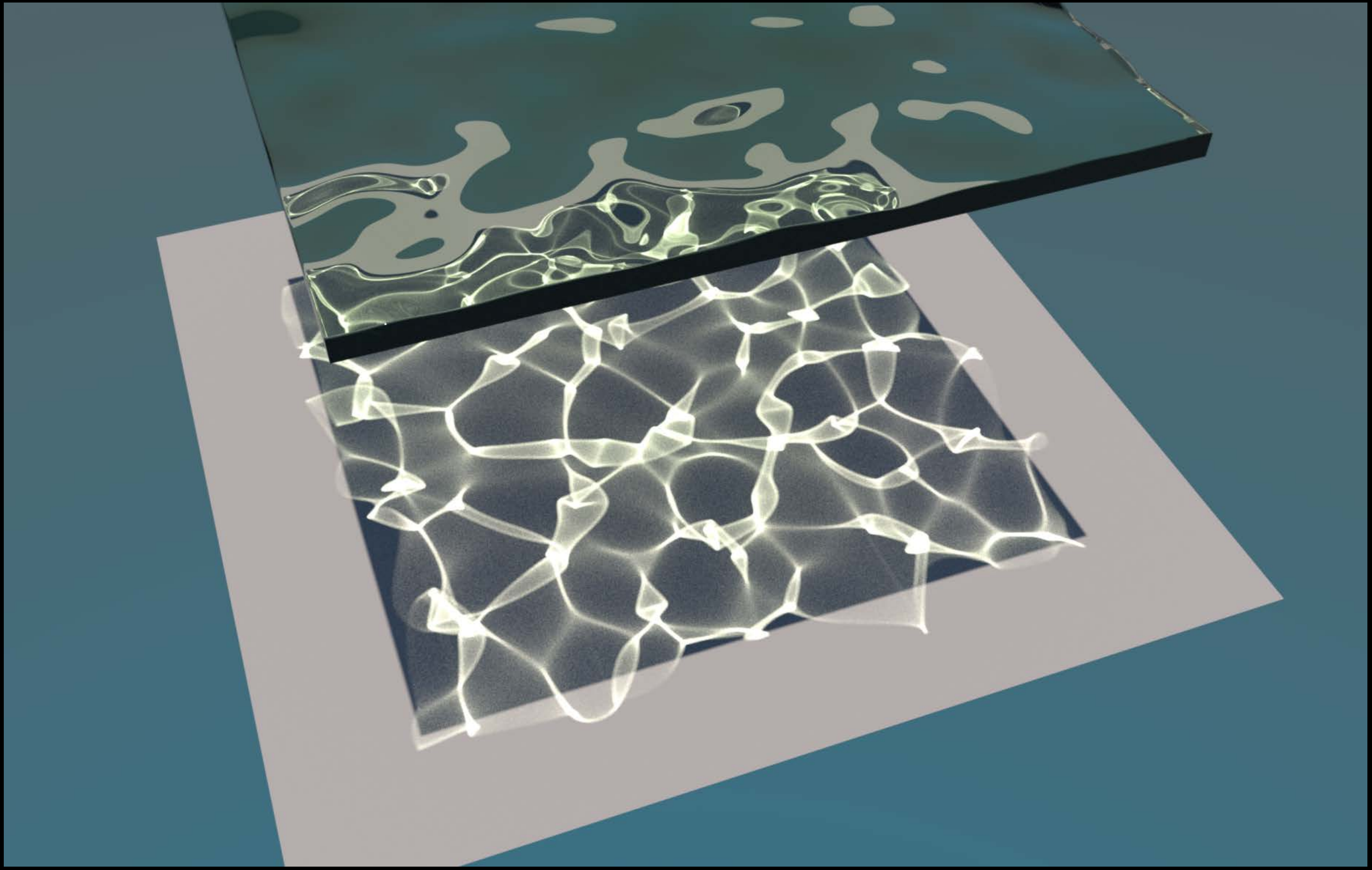
  i.e. by the fields **λ_1(q), λ_2(q), λ_3(q)**

$$\frac{\partial^2\Phi_v}{\partial q_i \partial q_j}(\vec{q}) \quad \rightarrow \quad \lambda_1(\vec{q}),\ \ \lambda_2(\vec{q}),\ \ \lambda_3(\vec{q})$$

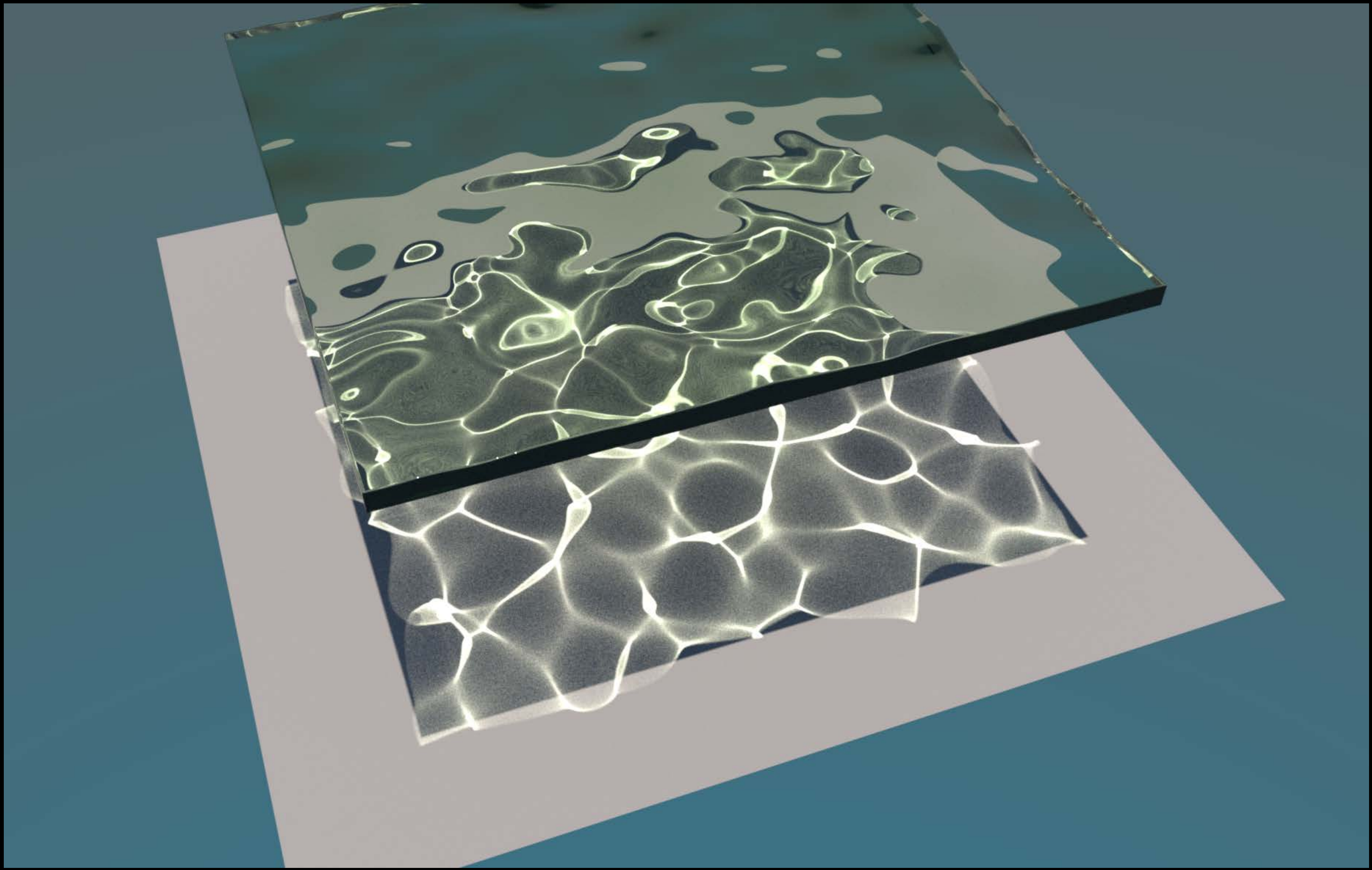


D_4^+
A_3^+
A_3^-
D_4^-
A_3^+
A_4
D_4^+
A_3^-
A_3^+

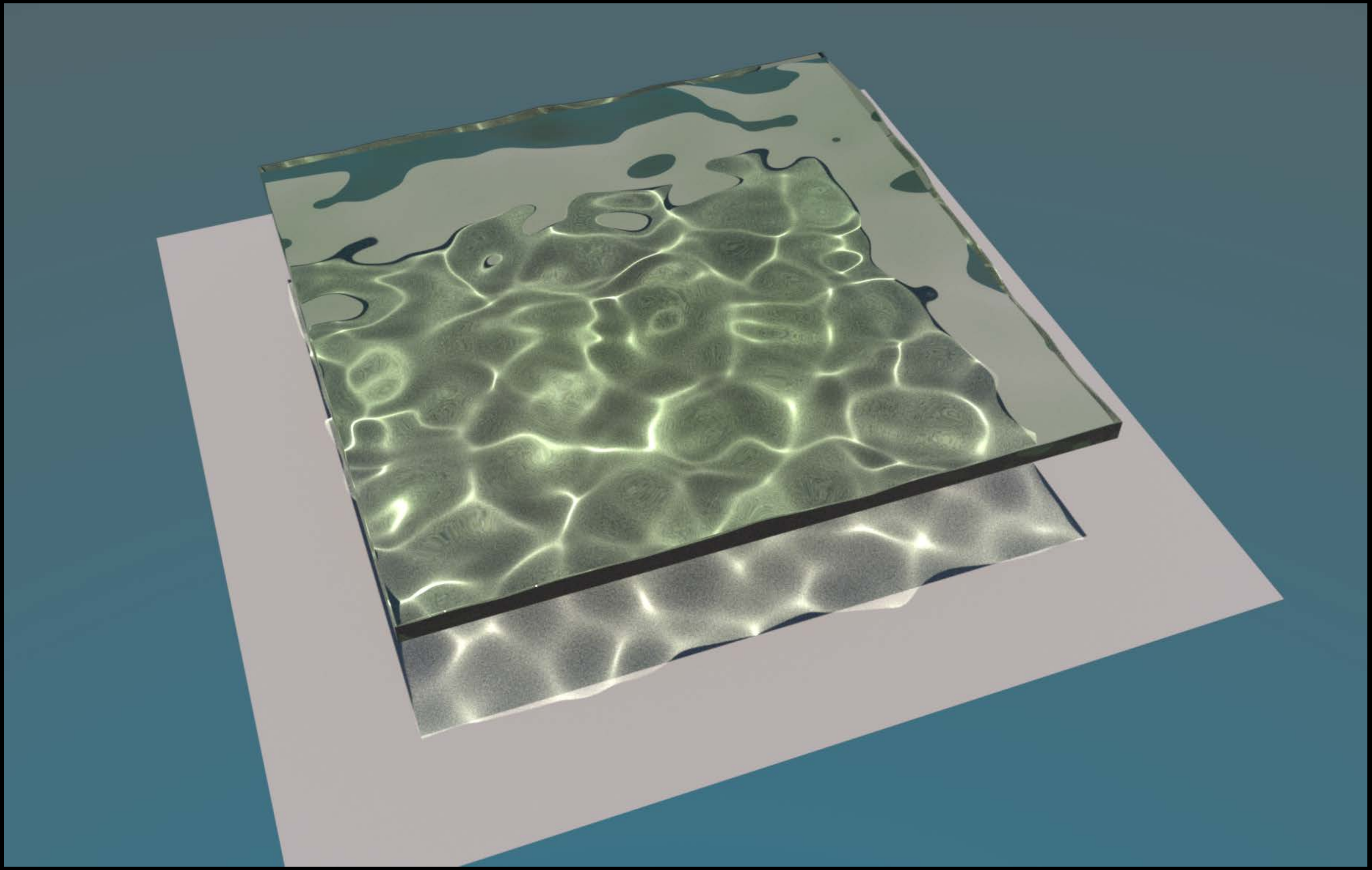
Caustics in Cosmic Structure



Caustics in Cosmic Structure



Caustics in Cosmic Structure

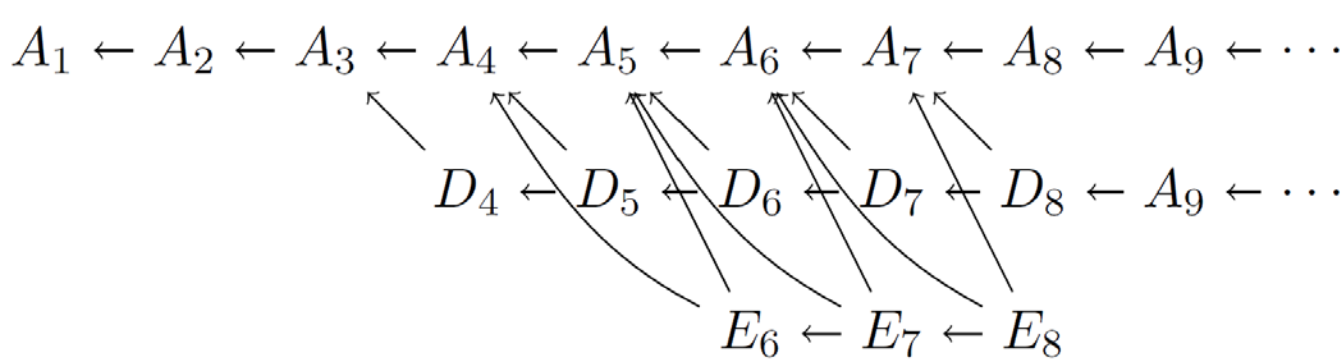


Cosmic Catastrophe Theory

- Mathematical singularity theory:
  Zeeman, Thom, Arnold

- Classification of singularities:
  Arnold (1972) developed classification of these Lagrangian catastrophes
  (up to local coordinate transformations)

- Application to Cosmology:
  Arnold, Shandarin & Zeldovich 1982



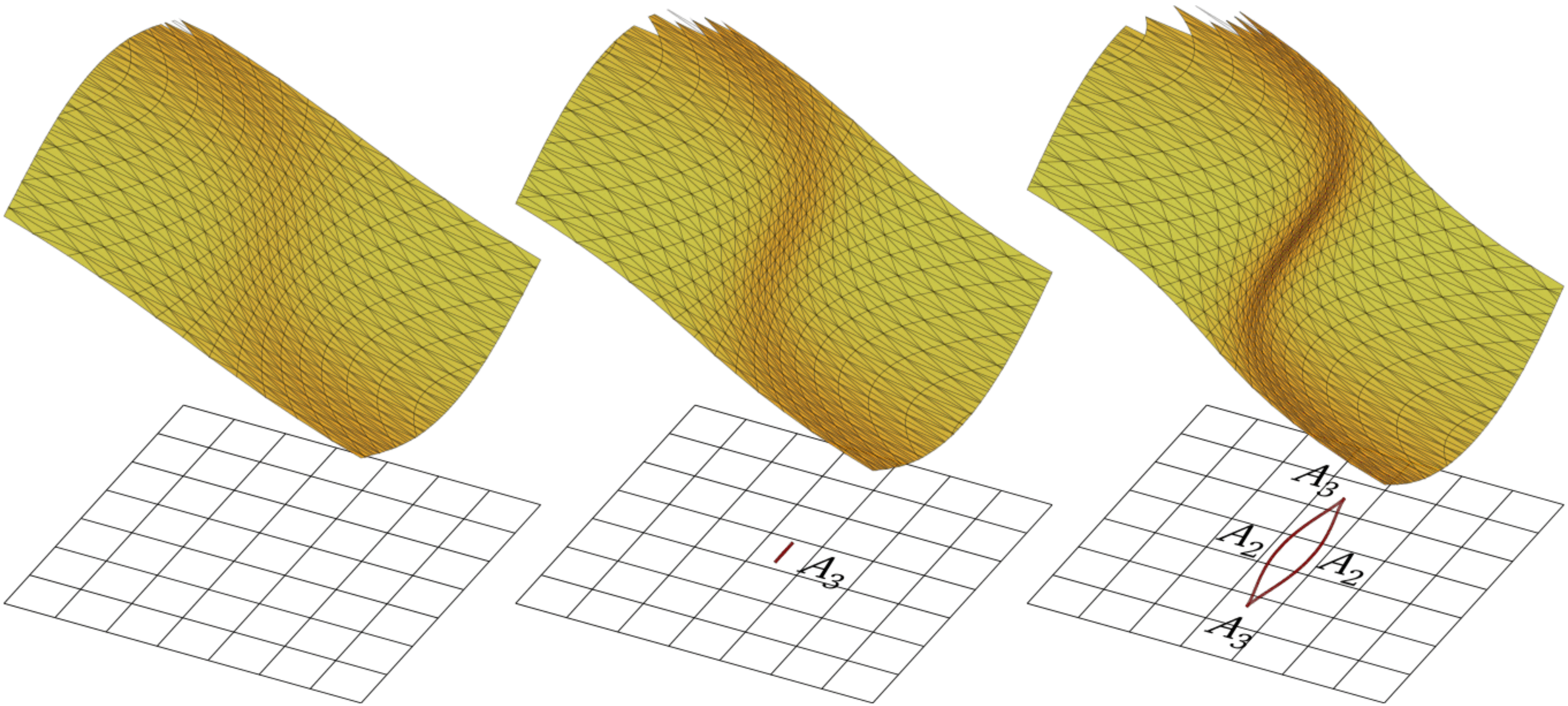
Cosmic Catastrophe Theory

- Classification of singularities:

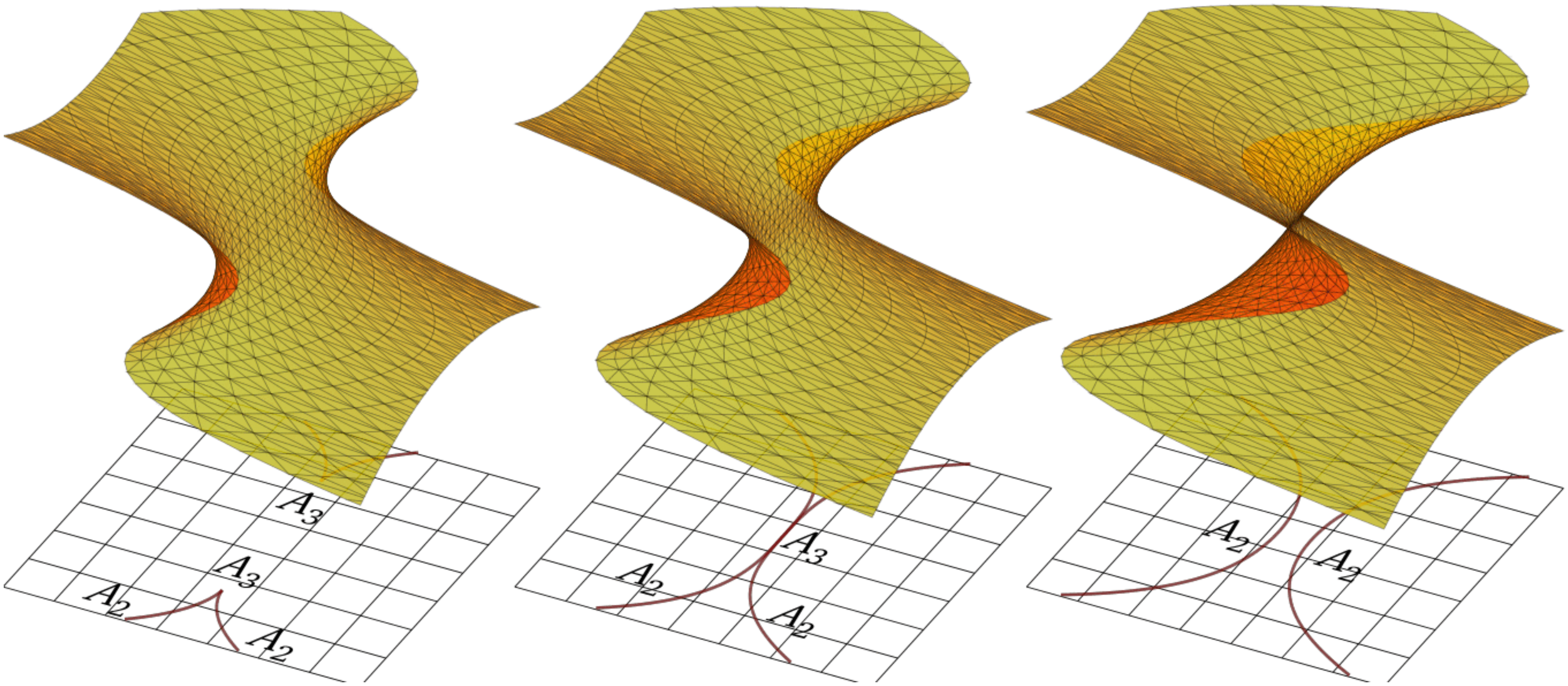
  Arnold (1972) developed classification of these Lagrangian catastrophes
  (up to local coordinate transformations)

1-D:	A2	fold catastrophe			
	A3	cusp catastrophe			
2-D:	A2	fold			
	A3	cusp			
	A4	swallow-tail	D4	umbilic	
3-D	A2				
	A3				
	A4		D4		
	A5		D5		E5

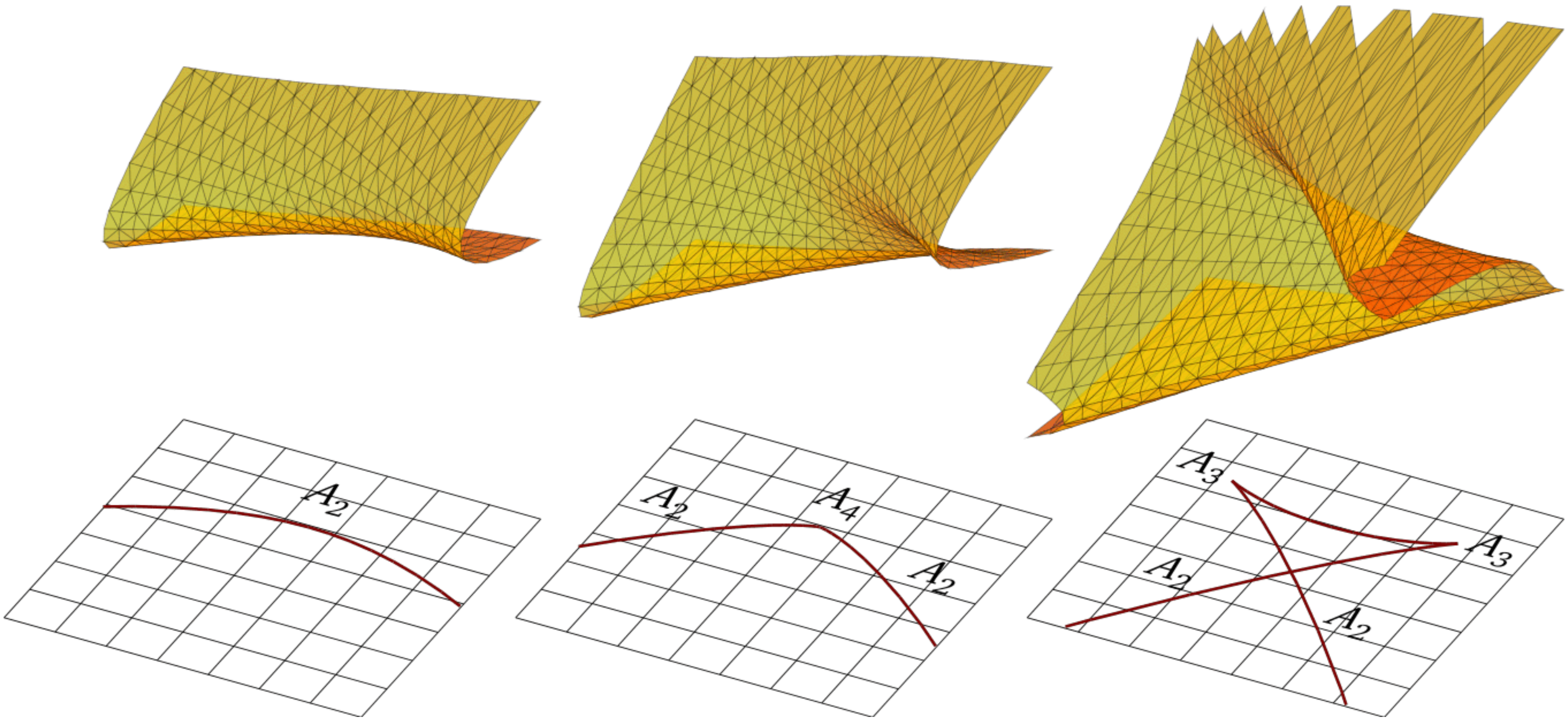
Fold & Cusp Singularities



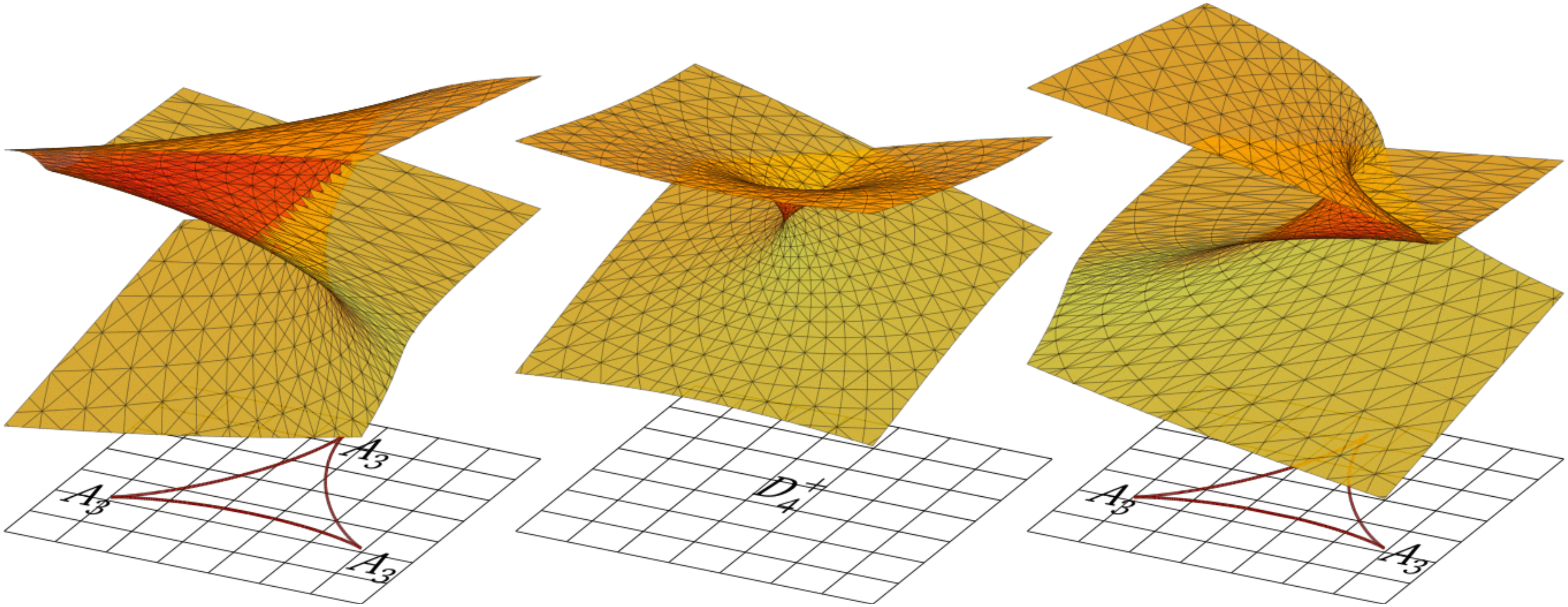
A_3: Cusp Annihilation



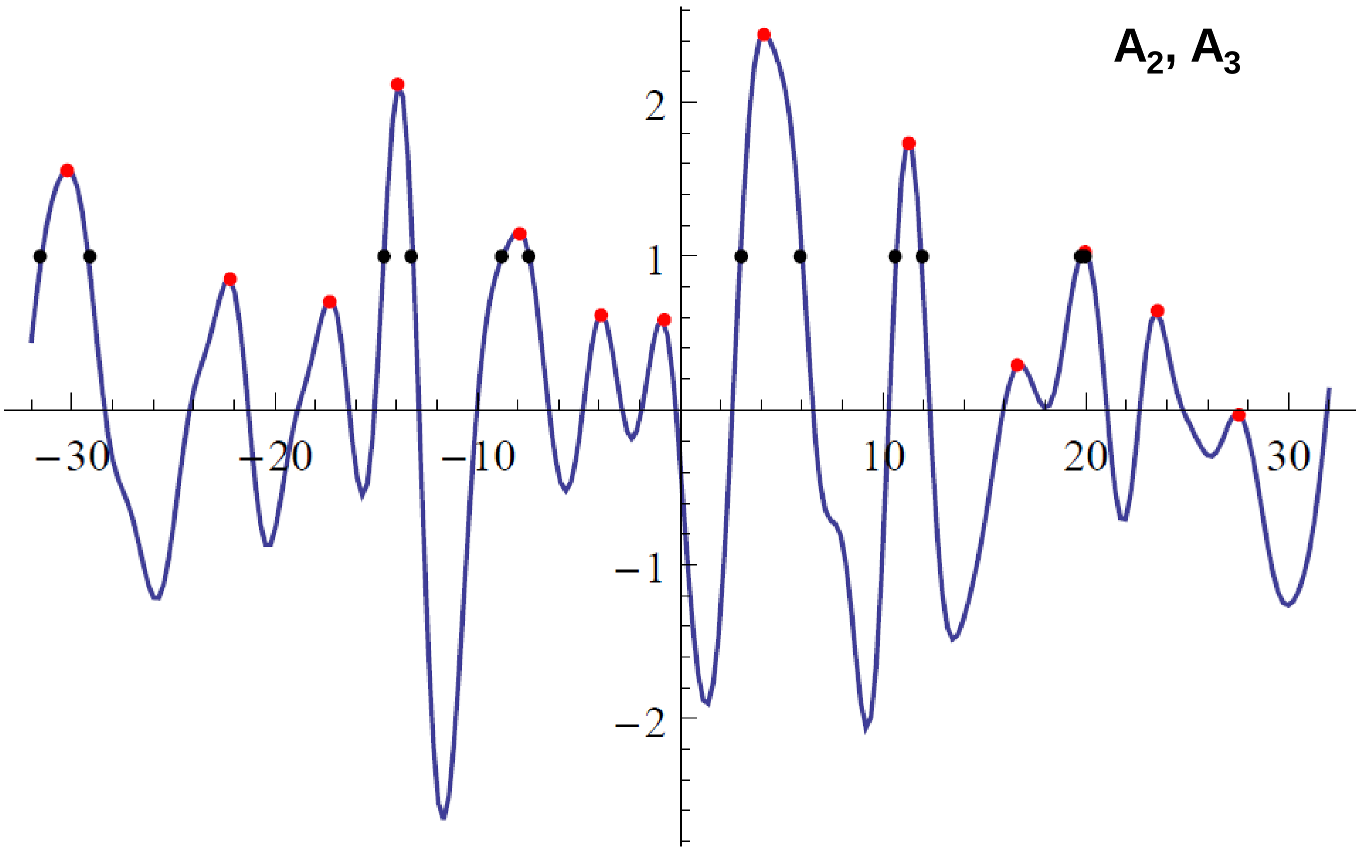
A_4: Swallow-Tail Singularities



D_4: Umbilic Singularities

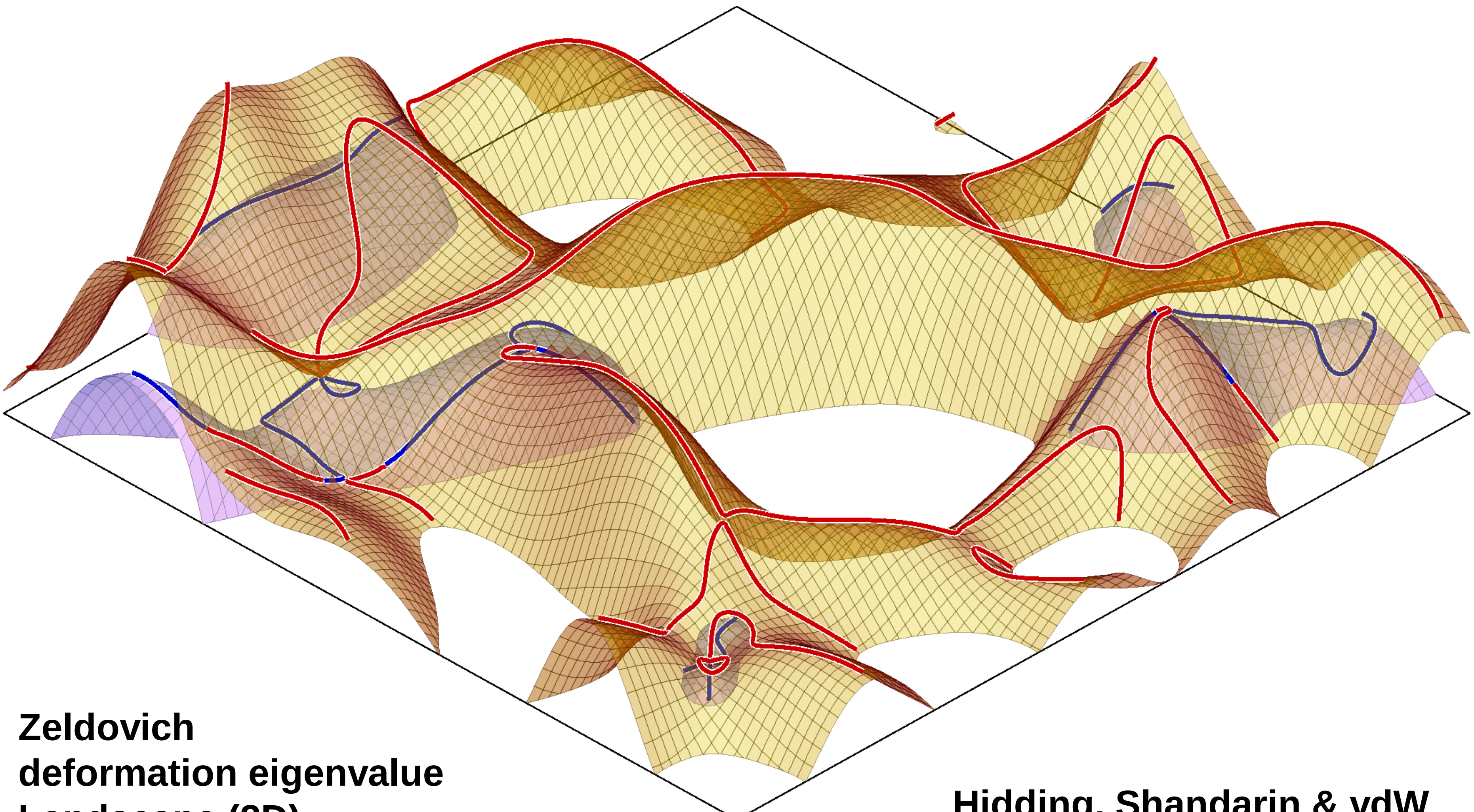


1-D Catastrophes



Feldbrugge 2014

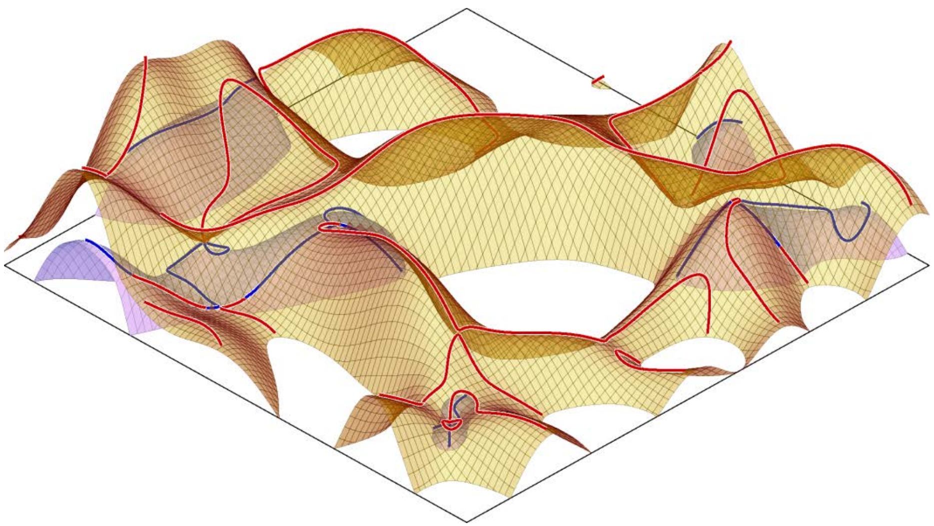
Singularities & Catastrophes

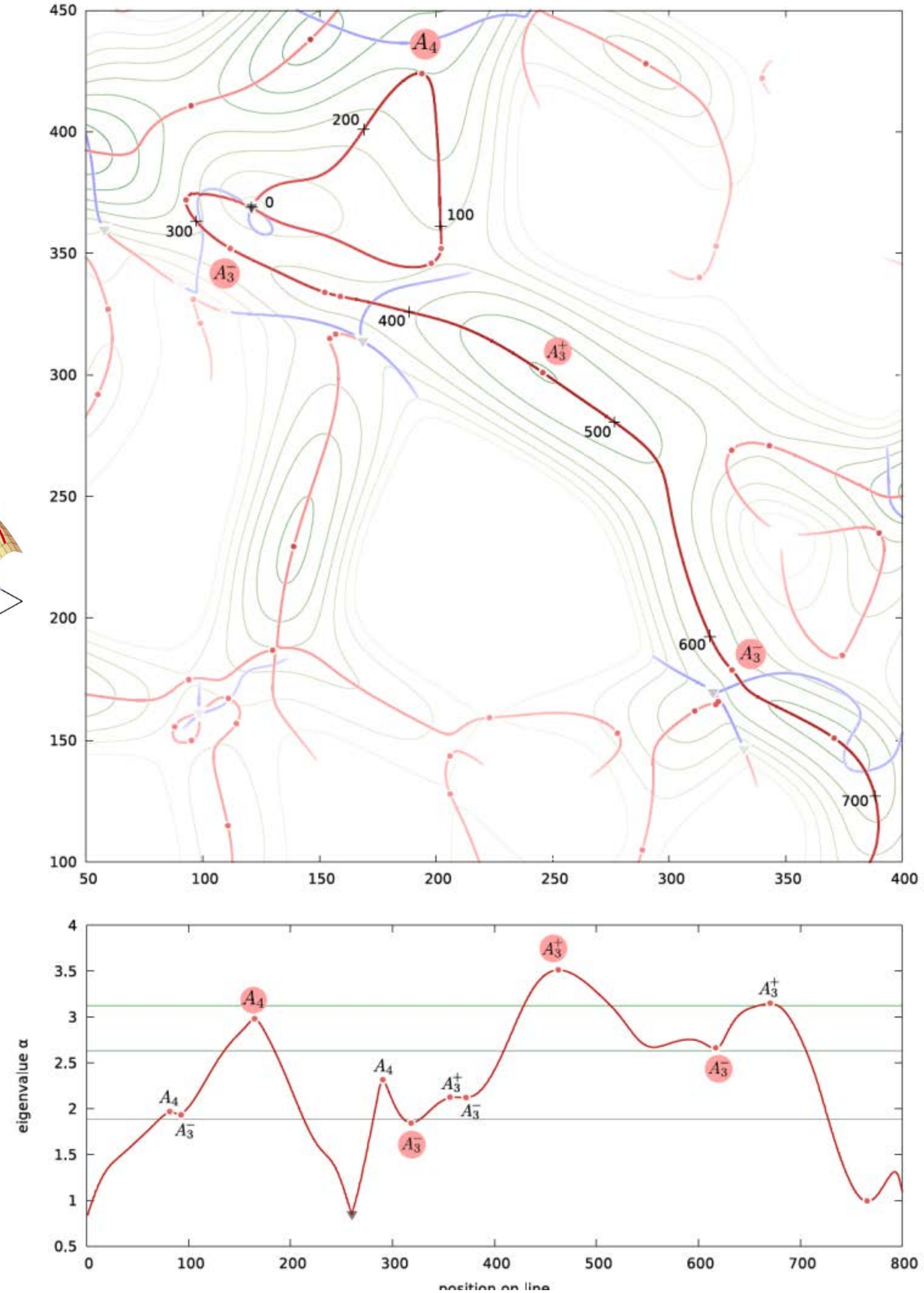


Zeldovich deformation eigenvalue Landscape (2D)

Hidding, Shandarin & vdW 2014

Eigenvalue Field

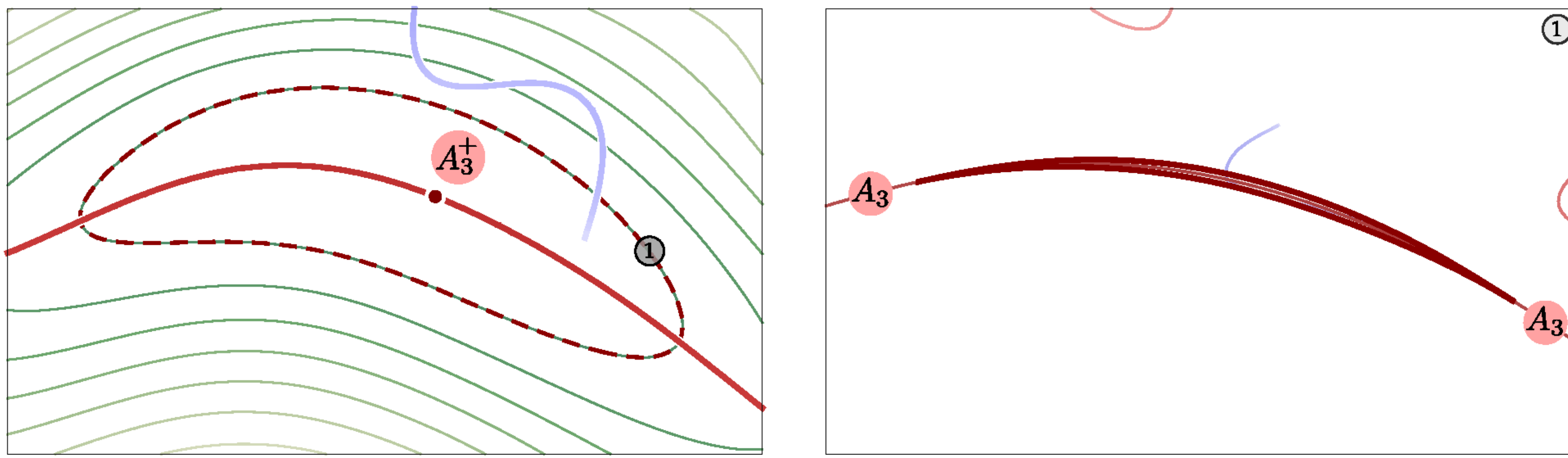




Hidding,
Shandarin & vdW 2014

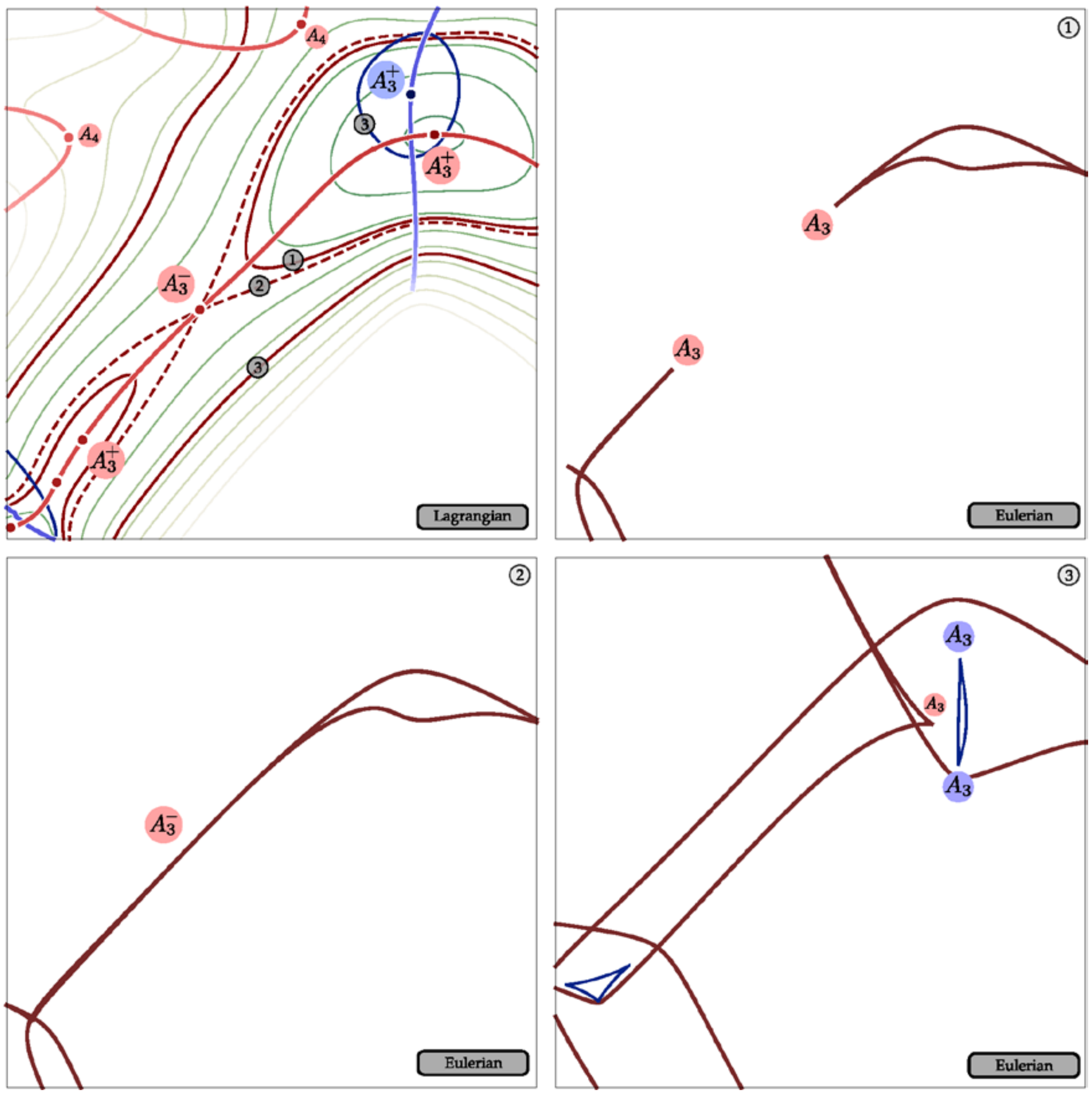
Fold Singularity: A_2 & A_3

Following the 1st eigenvalue λ_1 in time:
- as cosmic epoch D(t) proceeds, different values λ_1(q) pass through singularity
- Identifying the corresponding contours in Lagrangian space q
- Projected onto Eulerian fold structure



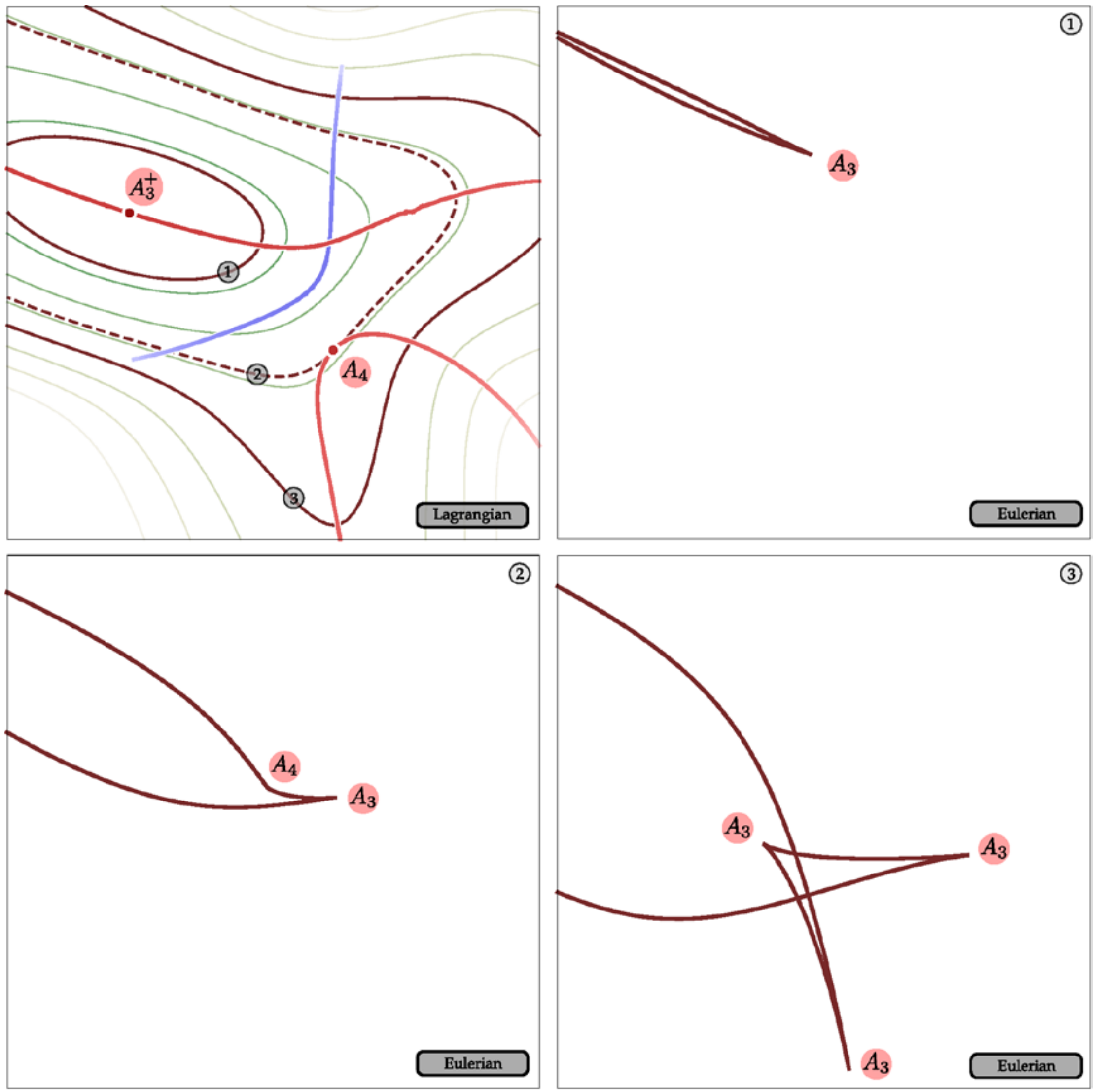
$$\rho(\vec{q},t) = \frac{\rho_u(t)}{\left(1 - D(t)\lambda_1(\vec{q})\right)\left(1 - D(t)\lambda_2(\vec{q})\right)\left(1 - D(t)\lambda_3(\vec{q})\right)}$$

Evolving Caustic: A_3



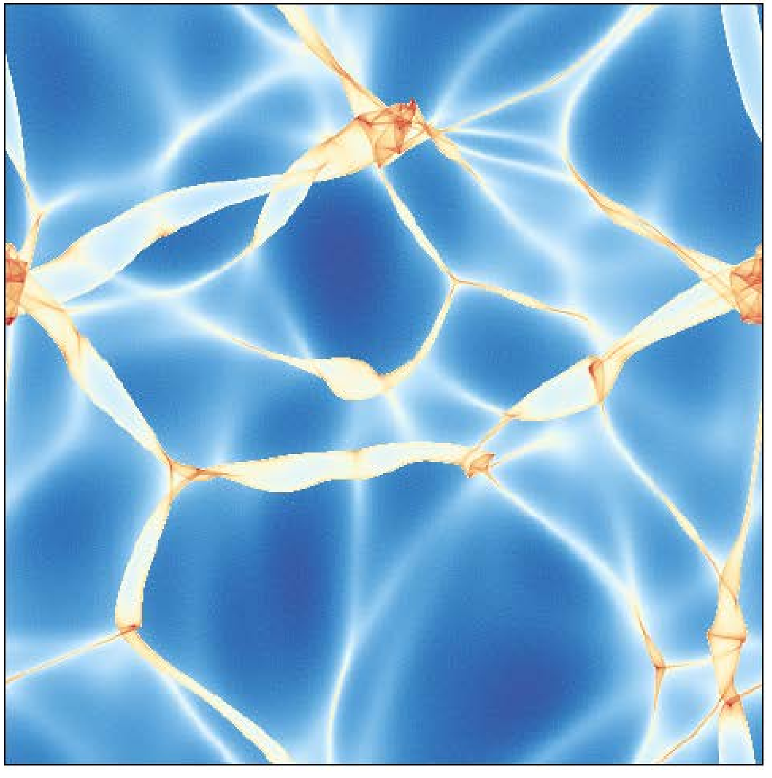
Hidding,
Shandarin & vdW 2014

Evolving Caustic: A_4

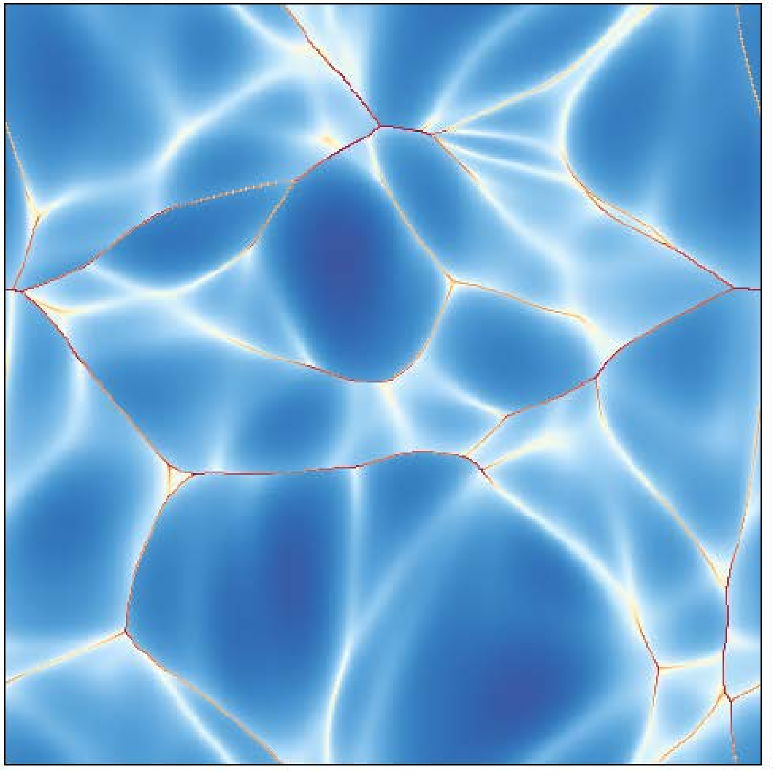


Hidding,
Shandarin & vdW 2014

Structure: Eulerian Space

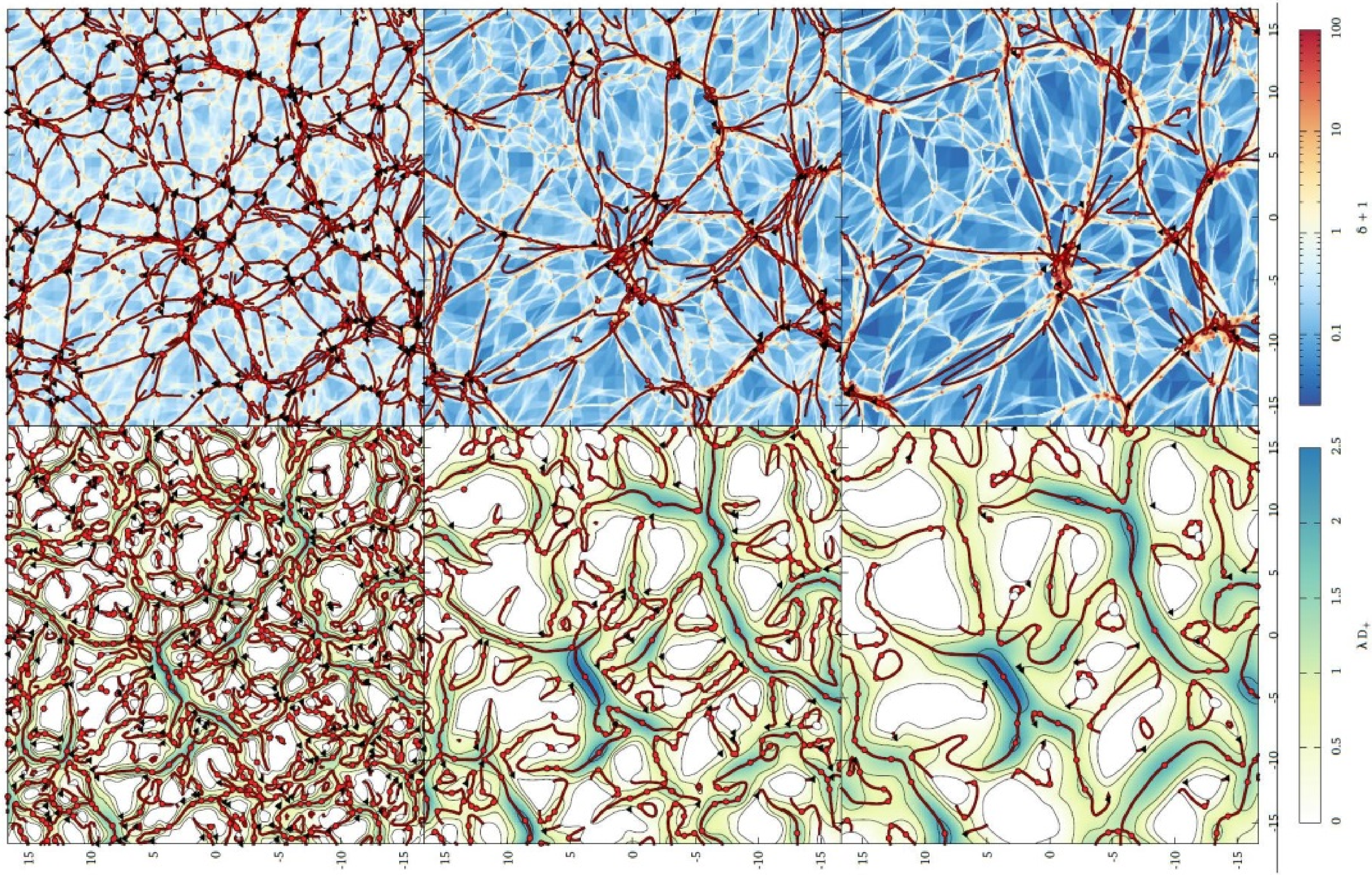


Zeldovich approximation



Adhesion approximation

Singularities: Lagrangian & Eulerian



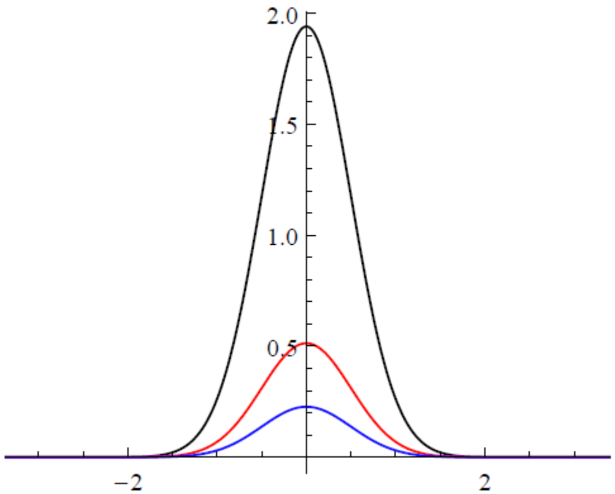
Caustics:
Number & Length Density

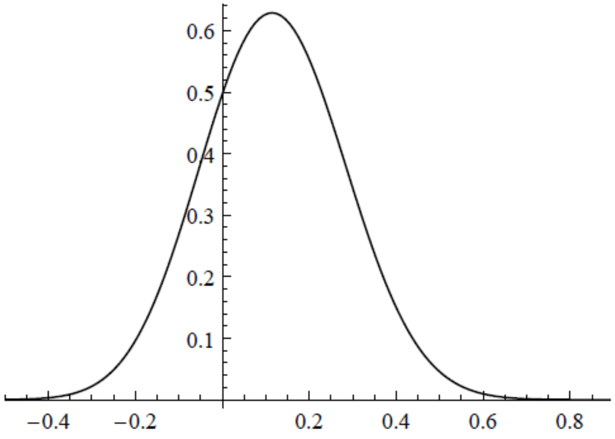
D_4 point density

$$\mathcal{N}_{D_4}(\lambda) = \int |\lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21}| \, p(\lambda_1 = \lambda, \lambda_2 = \lambda, \lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22}) \times \mathrm{d}\lambda_{11}\mathrm{d}\lambda_{12}\mathrm{d}\lambda_{21}\mathrm{d}\lambda_{22}$$

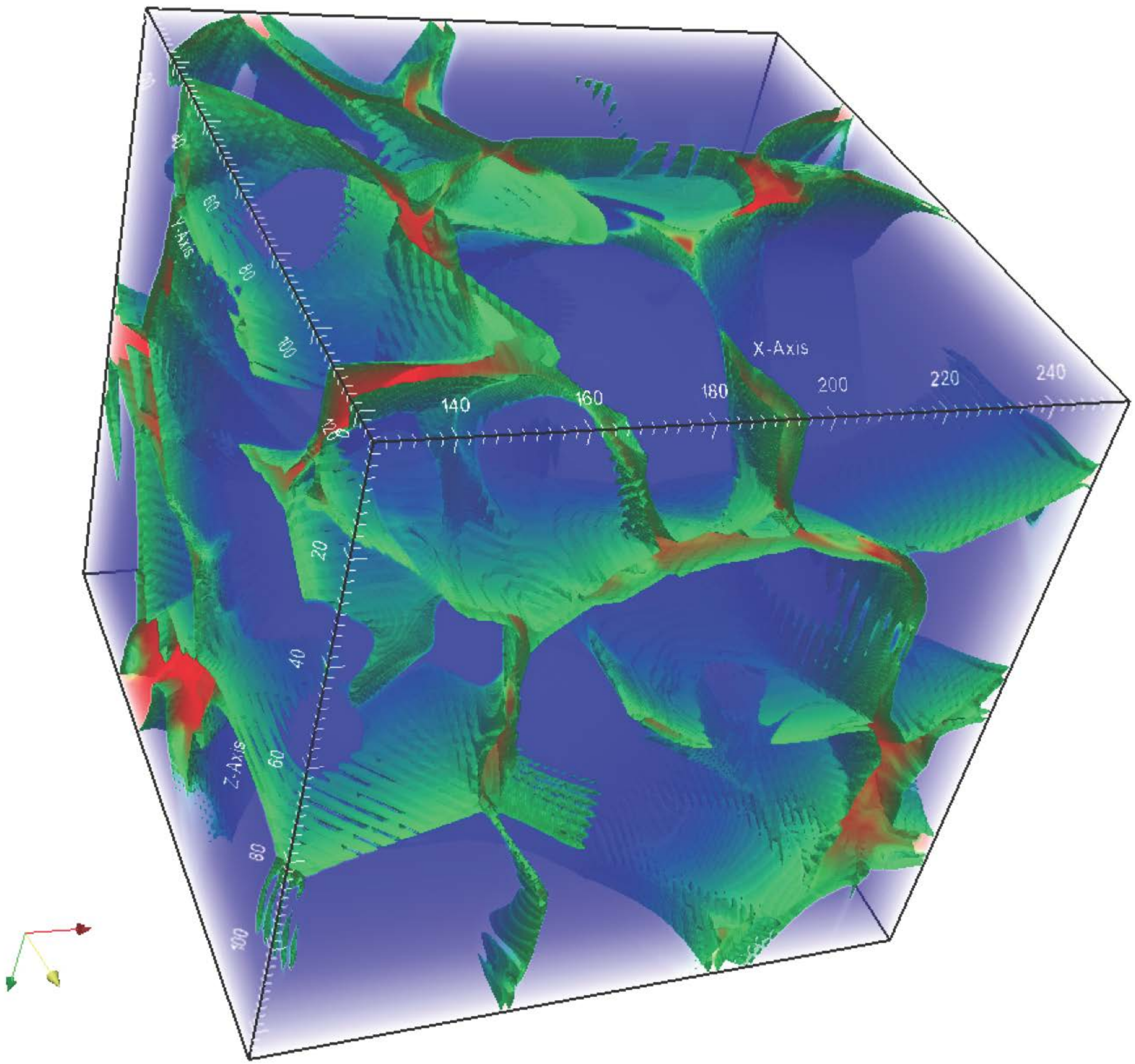
A_3 lines

$$\mathcal{L}_{A_3}(\lambda_1) = \pi \int \sqrt{\lambda_{111}^2 + \lambda_{112}^2} \, p(\lambda_1, \lambda_2, \lambda_{11} = 0, \lambda_{111}, \lambda_{112}) \times (\lambda_1 - \lambda_2)\mathrm{d}\lambda_{111}\mathrm{d}\lambda_{112}\mathrm{d}\lambda_2$$





Feldbrugge et al., 2014



Caustic Structure

3D

Conclusions

- Dynamics of
  cosmic structure formation needs
  description in 6-D phase space

- Rich structure in terms of the formation of caustics: defines the spine of the Cosmic Web.

- Classes of caustics can be identified via mathematical theory of catastrophes (Zeeman, Thom, Arnold)

- Tessellations prove a crucial step in the ability to study phase-space structure of the Cosmi Web.