List of talks and abstracts for the workshop on

Computational Algebraic Geometry @ FoCM'11

Hirotachi Abo, University of Idaho, USA : *New examples of defective secant varieties of Segre-Veronese varieties*

In 1995, Alexander and Hirschowitz finished classifying all the defective secant varieties of Veronese varieties (i.e., the secant varieties of Veronese varieties that do not have the expected dimension). This work completed the Waring-type for polynomials, which had remained unsolved for some time. There are corresponding conjecturally complete list of defective cases for Segre varieties and for Grassmann varieties. Very recently, the defectivity of of Segre-Veronese varieties with two factors was systematically studied and it was suggested that secant varieties of such Segre-Veronese varieties are not defective modulo a fully described list of exceptions. The secant defectivity of more general Segre-Veronese varieties is less well-understood.

In this talk, we explore higher secant varieties of Segre-Veronese varieties with three or more factors. The main goal of the talk is to prove the existence of defective secant varieties of three-factor and four-factor Segre-Veronese varieties embedded in a certain multi-degree. These defective secant varieties are previously unknown and are of importance in the classification of defective cases for Segre-Veronese varieties with three or more factors.

This is joint work with Chiara Brambilla.

Martín Avendaño, Texas A&M University, USA: *Randomized NP-completeness for p-adic rational roots of sparse polynomials in one variables*

Relative to the sparse encoding, we show that deciding whether a univariate polynomial has a p-adic rational root can be done in NP for most inputs. We also prove a sharper complexity upper bound of P for polynomials with suitable generic p-adic Newton polygon. The best previous complexity bound was EXPTIME. We then prove an unconditional complexity lower bound of NP-hardness with respect to randomized reductions, for general univariate polynomials. The best previous lower bound assumed an unproved hypothesis on the distribution of primes in arithmetic progression. We also discuss analogous results over R.

This is a joint work with Ashraf Ibrahim, Maurice Rojas and Korben Rusek.

Saugata Basu, Purdue University, USA : *Counting problems in computational algebraic geometry*

Toda proved in 1989 that the (discrete) polynomial time hierarchy, $\mathbf{PH}$, is contained in the class $\mathbf{P}^{\#\mathbf{P}}$, namely the class of languages that can be decided by a Turing machine in polynomial time given access to an oracle with the power to compute a function in the counting complexity class $\#\mathbf{P}$. This result which illustrates the power of counting is considered to be a seminal result in computational complexity theory. An analogous result (with a compactness hypothesis) in the complexity theory over the reals (in the sense of Blum-Shub-Smale real machines) was proved by Basu and Zell in 2009. Unlike Toda's proof in the discrete case, which relied on sophisticated combinatorial arguments, this proof is topological in nature in which the properties of the topological join is used in a fundamental way. However, the constructions used in a paper by Basu and Zell in 2009 were semi-algebraic – they used real inequalities in an essential way and as such do not extend to the complex case. In this talk, I will describe an extension of the above result to the complex case. A key role is played by the complex join of quasi-projective complex varieties. As a consequence we obtain a complex analogue of Toda's theorem. These results illustrate the central role of the Poincaré polynomial in algorithmic algebraic geometry, as well as, in computational complexity theory over the complex and real numbers – namely, the ability to compute it efficiently enables one to decide in polynomial time all languages in the (compact) polynomial hierarchy over the appropriate field.

Dan Bates, Colorado State University, USA : *Khovanskii-Rolle continuation for finding real solutions of polynomial systems*

Numerical homotopy methods can be used to compute all nondegenerate solutions of a polynomial system. To find only the nondegenerate real solutions of a polynomial system with homotopy methods, one must first compute all nondegenerate complex solutions. There are examples of systems with thousands of complex solutions but only a few real solutions, so this procedure can be inefficient.

Khovanskii-Rolle continuation is a numerical method to find only real solutions of a polynomial system via curve-following (related to homotopy methods, but without the homotopies). The method is rooted in fewnomial theory (particularly the Khovanskii-Rolle theorem) and Gale Duality. It constitutes the first known continuation method that produces only the real solutions without also producing the complex solutions, and its complexity is based on the number of real solutions rather than the number of complex solutions.

In this talk, I will provide a brief introduction to Gale duality and Khovanskii-Rolle continuation, particularly highlighting some of the current progress in making the algorithms more efficient and secure. This is joint work with J. Hauenstein, M. Niemerg, and F. Sottile.

Guillaume Chèze, University of Toulouse, France : *Computation of Darboux polynomials in polynomial time*

In this talk we will study planar polynomial differential systems of the form:

$$\frac{dX}{dt} = \dot{X} = A(X,Y),\ \frac{dY}{dt} = \dot{Y} = B(X,Y),$$

where $A, B \in \mathbb{Z}[X,Y]$ and $\deg A \leq d$, $\deg B \leq d$, $\|A\|_\infty \leq \mathcal{H}$ and $\|B\|_\infty \leq \mathcal{H}$. A lot of properties of planar polynomial differential systems are related to irreducible Darboux polynomials of the corresponding derivation: $D = A(X,Y)\partial_X + B(X,Y)\partial_Y$. Darboux polynomials are usually computed with the method of undetermined coefficients. With this method we have to solve a polynomial system. We show that this approach can give rise to the computation of an exponential number of reducible Darboux polynomials. Here we show that the Lagutinskii-Pereira's algorithm computes irreducible Darboux polynomials with degree smaller than N, with a polynomial number, relatively to d, $\log(\mathcal{H})$ and N, binary operations.

Aldo Conca, University of Genova, Italy : *Syzygies of Koszul algebras*

The goal of the talk is to explain recent results and conjectures that support the idea that the syzygies of Koszul algebras have many properties in common with the syzygies of algebras defined by monomials of degree two.

Wolfram Decker, University of the Saarland, Germany : *Integral Bases, Adjoint Ideals, and the Parametrization of Rational Curves*

We begin by presenting a new algorithm for finding an integral basis in an algebraic function field L of one variable. Supposing that the function field is realized as the function field of a plane algebraic curve, we proceed by localizing at the singular points of the curve. In a basic version of the integral basis algorithm, the local contributions to the integral basis are found by normalizing the corresponding local rings (in fact, this method also gives a new algorithm for computing the normalization in higher dimension, provided the singular locus has no embedded components). In a refined version of the integral basis algorithm, singularities whose Puiseux expansions show a certain behaviour are treated separately. In contrast to van Hoeij's algorithm, which requires the explicit computation of Puiseux expansions of high order at each of the singular points of the curve and power series expansions at other points, the new algorithm can avoid this in a large class of cases via Hensel lifting.

In a similar spirit, we also discuss new algorithms for computing the adjoint ideal of a curve, which encodes important geometric information, and for finding rational parametrizations of rational curves.

We should point out that the local computations can be performed in parallel. Combining this with modular zero-dimensional primary decomposition and zero-dimensional radical algorithms applied to the the singular locus, the SINGULAR implementation of the algorithms outperforms other implemented algorithms in a large number of cases.

This is joint work with Janko Böhm, Santiago Laplagne, and Frank Seelisch.

David Eklund, KTH Royal Institute of Technology, Stockholm, Sweden : *Dynamic intersections and numerical homotopy methods*

Fulton's and MacPherson's refined intersection product decomposes as a sum of terms associated to the so-called distinguished varieties of the intersection. Work of Severi and Lazarsfeld give an interesting dynamical interpretation of the distinguished varieties. Consider for example the intersection X of three surfaces in $\mathbb{P}^3$ and assume that the intersection consists of a curve C and some points. If this intersection is deformed in a general way to a finite intersection and then deformed back, some points in the general fibers will approach the isolated points of X and some points will go to C. There might be points on C that always attract points in this way for a general deformation, and those are called the distinguished points on C. Numerical homotopy methods exploit this kind of deformations to find roots of systems of polynomials. This talk will be an exposition on the link between the dynamic approach to intersections and numerical homotopy methods. I will also discuss how these ideas can be used to extract information about the solution set of a system of polynomials.

André Galligo, University of Nice, France : *Roots of the derivatives of random polynomials and characteristic polynomials of random matrices*

I will first recall old and recent results on distributions of roots of random polynomials and eigenvalues of random matrices. Then I will present original observations of patterns of roots of the derivatives of random polynomials and characteristic polynomials of random matrices; I will set some conjectures enforced by experiments and outline proofs of some claims.

Teresa Krick, University of Buenos Aires, Argentina : *On arithmetic implicitation problems and effective Nullstellensätze*

Joint work with Carlos D'Andrea and Martin Sombra, on precise bounds for the implicitation problem and the effective Nullstellensatz in their arithmetic aspects.

The implicitation problem consists in computing equations for an algebraic variety from a given rational parameterization of it. The typical case is when the variey is a hypersurface, in which case it is defined by a single "implicit equation" and the problem consists in computing it.

The Nullstellensatz establishes that if $f_1, ..., f_s$ in $k[x_1, ..., x_n]$ are polynomials with no common roots in the algebraic closure of the field k, then they satisfy a Bezout identity $1 = g_1 f_1 + ... + g_s f_s$ for some polynomials $g_1, ..., g_s$.

I will discuss degree and height bounds for these problems for the case when the polynomials are defined over the rational numbers.

Diane Maclagan, University of Warwick, USA : *The T-graph of Hilbert schemes*

The n-dimensional torus action on A^n extends to a torus action on any multigraded Hilbert scheme. The T-graph of this Hilbert scheme has vertices the fixed points of the torus action (corresponding to monomial ideals), and edges when there is a one-dimensional torus orbit whose closure contains the two fixed points. Little is understood about this graph, even in the case of the Hilbert scheme of d points in the plane, when the vertices correspond to partitions of d. I will describe some necessary conditions for the existence of an edge in these graphs, and some computations. This is joint work with Milena Hering (UConn).

Bernard Mourrain, INRIA Sophia Antipolis, France : *Tensor decompositions, truncated moment problems and applications*

In this presentation, we will describe some recent developments of the decomposition problem for symmetric and multi-homogeneous tensors. From a geometric point of view, we will see how algebraic varieties such as secant varieties, cactus varieties and the Hilbert scheme of points appear naturally in this framework. From an algorithmic point of view, we will show how solving truncated moment problems related to Hankel matrices help computing such a decomposition. An algorithm based on an extension of Sylvester approach will be described and illustrated on some geometric applications

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Luke Oeding, University of Firenze, Italy : *Toward a salmon conjecture*

By using a result from the numerical algebraic geometry package Bertini we show that (with extremely high probability) a set of degree six and degree nine polynomials cut out the secant variety $\sigma_4(\mathbb{P}^2 \times \mathbb{P}^2 \times \mathbb{P}^3)$. This, combined with an argument provided by Landsberg and Manivel, implies set-theoretic defining equations in degrees 5, 6 and 9 for a much larger set of secant varieties, including $\sigma_4(\mathbb{P}^3 \times \mathbb{P}^3 \times \mathbb{P}^3)$ which is of particular interest in light of the salmon prize offered by E. Allman for the ideal-theoretic defining equations. (joint work with Daniel J. Bates)

Giorgio Ottaviani, University of Firenze, Italy : *Algorithms for tensor decomposition*

We address the problem of decomposing tensors as sum of tensors of rank one. Some theoretical results about existence and uniqueness of this decomposition are considered. We mainly consider symmetric tensors.

Gregory Smith, Queen's University, Kingston, Ontario,, Canada : *Equations defining smooth subvarieties of a toric variety*

In algebraic geometry, the complexity of making effective calculations is subtly governed by the intrinsic geometry. In this talk, we will review bounds on the algebraic complexity for subvarieties of a projective space. We'll then motivate and present new results in which the ambient projective space is replaced by a smooth toric variety. In particular, we will discuss a linear bound (based on joint work with V. Lozovanu) on the multigraded regularity of a smooth subvariety.

Martín Sombra, ICREA and University of Barcelona, Spain : *Overdetermined systems of lacunary polynomials*

Filaseta, Granville and Schinzel have recently shown that the gcd of two lacunary polynomials can be computed in time quasi-linear in their degree. Their algorithm relies in an effective version of a result of Bombieri and Zannier on multiplicatively dependent points in algebraic varieties.

In this talk, I will present the following extension of the FGS result: a system of three lacunary polynomials in two variables can be reduced in time quasi-linear in its degree, to finite set of systems of two polynomials in two variables. This is a conditional result which depends on an effective version of the Zilber-Pink conjecture, extending the aforementioned result by Bombieri and Zannier.

This is joint work with Francesco Amoroso and Louis Leroux.

Seth Sullivant, North Carolina State University, USA : *Identifiability of Phylogenetic Mixture Models*

Phylogenetic models are statistical models used to analyze the evolutionary history between a collection of species. Their Zariski closures are algebraic varieties generalizing classical varieties like toric varieties, the Segre variety, and their secant varieties. Phylogenetic mixture models are more complicated models that can be interpreted as the joins of the basic phylogenetic models. The fundamental identifiability problem for phylogenetic models asks whether the mapping between parameters and probability distributions is one-to-one. I will discuss recent progress on this problem using tools from algebraic geometry, including tensor rank. This is joint work with Elizabeth Allman, Sonja Petrovic, and John Rhodes.

Agnes Szanto, North Carolina State University, USA : *Subresultants in Multiple Roots*

This is a joint work with Carlos D'Andrea and Teresa Krick. The topic is a continuation of our previous work on Poisson-like formulas for univariate and multivariate subresultants in terms of the roots of a polynomial system, as well as our work on the relationship between univariate subresultants and Sylvester double sums, provided that all these roots were *simple*, i.e. that the ideal generated by the input polynomials is radical and zero-dimensional. As it was pointed out by an anonymous referee, it would be interesting to work out these results for the case of *multiple* roots. This work is a first attempt in that direction. We successfully extended Poisson-like formulas for univariate and multivariate subresultants in the presence of multiple roots. We also obtained formulas for subresultants in roots in the univariate setting for some non-trivial extremal cases. However, it is still open how to generalize Sylvester double sums in the multiple roots case.
