Graph decompositions of small diameter

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Graph and tree-decomposition

Many problems of real life can be modeled and solved by **graphs**.
Often difficult **(NP-hard)**.

Prolific approach:

Dynamic programming using **Tree-decompositions** (widely studied during last 40 years).

Examples:

- **[Courcelle, 1990]** Every graph property definable in the monadic second-order logic of graphs can be decided in linear time on graphs of bounded treewidth.
- **[Bodlaender, Fomin, Lokshtanov, Penninkx, Saurabh, Thilikos, 2016]** (Meta) kernelization.

E.g., Max clique, Minimum Graph colorability, Gate Matrix Layout, Maximum Independent vertex Set, Minimum Fill-in, Minimum dominating set...

Tree-decompositions [Robertson, Seymour, 1984]

Idea: Decompose a graph into small pieces (**bags**), structured as a tree.

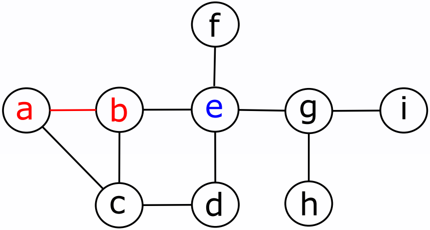
Tree-decomposition of $G = (V, E)$

A *tree-decomposition* T is a family $\mathcal{X} = \{X_t | t \in V(T)\}$ of bags s.t.:

- $\bigcup_{t \in V(T)} X_t = V(G)$;
- $\forall \{u, v\} \in E(G)$, $\exists t \in V(T)$ s.t. that $u, v \in X_t$;
- $\forall v \in V(G)$, $\{t \in V(T) \mid v \in X_t\}$ induces a tree.

Width: max. size of the bags minus 1.
Treewidth, $tw(G)$: min. width among all tree-decompositions.

A graph G:



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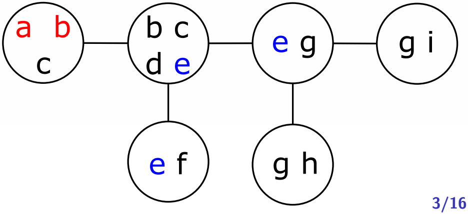

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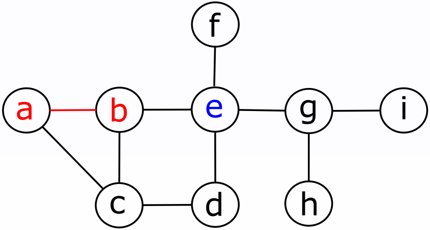

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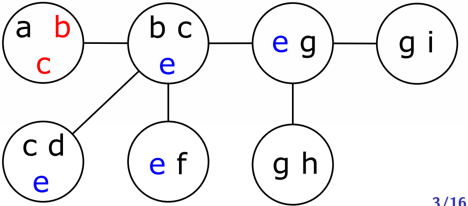

Treelength [Dourisboure, Gavoille, 2007]

Length: max. **diameter** of the bags.
Treelength, $tl(G)$: min. length among all tree-decompositions.

Applications:

TSP [Krauthgamer, Lee, 2006], Compact routing [Boguna, Papadopoulos, Krioukov, 2010]...

Remark:

Note that the red cycle $\{b, c, d, e\}$ can be used to prove that $tl(G) \geq 2$.

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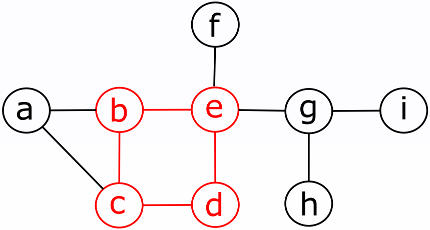

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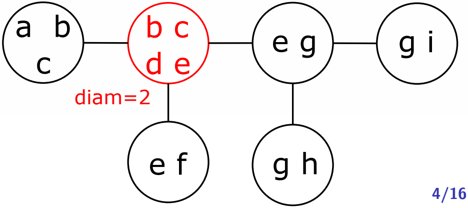

Path-decompositions

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Width: max. size of the bags minus 1.
Pathwidth, $pw(G)$: min. width among all path-decompositions.

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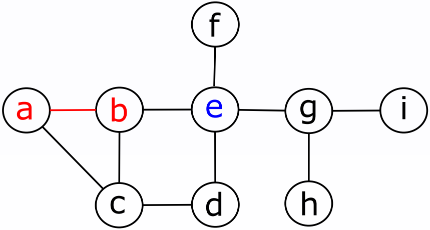

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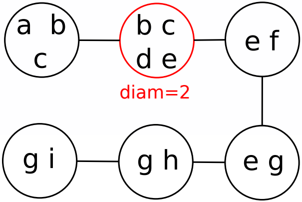


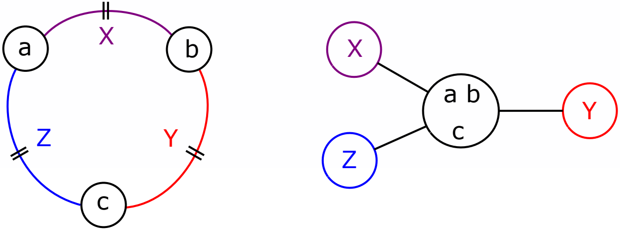
Difference between treewidth and treelength

Lemma :

$tl(K_n) = pl(K_n) = 1$ and $tw(K_n) = pw(K_n) = n-1$.

Lemma [Dourisboure,Gavoille, 2007]:

$tl(C_k) = \lceil \frac{k}{3} \rceil$ and $tw(C_k) = 2$. $pl(C_k) = \lfloor \frac{k}{2} \rfloor$ and $pw(C_k) = 2$.



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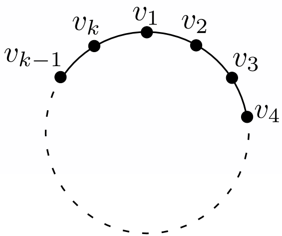
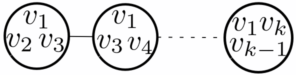




Tree-decomposition of cycle, a good lowerbound

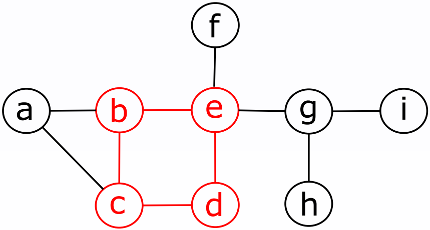
- $tl(C_k) = \lceil \frac{k}{3} \rceil$
- $pl(C_k) = \lfloor \frac{k}{2} \rfloor$

Lemma [Dourisboure,Gavoille, 2007]:

Let G be a graph and H an **isometric** (i.e. preserve distance) subgraph of G, then $tl(G) \geq tl(H)$.

Good lowerbound: $tl(G) \geq \lceil \frac{is(G)}{3} \rceil$
($is(G)$: largest size of an isometric cycle)

A graph G:



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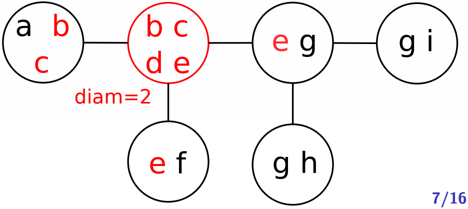

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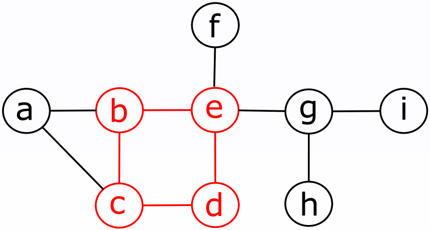
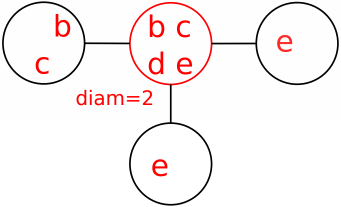


A tree-decomposition of C_4:



Computation of tree/path-decompositions

	Treewidth: $tw(G) \leq k$	**Treelength:** $tl(G) \leq k$	**Pathwidth:** $pw(G) \leq k$	**Pathlength:** $pl(G) \leq k$
k part of the input	NP-complete [Arnborg and al, 1987]	NP-complete	NP-complete [Arnborg and al, 1987]	NP-complete
k fixed parameter	Polynomial [Bodlaender, Kloks, 1996]	NP-complete [Lokshtanov, 2010]	Polynomial [Bodlaender, Kloks, 1996]	NP-complete [Ducoffe and al, 2020]
Approximation algorithm	$\sqrt{log(Opt)}$ [Feige and al 2008]	3 [Dourisboure, Gavoille, 2007]	$O(log(n)^{\frac{3}{2}})$ [Feige and al, 2008]	2 [Dragan and al, 2017]
Planar graph	Open since 40 years	Open	NP-complete [Monien, Sudborough, 1988]	Open

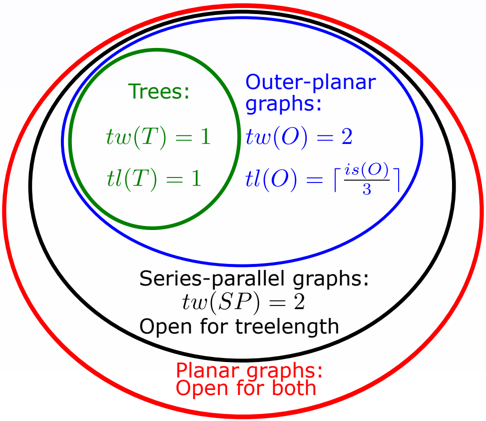
To approximate treewidth, use treelength ! [Coudert, Ducoffe, Nisse, 2016]

$tl(G) = \Theta(tw(G))$ in apex-free graph G with bounded largest isometric cycle.

=> Study of treelength/pathlength in planar graphs

Our Contributions

Sub-classes of planar graphs:



Treelength [T. Dissaux, G.Ducoffe, N. Nisse, S. Nivelle, 2021]

- $\frac{3}{2}$-Approximation Algorithm for SP graphs;
- Characterisation of SP graphs of treelength 2 (Forbiden isometric subgraphs).

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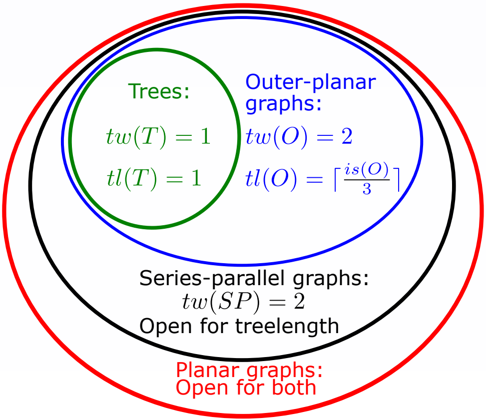

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Pathlength [T. Dissaux, N. Nisse, 2021]

- Linear time algorithm for trees;
- $pl(C_n) = \lfloor \frac{n}{2} \rfloor$;
- $(+1)$-approximation for Outerplanar graphs.

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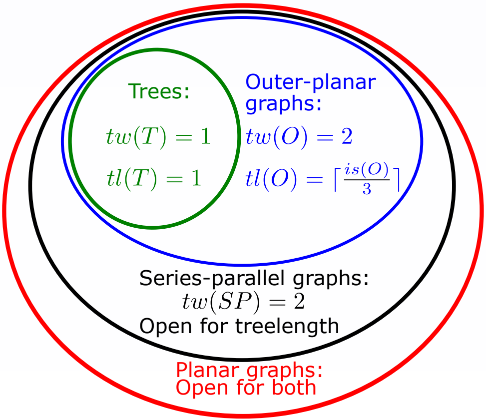

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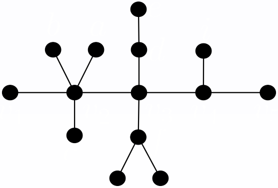
Pathlength of trees

Algorithm:

A Tree T and an optimal path-decomposition of T:



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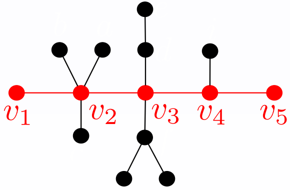

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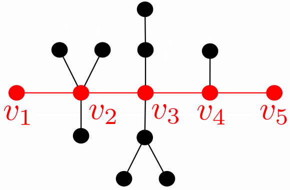

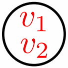

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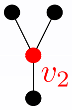

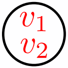

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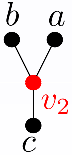

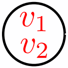

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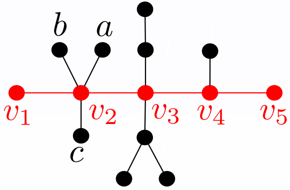

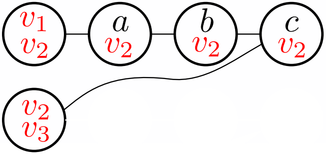

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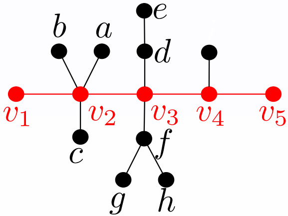

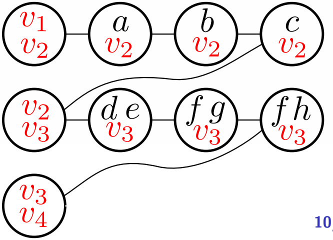

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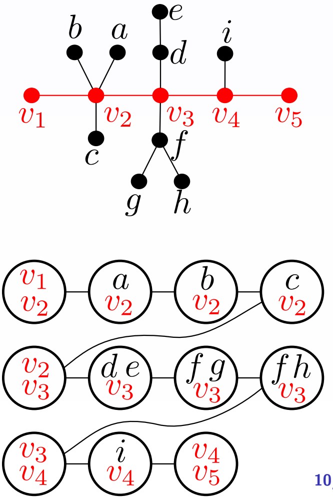

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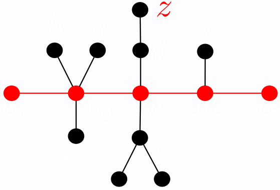

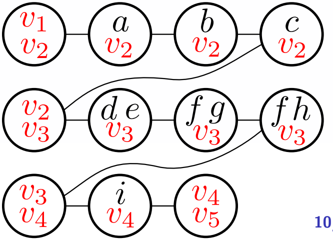

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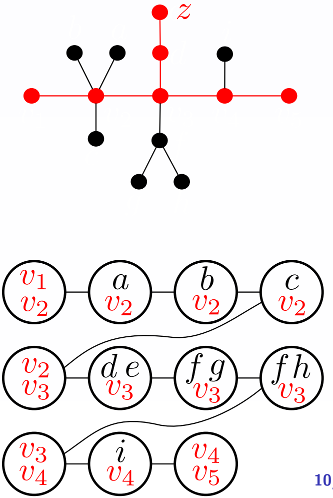

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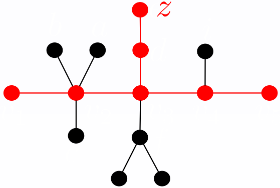

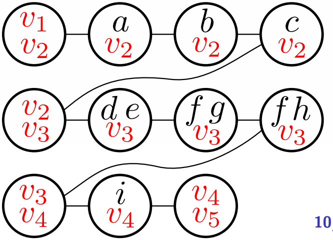

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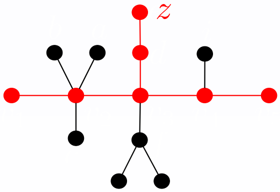

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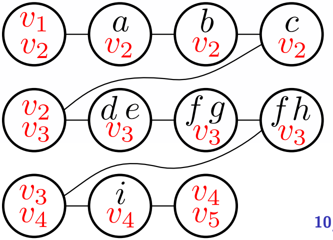
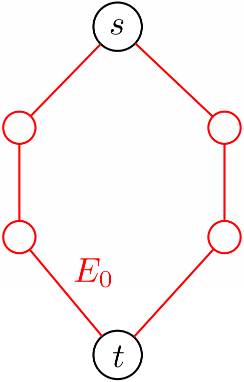


(+1)-Approximation of the pathlength of Outerplanar graphs

Outerplanar graphs

An outerplanar graph $G = (V, E)$ is a planar graph such that each vertex $v \in V$ is on the outer (unbounded) face.



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An outerplanar graph $G = (V, E)$ is a planar graph such that each vertex $v \in V$ is on the outer (unbounded) face.

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For any $k > 1$ and any Outerplanar graphs G, either the algorithm decides that $pl(G) > k$ or return a path-decomposition of length at most $pl(G) + 1$ in time $O(n^3(n + k^2))$.

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This algorithm is based on:

- the fact that every $(\{x, y\})$ path-decomposition $D = (X_1, \ldots, X_p)$ of G contains an edge x in X_1 and an edge $y \in X_p$.
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Restriction for the talk: The pathlength of 2-connected outerplanar graphs is much simpler.

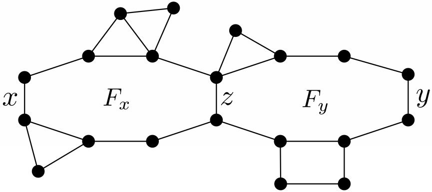
Characterization: "separated"

"Separated"

If there exists $z \in E_{int}$ such that z separates x and y, then a $\{x, y\}$-path-decomposition can be obtained from a $\{x, z\}$-path-decomposition and a $\{z, y\}$-path-decomposition.

Let G_x (resp. G_y) be the connected component of $G \setminus \{z\}$ containing x (resp. y) union z.

A 2-connected outerplanar graph G:



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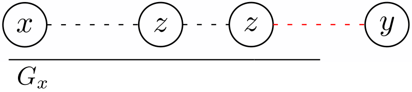

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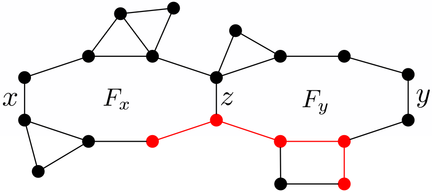

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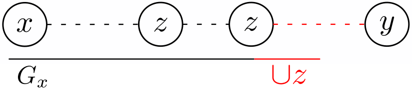

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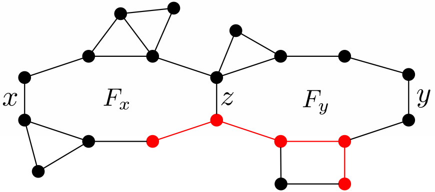
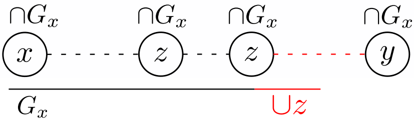


A $\{x, z\}$-path-decomposition of G_x of length at most $pl(G)$:



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For any component C of $G \setminus F$, there exists $1 \leq a_C < b_C \leq p$ such

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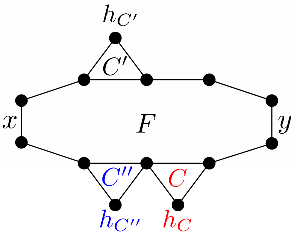

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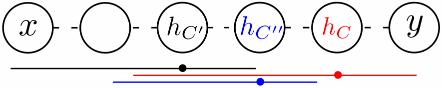

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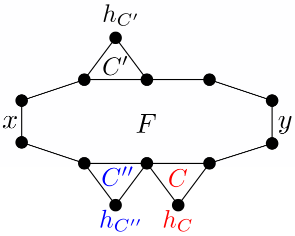

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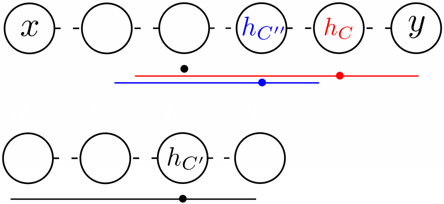

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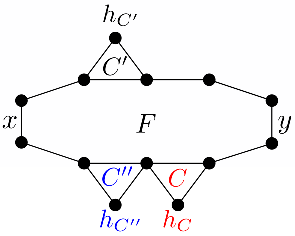

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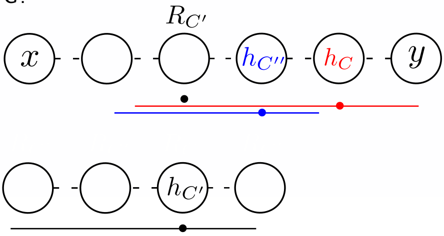

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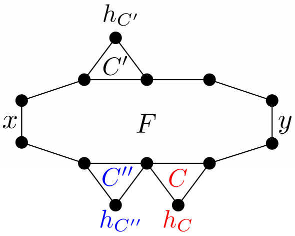

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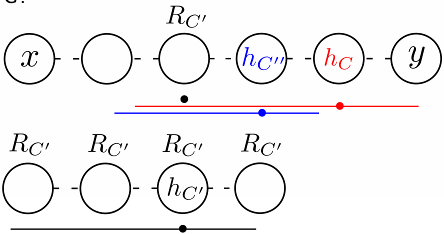

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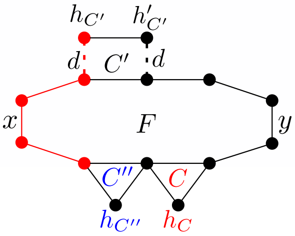

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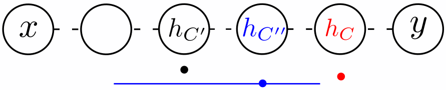

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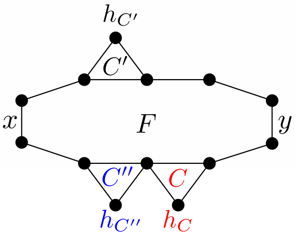

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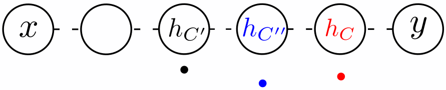

Characterization: "LtR" (Left to Right)

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For any two component C and C' in C_{up} (resp. C_{down}), if $N(C)$ is closer to x than $N(C')$, then $b_C < a'_C$, i.e. C appears before C' in the $\{x, y\}$-path-decomposition.

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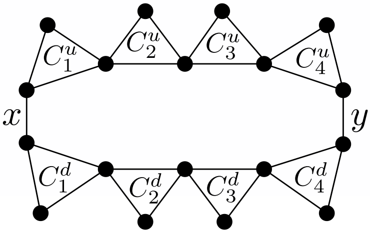

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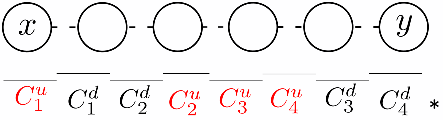

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Let $D[u, v]$ be a "partial" contiguous LtR path-decomposition of G such that the last bag contains u and v.

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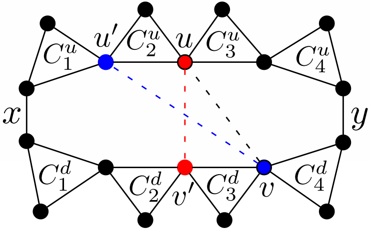

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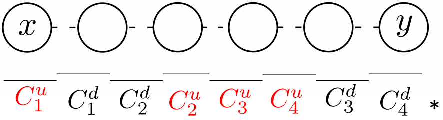
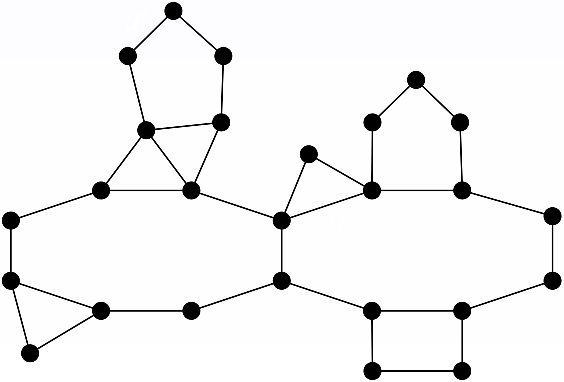


(+1)-Approximation

Algorithm:



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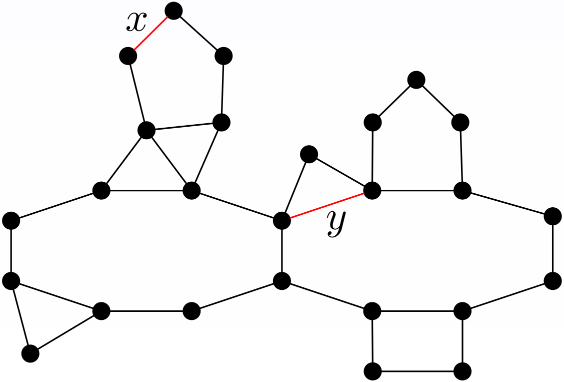
Algorithm: $O(n^2)$

- First, fix $x, y \in E$ ($|E| = O(n)$ in planar graphs): $O(n^2)$;



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- First, fix $x, y \in E$ ($|E| = O(n)$ in planar graphs): $O(n^2)$;
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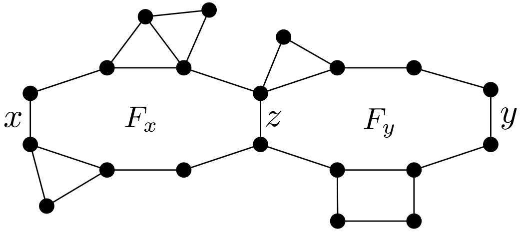

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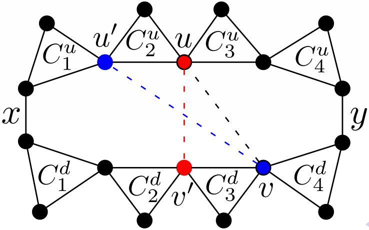
- Third, compute a greedy path-decomposition for every component of $G \setminus F$: $O(n)$;

Lemma "greedy":

For any 2-connected outerplanar graph G and any $x \in E(G)$, a $\{x, x\}$-path-decomposition of G can be computed in time $O(n)$.



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Algorithm: $O(n^3(n+k^2))$

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- Finally, it remains to compare the computed path-decompositions and keep the one that has minimum length.

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