A Review of the Graph Burning Problem

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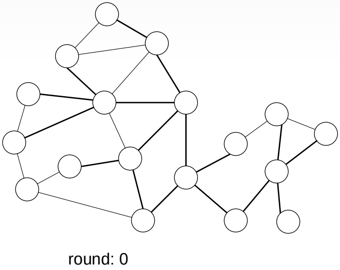
(Joint work with A. Bonato, A. Miller, M., and K. Zhang)

May 18th, 2022

GRASTA, Porquerolles, France

Graph Burning Problem

- Given an undirected graph G, the goal is to **burn** in a minimum number of **rounds** [Bonato et al., 2014].



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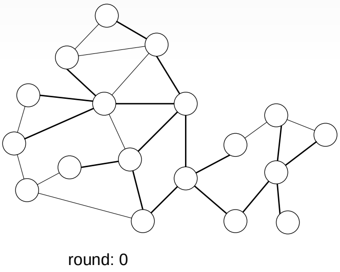

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- At each given round:
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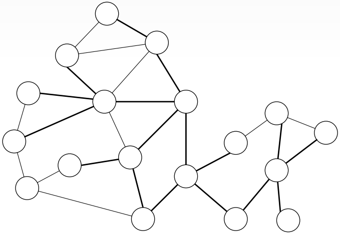

round: 0

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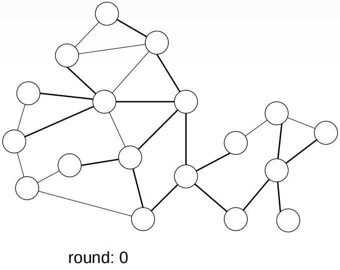

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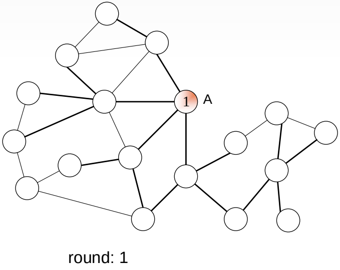

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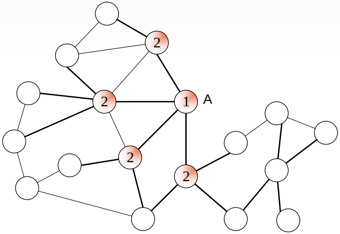

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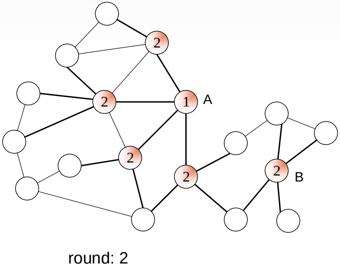

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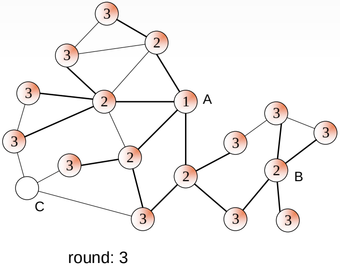

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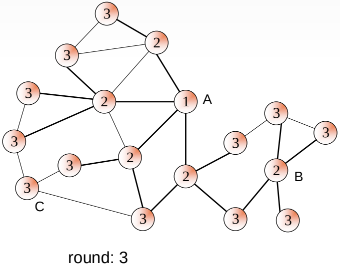

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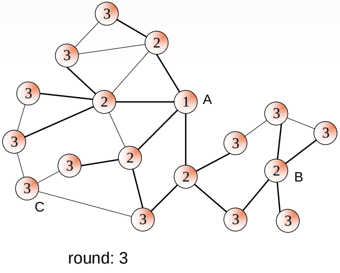

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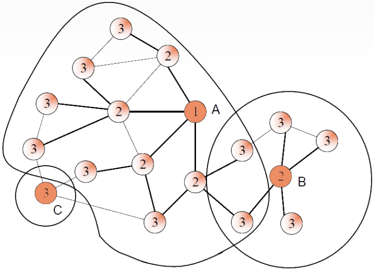

- Decision problem:
  - Can we burn G in k rounds?
  - Equivalently, can we cover the graph with "disks" of radii $0, 1, 2, \ldots, k-1$?

Motivation

- How an adversary can "contaminate" a network.
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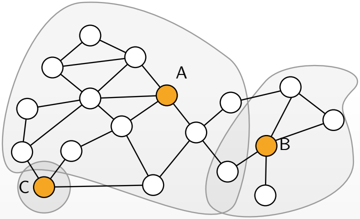
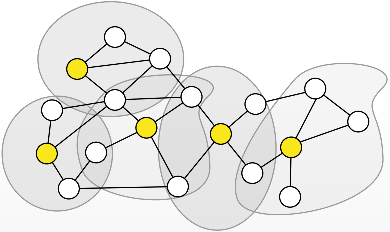
- The **burning number** is the smallest number of rounds to burn a network
  - It measures how vulnerable a network is against adversarial “attacks”.

Burning vs. k-center

- The burning problem is related to the k-**center problem.**
  - k-center: given a parameter k, cover all vertices with k disks of minimum **uniform** radii.
  - burning: cover all vertices with disks of radii $0, 1, \ldots, k-1$ for minimum k.



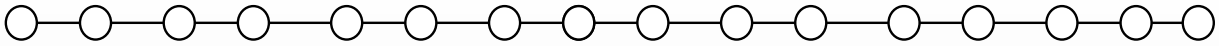
burning



k-center with $k = 5$

Burning Paths

- A path P_n of length n can be covered with disks of radii $0, 1, 2, \ldots, \lceil\sqrt{n}\rceil$ [Bonato et al. 2014].



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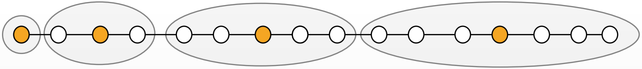

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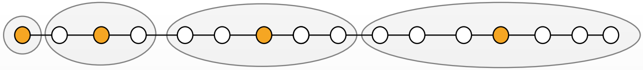

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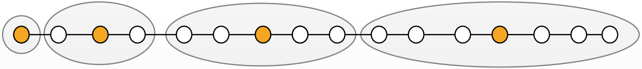

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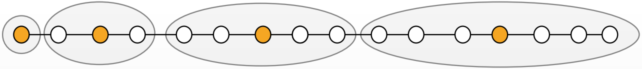

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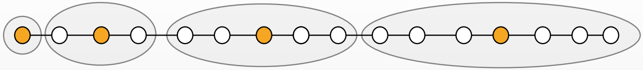
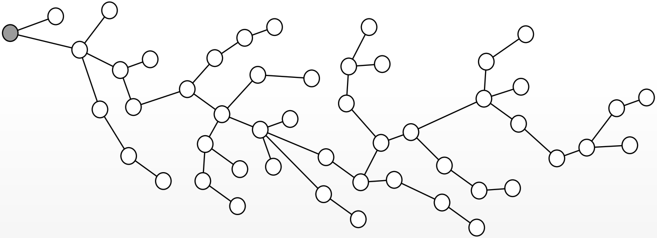
  - The burning number of any connected graph is at most $\sqrt{4n/3} + o(\sqrt{n})$ rounds [Bonato and S.K., 2021], [Bastide, Bonamy, Bonato, Charbit, S.K., Pierron, Rabie, 2021].



Burning Upper Bounds

- **Observation:** to prove $f(n)$ rounds are sufficient to burn **any** graph of size n, it suffices to prove $f(n)$ rounds are sufficient to burn **trees**.



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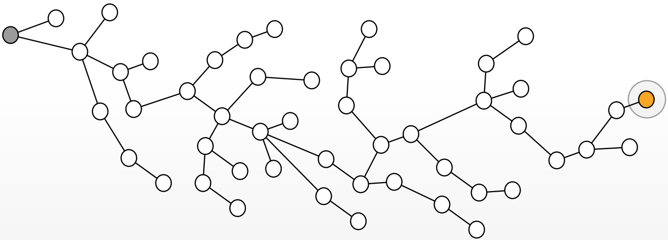
- Given (arbitrarily rooted) tree T of size n, it is easy to burn T in $\approx \sqrt{2n}$ rounds.
  - Process disks of radii $\{0, 1, \ldots, \lceil \sqrt{2n} \rceil - 1\}$ in **any** order and use a disk of radius r to cover the "deepest" subtree of height r.



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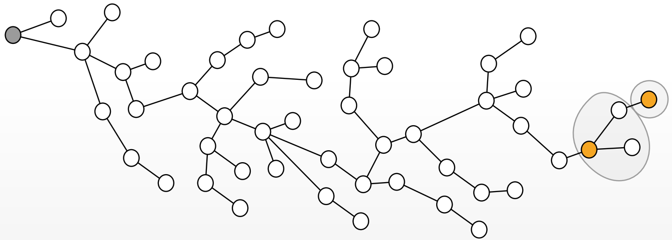

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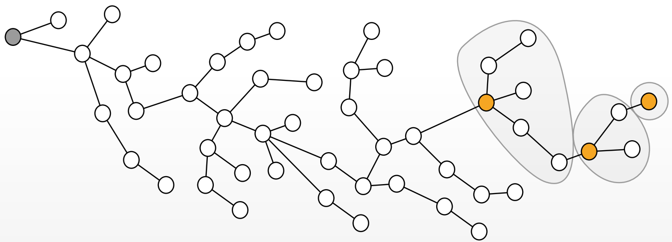

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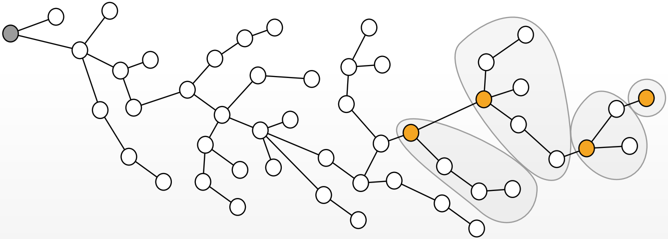
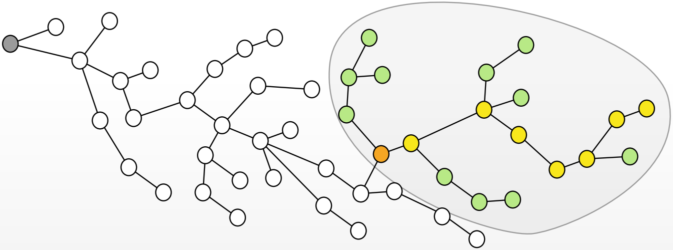


Burning Upper Bounds

- **Improvement:** use a disk of radius r to cover at least $1.5r$ vertices.
    - At each iteration, use either the disk of the largest radius x (if the deepest subtree of height x covers $1.5x$ vertices) or otherwise a disk of radius r, where r depends on the structure of the tree.
    - E.g., we want to burn this graph with $n = 48$ in $\sqrt{4n/3} = 8$ rounds.



Burning conjecture summary

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- The burning conjecture holds for graphs of minimum degree 4 [Bastide, Bonamy, Bonato, Charbit, S.K., Pierron, Rabie, 2021].
- And graph families as spider trees, caterpillars, etc. (see the survey by [Bonato, 2020]).

Computational Complexity

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- Finding the optimal schedule is NP-hard for disjoint set of paths, trees, other graph families [Bessy et al., 2017].
- It is claimed that the problem is APX-hard [Mondal et al., 2021] (no $(1+\epsilon)$-approximation exists assuming $P \neq NP$).

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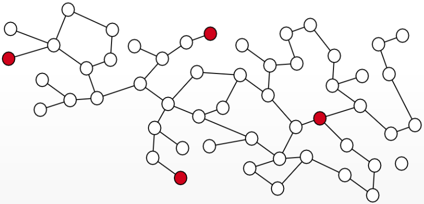
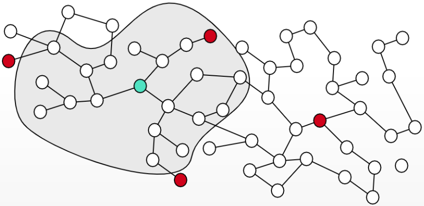


Approximation Algorithms

- If there are r vertices of pairwise distance $\geq 2r-1$ in a graph G, then G cannot be burned in less than r rounds.
- Example: suppose there are $r=4$ vertices of pairwise $2r-1=7$ in a graph G.
  - It is not possible to cover G with 3 disks of radii 3.
  - Therefore it is not possible to cover G with 3 disks of radii $0, 1, 2$.



Constant Approximation Algorithm

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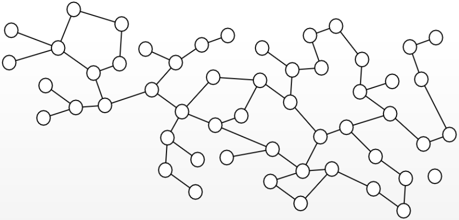
- The smallest value of g^* for which Burn-Guess returns a schedule gives a burning scheme that completes in $3g^* - 3$ while the optimal schedule will require $g^* - 1$ rounds to complete.
  - Approximation ratio of at most 3.

Burn-Guess Process

- Initially empty sets S of "centers" and L of "labeled vertices".



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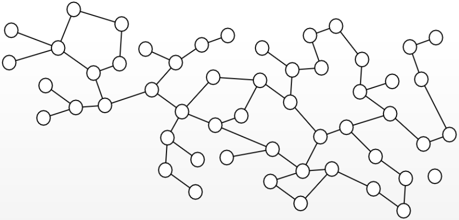

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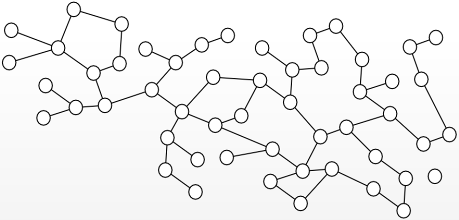

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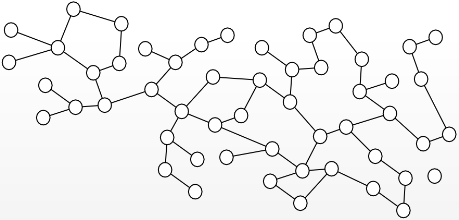

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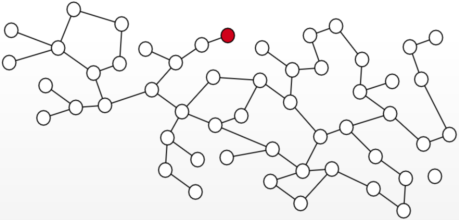

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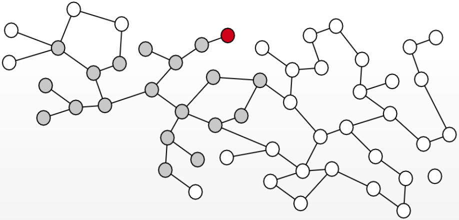

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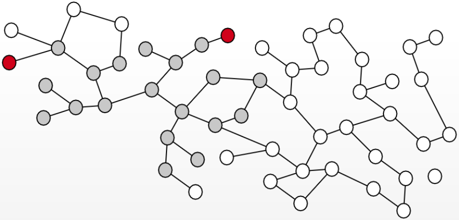

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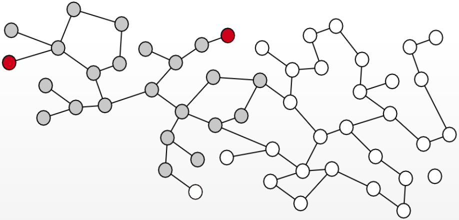

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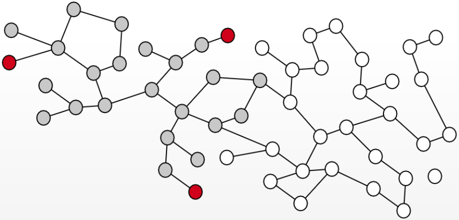

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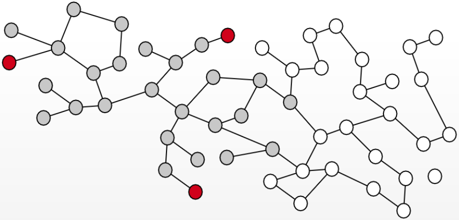

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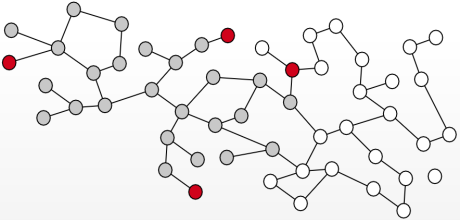

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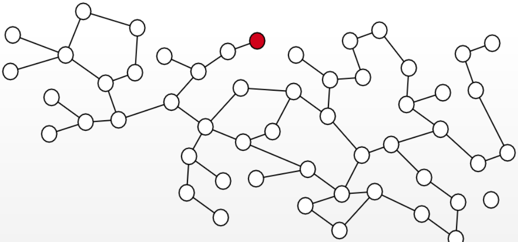

Burn-Guess Process

- Initially empty sets S of "centers" and L of "labeled vertices".
- Take an arbitrary unlabeled vertex u, add it to S and add all unlabeled vertices within distance $2g - 2$ of u to L.
  - If the number of centers becomes g, then return Bad-Guess.
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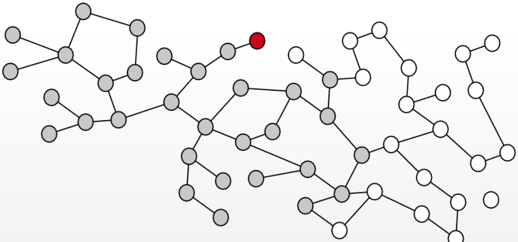

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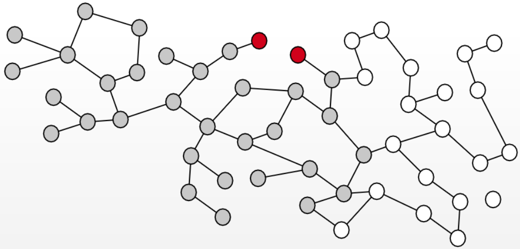

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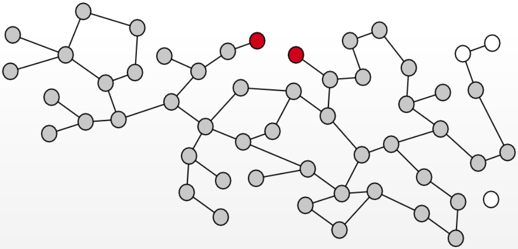

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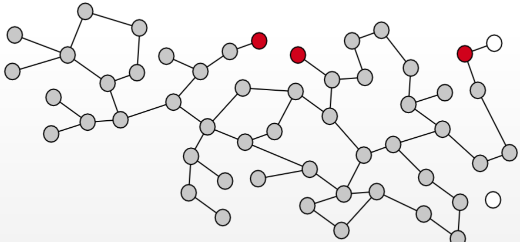

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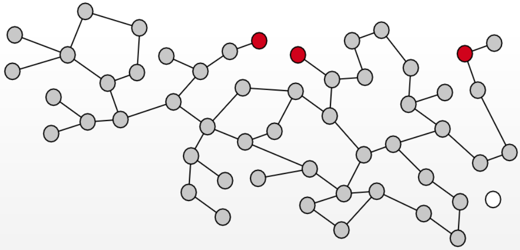
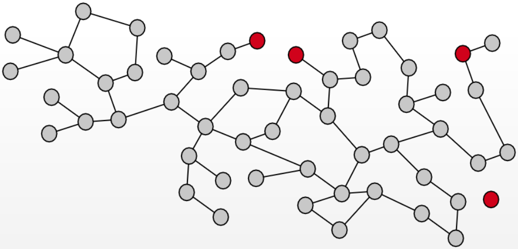


Burn-Guess Process

- Initially empty sets S of "centers" and L of "labeled vertices".
- Take an arbitrary unlabeled vertex u, add it to S and add all unlabeled vertices within distance $2g-2$ of u to L.
  - If the number of centers becomes g, then return Bad-Guess.
  - If all vertices are added to L, return an arbitrary ordering of centers as the burning scheme (which completes in at most $(g-1)+(2g-2)=3g-3$ rounds).
- E.g., here $g=4$ and later we look at $g=5$.



General Graph Summary

- There is a polynomial algorithm with approximation ratio of 3 for burning any graph $G = (V, E)$ [Bessy & Rautenbach, 2016] [Bonato & S.K., 2019].
- What about graph families? can we get better approximation ratio for families of graphs?

Burning Trees

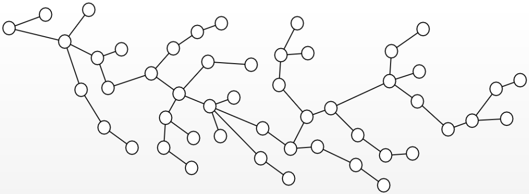
- It is possible to achieve an approximation factor of 2.

Burning Trees

- It is possible to achieve an approximation factor of 2.
- Burn-Guess-Tree (τ, g) returns either a schedule that completes in at most $2g - 2$ rounds or 'Bad-Guess', which means burning cannot complete in $g - 1$ rounds.

Trees

- Burn-Guess-Tree treats τ as a rooted tree:



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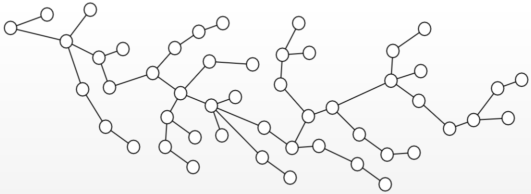

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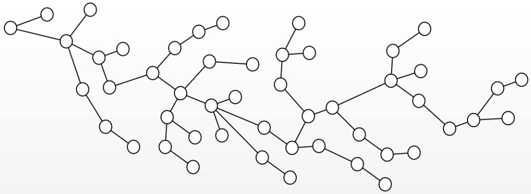

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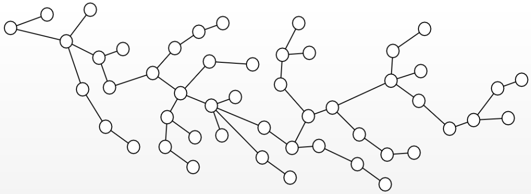

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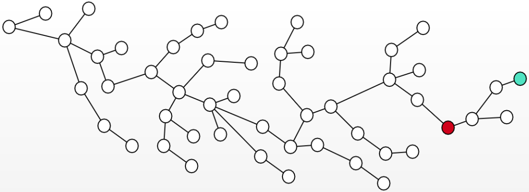

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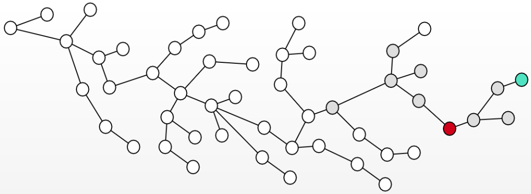

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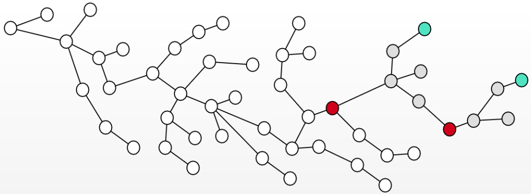

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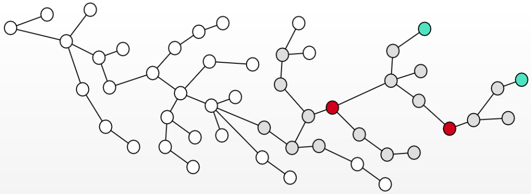

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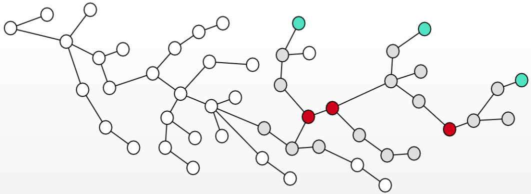

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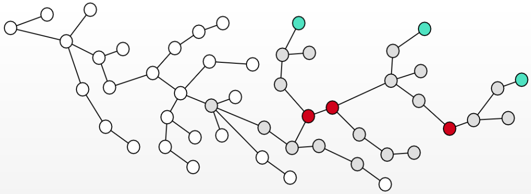

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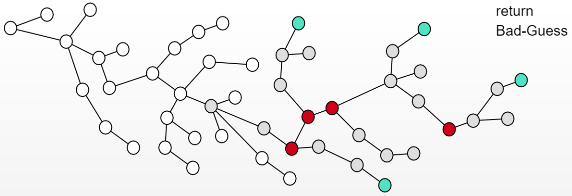

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  - When $|T| = g$, return Bad-Guess.
  - When all vertices are labeled, return any ordering of C as the burning schedule. All nodes are within distance $g-1$ of g centers.
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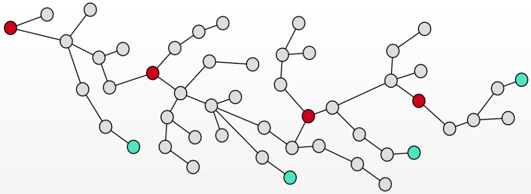

Burning Trees Summary

- There is a polynomial algorithm with approximation ratio of 2 for burning any tree [Bonato & S.K., 2019].
- **Open question:** what is the best approximation factor attainable for trees? or graphs of bounded treewidth? is it possible to get an PTAS?

Forests of Disjoint Paths

- The burning problem is NP-hard when the input graph is a forest of disjoint paths [Bessy et al., 2017].
  - Given disks of radii $0, 1, \ldots, k-1$, it is not clear which disk should be assigned to which path.
- If there are $\Theta(1)$ disjoint paths, there is a polynomial-time algorithm that generates an optimal burning scheme [Bonato and S.K., 2019].
  - Apply a dynamic programming approach!

Forests of Disjoint Paths

- Given any positive value ϵ, there is a fully polynomial-time approximation algorithm (FPTAS) that generates a burning scheme that completes within a factor $1 + \epsilon$ of an optimal scheme [Bonato and S.K., 2019].

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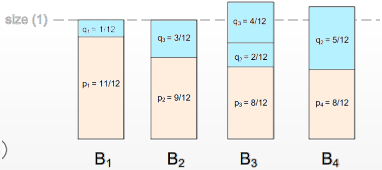
  - **Bin covering:** "cover" a maximum number of bins of unit size with a given multi-set of items with sizes in $(0, 1]$.



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- Reduction: Given a path forest G with b paths generate an instance of the bin covering problem such that G can be burned in k rounds iff it is possible to cover b bins.

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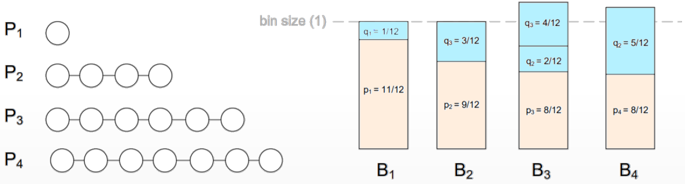
  - Think of paths as uniform "bins" that need to be "covered" by items (disks) of radii $0, 1, \ldots, k-1$.

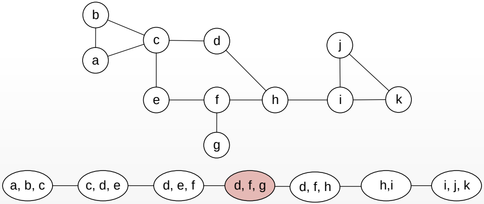


Burning Forests of Disjoint Paths Summary

- There is a fully polynomial-time approximation scheme (FPTAS) for burning any forest of disjoint paths [Bonato and S.K., 2019].
- The complexity of the problem is settled for forests of disjoint paths.

Tree Decomposition & Burning

- In a Robertson-Seymour path decomposition:
  - Path-length [Dourisboure and Gavoille, 2007] is the max **distance** of vertices in any bag.
  - The graph below has path-width 2 and path-length 3.



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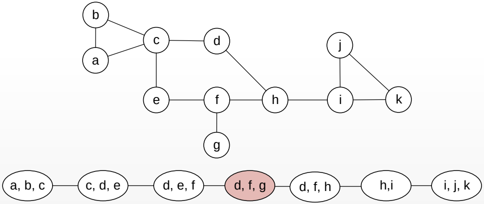
- The burning number of a graph with **path-length** *pl* and diameter d is at most $\lceil \sqrt{d} \rceil + pl$ [S.K., A. Miller, and K. Zhang, 2020].



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  - The graph below has path-width 2 and path-length 3.
  - A graph has path-length 1 if and only if it is an interval graph.

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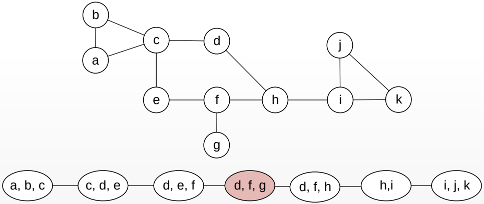

Burning of Graph Families

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  - Otherwise, burn the graph in $\sqrt{d} + pl$ rounds, getting an approximation factor at most $\frac{\sqrt{d}+pl}{\sqrt{d}} = 1 + o(1)$.
- There is an approximation algorithm with factor $2 + o(1)$ for burning any graph G of constant tree-length [S.K. et al., 2020].

Burning Graph Complexity

Graph family	Apx. Factor	Details
general graphs	3	[Bonato and S.K., 2019]
trees	2	[Bonato and S.K., 2019]
cacti	2.75	[S.K. and Shabani, 2021]
forests of disjoint paths	$1 + \epsilon$ (FPTAS)	[Bonato and S.K., 2019]
graphs of bounded path-length	$1 + o(1)$	[S.K. et al., 2020]
graphs of bounded tree-length	$2 + o(1)$	[S.K. et al., 2020]

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Ongoing work.