Searching with turn cost and related problems

Spyros Angelopoulos

CNRS-UPMC

Joint work with

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CNRS | CNRS | Univ. of Waterloo

Outline of the presentation

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- Setting : A searcher that must locate a fixed target
  Starting point: environment = set of rays
- Objective : As quickly as possible -> performance guarantees
- Variant : Turning direction incurs a fixed cost

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  ★ Connections with problems in AI

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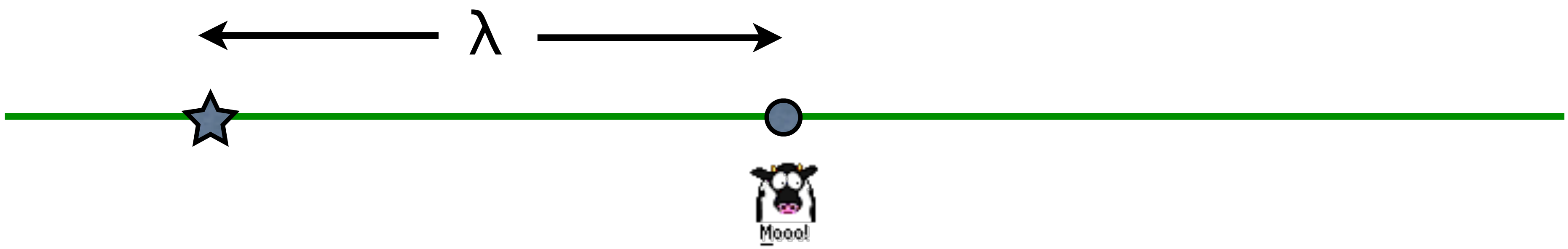

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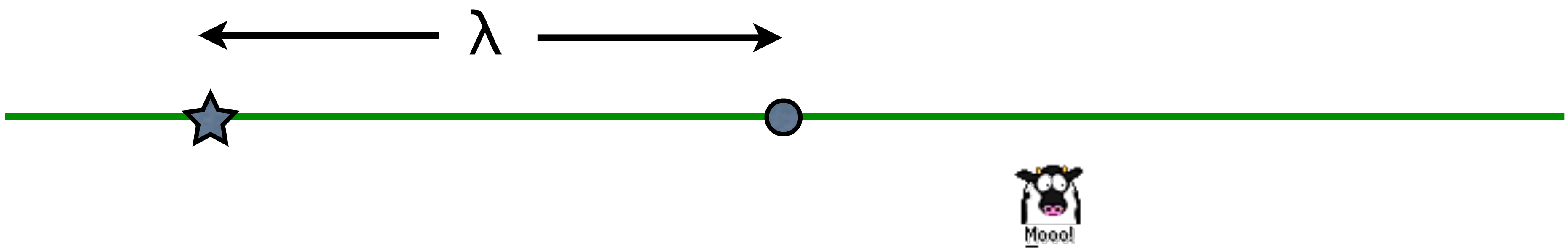

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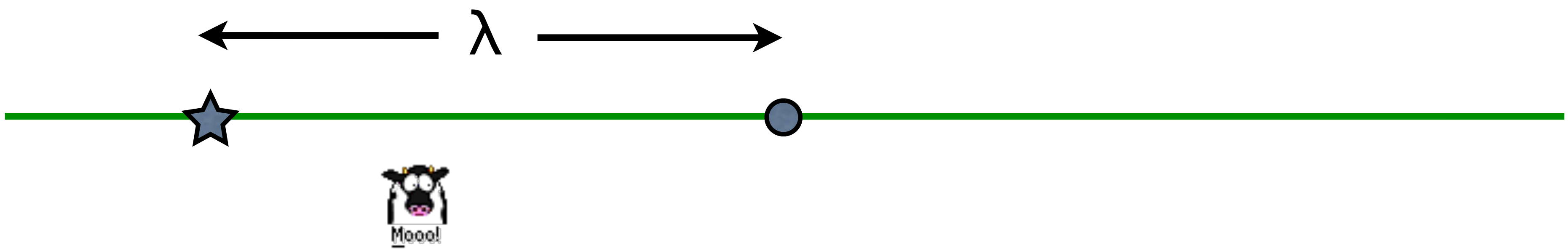

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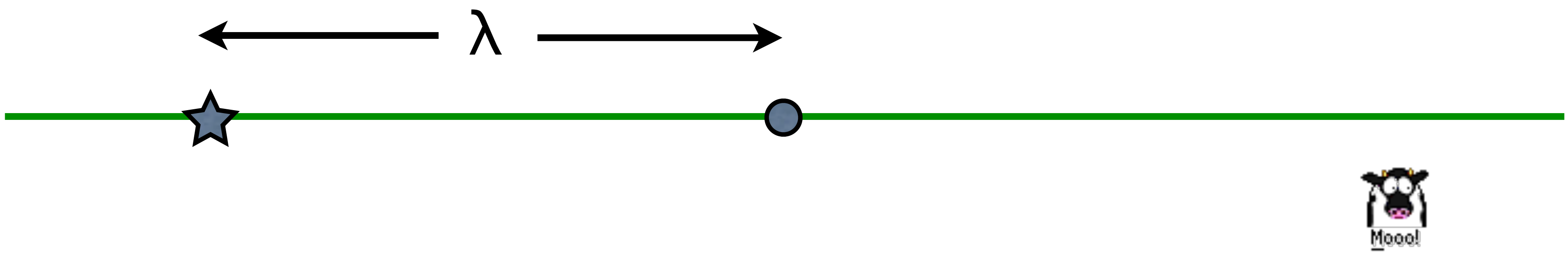

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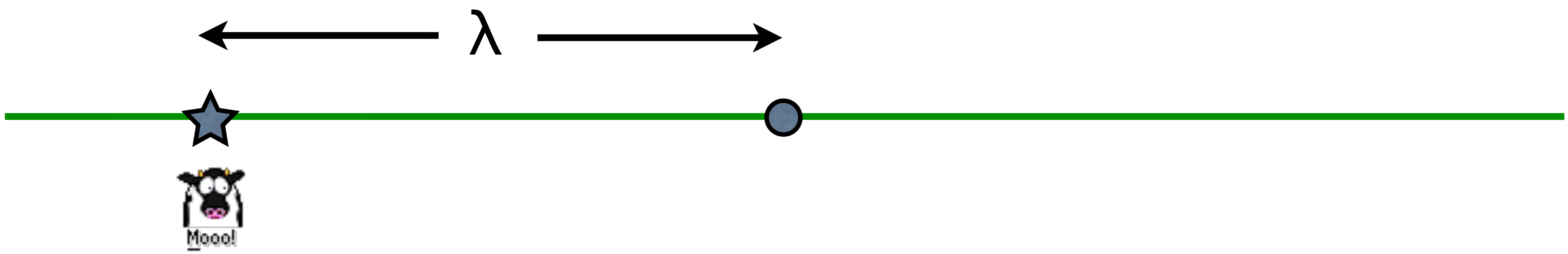

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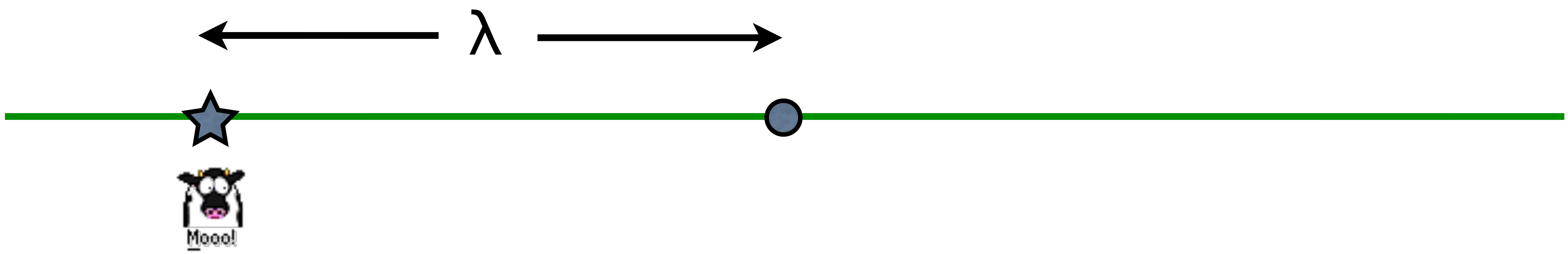

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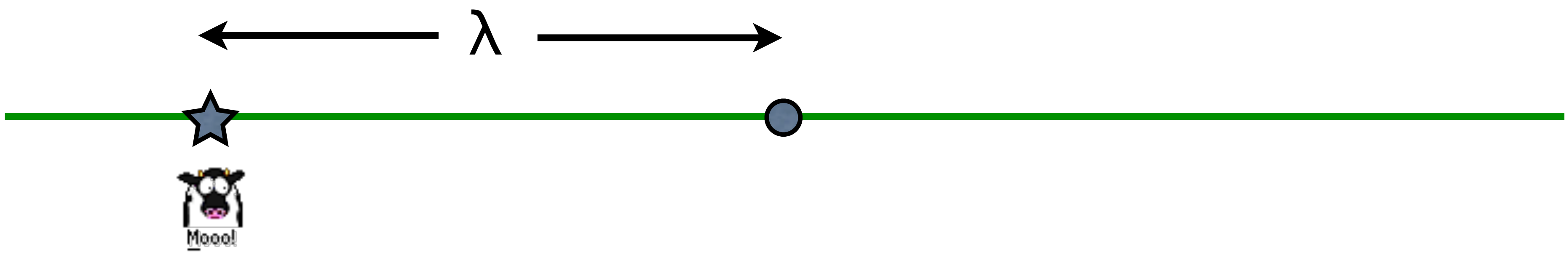

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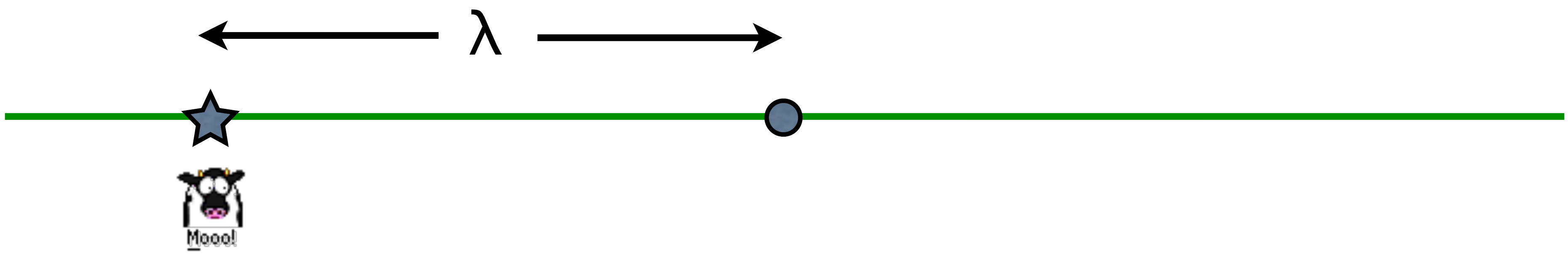

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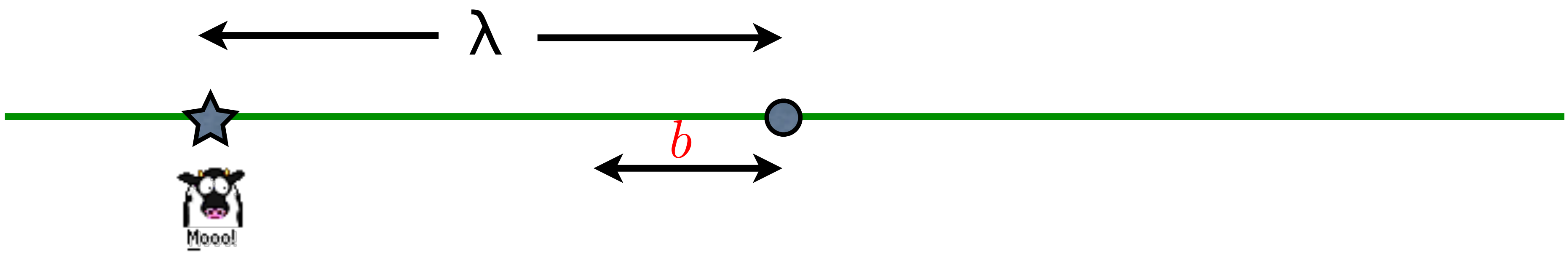

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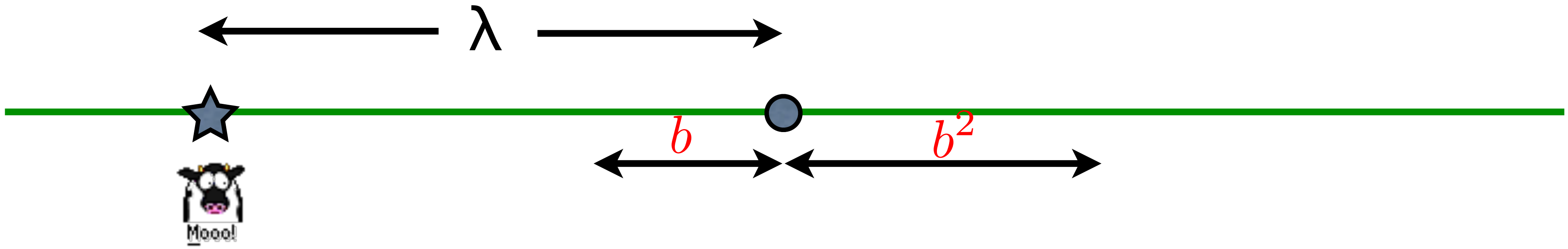

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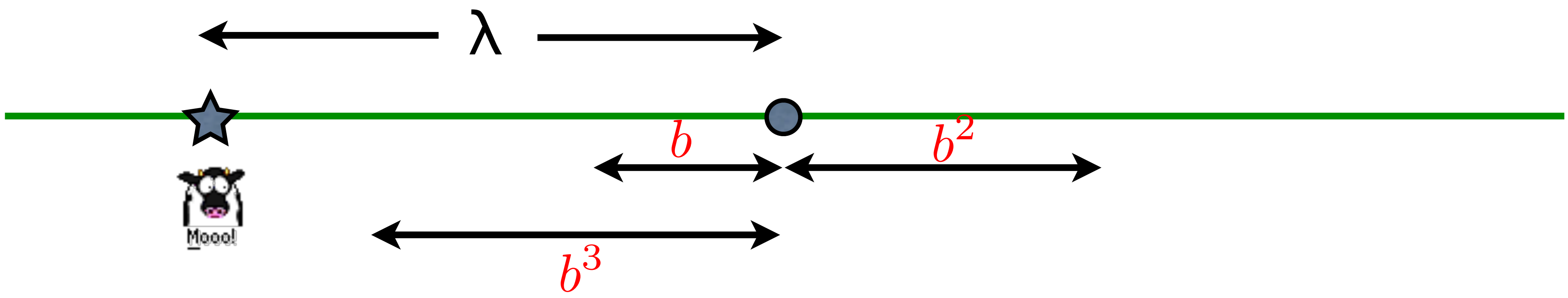

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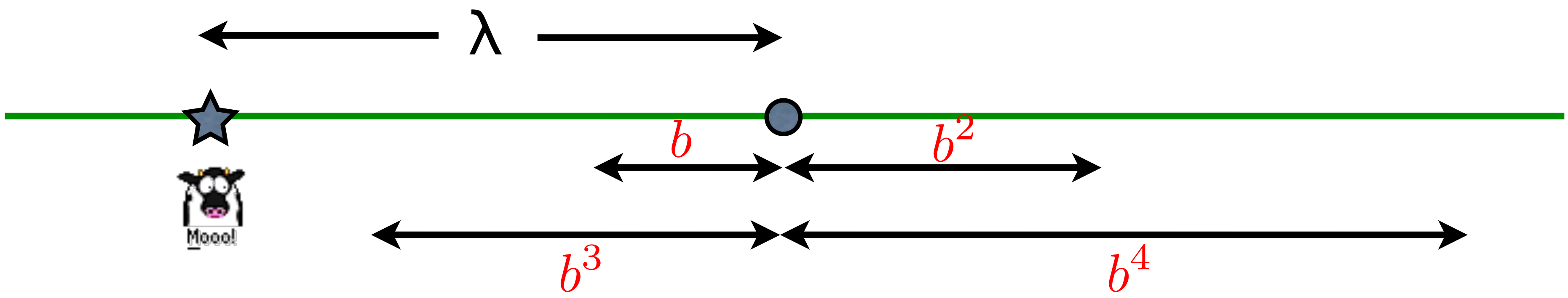

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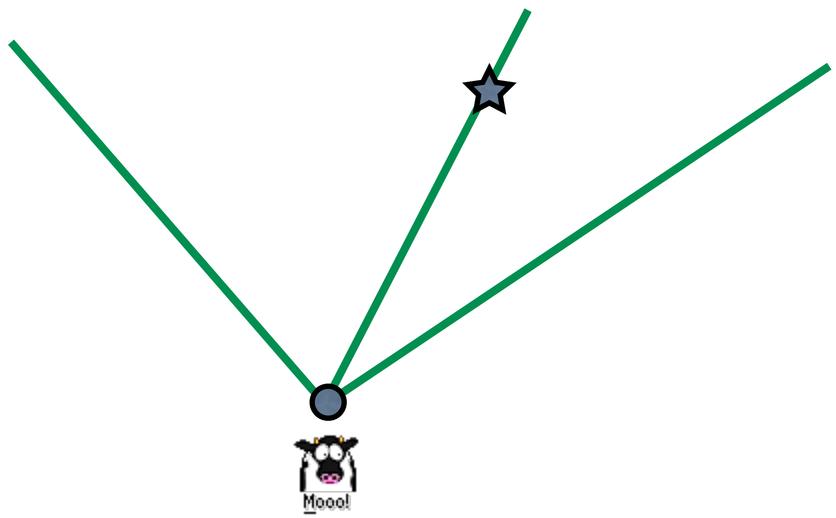

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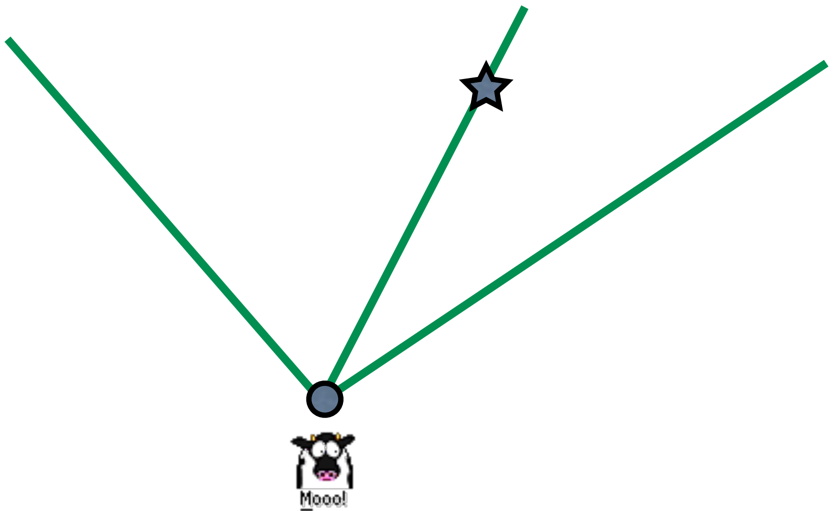

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Competitive ratio = $\sup_{\star}$ (search cost) $/\lambda$

Optimal strategy : geometric search [Gal 72]

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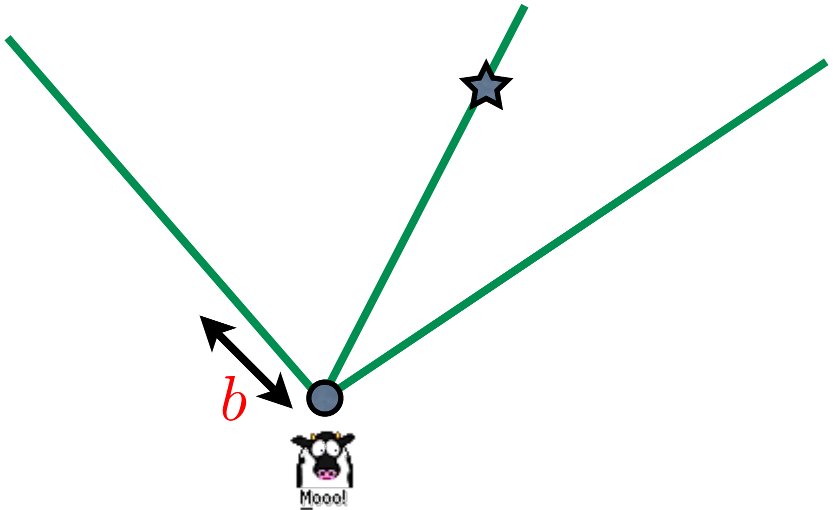

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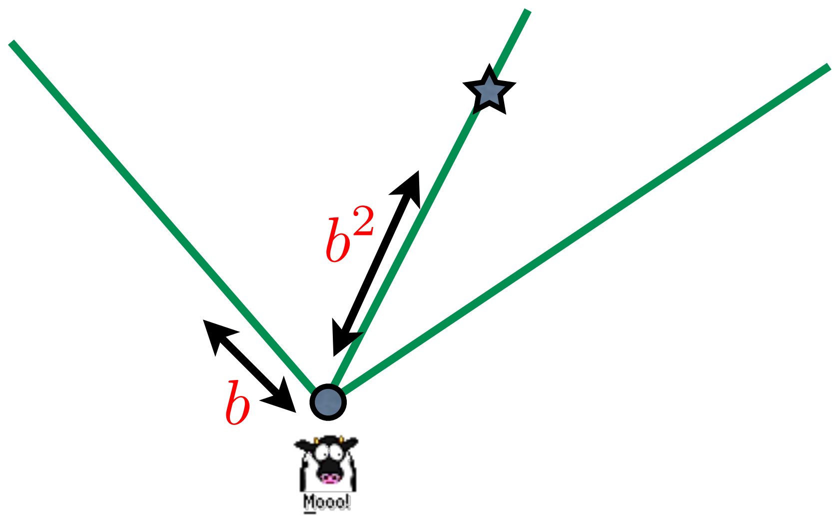

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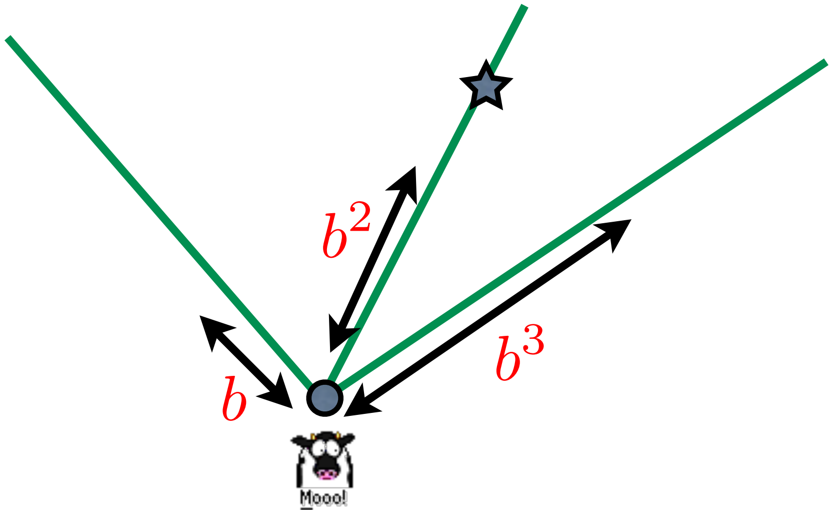

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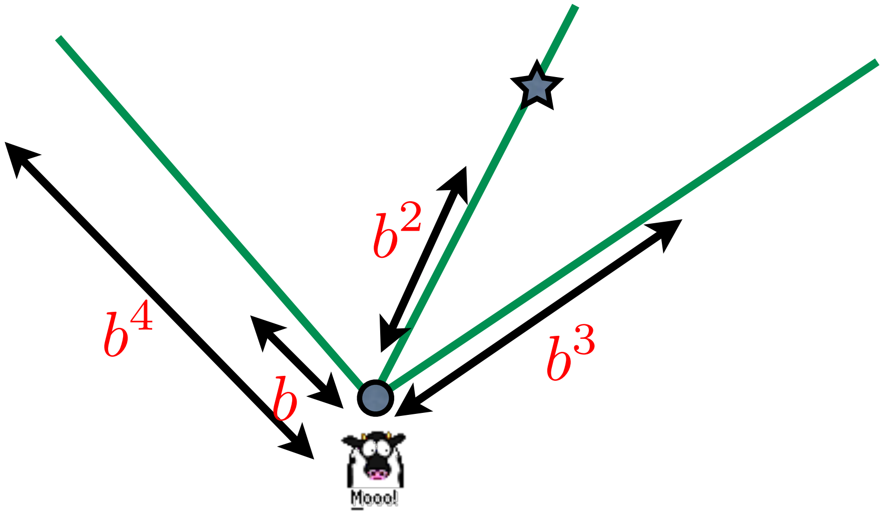

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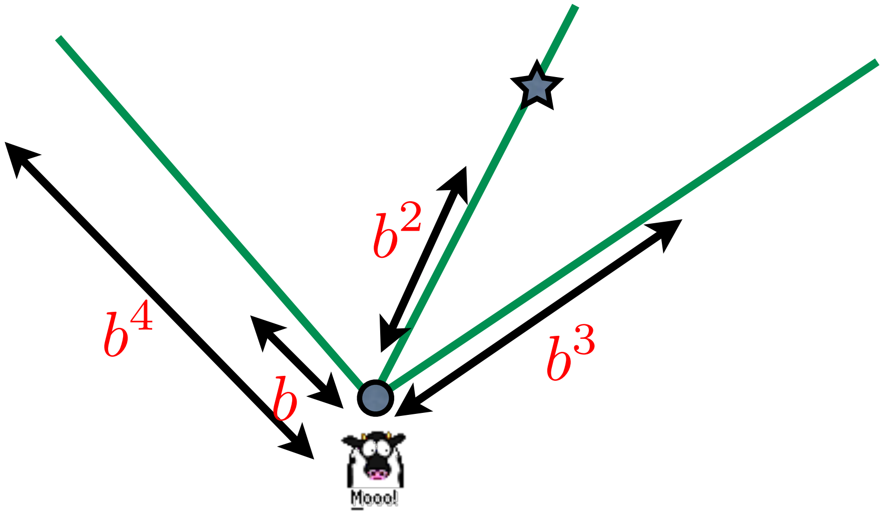

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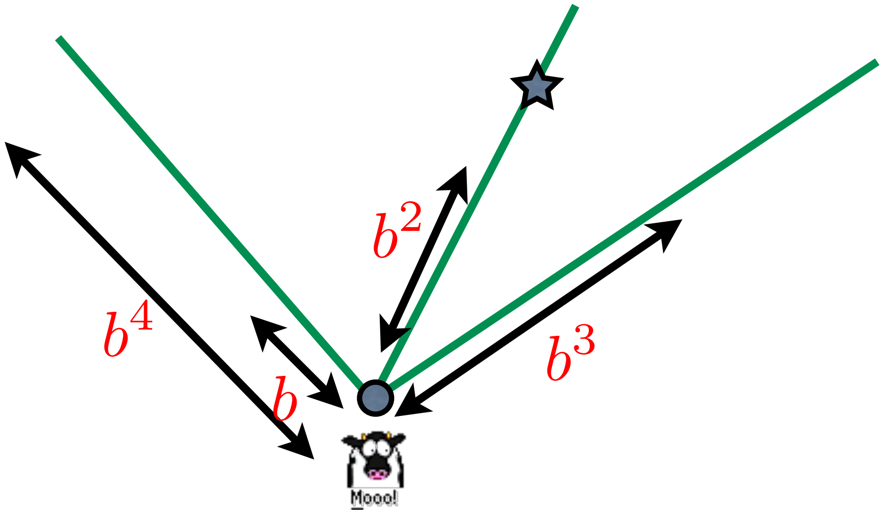

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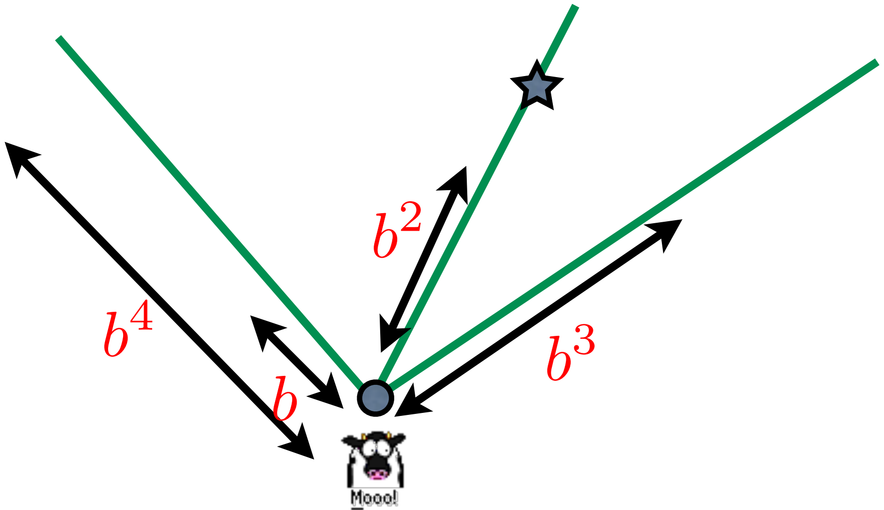

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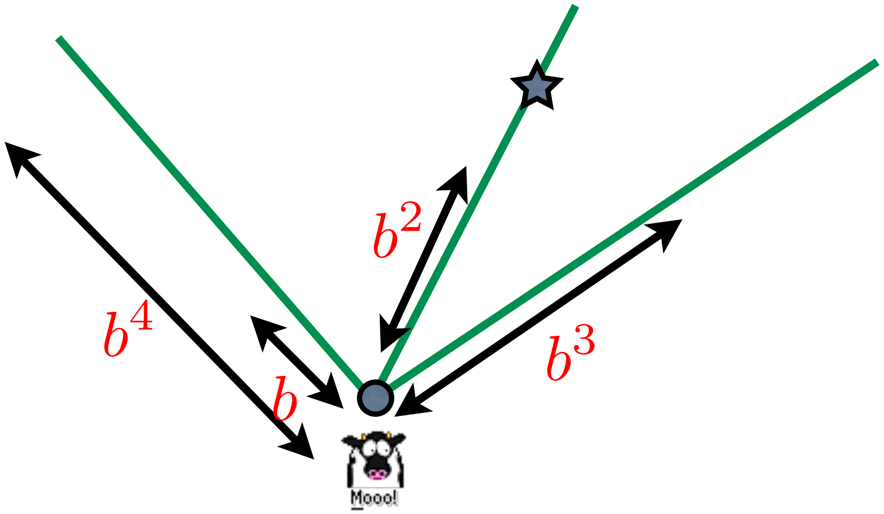

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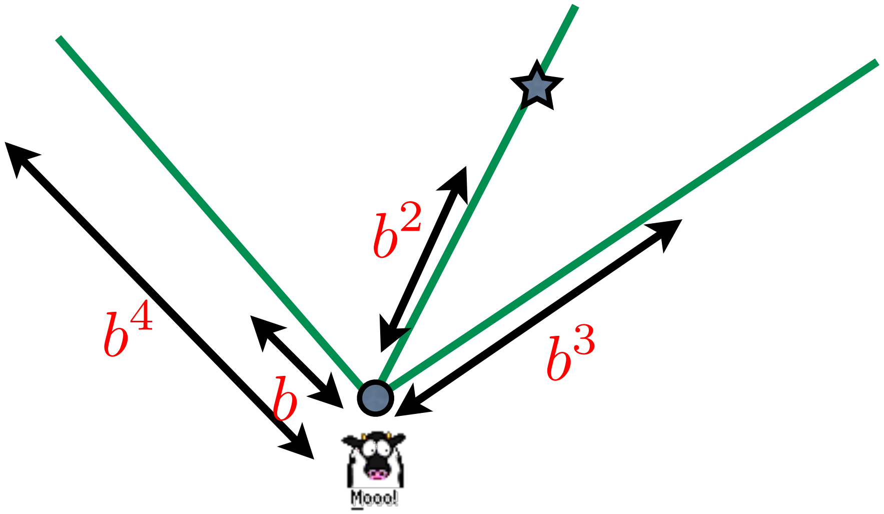

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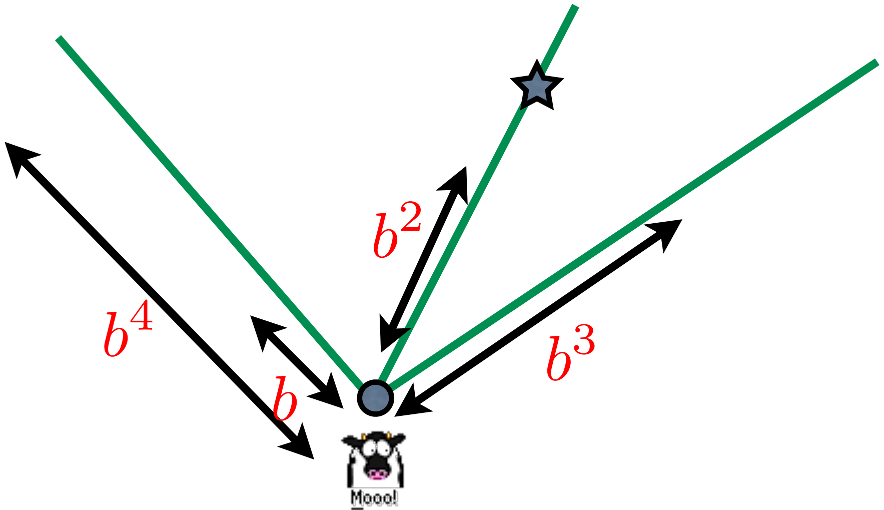

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Textbook by Alpern and Gal: The Theory of Search Games and Rendezvous

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- In preparation : More connections between searching and interruptible algorithms

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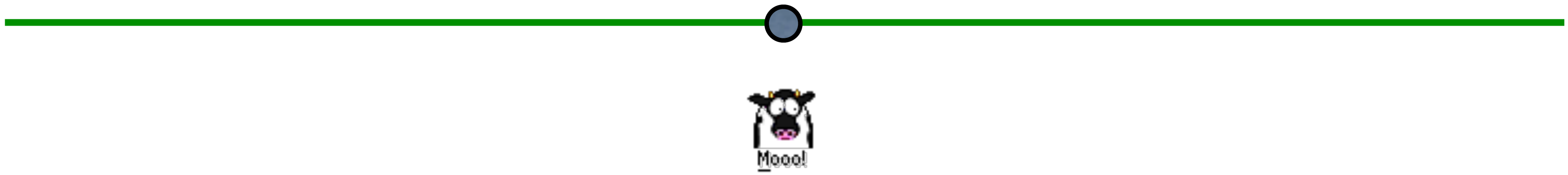

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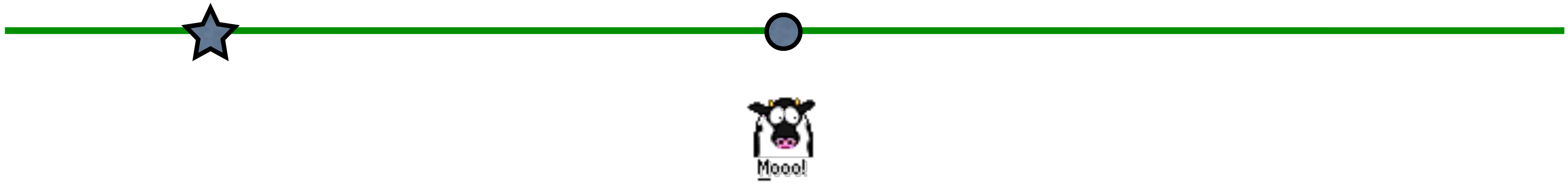

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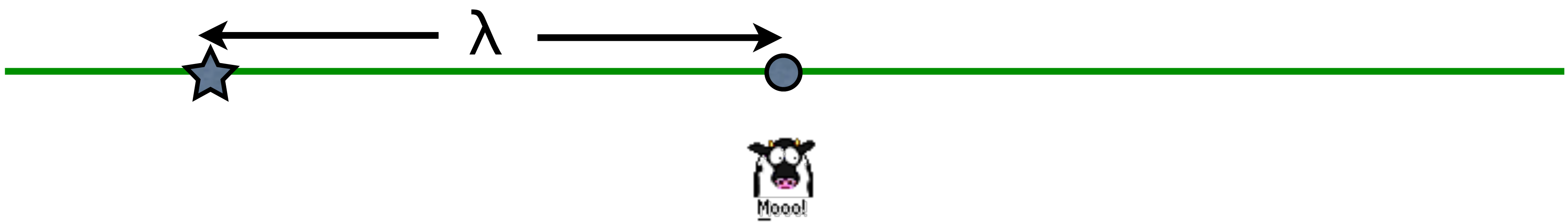

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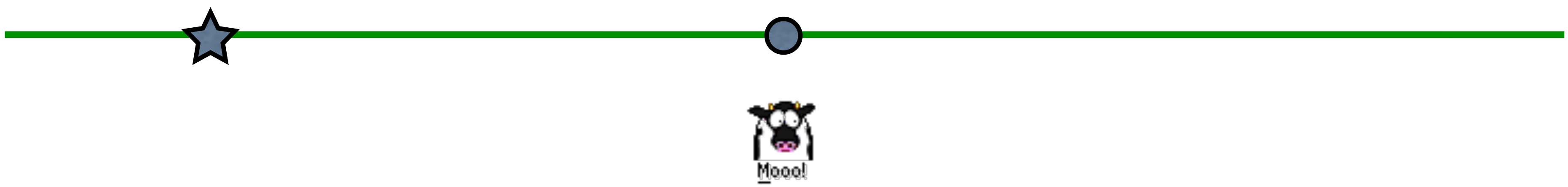

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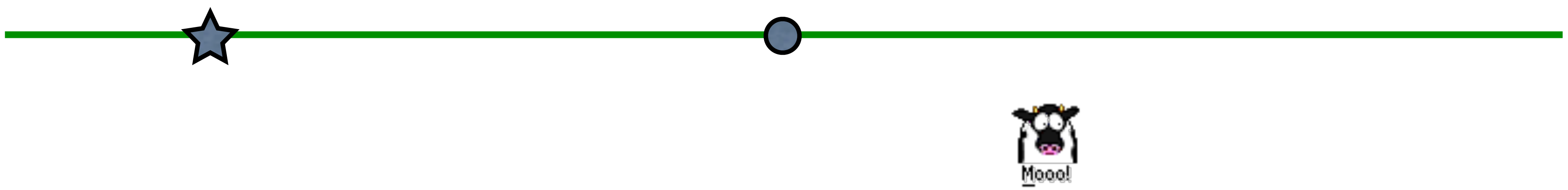

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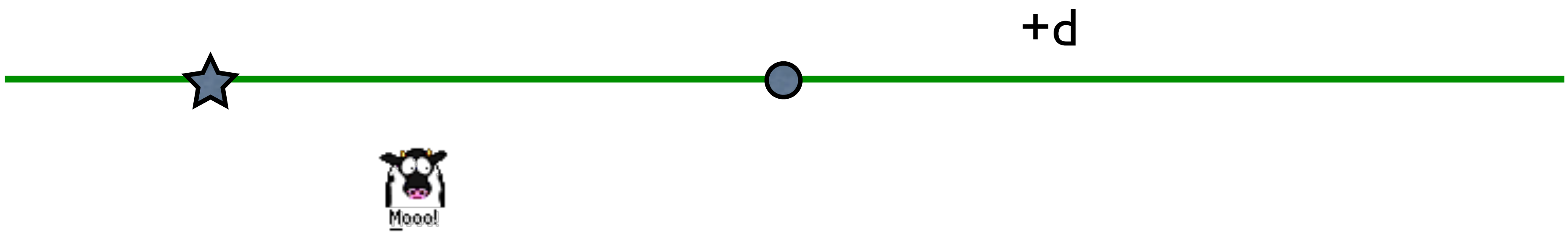

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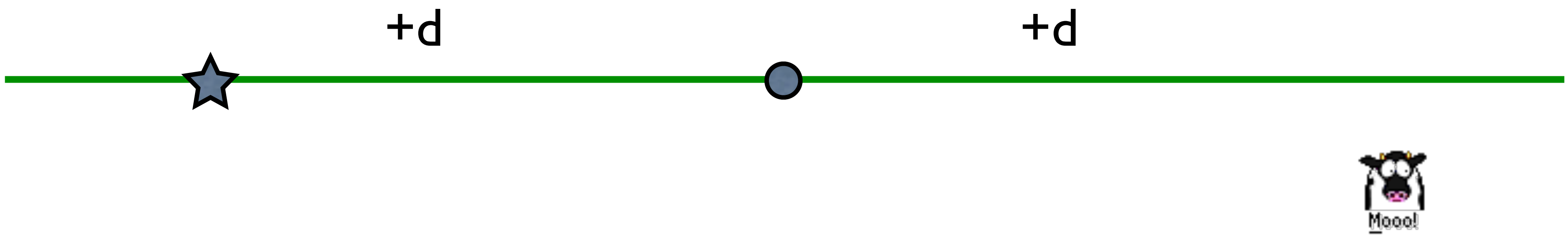

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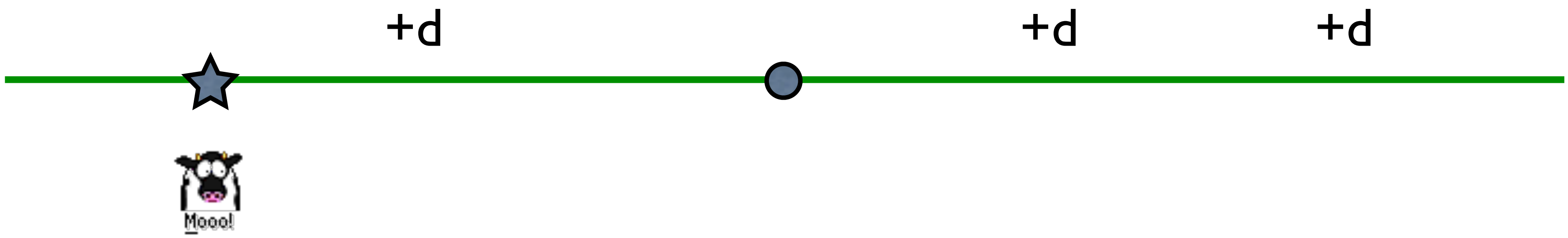

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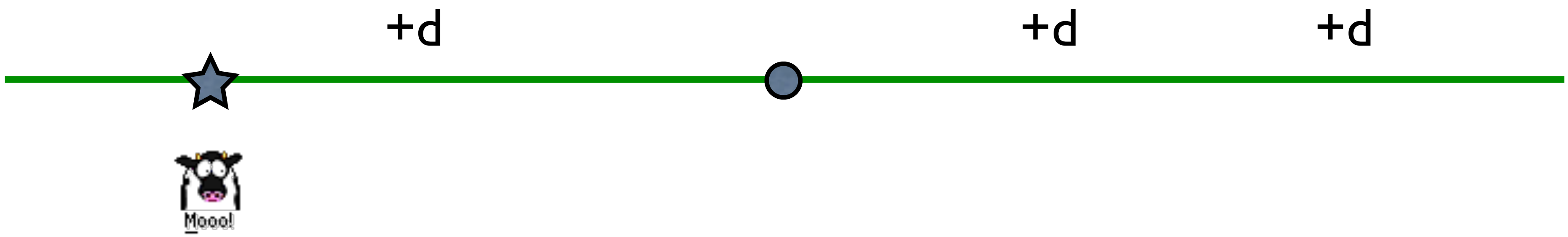
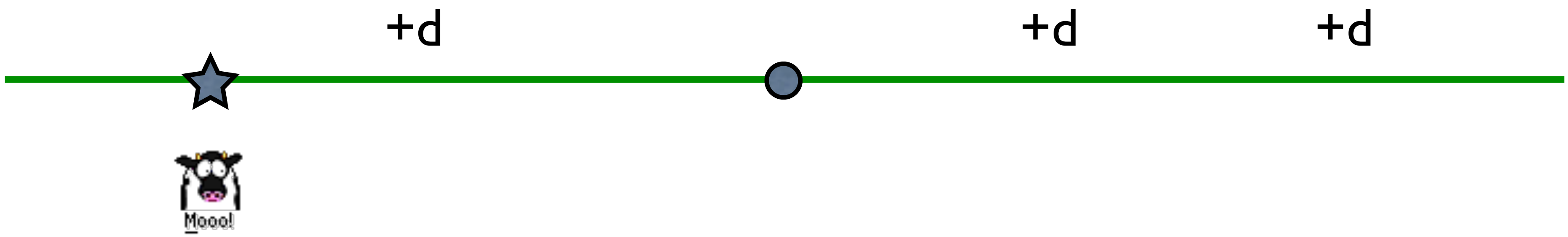


- Total cost = Distance traversed + overall turn cost

Online searching with turn cost



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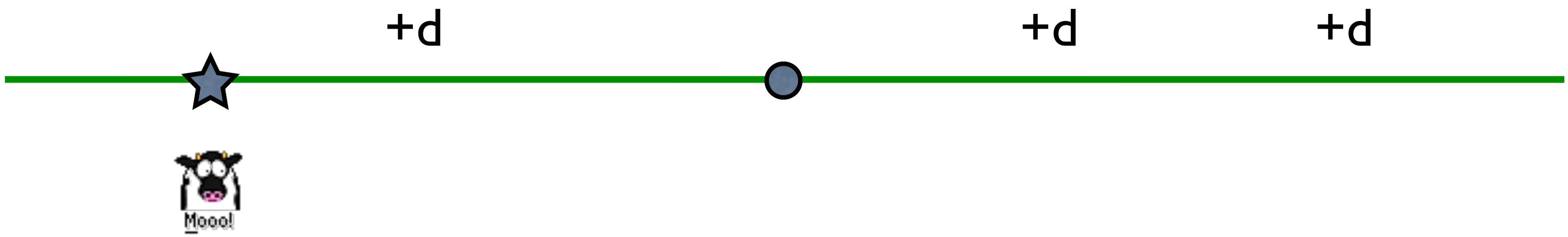

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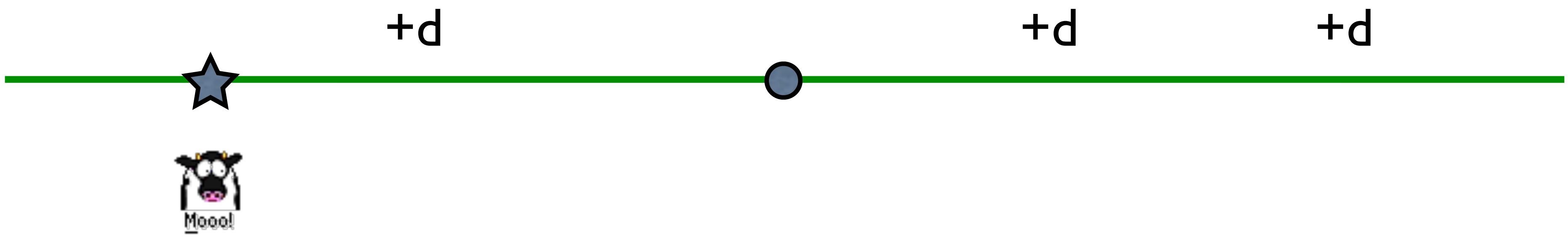

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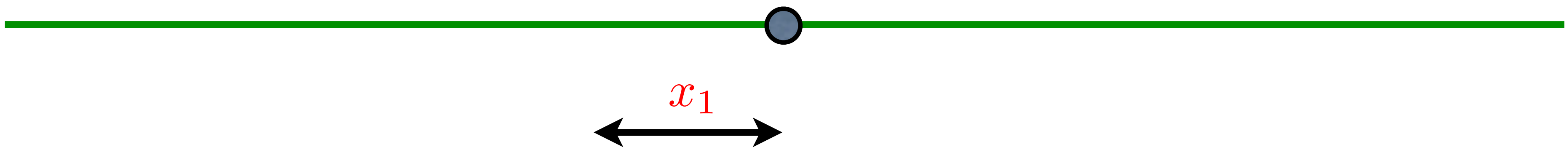

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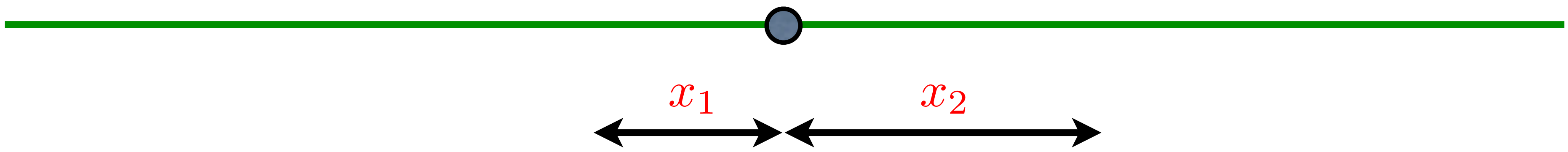

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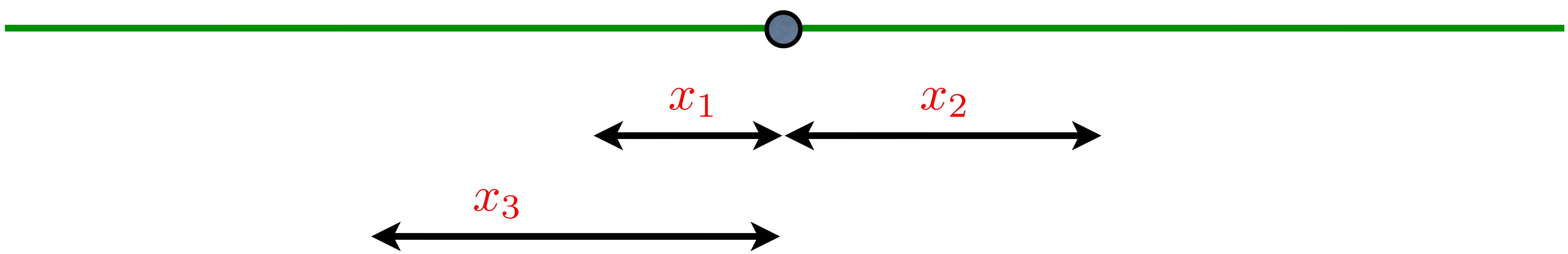

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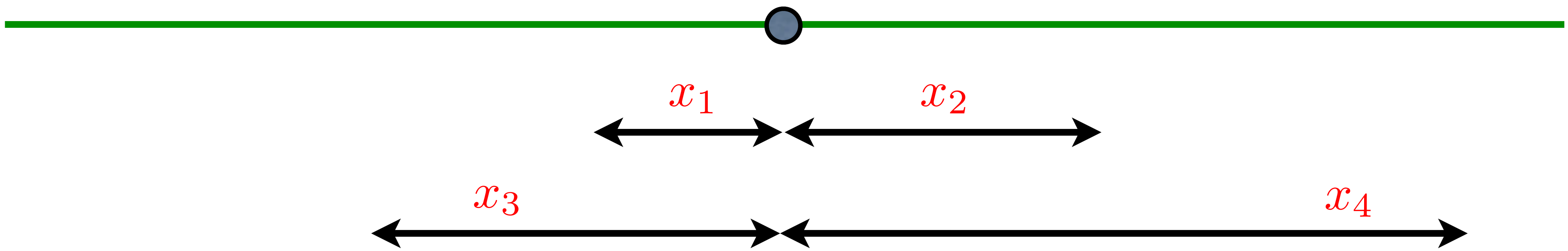

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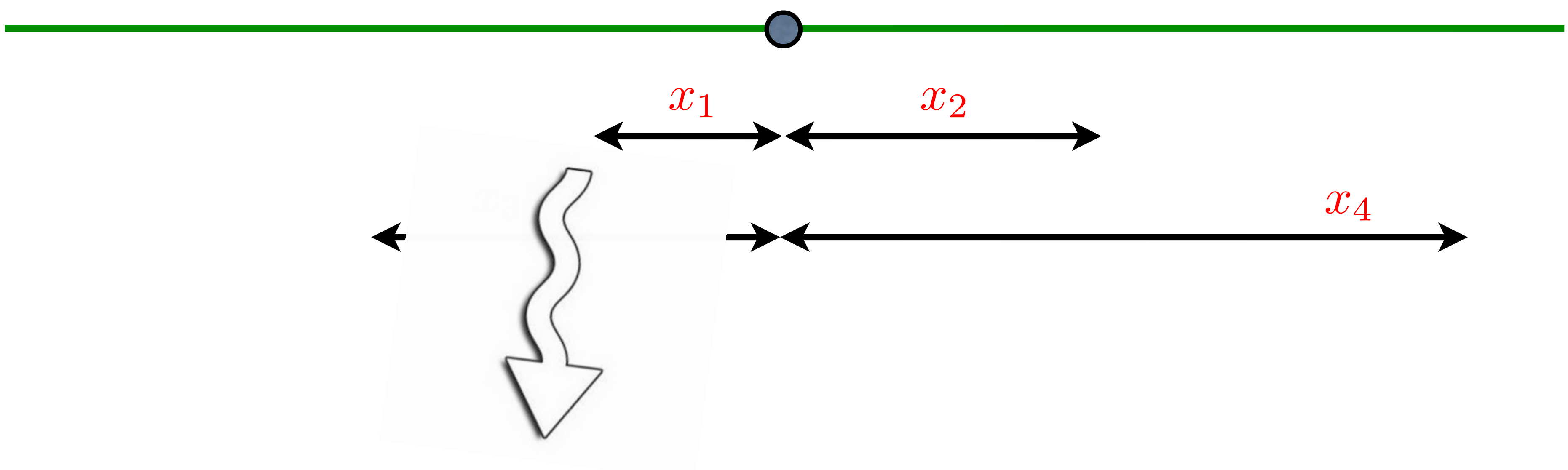

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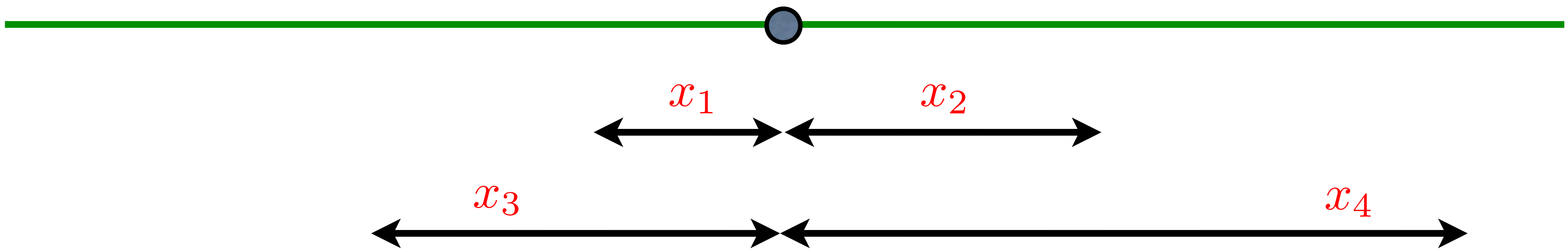

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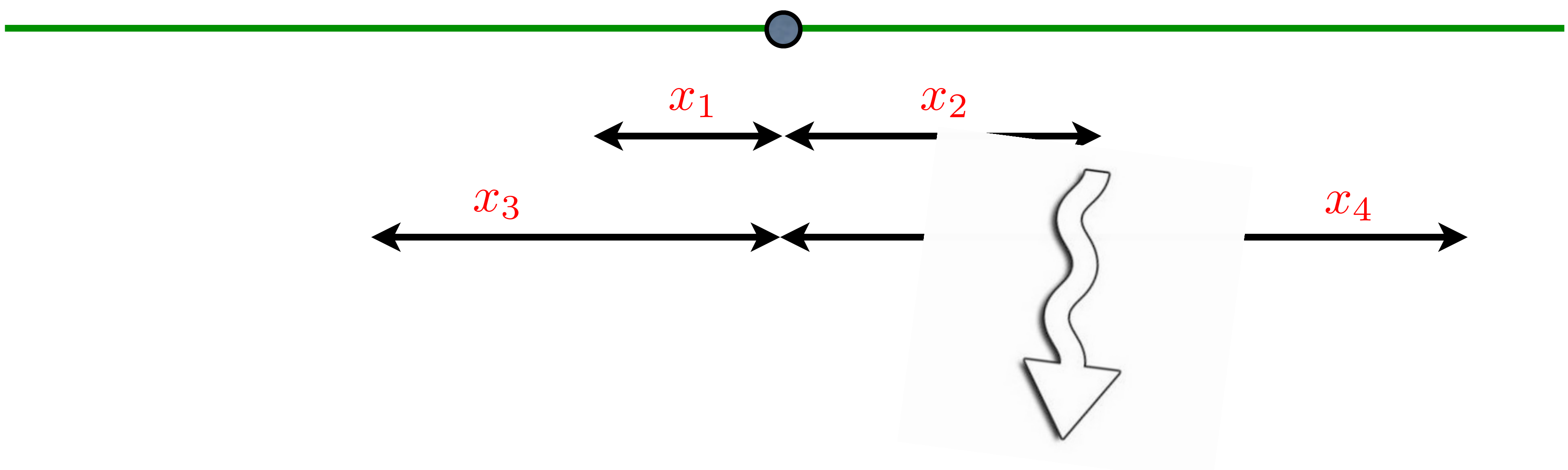

$$2x_1 + 2x_2 + 2x_3 + x_2 + 3d \leq 9x_2 + B$$

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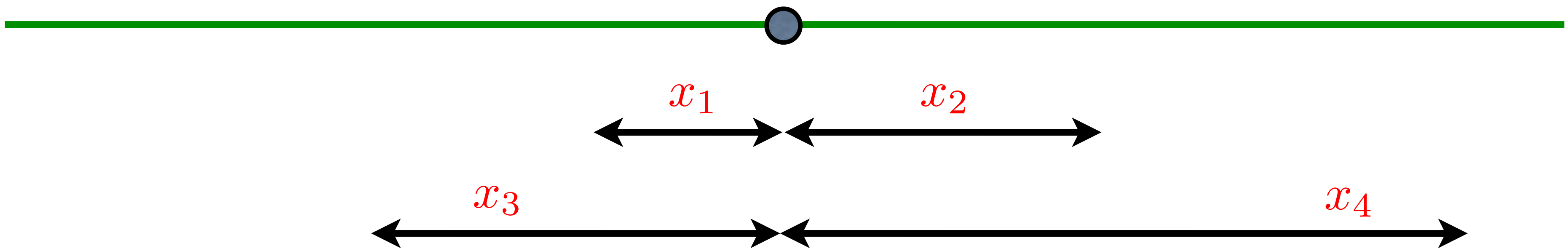

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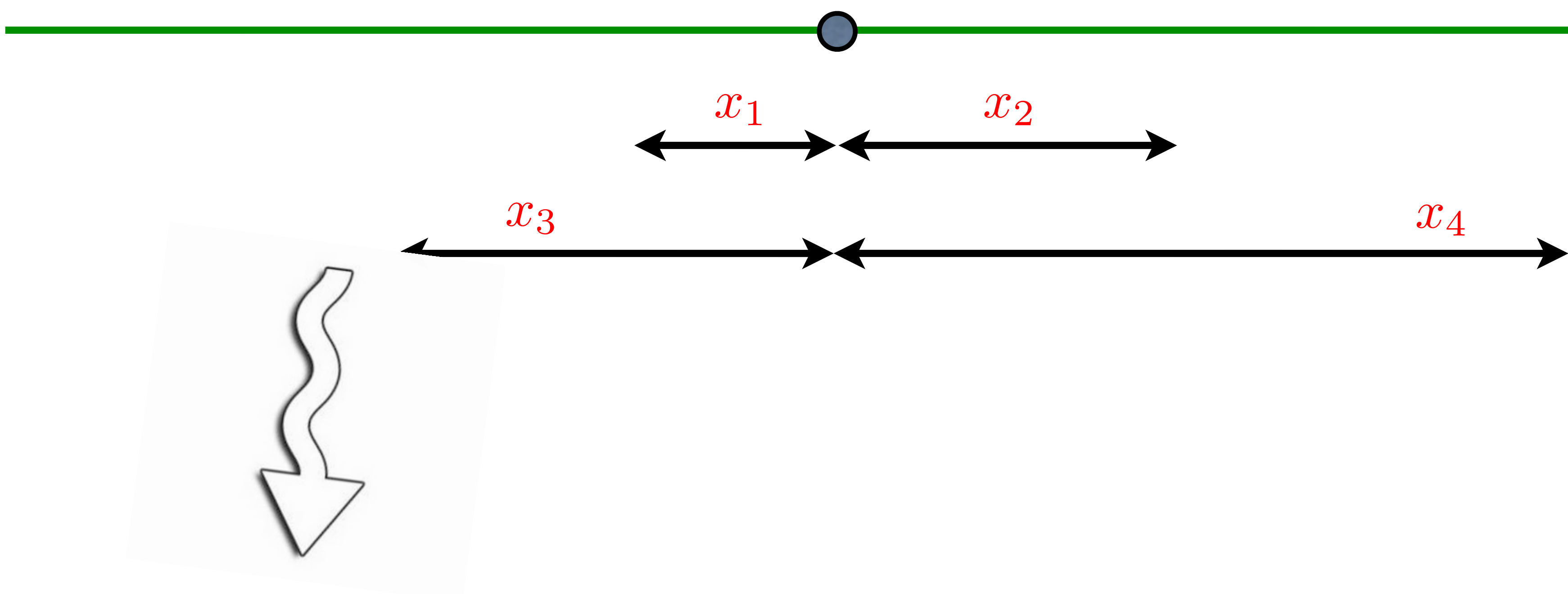

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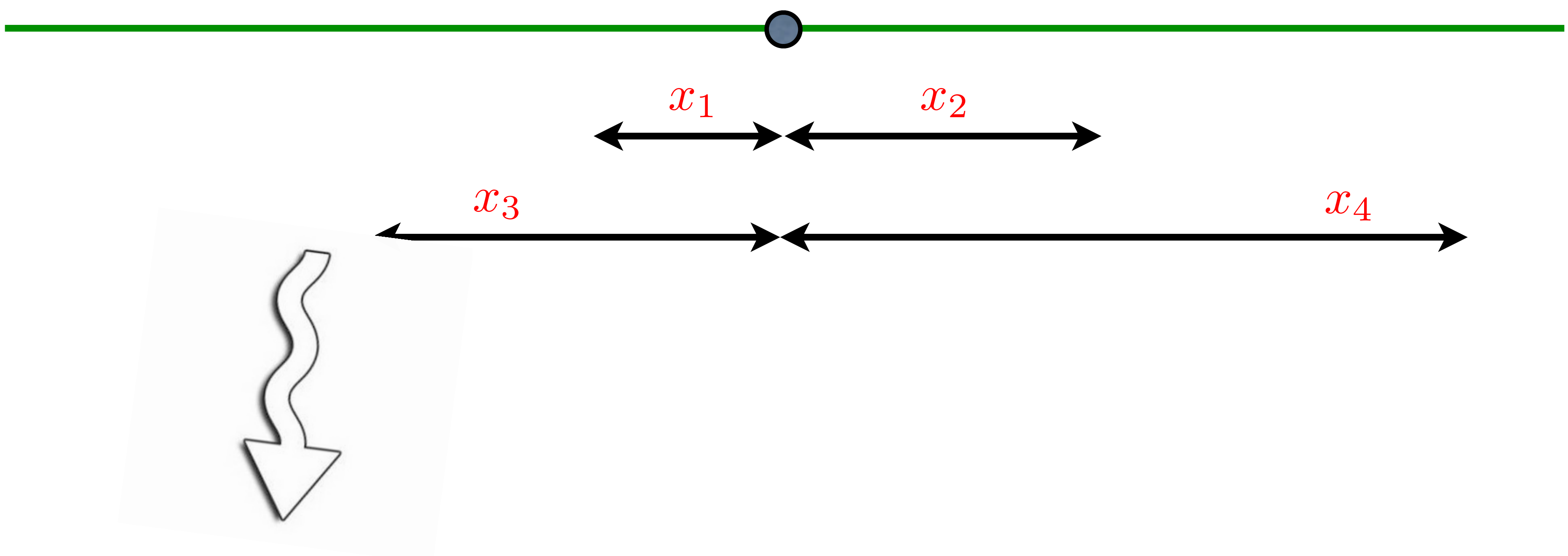

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Infinite LP : Min B subject to an infinite number of constraints

Ray searching with turn cost

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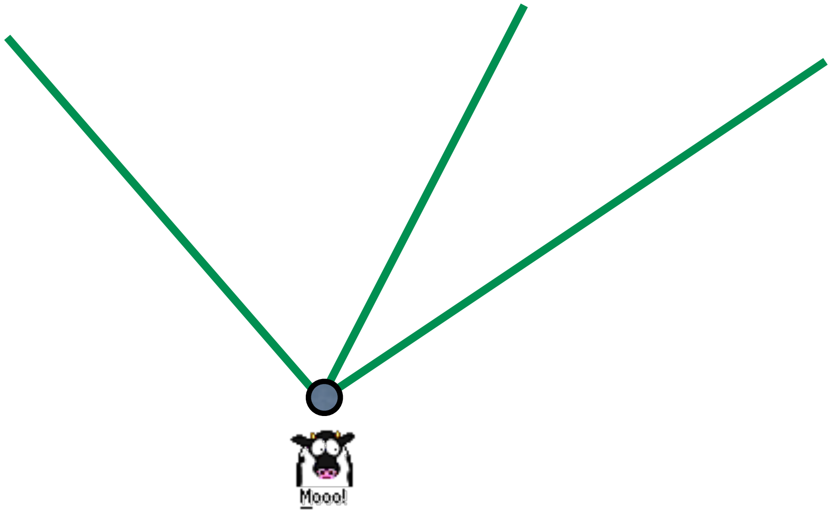

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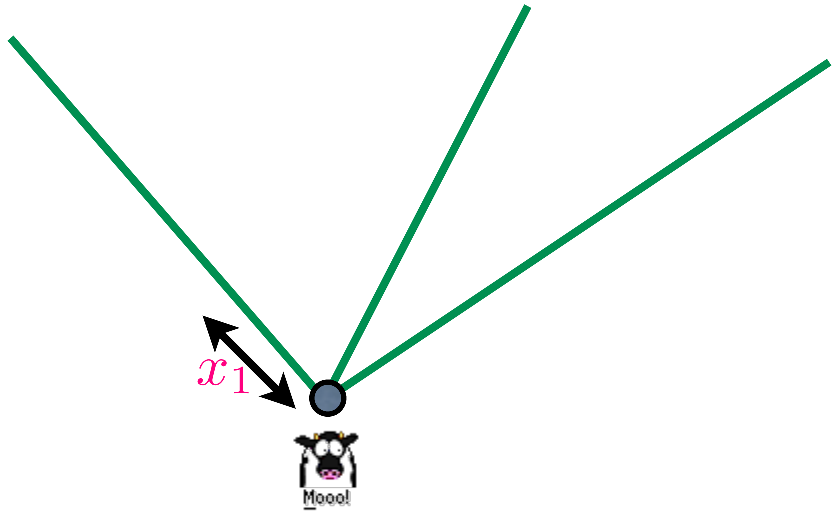

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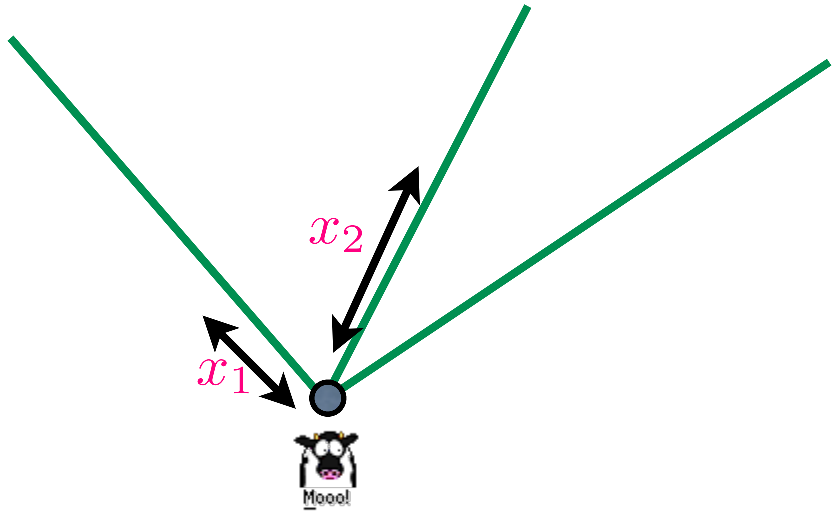

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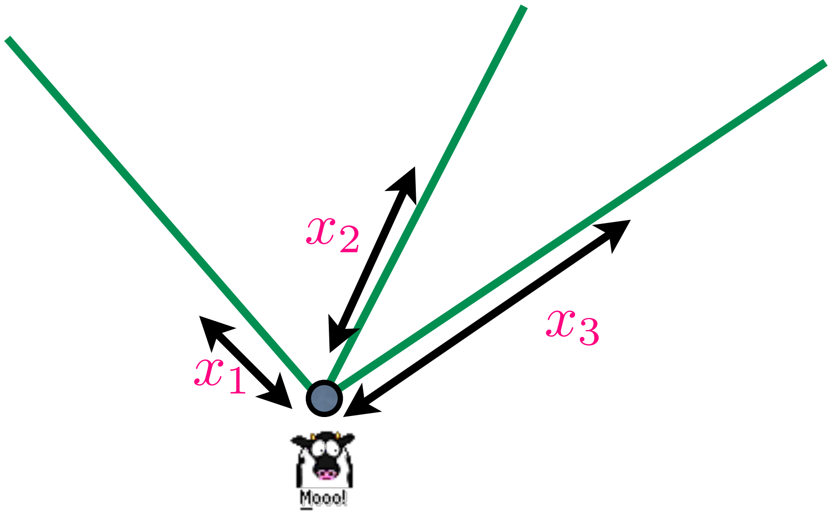

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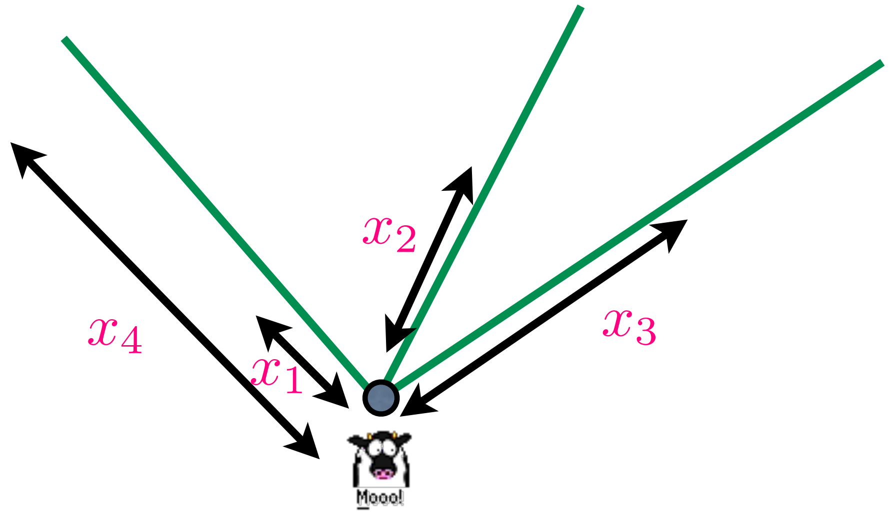

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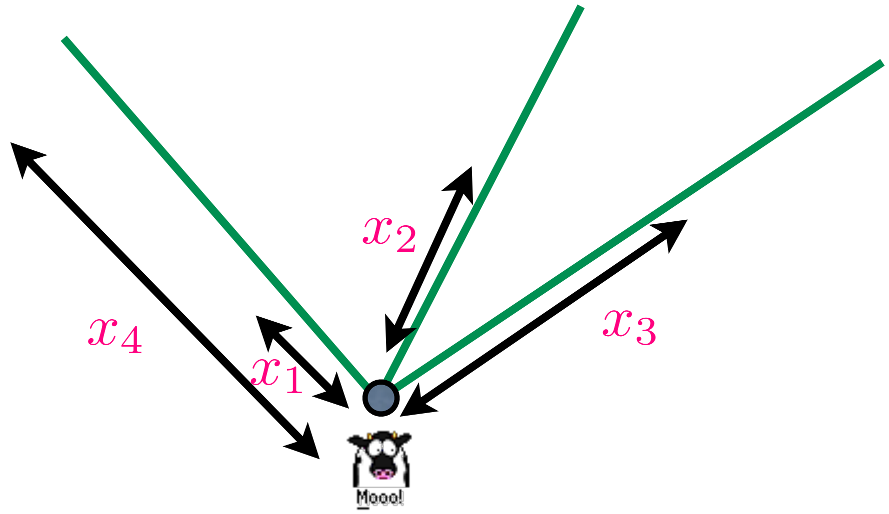

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$$\text{where } M = \frac{b^m}{b-1} \text{ and } b = \frac{m}{m-1}$$

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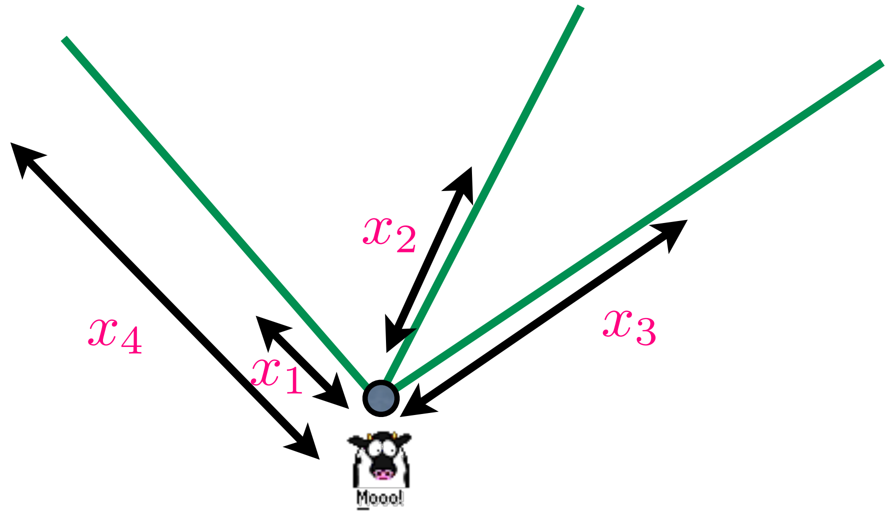

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where $M = \frac{b^m}{b-1}$ and $b = \frac{m}{m-1}$

Using the strategy $x_i = \frac{d}{2}(b^i - 1)$ yields $B = (M - m)d$

Extension to ray searching (Demaine et al.)

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We can obtain an infinite family of LP formulations

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\begin{aligned}
\min \quad & B && (P_1)\\
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with corresponding dual LPs

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\max \quad & \left((m-1)z + \sum_{i=0}^{k} y_i(m+i)\right) d & (D_1) \\
\text{s.t.} \quad & z + \sum_{i=0}^{k} y_i \leqslant 1 & \\
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THE REST OF YOUR LIFE...
AVERAGE
MEMORABLE

Dealing with the infinite LP

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Demaine et al. focus on the infinite dual LP

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which yields an objective value of (M-m)d (which is optimal)

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- One can show that a different feasible solution to the infinite dual LP yields an objective of $Md > (M-m)d = \text{upper bound}$
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- Finding the best dual solution : establishing some properties of the linear recurrence **y**

- More precisely: for which initial data does **y** become eventually negative?

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- Quality of output improves as a function of time
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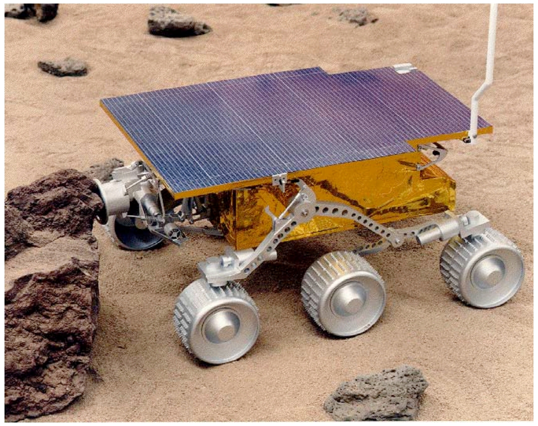
From contract to interruptible algorithms

- Main goal: "Black-box" techniques for turning every contract algorithm to its interruptible version
- Establish measures of how good this simulation is
- Find efficient simulations in this measure
- Survey: "Using anytime algorithms in intelligent systems" (Zilberstein)



"Plan synthesis must have anytime, incremental characteristics. It should be possible to stop a plan synthesis algorithm at any time during its execution and expect useful results. One should expect the "quality" of the results to improve continuously as a function of time."

John Bressina and Mark Drummond
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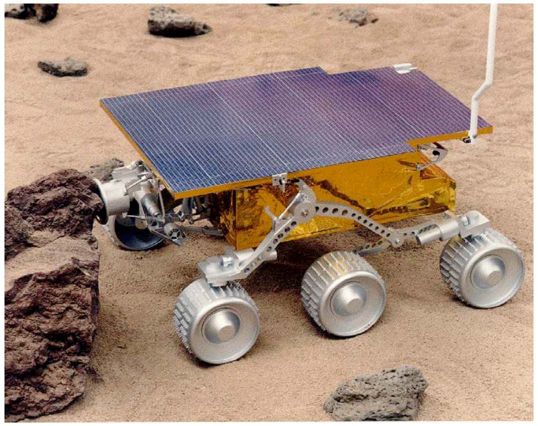

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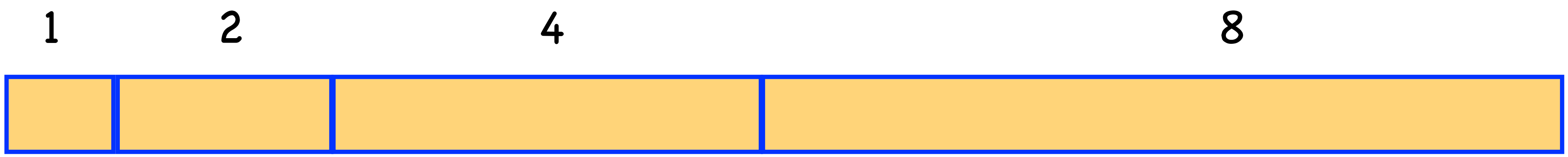

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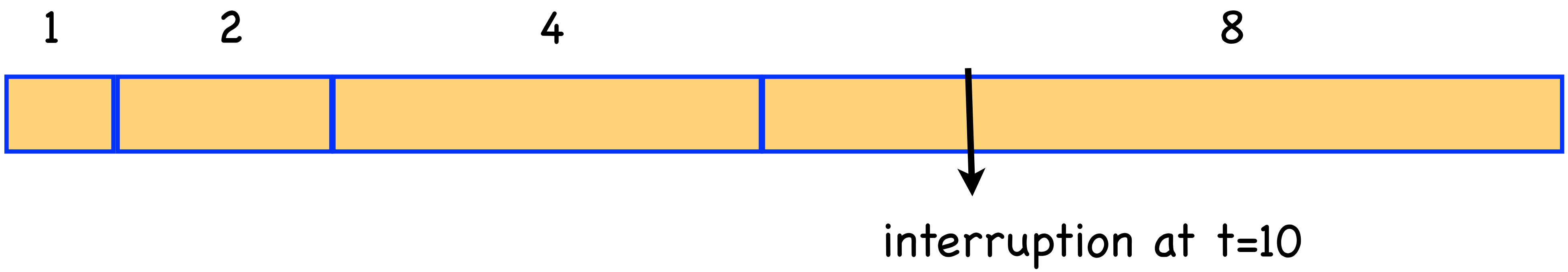

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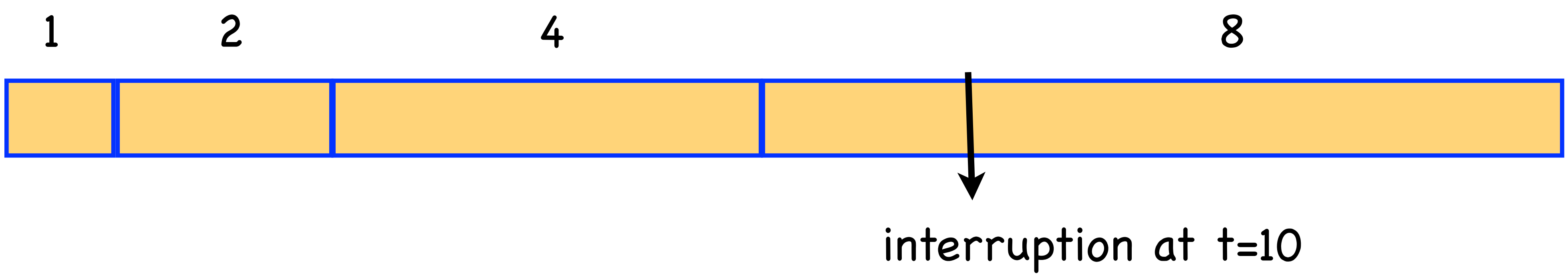

- In hindsight we could have run the algorithm for 10 units, but the best we achieved is a running time of 4
- Inefficiency = $\max_t \frac{t}{\text{longest contract finished by } t}$

Conclusion and outlook

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Thank you!